

$$f(x) = \frac{x}{3} + \arccos \frac{9-x^2}{9+x^2}$$

$$1) 0x \in (-\infty; +\infty)$$

$$0x:$$

$$y=0$$

$$-\frac{x}{3} = \arccos \frac{9-x^2}{9+x^2}$$

$$x = -6,993$$

$$x=0$$

$$0y:$$

$$x=0$$

$$y=0$$

$$-6,993 + 0 +$$

$$2) y(x) = \arccos \frac{9-x^2}{9+x^2} \quad y(x) = -y(x)$$

Функция общего типа

3) Функция не имеет вертикального асимптота

$$k = \lim_{x \rightarrow \infty} \frac{\left(\frac{x}{3} + \arccos \frac{9-x^2}{9+x^2}\right)}{x} = \lim_{x \rightarrow \infty} \frac{x + 3\pi}{3x} = \frac{1}{3}$$

$$b = \lim_{x \rightarrow \infty} \left(\frac{x}{3} + \arccos \frac{9-x^2}{9+x^2} - \frac{x}{3} \right) = \pi$$

$$x \rightarrow -\infty \quad k = \frac{1}{3}, b = \pi$$

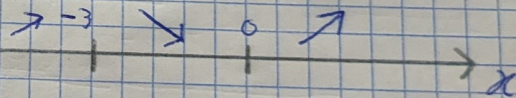
Зонаклонная асимптота: $y = \frac{x}{3} + \pi$

$$4) y' = \frac{1}{3} - \frac{1}{\sqrt{1-\left(\frac{9-x^2}{9+x^2}\right)^2}} \cdot \left(\frac{9-x^2}{9+x^2}\right)' = \frac{1}{3} - \frac{1}{\sqrt{\frac{2x^2}{(9+x^2)^2}}} \cdot \frac{-2x \cdot 18}{(9+x^2)^2} = \frac{1}{3} + \frac{6x}{x(9+x^2)}$$

$$\frac{1}{3} + \frac{6x}{x(9+x^2)} = 0 \quad -18x = |x|(9+x^2) \quad 9|x| + |x|x^2 + 18x = 0$$

$$x > 0 \quad x^2 + 27x = 0 \quad x = 0, x = -27$$

$$x < 0 \quad x^2 - 9x = 0 \quad x = 0, x = 9$$



$$y(0) = 0; \quad y(-3) = -1 + \arccos \frac{9-9}{9+9} = -1 + \arccos 0 = \frac{\pi}{2} - 1 \approx 0,571$$

$M(0;0)$ - локальный минимум

$M(-3; 0,571)$ - локальный максимум

$M(0;0)$ и $M(-3; 0,571)$ - глобальные экстремумы

$$5) (|x|(9+x^2))' = 2x|x| + \frac{x}{|x|}(9+x^2) = \frac{3x^3 + 9x}{|x|}$$

$$y'' = \left(\frac{1}{3} + \frac{6x}{|x|(9+x^2)} \right)' = 6 \cdot \left(\frac{x' \cdot |x|(9+x^2) - x \cdot (|x|(9+x^2))'}{(|x|(9+x^2))^2} \right) = 6 \cdot \left(\frac{|x|(9+x^2) - \frac{3x^4 + 9x^2}{|x|}}{x^2(9+x^2)^2} \right)$$

$$= \frac{12|x|}{(9+x^2)^2}$$

$$= \frac{12|x|}{(9+x^2)^2} = 0 \quad x=0$$

$M(0;0)$ - точка перегиба

6)

x	$-\infty$	$-6,993$	$-0,999$	-3	-3	0	0
y	$-$	0	$+$	$0,571$	$+$	0	$+$
y''	\nearrow	$0,23$	\nearrow	0	\searrow	\times	\nearrow
y'''	\frown	$-0,33$	\frown	$-0,25$	\frown	0	\frown

7)

