Parallel programming. Task 10.

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1 Task statement

Consider boundary value problem:

$$y'' = f(x, y); \quad y(x_0) = a; \quad y(x_1) = b \tag{1}$$

The task is to obtain solution via Numerov approximation:

$$y_{m+1} - 2y_m + y_{m-1} = h^2 \cdot [f_m + \frac{1}{12}(f_{m+1} - 2f_m + f_{m-1})] \tag{2}$$

2 Obtaining solution

2.1 Newton's linearization

First of all, using Numerov approximation we can construct the system of $N-2$ non-linear equations.

$$\begin{cases} g_1 = y_2 - 2y_1 + a - h^2 \cdot [f_1 + \frac{1}{12}(f_2 - 2f_1 + f(a))] = 0 \\ \dots \\ g_m = y_{m+1} - 2y_m + y_{m-1} - h^2 \cdot [f_m + \frac{1}{12}(f_{m+1} - 2f_m + f_{m-1})] = 0 \\ \dots \\ g_{N-2} = b - 2y_{N-2} + y_{N-3} - h^2 \cdot [f_{N-2} + \frac{1}{12}(f(b) - 2f_{N-2} + f_{N-3})] = 0 \end{cases} \tag{3}$$

The solution can be found using Newton's linearization:

$$\mathbf{y}^{k+1} = \mathbf{y}^k - [G'(\mathbf{y}^k)]^{-1} \cdot \mathbf{g}(\mathbf{y}^k) = \mathbf{y}^k - \mathbf{p}(\mathbf{y}^k) \tag{4}$$

Where G is Jacobi matrix and $\mathbf{p}(\mathbf{y}^k)$ can be obtained from:

$$G(\mathbf{y}^k) \cdot p = \mathbf{g}(\mathbf{y}^k) \tag{5}$$

The matrix G has the form:

$$\begin{pmatrix} \frac{\partial g_1}{\partial y_1} & \frac{\partial g_1}{\partial y_2} & \dots & \frac{\partial g_1}{\partial y_{N-2}} \\ \frac{\partial g_2}{\partial y_1} & \frac{\partial g_2}{\partial y_2} & \dots & \frac{\partial g_2}{\partial y_{N-2}} \\ \dots & \dots & \dots & \dots \\ \frac{\partial g_{N-2}}{\partial y_1} & \frac{\partial g_{N-2}}{\partial y_2} & \dots & \frac{\partial g_{N-2}}{\partial y_{N-2}} \end{pmatrix} \tag{6}$$

With Numerov approximation this matrix become tridiagonal.

$$\frac{\partial g_i}{\partial y_i} = -2 - h^2 \cdot \frac{5}{6} f'_y(x_i, y_i), \quad i \in [1, N-2] \tag{7}$$

$$\frac{\partial g_i}{\partial y_{i-1}} = 1 - h^2 \cdot \frac{1}{12} f'_y(x_{i-1}, y_{i-1}), \quad i \in [2, N-2] \tag{8}$$

$$\frac{\partial g_i}{\partial y_{i+1}} = 1 - h^2 \cdot \frac{1}{12} f'_y(x_{i+1}, y_{i+1}), \quad i \in [1, N-3] \tag{9}$$

Others 0.

2.2 Solving linear equation