

Qa.

Theoretical complexity for Insert is $O(\log n)$ because it has to maintain the minmaxheap property, so complexity for build would be $O(n \log n)$. Worst case for my Build is also $O(n \log n)$, because it inserts (which is $O(1)$), heapifies (which is $O(\log n)$), and does this n times, which makes it $O(n \log n)$.

Complexities for FindMin and FindMax are $O(1)$, as the min and max values will always be at the top of the heap. The worst cases for my code are also $O(1)$ because it just checks the first few nodes.

Complexities for DeleteMin and DeleteMax are $O(\log n)$ because they have to maintain the minmaxheap property. My algorithms have this same complexity because they remove the max (which is $O(1)$), replace the max with second largest (which is also $O(1)$), fills new hole with last node in level order ($O(1)$), and then heapifies (which is $O(\log n)$).

Qb.

Worst case for build is $O(n \log n)$, because it would do a heapify n times. However, my times in this screenshot seem to be linear, increasing by about .007-.009 seconds between each input size. I imagine this is because some nodes that are inserted don't need to be heapified, and a heapify at most needs to traverse half the layers of the tree. However, I could not provide a solid proof on why my times appear linear.

Since the find functions are always $O(1)$, they will remain about the same times no matter the input size. Any variation shown by the test is due to rounding to the nearest microsecond.

For the Deletes, which are $O(\log n)$, $\log_2(500000)$ is about 19, and $\log_2(100000)$ is about 17. A difference of 2 levels between the smallest and largest input size makes the appearance of constant time make sense. There is not enough of a difference in the amount of nodes being traversed to make a noticeable difference in runtime.

Input size	100,000	200,000	300,000	400,000	500,000
Build (s)	0.009620	0.015928	0.023950	0.033603	0.040089
FindMin (s)	0.000001	0.000001	0.000000	0.000001	0.000001
FindMax (s)	0.000001	0.000001	0.000001	0.000001	0.000001
DeleteMin (s)	0.000002	0.000001	0.000001	0.000002	0.000002
DeleteMax (s)	0.000002	0.000001	0.000002	0.000002	0.000002