Final Project of Numerical Methods for PDE

Finite Volume Method for Euler Equation

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Abstract

Keywords:

1 Problem Description

- Mass Conservation

$$\rho_t + (\rho u)_x = 0.$$

- Momentum Conservation

$$(\rho u)_t + (\rho u^2 + p)_x = 0.$$

- Energy Conservation

$$E = \rho e + \frac{1}{2}\rho u^2.$$

$$E_t + (u(E+p))_x = 0.$$

1.1 Riemann Problem

1.1.1 Exact Riemann Problem

1.1.2 Approximate Riemann Problem

1.2 Conservative Flux

1.3 Lax-Friedrichs Numerical Flux

1.4 Refined Lax-Friedrichs Numerical Flux

1.4.1 Roe Numerical Flux

2 Methods

2.1 Finite Volume Method

The governing equation could be written into matrix form:

$$\frac{\partial \mathbf{V}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{V})}{\partial x} = 0$$

$$\mathbf{V} = \begin{bmatrix} \rho \\ \rho u \\ \rho E \end{bmatrix}, \quad \mathbf{F}(\mathbf{V}) = \begin{bmatrix} \rho u \\ \frac{1}{2}\rho u^2 + pb \\ (E+p)u \end{bmatrix}$$

3 Error Analysis

4 Conclusion

References