# Neural Radiance Fields to Implementation

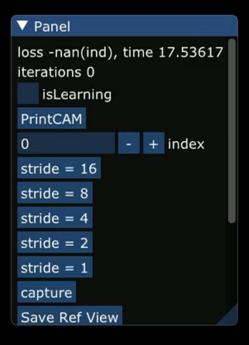
IF YOU ONLY KNEW THE POWER OF THE DARK SIDE

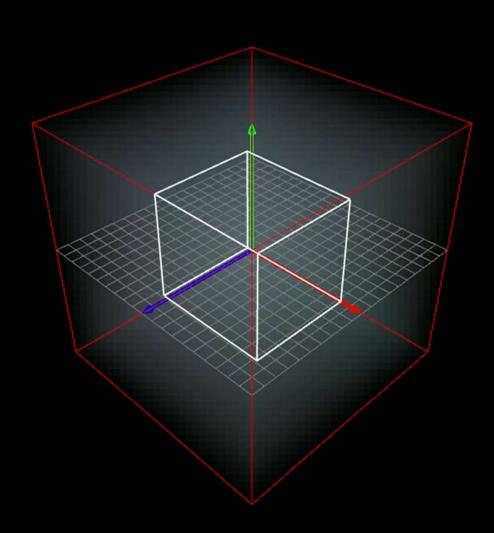
#### What's Neural Radiance Fields?

- A novel volumetric scene representation
  - Location( X, Y, Z ) and Direction(  $\theta$ ,  $\phi$  ) to Radiance and volume density ( R, G, B,  $\sigma$  )
  - Neural-based volume encoding fully-connected neural network
- Constructed from sparse reference inputs such as photos
- View synthesizing via volume ray marching

# Inputs

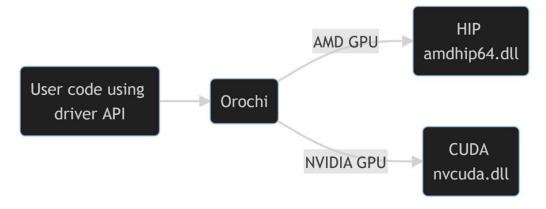






#### Environment

- Radeon Pro W6800 GPU
- CUDA (HIP via Orochi)
- Windows
- Works also on NV GPU

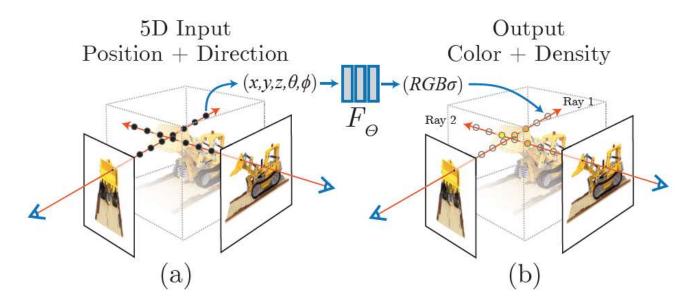


https://gpuopen.com/learn/introducing-orochi/ https://github.com/GPUOpen-LibrariesAndSDKs/Orochi

# Deep dive into NeRF

### Scene Representation as Volume

- A fancy function F Location( X, Y, Z ) and Direction(  $\theta$ ,  $\phi$  ) to Radiance and volume density ( R, G, B,  $\sigma$  )
- F is a fully-connected neural network!

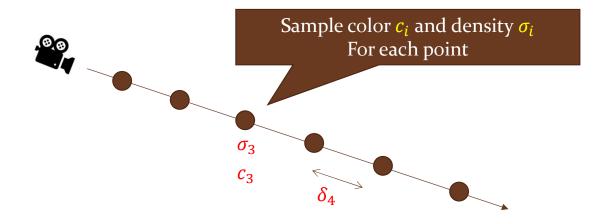


NeRF paper - Fig. 2

# Volume Rendering

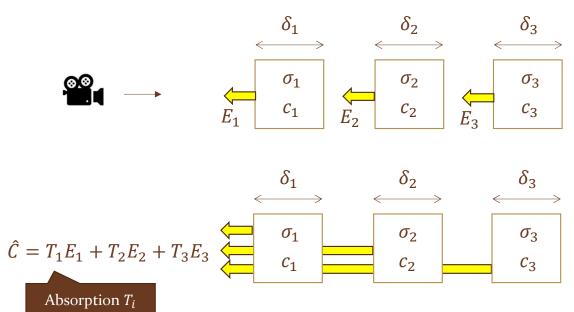
- Ray marching sample generation
  - Arbitrary sample strategy is applicable
    - Uniform
    - Proportional to a distance from its camera
    - NDC( normalized device coordinates )

Still not sure what's the best So let me skip this topic for now



# Volume Rendering

• Each subvolume emits some radiance & absorbed by the front subvolume



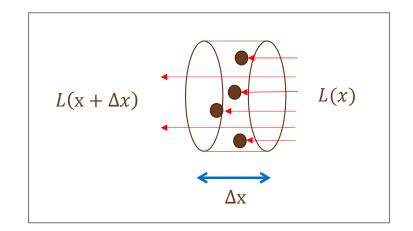
# Absorption

$$\frac{L(x + \Delta x) - L(x)}{\Delta x} = -\sigma L(x)$$



$$\frac{dL(x)}{dx} = -\sigma L(x)$$

$$L(x) = \exp(-\sigma x)L(0)$$



# Absorption

$$\hat{C} = T_1 E_1 + T_2 E_2 + T_3 E_3$$

$$\begin{array}{c} \delta_1 \\ \leftarrow \\ \end{array}$$

$$\begin{array}{c} \delta_2 \\ \leftarrow \\ \end{array}$$

$$\begin{array}{c} \sigma_2 \\ c_2 \\ \end{array}$$

$$\begin{array}{c} \sigma_3 \\ c_3 \\ \end{array}$$

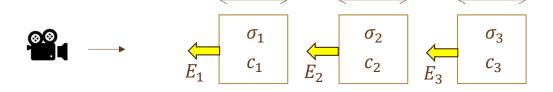
$$\begin{split} T_1 &= 1 \\ T_2 &= \exp(-\sigma_1 \delta_1) \\ T_3 &= \exp(-\sigma_1 \delta_1) \exp(-\sigma_2 \delta_2) = \exp(-\sigma_1 \delta_1 - \sigma_2 \delta_2) \end{split}$$



$$T_i = \prod_{j=1}^{i-1} \exp(-\sigma_i \delta_i) = \exp(-\sum_{j=1}^{i-1} \sigma_i \delta_i)$$

# Emission

• What about  $E_i$ ?



### **Emission**

$$\frac{E(\mathbf{x} + \Delta x) - E(\mathbf{x})}{\Delta x} = \sigma c$$

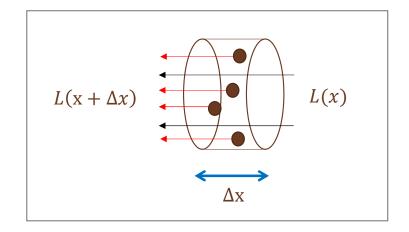


$$E = \int_0^x \sigma c \exp(-\sigma x) dx$$

$$= \sigma c \int_0^x \exp(-\sigma x) dx$$

$$= \sigma c (\frac{1 - \exp(-\sigma x)}{\sigma})$$

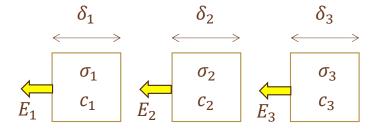
$$= c(1 - \exp(-\sigma x))$$



### **Emission**

• What about  $E_i$ ?





$$E_1 = c_1(1 - \exp(-\sigma_1 \delta_1))$$

$$E_2 = c_2(1 - \exp(-\sigma_2 \delta_2))$$

$$E_3 = c_3(1 - \exp(-\sigma_3 \delta_3))$$



$$E_i = c_i(1 - \exp(-\sigma_i \delta_i))$$

# Volume Rendering

• The final form

$$\hat{C} = \sum_{i=1}^{N} T_i \underbrace{(1 - \exp(-\sigma_i \delta_i)) c_i}_{\text{Emission}}$$

$$T_i = \exp(-\sum_{i=1}^{l-1} \sigma_i \delta_i)$$
Absorption



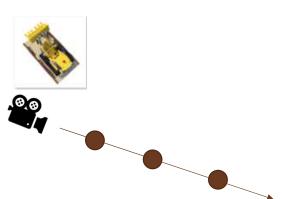
results in the MLP being evaluated at continuous positions over the course of optimization. We use these samples to estimate  $C(\mathbf{r})$  with the quadrature rule discussed in the volume rendering review by Max  $\boxed{26}$ :

$$\hat{C}(\mathbf{r}) = \sum_{i=1}^{N} T_i (1 - \exp(-\sigma_i \delta_i)) \mathbf{c}_i, \text{ where } T_i = \exp\left(-\sum_{j=1}^{i-1} \sigma_j \delta_j\right),$$
(3)

where  $\delta_i = t_{i+1} - t_i$  is the distance between adjacent samples. This function for calculating  $\hat{C}(x)$  from the set of (x, x) values is trivially differentiable and

# Training

• Loss function  $\mathcal{L}$  is based on the color difference



Example ) Simple squared error:

$$\mathcal{L} = \frac{1}{2} (\hat{C}_r - C_r)^2 + \frac{1}{2} (\hat{C}_g - C_g)^2 + \frac{1}{2} (\hat{C}_b - C_b)^2$$

$$\frac{d\mathcal{L}}{d\hat{C}_r} = \hat{C}_r - C_r \qquad \frac{d\mathcal{L}}{d\hat{C}_g} = \hat{C}_g - C_g \qquad \frac{d\mathcal{L}}{d\hat{C}_b} = \hat{C}_b - C_b$$

Remark:

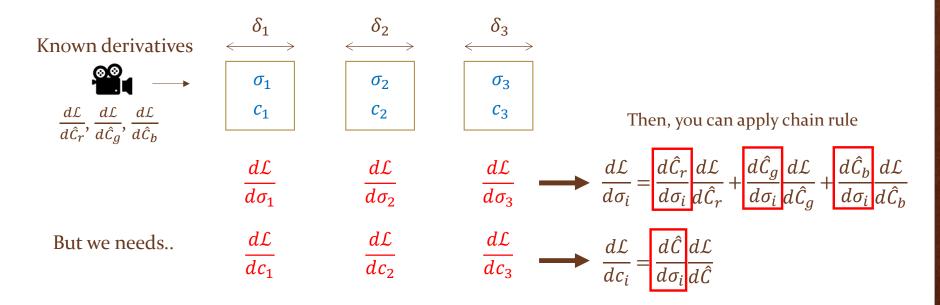
Instant NGP repository uses "Huber Loss" and it is more stable and faster to converge.

# $\sigma_1$

# Training

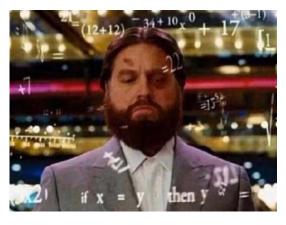


- We need all derivatives of the learnable parameters
  - However, we only know  $\frac{d\mathcal{L}}{d\hat{c}_r}$ ,  $\frac{d\mathcal{L}}{d\hat{c}_g}$ ,  $\frac{d\mathcal{L}}{d\hat{c}_b}$ , not the derivatives of these parameters



$$\hat{C}(\mathbf{r}) = \sum_{i=1}^{N} T_i (1 - \exp(-\sigma_i \delta_i)) \mathbf{c}_i, \text{ where } T_i = \exp\left(-\sum_{j=1}^{i-1} \sigma_j \delta_j\right), \quad (3)$$

where  $\delta_i = t_{i+1} - t_i$  is the distance between adjacent samples. This function for calculating  $\hat{C}(\mathbf{r})$  from the set of  $(\mathbf{c}_i, \sigma_i)$  values is trivially differentiable and reduces to traditional alpha compositing with alpha values  $\alpha_i = 1 - \exp(-\sigma_i \delta_i)$ .



NeRF paper – Eq. 3

Me when seeing that the paper explain it as trivial

Derivative 
$$\frac{d\hat{C}}{dc_i}$$

$$\hat{C} = T_1(1 - \exp(-\sigma_1 \delta_1))c_1 + T_2(1 - \exp(-\sigma_2 \delta_2))c_2 + T_3(1 - \exp(-\sigma_3 \delta_3))c_3$$

Fortunately, these derivatives are trivial indeed as follows:

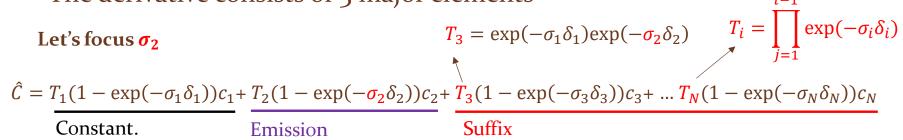
$$\frac{d\hat{C}}{dc_1} = T_1(1 - \exp(-\sigma_1 \delta_1))$$

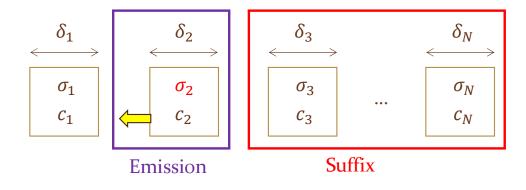
$$\frac{d\hat{C}}{dc_2} = T_2(1 - \exp(-\sigma_2 \delta_2))$$

$$\frac{d\hat{C}}{dc_3} = T_3(1 - \exp(-\sigma_3 \delta_3))$$

# Derivative $\frac{d\hat{c}}{d\sigma_i}$

• The derivative consists of 3 major elements





# Derivative $\frac{d\hat{C}}{d\sigma_i}$ - Suffix

• Let's define suffix  $S_i$ 

$$T_{3} = \exp(-\sigma_{1}\delta_{1})\exp(-\sigma_{2}\delta_{2})$$
• Let's define suffix  $S_{j}$ 

$$\hat{C} = T_{1}(1 - \exp(-\sigma_{1}\delta_{1}))c_{1} + T_{2}(1 - \exp(-\sigma_{2}\delta_{2}))c_{2} + T_{3}(1 - \exp(-\sigma_{3}\delta_{3}))c_{3} + \dots T_{N}(1 - \exp(-\sigma_{N}\delta_{N}))c_{N}$$

$$S_2 = T_3(1 - \exp(-\sigma_3 \delta_3))c_3 + \dots T_N(1 - \exp(-\sigma_N \delta_N))c_N$$

$$S_j = \sum_{i=j+1}^N T_i(1 - \exp(-\sigma_i \delta_i))c_i$$

Key insight:

All terms in  $S_2$  contains just an  $\exp(-\sigma_2 \delta_2)$  term inside T. Thus, the derivative is

$$S_2 = \exp(-\sigma_2 \delta_2)(Constant)$$

$$\frac{dS_2}{d\sigma_2} = -\delta_2 \exp(-\sigma_2 \delta_2)(Constant) = -\delta_2 S_2$$

# Derivative $\frac{d\hat{c}}{d\sigma_i}$ - Suffix

• Suffix  $S_i$  is cheaply available via  $\hat{C}$ 

```
float3 color = make_float3( 0.0f, 0.0f, 0.0f );
for( int yi = eval_beg; yi < eval_end; yi++ )
{
    float dt = ...;
    float sigma = ...
    float3 c = ...
    float a = 1.0f - exp( -sigma * dt );
    float coefficient = T * a;
    color += coefficient * c;

    T *= ( 1.0f - a );
    float3 S = C_hat - color;
}</pre>
```

# Derivative $\frac{d\hat{c}}{d\sigma_i}$ - Emission

• Almost trivial to get the derivative of the emission

$$\hat{C} = T_1(1 - \exp(-\sigma_1 \delta_1))c_1 + \frac{T_2(1 - \exp(-\sigma_2 \delta_2))c_2}{T_3(1 - \exp(-\sigma_3 \delta_3))c_3 + \dots T_N(1 - \exp(-\sigma_N \delta_N))c_N}$$
Emission

$$\hat{C}_{emission} = T_2 (1 - \exp(-\sigma_2 \delta_2)) c_2 = T_2 c_2 - T_2 c_2 \exp(-\sigma_2 \delta_2)$$

$$\frac{d\hat{C}_{emission}}{d\sigma_2} = T_2 c_2 \delta_2 \exp(-\sigma_2 \delta_2) = T_3 c_2 \delta_2$$

$$\uparrow$$

$$T_{i+1} = T_i \exp(-\sigma_i \delta_i)$$

# Derivative of $\sigma_i$ - Complete Form

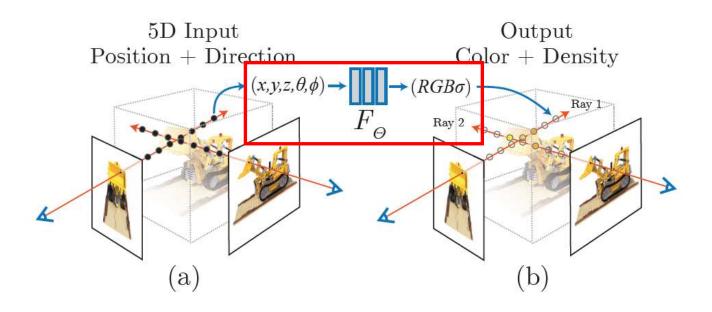
• Simple and very easy to evaluate ©

$$\frac{d\hat{C}}{d\sigma_i} = \delta_i (T_{i+1}c_i - S_i)$$
 Propagate this to the neural network

#### Intuitive explanation

Operation	Result
Increase $\sigma$	Make it more opaque then suppress the subsequent color contributions.
Decrease $\sigma$	Make it more transparent then take more contributions from the subsequent colors.

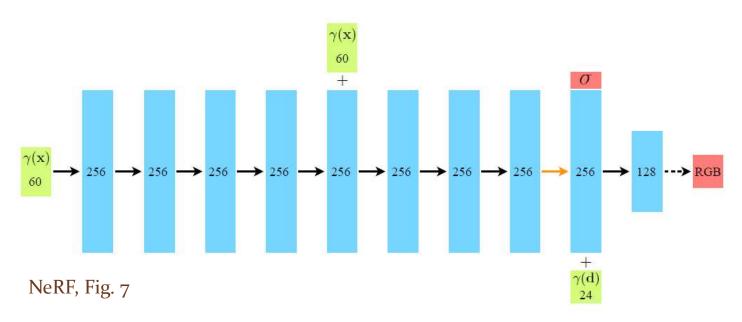
### Neural network



NeRF paper - Fig. 2

# Neural network – the original proposal

- 256 dimension, 9 layers + 128 dim layer
- Expensive to evaluate and train

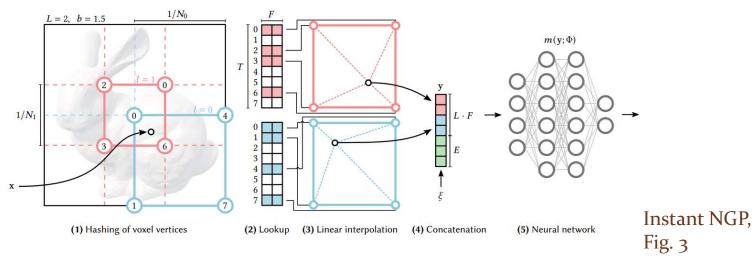


# Neural network – Instant Neural Graphics Primitives with a Multiresolution Hash Encoding

- A novel input preprocess technique for neural network
- Reduce network size without sacrificing quality
  - Produce even better quality
- Simple to implement

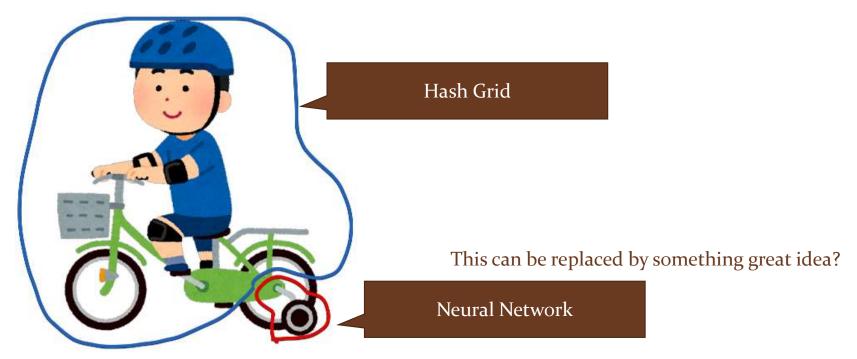
#### The idea

- Define grids with different resolutions for the input space
- Each grid corner is assigned to elements on the hash table
- Feed the linearly interpolated value to the neural network



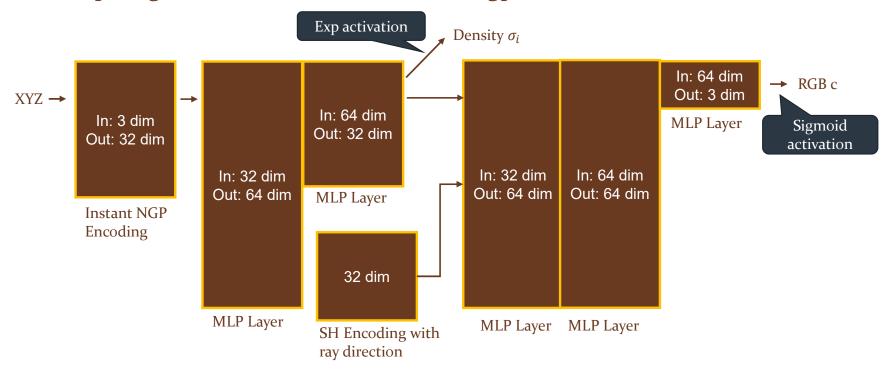
# Random thoughts

Then, the role of the Neural Network is just a tiny adjustment?



#### The network architecture

• https://github.com/NVlabs/instant-ngp



# GPU implementation

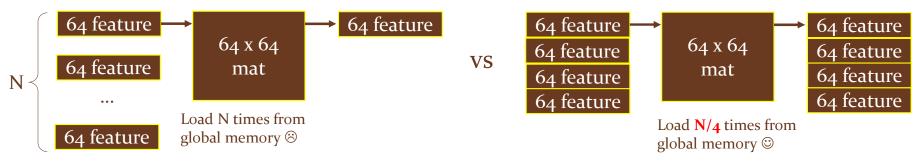
- Almost of the calculation can be done with matrix multiplications
- A smaller network architecture such as for NeRF is suitable for the GPU

# Thread-group matrix multiplications

Explained at "Real-time Neural Radiance Caching for Path Tracing"

But why?

- Feature vectors and weighting matrix on regs such as 64 wide consume too many vector registers ☺
- Batch multiplications can reduce the bandwidth for weighting matrix ☺
  - Weighting matrix can be large



Goal: 
$$\begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ x_{31} & x_{32} \end{pmatrix} \begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{pmatrix} =$$
 
$$\begin{pmatrix} x_{11}w_{11} + x_{12}w_{21} & x_{11}w_{12} + x_{12}w_{22} & x_{11}w_{13} + x_{12}w_{23} \\ x_{21}w_{11} + x_{22}w_{21} & x_{21}w_{12} + x_{22}w_{22} & x_{21}w_{13} + x_{22}w_{23} \\ x_{31}w_{11} + x_{32}w_{21} & x_{31}w_{12} + x_{32}w_{22} & x_{31}w_{13} + x_{32}w_{23} \end{pmatrix}$$

Note:

Feature Vector is on shared memory Weighting Matrix is on global memory



1. The output matrix is separately assigned to threads

Thread 0	Thread 1	Thread 2
0	0	0
0	0	0
0	0	0

Goal: 
$$\begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ x_{31} & x_{32} \end{pmatrix} \underbrace{ \begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{pmatrix} }_{ w_{21} = 1} =$$
 
$$\begin{pmatrix} x_{11}w_{11} + x_{12}w_{21} & x_{11}w_{12} + x_{12}w_{22} & x_{11}w_{13} + x_{12}w_{23} \\ x_{21}w_{11} + x_{22}w_{21} & x_{21}w_{12} + x_{22}w_{22} & x_{21}w_{13} + x_{22}w_{23} \\ x_{31}w_{11} + x_{32}w_{21} & x_{31}w_{12} + x_{32}w_{22} & x_{31}w_{13} + x_{32}w_{23} \end{pmatrix}$$

2. Load a weight from the global memory for each thread

Thread 0	Thread 1	Thread 2
0	0	0
0	0	0
0	0	0
<i>w</i> <sub>11</sub>	$w_{12}$	$w_{12}$

Goal: 
$$\begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ x_{31} & x_{32} \end{pmatrix} \begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{pmatrix} =$$
 
$$\begin{pmatrix} x_{11}w_{11} + x_{12}w_{21} & x_{11}w_{12} + x_{12}w_{22} & x_{11}w_{13} + x_{12}w_{23} \\ x_{21}w_{11} + x_{22}w_{21} & x_{21}w_{12} + x_{22}w_{22} & x_{21}w_{13} + x_{22}w_{23} \\ x_{31}w_{11} + x_{32}w_{21} & x_{31}w_{12} + x_{32}w_{22} & x_{31}w_{13} + x_{32}w_{23} \end{pmatrix}$$

3. Load a column of the feature vectors from shared memory, and do multiply-add operations

Thread 0	Thread 1	Thread 2
$x_{11}w_{11}$	$x_{11}w_{12}$	$x_{11}w_{13}$
$x_{21}w_{11}$	$x_{21}w_{12}$	$x_{21}w_{13}$
$x_{31}w_{11}$	$x_{31}w_{12}$	$x_{31}w_{13}$
$w_{11}$	$w_{12}$	$w_{12}$

Goal: 
$$\begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ x_{31} & x_{32} \end{pmatrix} \begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{pmatrix} =$$
 
$$\begin{pmatrix} x_{11}w_{11} + x_{12}w_{21} & x_{11}w_{12} + x_{12}w_{22} & x_{11}w_{13} + x_{12}w_{23} \\ x_{21}w_{11} + x_{22}w_{21} & x_{21}w_{12} + x_{22}w_{22} & x_{21}w_{13} + x_{22}w_{23} \\ x_{31}w_{11} + x_{32}w_{21} & x_{31}w_{12} + x_{32}w_{22} & x_{31}w_{13} + x_{32}w_{23} \end{pmatrix}$$

4. Load a weight from the global memory for each thread

Thread 0	Thread 1	Thread 2
$x_{11}w_{11}$	$x_{11}w_{12}$	$x_{11}w_{13}$
$x_{21}w_{11}$	$x_{21}w_{12}$	$x_{21}w_{13}$
$x_{31}w_{11}$	$x_{31}w_{12}$	$x_{31}w_{13}$
$W_{21}$	$W_{22}$	$W_{22}$

Goal: 
$$\begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ x_{31} & x_{32} \end{pmatrix} \begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{pmatrix} =$$
 
$$\begin{pmatrix} x_{11}w_{11} + x_{12}w_{21} & x_{11}w_{12} + x_{12}w_{22} & x_{11}w_{13} + x_{12}w_{23} \\ x_{21}w_{11} + x_{22}w_{21} & x_{21}w_{12} + x_{22}w_{22} & x_{21}w_{13} + x_{22}w_{23} \\ x_{31}w_{11} + x_{32}w_{21} & x_{31}w_{12} + x_{32}w_{22} & x_{31}w_{13} + x_{32}w_{23} \end{pmatrix}$$

5. Load a column of the feature vectors from shared memory, and do multiply-add operations

Thread 0 $x_{11}w_{11} + x_{12}w_{21}$ $x_{21}w_{11} + x_{22}w_{21}$ $x_{31}w_{11} + x_{32}w_{21}$	Thread 1 $x_{11}w_{12} + x_{12}w_{22}$ $x_{21}w_{12} + x_{22}w_{22}$ $x_{31}w_{12} + x_{32}w_{22}$	Thread 2 $x_{11}w_{13} + x_{12}w_{23}$ $x_{21}w_{13} + x_{22}w_{23}$ $x_{31}w_{13} + x_{32}w_{23}$
731111 1 7321121	731.12 . 732.722	731.13 . 732.723

 $w_{21}$ 

 $w_{22}$ 

 $w_{22}$ 

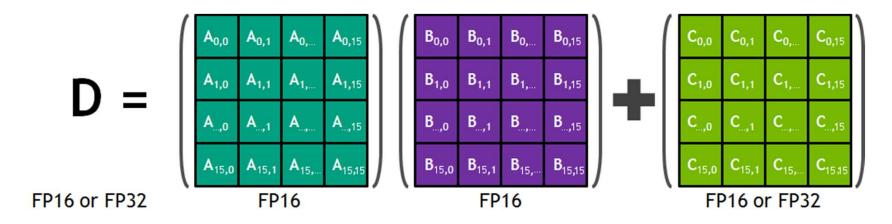
Done!

Roughly  $4x \sim 5x$  faster than the naive "an item per thread" approach.

Vector register allocation is modest ☺

# Thread-group matrix multiplications ( Hardware )

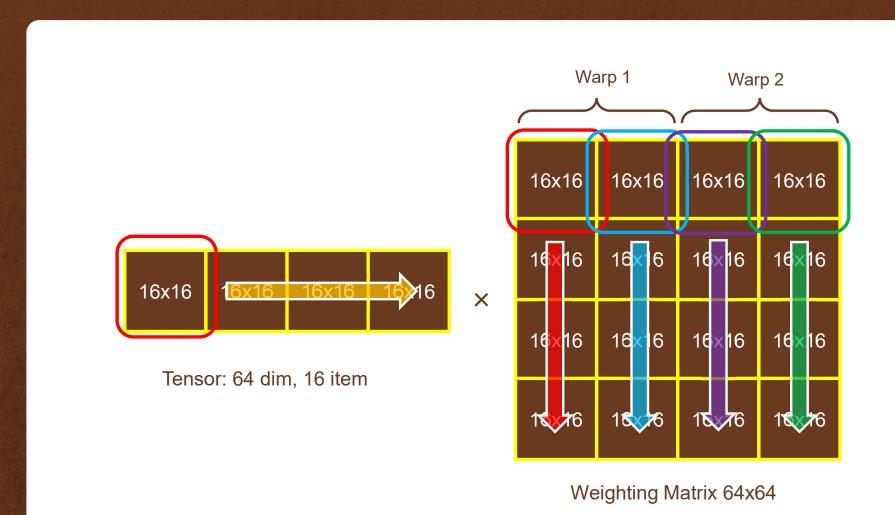
- WMMA on NV platform
- Fixed size matrix multiplications for each warp e.g. 16x16 matrix



https://developer.nvidia.com/blog/programming-tensor-cores-cuda-9/

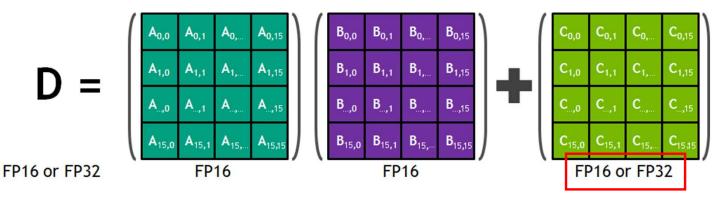
#### The idea is simple:

• Separate a matrix multiplication into smaller matrix multiplications



#### Remark

- Roughly 1.3x faster than the software on my nerf implementation
  - Not significant, because hash grid encoding is also bottleneck
  - May need more investigations
- The accumulation can be done with FP32
  - Depends on applications, but I couldn't find any numerical issue



### Summary

- The volume formulation on NeRF is simple and powerful
  - The implementation is not rocket science
- GPU implementation makes sense
  - Thread-group matrix multiplications achieve great performance
    - Even without dedicated HW
  - WMMA can improve more
    - You have any ideas? Please let me know

Questions?

#### References

- Ben Mildenhall, Pratul P. Srinivasan, Matthew Tancik, Jonathan T. Barron, Ravi Ramamoorthi and Ren Ng, "NeRF: Representing Scenes as Neural Radiance Fields for View Synthesis"
- Thomas Müller, Fabrice Rousselle, Jan Novák, and Alex Keller, "Real-time Neural Radiance Caching for Path Tracing"
- Thomas Müller, Alex Evans, Christoph Schied, and Alexander Keller "Instant Neural Graphics Primitives with a Multiresolution Hash Encoding"
- "Instant NGP" repository: <a href="https://github.com/NVlabs/instant-ngp">https://github.com/NVlabs/instant-ngp</a>
- Takahiro Harada and Aaryaman Vasishta, "Introducing Orochi" <a href="https://gpuopen.com/learn/introducing-orochi/">https://gpuopen.com/learn/introducing-orochi/</a>
- "Orochi" repository: <a href="https://github.com/GPUOpen-LibrariesAndSDKs/Orochi">https://github.com/GPUOpen-LibrariesAndSDKs/Orochi</a>
- Jeremy Appleyard and Scott Yokim, "Programming Tensor Cores in CUDA 9": https://developer.nvidia.com/blog/programming-tensor-cores-cuda-9/