Confidence interval and Hypothesis Testing

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IITU 2021

COURSE SCHEDULE

week	Mid Term (weeks 01-07)	End Term (weeks 08-14)	week
01	Intro: Data Science Area and open source tools for Data Science	Post Feedback Week	08
02	NumPy package for data science	Correlation and Covariance	09
03	Pandas package for data science	Sampling and Estimation	10
04	Visualization with matplotlib	Hypothesis testing	11
05	Statistics: Distribution – Normal	Decision Tree	12
06	Exploratory Data Analysis (EDA)	Linear Regression	13
07	**Summary for 6 weeks QA session**	**Summary for 6 weeks QA session**	**14**
15	**Course summary**		

PREVIOUSLY

- Correlation
- Sampling and Estimation

IDEA

- The main question here is: **are the effects** we see in a **sample likely** to appear in the **larger population**?
- Or if it might appear in the sample by chance(случайно).

CONFIDENCE INTERVAL

CONFIDENCE INTERVAL

- A confidence interval is how much **uncertainty** there is with any statistic.
- Confidence intervals are often used with **a margin of error(погрешность)**.
- It tells you how confident you can be that the results from a poll or survey reflect what you would expect to find if it were possible to survey the entire population.

CONFIDENCE LEVELS

Widely used confidence levels:

- **90%**
- **95%**
- **99%**

https://www.khanacademy.org/math/statistics-probability/confidence-intervals-one-sample

CONFIDENCE INTERVAL

- Confidence interval **for mean**

$$\bar{x} \pm z_{\frac{\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right) \qquad \bar{\mathrm{X}} \pm t\frac{\mathrm{s}}{\sqrt{\mathrm{n}}}$$

- Confidence interval **for proportion**

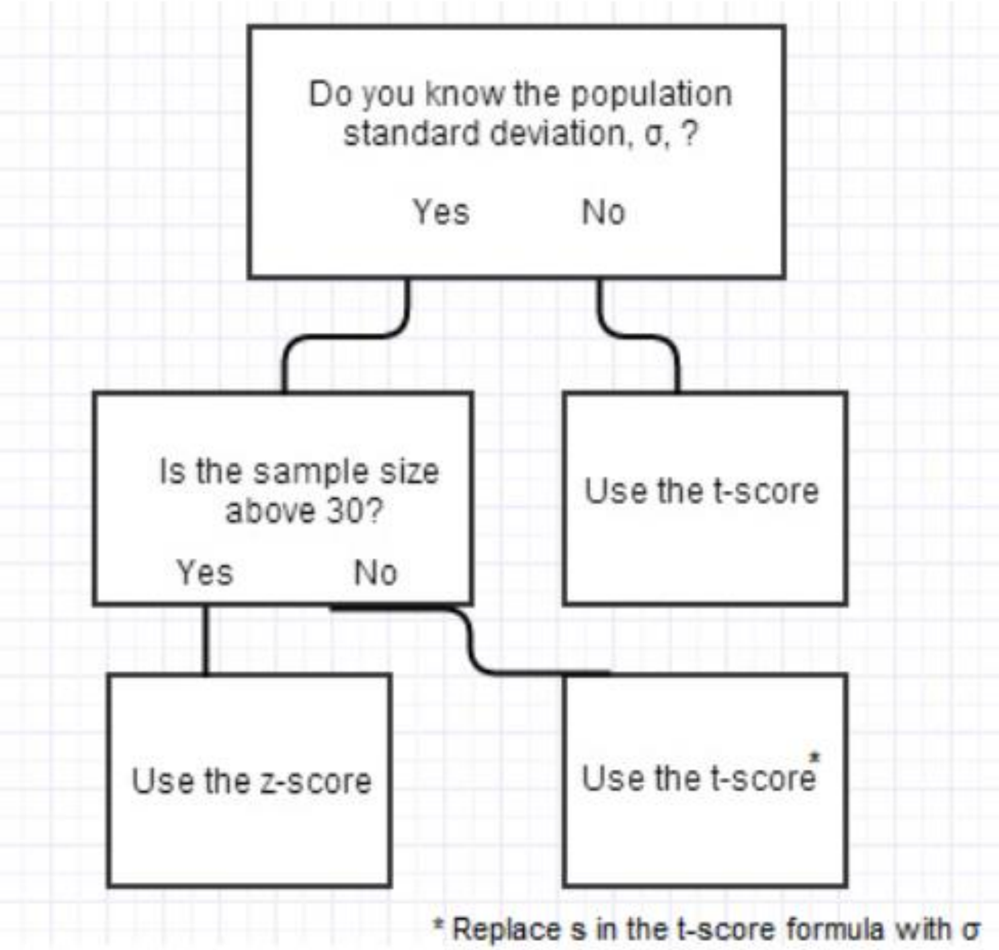
$$\hat{p} \pm z^{*}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Z-score and T-score

- **Z-score** Simply put, a z-score (also called a standard score) gives you an idea of how far from the mean a data point is. But more technically it's a measure of how many standard deviations below or above the population mean a raw score is.
- **t-score**
- **z-score** vs. **t-score**

$$z_i = \frac{x_i - \bar{x}}{s}$$

Z-score and T-score



CONFIDENCE LEVELS

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

CONFIDENCE LEVELS

HYPOTHESIS TESTING

HYPOTHESIS TESTING

- The goal of classical hypothesis testing is to answer the question, "**Given a sample and an apparent effect, what is the probability of seeing such an effect by chance?**"

HYPOTHESIS TESTING

-Fisher null hypothesis testing,

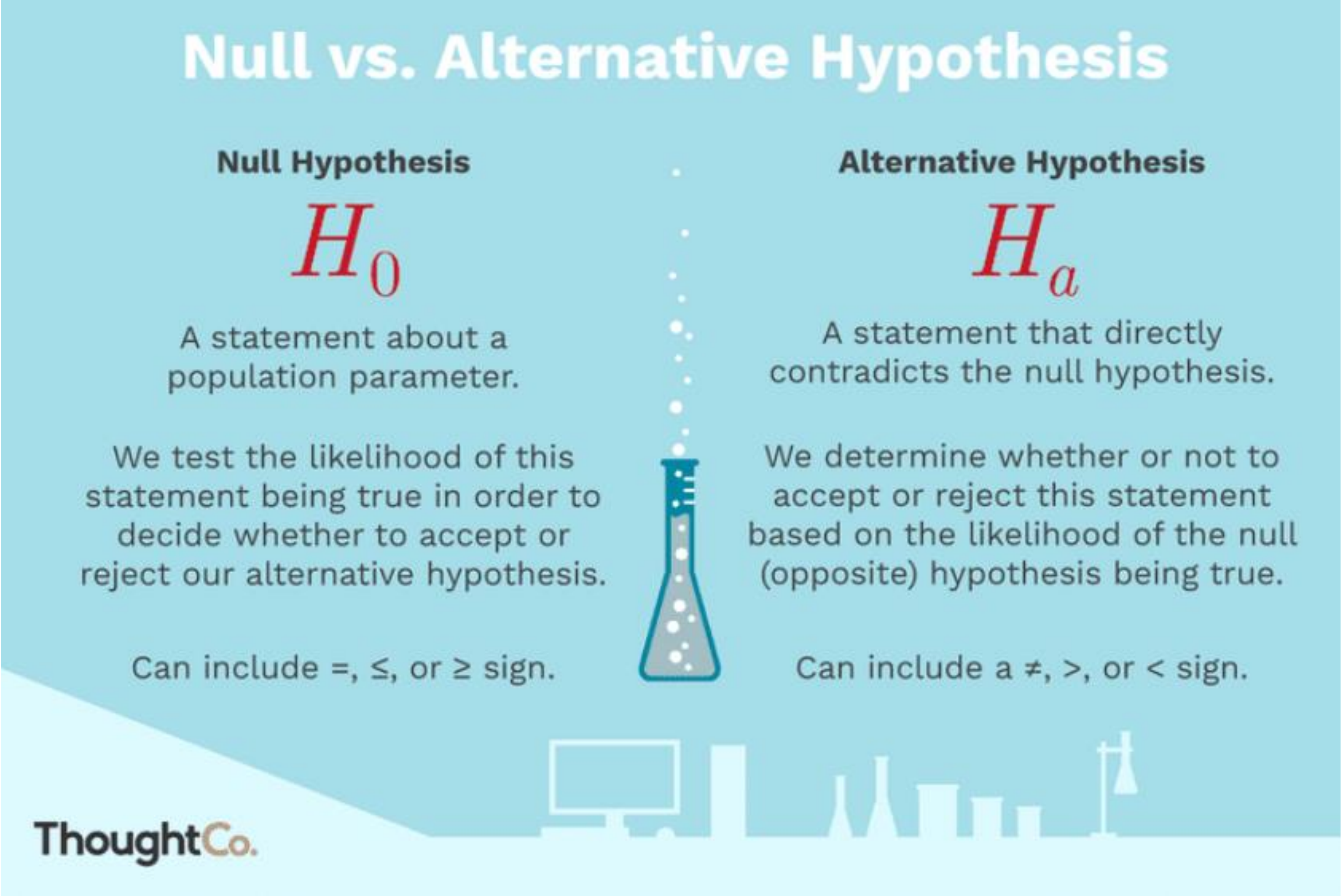
-Newman-Pearson decision theory,

-Bayesian inference

HYPOTHESIS TESTING

- Hypothesis testing
  - T Test
  - **Z Test**
  - ANOVA Test
  - **Chi-Square Test**

CLASSICAL HYPOTHESIS TESTING



https://towardsdatascience.com/hypothesis-testing-in-machine-learning-using-python-a0dc89e169ce

CLASSICAL HYPOTHESIS TESTING

STEP 01:

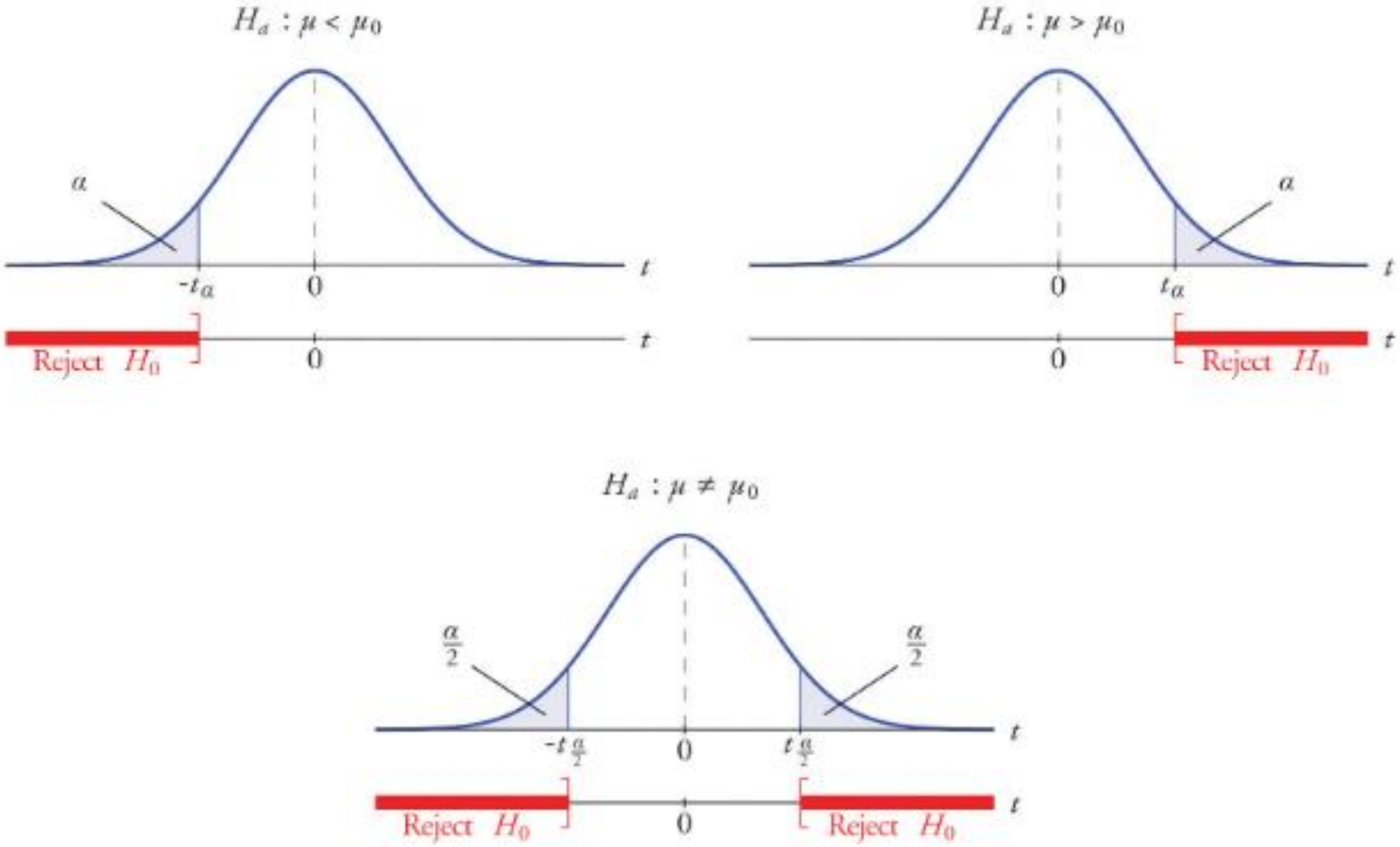
- Quantify the size of the apparent effect by choosing a **test statistic**.

CLASSICAL HYPOTHESIS TESTING

STEP 02:

- Define **a null hypothesis**, which is a model of the system based on the assumption that the apparent effect is not real.

CLASSICAL HYPOTHESIS TESTING



CLASSICAL HYPOTHESIS TESTING

STEP 03:

- Compute a **p-value/z-score**, which is the probability of seeing the apparent effect if the null hypothesis is true.

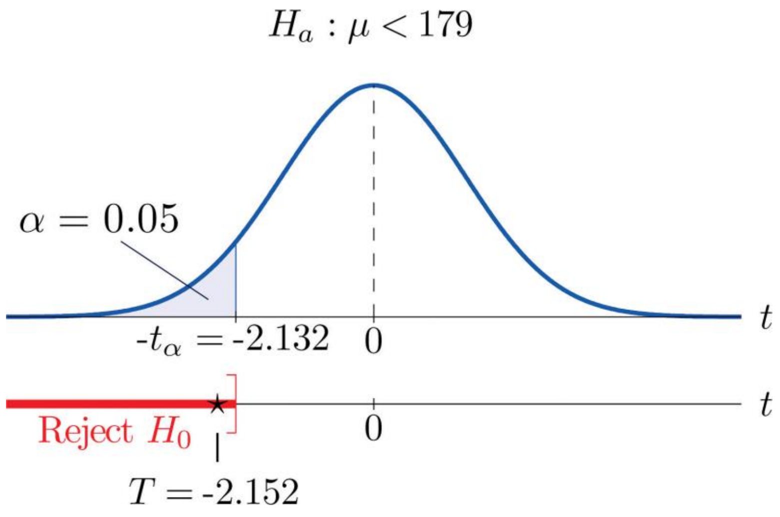
$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{N}}}$$

$$T = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{{s_1}^2}{n_1} + \frac{{s_2}^2}{n_2}}}$$

CLASSICAL HYPOTHESIS TESTING

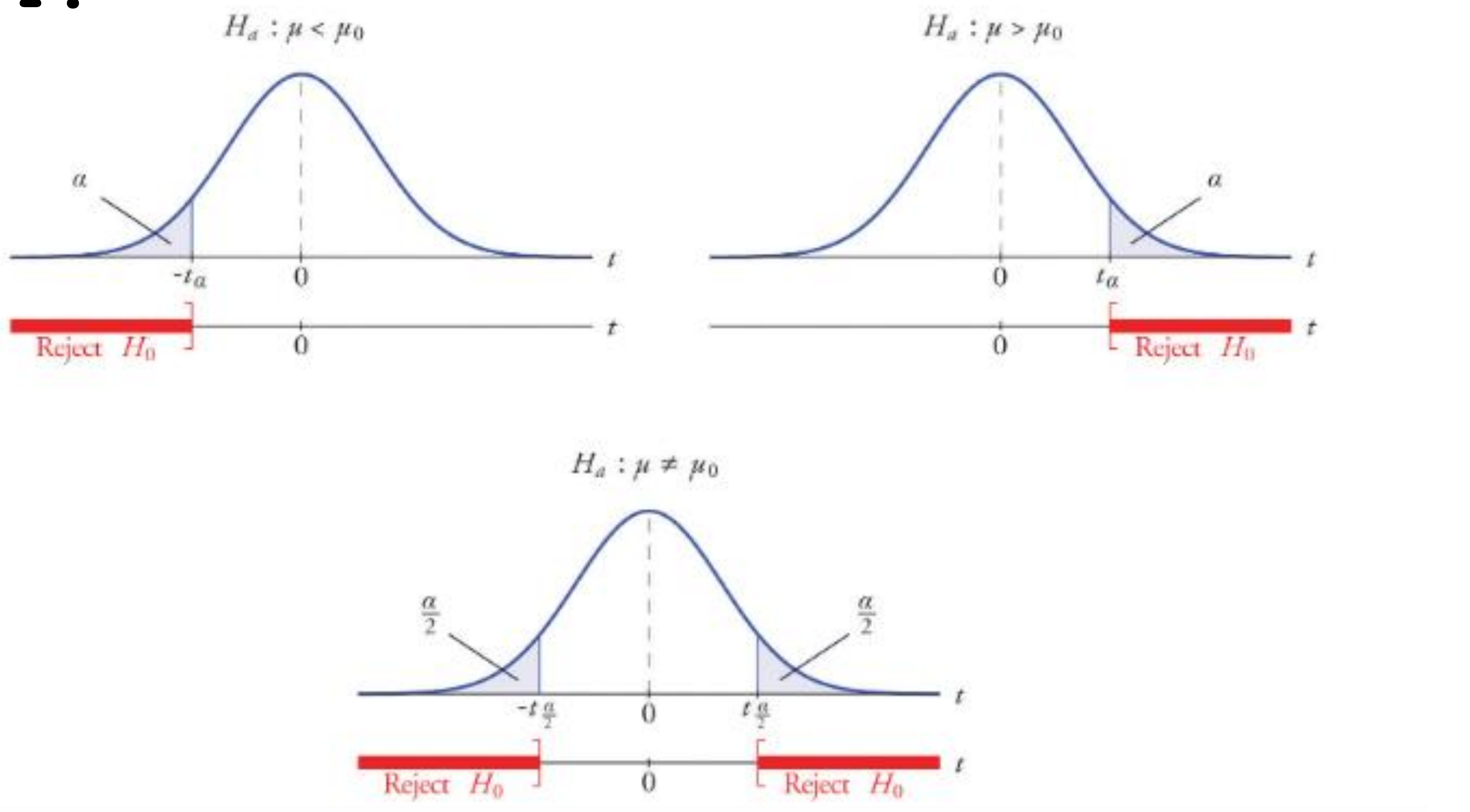
STEP 04:

- Interpret the result. **If the p-value/z-score** is low or too high, the effect is said to be statistically significant



CLASSICAL HYPOTHESIS TESTING

STEP 04:



Z TEST

Example (Z-test)

Sample of heights

x = [1.60, 1.59, 1.69, 1.71, 1.55, 1.68, 1.67, 1.69, 1.70, 1.73, 1.63]

$\bar{x}$ **= 1.66**

S = 0.056

n = 11 (11<30 – we will use t-score)

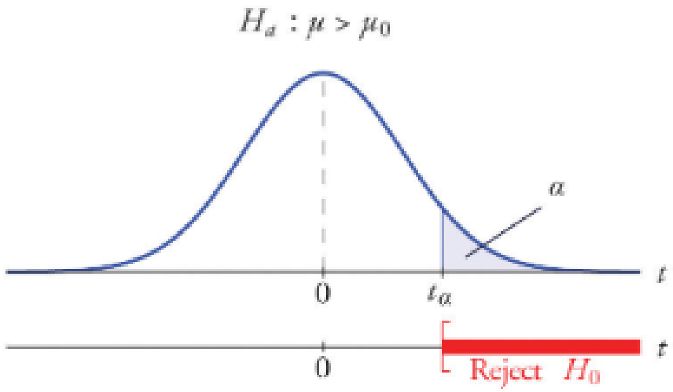
Z TEST

Hypothesizes: We heard that usually girls' height is **1.62 meters.**

$H_0: \mu = 1.62$ **we assume that effect in our dataset is by chance**

$H_a: \mu > 1.62$ **we assume that effect in our dataset is not occasionally**

$\alpha = 0.05$ (test is one side - **right sided**)



Z TEST

$H_0: \mu = 1.62$

$H_a: \mu > 1.62$

$\alpha = 0.05$

Z TEST

$\alpha = 0.05$ (test is one side – **right sided**)

$T_{\alpha=0.05} = 1.812$

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587

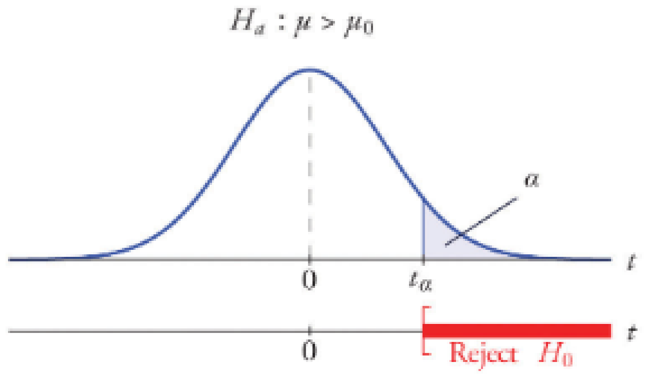
Z TEST

Let's calculate t score for our hypothesis -

$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{N}}} \qquad \frac{1.66 - 1.62}{\frac{0.056}{\sqrt{11}}} = 2,35$$

t_α = 1,812 $and\ t_{H0} = 2,35$

H0 – should be rejected



Chi-squared tests

Chi-squared tests are commonly used to test proportions.

When you have two categorical variables from a single population.

It is used to determine whether there is a significant association between the two variables.

Chi-squared tests

For example, in an election survey, voters might be classified by gender (male or female) and voting preference (Democrat, Republican, or Independent). We could use a chi-square test for independence to determine whether gender is related to voting preference

Chi-squared tests

$$\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$

Where **Oi are the observed** frequencies and **Ei are the expected** frequencies

Chi-squared test, EXPLAINED

Let’s say we have a standardized test with possible answers A, B, C and D.

Are the chances to hit the right answer equally distributed?

Chi-squared test, EXPLAINED

Let’s say we have a standardized test with possible answers **A, B, C and D.**

I have an intuition that A has more chances to be right. **Let’s say:**

A = 20%

B = 20%

C = 25%

D = 35%

Chi-squared test, EXPLAINED

Let's say we have a standardized test with possible answers A, B, C and D.

I have an intuition that A has more chances to be right. Let's say:

A = 20%

B = 20%

C = 25%

D = 35%

HOW CAN WE TEST THIS?

Chi-squared test, EXPLAINED

The null hypotheses is

H0: they are equally distributed

Ha: they are NOT equally distributed

A = 25%

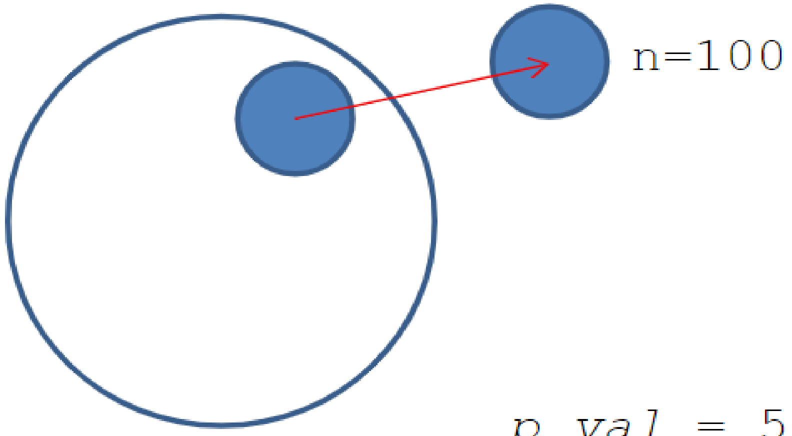
B = 25%

C = 25%

D = 25%

Chi-squared test, EXPLAINED

Let's say we take a sample from a population:



Correct choice	Expected #	Actual #
A	25	20
B	25	20
C	25	25
D	25	35

p_val = 5%

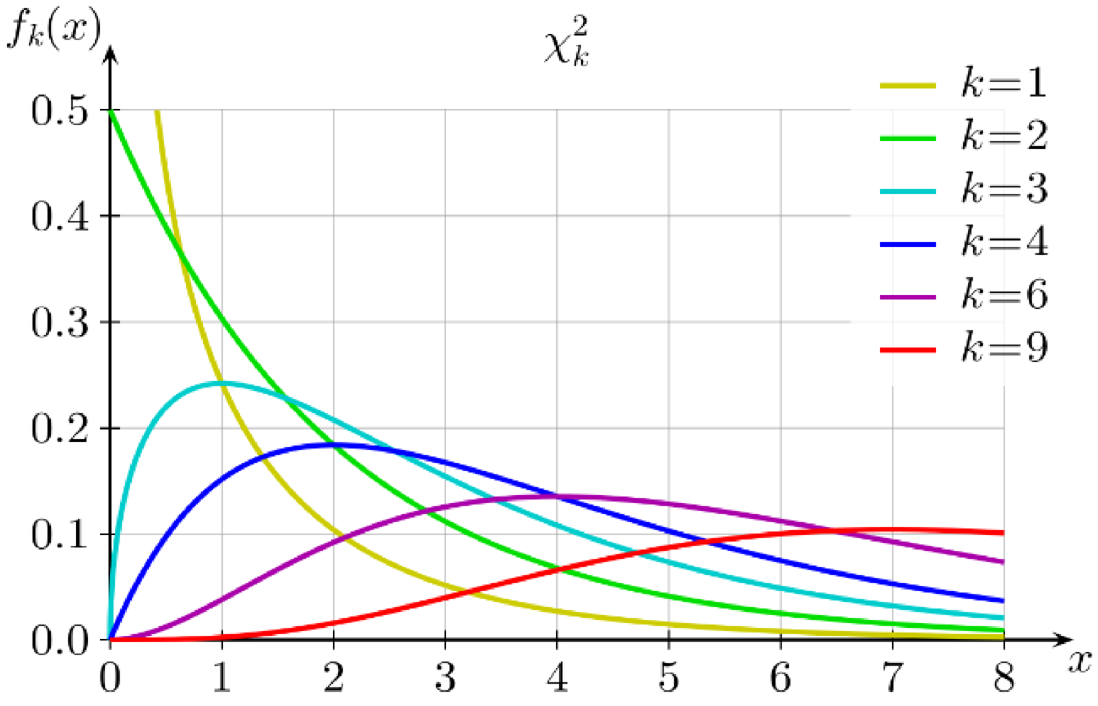
$$\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i} = \frac{(20-25)^2}{25} + \frac{(20-25)^2}{25} + \frac{(25-25)^2}{25} + \frac{(25-35)^2}{25} = 6$$

Chi-squared test, EXPLAINED

Correct choice	Expected #	Actual #
A	25	20
B	25	20
C	25	25
D	25	35

p_val = 5%

$\chi = 6$

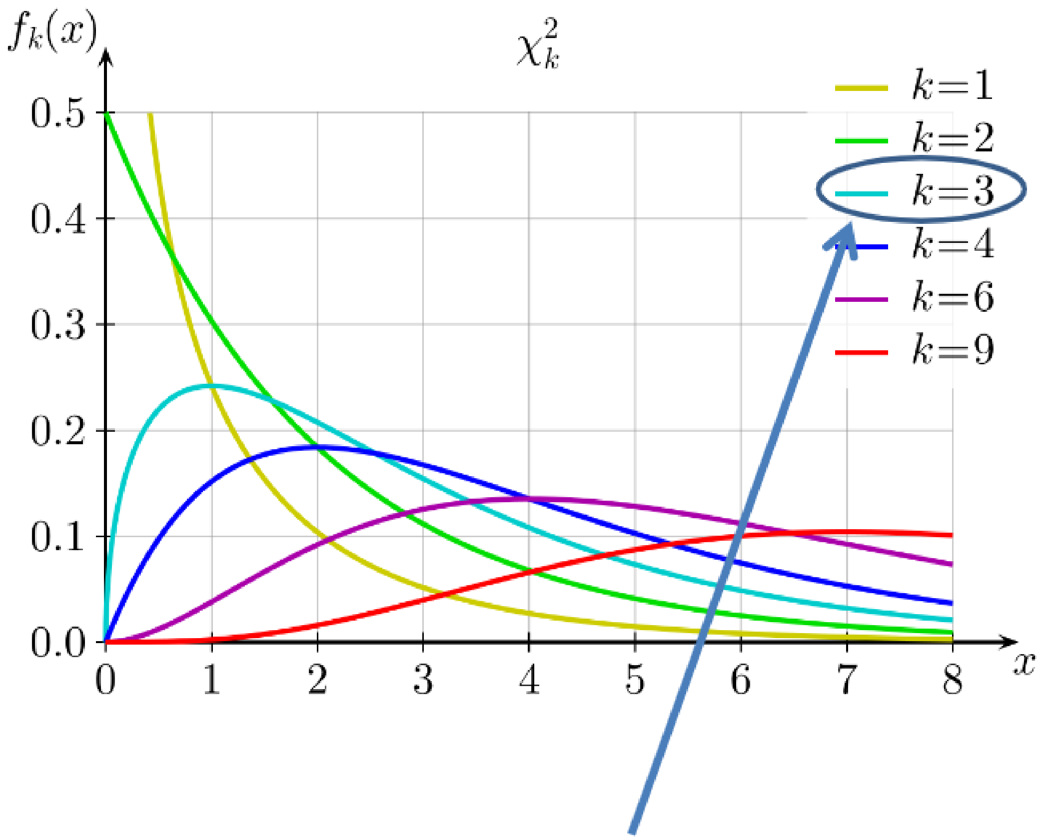


Chi-squared test, EXPLAINED

Correct choice	Expected #	Actual #
A	25	20
B	25	20
C	25	25
D	25	35

p_val = 5%

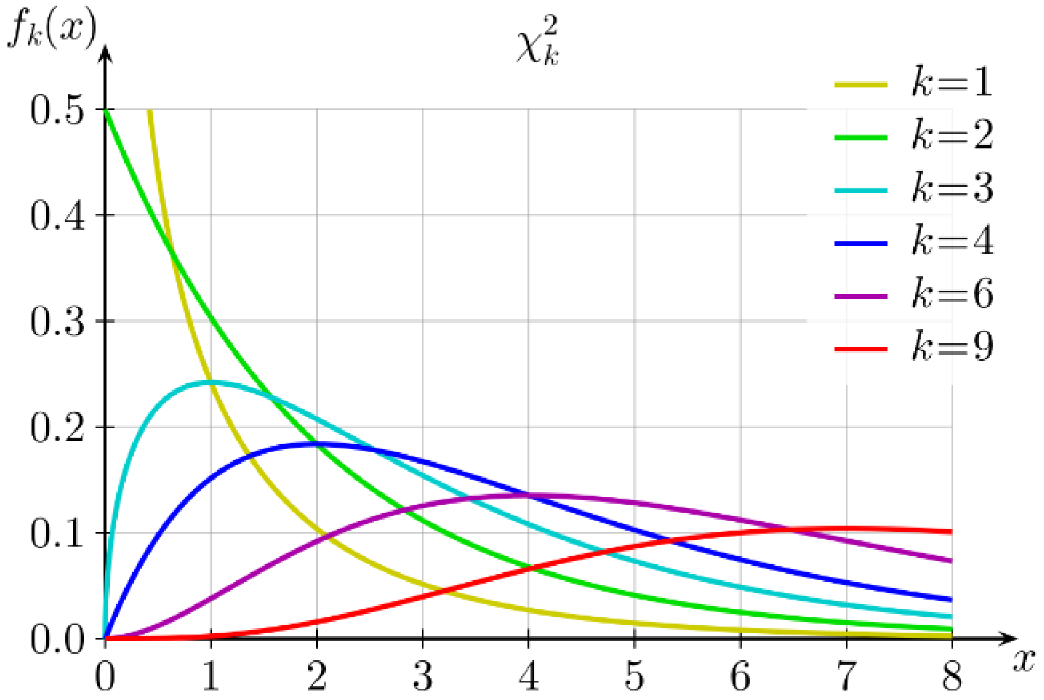
$\chi = 6$



We have 4 possible outcomes, so our degree of freedom is 3

Chi-squared test, EXPLAINED

Correct choice	Expected #	Actual #
A	25	20
B	25	20
C	25	25
D	25	35



p_val = 5%

$\chi = 6$

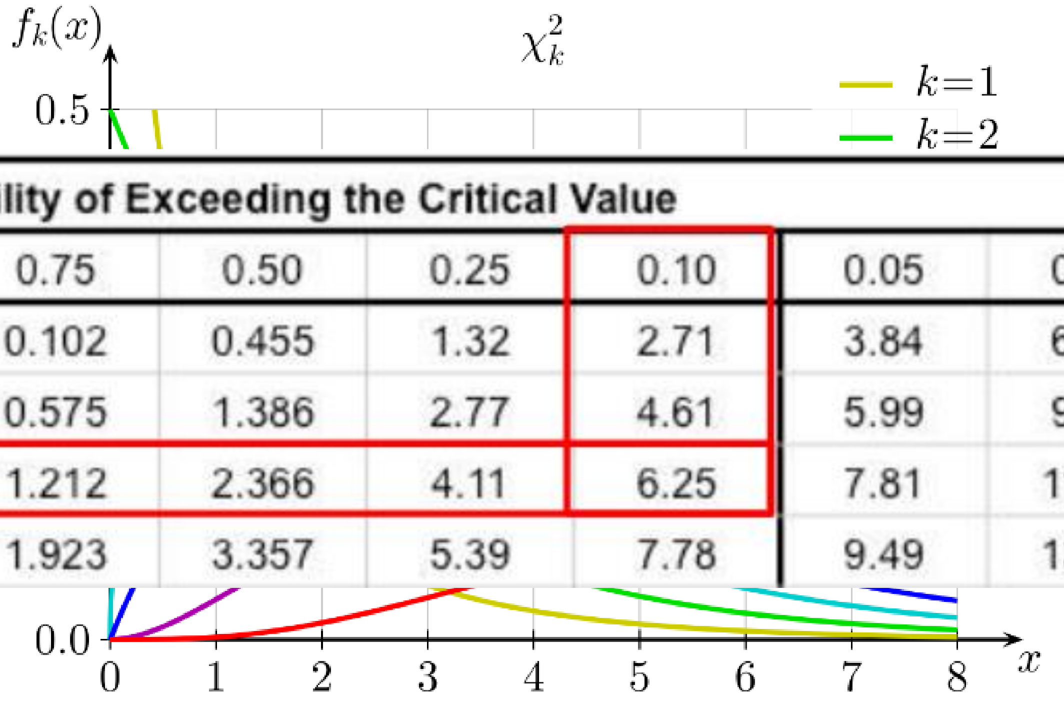
Now we ask: what is the probability of getting $\chi = 6$ given that our d.o.f is 3?

Chi-squared test, EXPLAINED

Degree of Freedom	Probability of Exceeding the Critical Value								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09
6	0.872	1.635	2.204	3.455	5.348	7.84	10.64	12.59	16.81
7	1.239	2.167	2.833	4.255	6.346	9.04	12.02	14.07	18.48
8	1.647	2.733	3.490	5.071	7.344	10.22	13.36	15.51	20.09
9	2.088	3.325	4.168	5.899	8.343	11.39	14.68	16.92	21.67
10	2.558	3.940	4.865	6.737	9.342	12.55	15.99	18.31	23.21
11	3.053	4.575	5.578	7.584	10.341	13.70	17.28	19.68	24.72
12	3.571	5.226	6.304	8.438	11.340	14.85	18.55	21.03	26.22
13	4.107	5.892	7.042	9.299	12.340	15.98	19.81	22.36	27.69
14	4.660	6.571	7.790	10.165	13.339	17.12	21.06	23.68	29.14
15	5.229	7.261	8.547	11.037	14.339	18.25	22.31	25.00	30.58
16	5.812	7.962	9.312	11.912	15.338	19.37	23.54	26.30	32.00
17	6.408	8.672	10.085	12.792	16.338	20.49	24.77	27.59	33.41
18	7.015	9.390	10.865	13.675	17.338	21.60	25.99	28.87	34.80
19	7.633	10.117	11.651	14.562	18.338	22.72	27.20	30.14	36.19
20	8.260	10.851	12.443	15.452	19.337	23.83	28.41	31.41	37.57
22	9.542	12.338	14.041	17.240	21.337	26.04	30.81	33.92	40.29
24	10.856	13.848	15.659	19.037	23.337	28.24	33.20	36.42	42.98
26	12.198	15.379	17.292	20.843	25.336	30.43	35.56	38.89	45.64
28	13.565	16.928	18.939	22.657	27.336	32.62	37.92	41.34	48.28
30	14.953	18.493	20.599	24.478	29.336	34.80	40.26	43.77	50.89
40	22.164	26.509	29.051	33.660	39.335	45.62	51.80	55.76	63.69
50	27.707	34.764	37.689	42.942	49.335	56.33	63.17	67.50	76.15
60	37.485	43.188	46.459	52.294	59.335	66.98	74.40	79.08	88.38
	Not Significant							*Significant*	

Chi-squared test, EXPLAINED

Correct choice	Expected #	Actual #
A	25	20



Degree of Freedom	Probability of Exceeding the Critical Value								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28

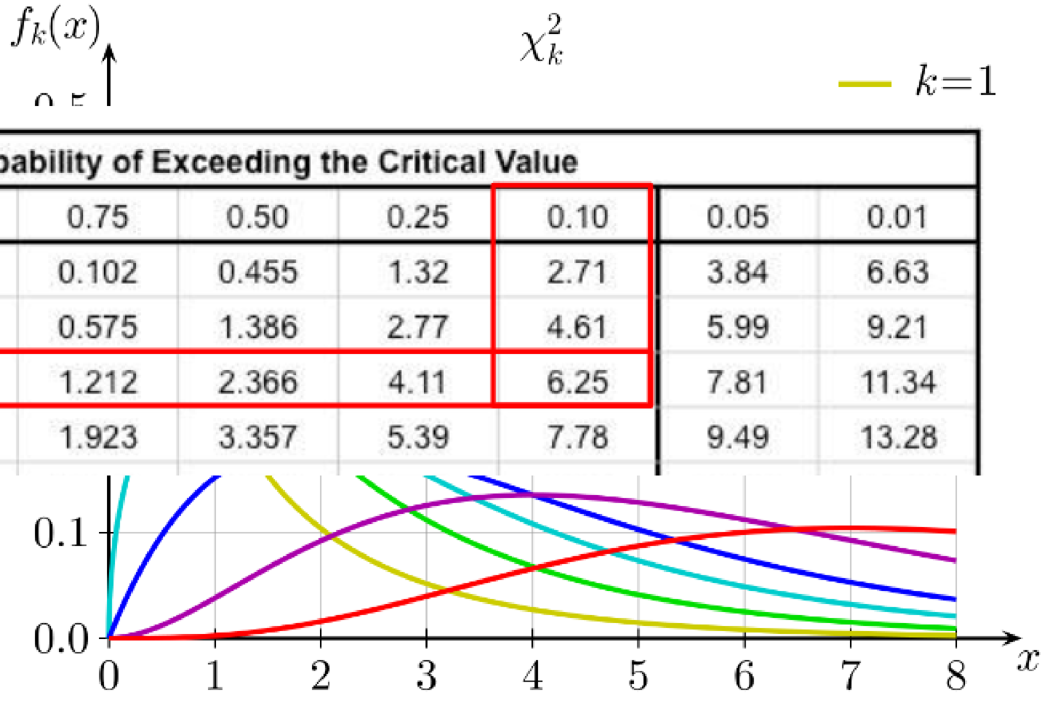
$\chi = 6$

Now we ask: what is the probability of getting $\chi = 6$ given that our d.o.f is 3?

Chi-squared test, EXPLAINED

Correct choice	Expected #	Actual #

Degree of Freedom	Probability of Exceeding the Critical Value								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28



p_val = 5%

$\chi = 6$

Now we ask: what is the probability of getting $\chi = 6$ or more, given that our d.o.f is 3?
The table indicates that the probability is 10%. Which is higher that 5%.

Chi-squared test, EXPLAINED

We cannot reject our null hypothesis, because the p =10%.Our sample **more likely occurred by chance**.

READINGS

- https://towardsdatascience.com/hypothesis-testing-in-machine-learning-using-python-a0dc89e169ce
- https://en.wikipedia.org/wiki/Chi-squared_test
- https://www.statsmodels.org/stable/generated/statsmodels.stats.weightstats.ztest.html
- https://www.statisticshowto.com/probability-and-statistics/hypothesis-testing/t-score-vs-z-score/