Lineare Algebra Nachbereitungsaufgabe 11

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(a) Eigenwerte von $A_{m,n}$:
$X(A_{m,n}) = det\begin{pmatrix} 2-\lambda & m & 0 \\ 0 & 1-\lambda & n \\ 0 & n & 1-\lambda \end{pmatrix} = (2-\lambda)((1-\lambda)^2 - n^2)) = (2-\lambda)(1-\lambda-n)(1-\lambda+n) \Rightarrow$ Eigenwerte:
$\lambda_1 = 2; \lambda_2 = 1-n; \lambda = 1+n$

(b) (1) $n = 0 : Eig_2 = Ker\begin{pmatrix} 0 & m & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = Span\{(1,0,0)^T\}$

$Eig_{1-n} = Eig_1 = Ker\begin{pmatrix} 1 & m & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = Span\{(m,-1,0)^T, (0,0,1)^T\}$

$Eig_{1+n} = Eig_1$

(2) $n = 1 : Eig_2 = Ker\begin{pmatrix} 0 & m & 0 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix} = Span\{(1,0,0)^T\}$

$Eig_{1-n} = Eig_0 = Ker\begin{pmatrix} 2 & m & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} = Span\{(\frac{m}{2},-1,1)^T\}$

$Eig_{1+n} = Eig_2$

(3) $n = -1 : Eig_2 = Ker\begin{pmatrix} 0 & m & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{pmatrix} = Span\{(1,0,0)^T\}$

$Eig_{1-n} = Eig_2$

$Eig_{1+n} = Eig_0 = Ker\begin{pmatrix} 2 & m & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = (0,0,0)^T$

(4) $n \neq 0; n \neq \pm 1 : Eig_2 = Ker\begin{pmatrix} 0 & m & 0 \\ 0 & -1 & n \\ 0 & n & -1 \end{pmatrix} = Span\{(1,0,0)^T\}$

$Eig_{1-n} = Ker\begin{pmatrix} 1+n & m & 0 \\ 0 & n & n \\ 0 & n & n \end{pmatrix} = Span\{(\frac{m}{1+n},-1,1)^T\}$

$Eig_{1+n} = Ker\begin{pmatrix} 1-n & m & 0 \\ 0 & -n & n \\ 0 & n & -n \end{pmatrix} = Span\{(\frac{-m}{1-n},1,1)^T\}$

(c) $m = 9; n = 4 :$
$A = \begin{pmatrix} 2 & 9 & 0 \\ 0 & 1 & 4 \\ 0 & 4 & 1 \end{pmatrix}$

$X(A) = det\begin{pmatrix} 2-\lambda & 9 & 0 \\ 0 & 1-\lambda & 4 \\ 0 & 4 & 1-\lambda \end{pmatrix} = (2-\lambda)((1-\lambda)^2 - 16) = -(2-\lambda)((5-\lambda)(3+\lambda)) \Rightarrow$
$\Rightarrow \lambda_1 = -3; \lambda_2 = 2; \lambda_3 = 5 \Rightarrow$ A ist diagonalisierbar

$Eig_{-3} = Ker\begin{pmatrix} 5 & 9 & 0 \\ 0 & 4 & 4 \\ 0 & 4 & 4 \end{pmatrix} = Span\{(9,-5,5)^T\}$

$Eig_2 = Ker\begin{pmatrix} 0 & 9 & 0 \\ 0 & -1 & 4 \\ 0 & 4 & -1 \end{pmatrix} = Span\{(1,0,0)^T\}$

$$Eig_5 = Ker\begin{pmatrix} -3 & 9 & 0 \\ 0 & -4 & 4 \\ 0 & 4 & -4 \end{pmatrix} = Span\{(3,1,-1)^T\}$$

$$D = \begin{pmatrix} -3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

$$S = \begin{pmatrix} 9 & 1 & 3 \\ -5 & 0 & 1 \\ 5 & 0 & -1 \end{pmatrix}$$

$det(S) = 1((-5)(-1) - 5) = 0 \Rightarrow S^{-1}$ existiert nicht