Semester Project – All Sections BCS

Problem Statement

Quantitative and Qualitative Analysis of modified Kadomtsev-Petviashvili equation:

Consider the KP equation derived for characterized parameter values:

$$\frac{\partial}{\partial\xi}\left[\frac{\partial\phi_1}{\partial\tau}+A\frac{\partial(\phi_1\phi_2)}{\partial\xi}-D\phi^2{}_1\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial\phi_1}{\partial\xi}\right]+C\frac{\partial^2\phi_1}{\partial X^2}=0, \tag{1}$$

where $A=-\frac{(3-q)\mathrm{V}^2-6}{4V}, B=\frac{V}{\beta(1+q)}, C=\frac{V}{2},$

$$D=\frac{3V}{2a}(k-\frac{3b}{V^2}+\frac{2a}{V^4}), a=\frac{q+1}{2}, b=\frac{(q+1)(3-q)}{8}, k=\frac{(q+1)(3-q)(5-3q)}{48}, V=\frac{\sqrt{\delta}}{\sqrt{a}}$$

For a particular choice of these parameters; like $q=1$ and $V=\sqrt{3}$ the coefficient A takes the value zero. Eq. (2) is what is called mKP equation:

$$\frac{\partial}{\partial\xi}\left[\frac{\partial\phi_1}{\partial\tau}-D\phi^2{}_1\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial\phi_1}{\partial\xi}\right]+C\frac{\partial^2\phi_1}{\partial X^2}=0. \tag{2}$$

Seek travelling wave solution of this equation. For that, define a new variable as follows:

$$\chi=\alpha(l\xi+mX-U\tau)$$

where l and m are direction cosines of the angles made by the wave propagation with x axis and z axis respectively. Here U is the speed of the wave. Taking $\psi(\chi)=\phi_1(\xi,\mathrm{X},\tau)$ in Eq. (2), and integrating twice, we obtain

$$(Cm^2-lU)\psi-\frac{Dl^2}{3}\psi^3+B\alpha^2l^4\frac{d^2\psi}{d\chi^2}=0. \tag{3}$$

This second order ode can be expressed into system of first order differential equations

Project requirements

a) Perform the quantitative analysis (finding solutions analytically) of Eq. (3). For $q = 1.901$ and $q = 1.301$ with $\alpha = 12, \beta = 0.52, \delta = 4.3396, l = 0.34$ and $U = 4.9$.
b) Perform the quantitative analysis (finding solutions analytically) of Eq. (3). For $q = 1.101$ and $q = 1.201$ with $\alpha = 3.35, \beta = 0.381, \delta = 1.34, l = 0.2$ and $U = 2.4$
c) Reduce this second order differential Eqs. into system of differential equations.
d) Perform the qualitative analysis (finding equilibrium points, eigenvalues and drawing phase portraits) of Eq. (3) keeping the parameter values same as in part (a) and (b).
e) Plot the solutions obtained analytically.
f) Interpret the phase diagrams.

Sr. No.	Deliverable	Marks
1.	**Objectives and Introduction:** Objectives and introduction of the problem. In this section briefly introduce the problem and the methodology that will be adopted by you to solve the problem.	10
2.	**Analytical Solution:** A step-by-step analytical solution (by-hand solution). Clearly state the assumptions and values that you use for the solution.	20
3.	**MATLAB Code:** Complete and well commented MATLAB code. This section must include the explanation of the commands, functions, and toolboxes used.	15
4.	**MATLAB Solution and Results:** Step-by-step example demonstrating the MATLAB solution. Clear retraceable steps should be listed to obtain the presented solution.	20

	Also, present detailed results and discussion in this section. Do not just paste the graphs or screenshot of the command window. Compare your by-hand and MATLAB solutions, and present physical interpretation of your results and graphs.	
5.	**Flowchart:** Flowchart of the solution methodology	5
6.	**Conclusions:** In this section, include conclusions related to this assignment. The conclusion section stands independently from the report and gives the reader a comprehensive idea of the project; thus, the conclusion section should briefly explain the problem, solution methodology, results, and analysis. The conclusion section is not very large and typically consists of 1-2 paragraphs. The conclusions section can also include bullet points.	5

Each report element should be documented under a separate heading. Report must not exceed 12 A4 size pages including table of contents as well as a single title page with project title, student names, ids, section, and name of the course. 3 marks will be deducted from obtained marks for every extra page. Each page should be numbered. The report should be written in Calibri or Times New Roman typeface only. The size of the font should be 12. The size of first and second level of headings should be 14 bold, and 12 bold, respectively. The alignment of the report should be justified, while pictures and tables should be center aligned with relevant captions. The option to align the text left, right, center, and justify can be found under paragraph options on *Home* tab. Line and paragraph spacing should be set as 1.5.

Project Submission Guidelines

This project is an open-ended problem designed to demonstrate the application of differentiation and optimization in real life. The open-ended nature of the problem means that this problem can be solved in more than one way using various techniques and methodologies, some of these techniques have been covered in this course. You are free to adopt any technique and solution methodology to solve this problem. Solution techniques and methodologies that are not part of the course outline can also be used to solve the problem. However, you are required to take approval of such a solution technique before starting the project. You will have to do extensive research to completely solve the problem. Project guidelines are summarized below:

- This is an individual project and carries 75 marks.
- Plagiarized work (from internet or fellow students) will result in zero marks.
- Deadline for complete project submission of hardcopy and on **google classroom** (one MS Word file and one pdf of the same Word file including all the codes and by-hand solutions) is **<Date and Time>**. Do not submit your project in a .zip or .rar format. You can submit additional files such as .m files, however, the single PDF and MS Word file must also include all these files.
- Name of your project report file must be as per following format: **ID1_ID2_ID3_MT1006_Project_Section**.
- Do not submit your project via email, it will not be considered.
- Late submissions will not be considered