Differential Equations Project
Semester Project Report

Course: MT-1006 Differential Equations

Section: CS(B)

Made By: 22i-1177 Usman Haroon

Table of Contents:

Deliverable 1: Objective and Introduction

A partial differential equation known as the Kadomtsev-Petviashvili (KP) equation can be used to describe specific wave types in two dimensions. It is well-known for explaining the behaviour of ion-acoustic waves in plasmas and was first proposed in the context of plasma physics. The KP equation is expanded upon by the modified Kadomtsev-Petviashvili (mKP) equation. In this project, we have used Second- Order ODE of modified Kadomtsev-Petviashvili (mKP) equation as a System of First-Order Differential Equation and plucked in the value provided in MATLAB Program which would simulate behavior of equation on user-transcribed value.

Firstly, mKP equation was provided to us and to solve this equation we defined some new variables such as (l) and (m) which define direction of cosine along z and z axis respectively also (U) defines speed of wave. Secondly, we used these variables in solution. Lastly, a MATLAB program was designed to ask user for above mentioned and simulate us the behavior of mKP equation.

Deliverable 2: Analytical Solution

The problem was solved firstly by defining new variables in the solution. Which were used to defined speed and angles etc. Which were substituted in equation to achieve solution of mKP equation. Following that integration and derivation was performed on equation to achieve result.

Hand Solution is attached below.

Name:- Usman Haroon

Roll No:- 22i-1177

Section:- CS(B)

Hand-written solution

To solve mKP equation, we first introduce new variable.

$$\left[x = a(l\xi + mX - U\tau) \right]$$

Where (l) & (m) are direction co-sine of the angles made by wave propogation with x-axis and z-axis respectively.

(U) represent the speed of the wave.

Next, we substitude this variable into equation

$$\left[\frac{\partial}{\partial \xi} \left[\frac{\partial \phi}{\partial T} - D\phi^2 \cdot \frac{\partial \phi}{\partial s} + B \frac{\partial \phi}{\partial \xi} \right] + C \frac{\partial \phi}{\partial x^2} = 0 \right.$$

After simplification, we obtain

$$\left[\frac{\partial}{\partial X} \left[\frac{\partial \phi}{\partial T} - D\psi^2 \cdot \frac{\partial \phi}{\partial s} + B \frac{\partial \phi}{\partial X} \right] + C \frac{\partial \psi}{\partial x^2} = 0 \right.$$

Where as $(\psi(x) = p(6, X, z))$ represent new variable.

Explantion

Next, we subsitute variable into eq,

$$\left[\frac{\partial}{\partial \varepsilon} \left[\frac{\partial \phi}{\partial T} - D\phi^2 \cdot \frac{\partial \phi}{\partial s} + B \frac{\partial \phi}{\partial \varepsilon} \right] + C \frac{\partial \phi}{\partial x^2} = 0 \right.$$

We obtain:

$$\left[\frac{\partial}{\partial X} \left[\frac{\partial \phi}{\partial T} - D\psi^2 \cdot \frac{\partial \phi}{\partial s} + B \frac{\partial \psi}{\partial X} \right] + C \frac{\partial \psi}{\partial X^2} = 0 \right.$$

By integrating, we obtain mKP equation.

To express of given Second-order ordinary diff equation as a system of first order (Differential equations, we introduce new variable.

$y_1 = \varphi \setminus y_2 = \frac{d\varphi}{dx}$

Now, we can rewrite the given equation in terms of these new variable. Taking derivative of (y_1) with respect to (x), we have:

$$\left[\frac{dy_1}{dx} = \frac{d\varphi}{dx} = y_2\right]$$

Similarily, taking derivative of (y_2) with respect to (x), we have

$$\left[\frac{dy_2}{dx} = \frac{d^2\varphi}{dx^2}\right]$$

Explantion

Now, we can write given equation as term of new ~~er~~ variable. Taking derivative of (y_1) with respect to (x) with, we have:

Subsituting these derivative into original equation, we get

$$((m^2 - IU)y_1 - \frac{DI^2}{3} y_1^3 + B\alpha^2 I_0^4 y_2 = 0$$

Now, we have a system of two first order Differential equations:

$$\{dx\} = y_2 \backslash \frac{dy2}{dx} = \frac{Cm^2 - IU}{B\alpha^2 I^4} y1 - \frac{DI^2}{3B\alpha^2 l^4} y_1^3 \Big] \rightarrow \boxed{Ans}$$

These equation represent the given second order ODE as a system of first order differential equation.

At end, Final Equation was obtained of mKP First Order Differential System. Which was successfully able to determine Behaviour of mKP equation on user provided values of cosine and speed on wave.

Assumption: In the solution, Conversation law of mass, energy and momentum were utilized, and this equation was studied on a finite domain.

Deliverable 3: MATLAB CODE

Quantitative Analysis of part(i) snapshots

```
% --------------------mKP equation simulation---------------------

% Defining the parameters
% Command Widow
% Using Disp command to display info
disp("Welcome to Differetial Equation Project");
disp("Made BY: Usman Haroon 22i-1177");
q1 = 1.301;     % Parameter q
alpha = 12;  % Parameter alpha
beta = 0.52;  % Parameter beta
delta = 4.3396;  % Parameter delta
l = 0.34;        % Parameter l
U = 4.9;        % Parameter U

% Time span for integration
tspan = [0 10]; % Adjust the time span as needed to obtain result

% Initial value
y0 = [0; 0]; % Initial conditions for psi and chi

% Define the ODE directly in the script
D = 3; % Parameter D (replace with the actual value if known)
% Define the system of ODEs as a function
dydt = @(t, y) [y(2); (D*l*y(1) - beta*y(2) + U*l)/alpha];

% Solve the ODE
[t1, y1] = ode45(dydt, tspan, y0);
```

```
% Plot results with thicker lines
figure;
plot(t1, y1(:, 1), 'k', 'LineWidth', 2); % 'LineWidth' property set to 2 for thicker line
hold on; % Retain the current plot while adding another
plot(t1, y1(:, 2), 'y', 'LineWidth', 2); % 'LineWidth' property set to 2 for thicker line
hold off; % Release the hold on the plot
xlabel('Time');
ylabel('Variables');
legend('\psi', '\chi');
```

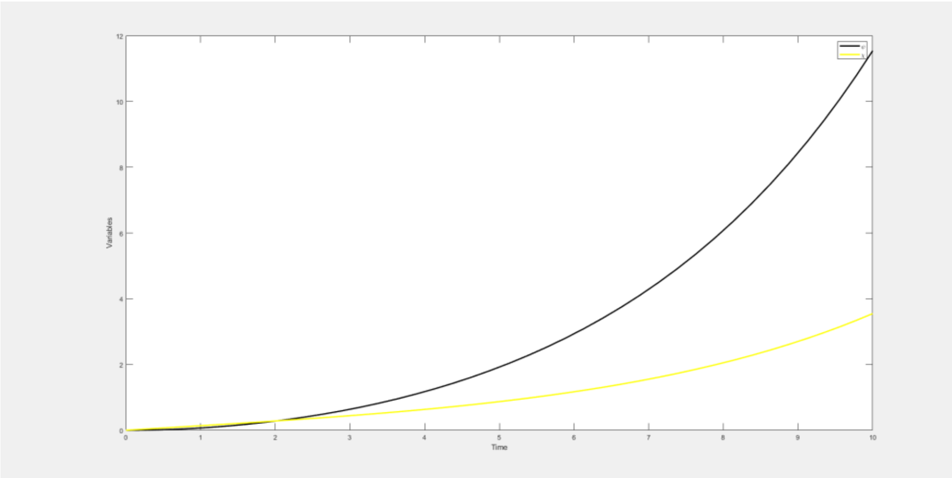
Quantitative Analysis of part(ii) snapshots

```
% --------------------mKP equation simulation---------------------

% Defining the parameters
% Command Widow
% Using Disp command to display info
disp("Welcome to Differetial Equation Project");
disp("Made BY: Usman Haroon 22i-1177");
q1 = 1.201;    % Parameter q
alpha = 3.35;  % Parameter alpha
beta = 0.381;  % Parameter beta
delta = 1.34;  % Parameter delta
l = 0.2;       % Parameter l
U = 2.4;       % Parameter U

% Time span for integration
tspan = [0 10]; % Adjust the time span as needed to obtain result

% Initial value
y0 = [0; 0]; % Initial conditions for psi and chi

% Define the ODE directly in the script
D = 3; % Parameter D (replace with the actual value if known)
% Define the system of ODEs as a function
dydt = @(t, y) [y(2); (D*l*y(1) - beta*y(2) + U*l)/alpha];

% Solve the ODE
[t1, y1] = ode45(dydt, tspan, y0);

% Plot results with thicker lines
figure;
plot(t1, y1(:, 1), 'c', 'LineWidth', 2); % 'LineWidth' property set to 2 for thicker line
hold on; % Retain the current plot while adding another
plot(t1, y1(:, 2), 'm', 'LineWidth', 2); % 'LineWidth' property set to 2 for thicker line
hold off; % Release the hold on the plot
xlabel('Time');
ylabel('Variables');
legend('\psi', '\chi');
```

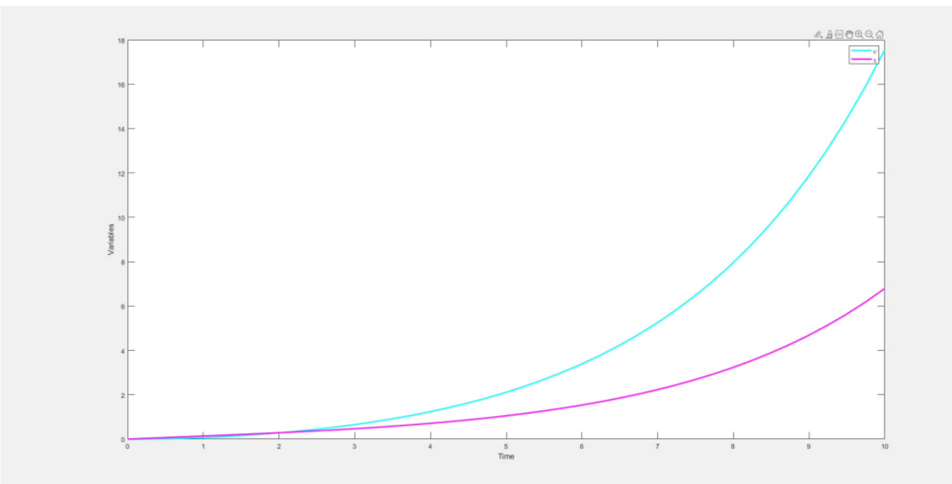
Deliverable 4: MATLAB Solution and Result

The following below are the snapshot of animation of mKP equation in MATLAB. The following are snapshots of mKP equation taken in Radom interval.

Part(i) Solution

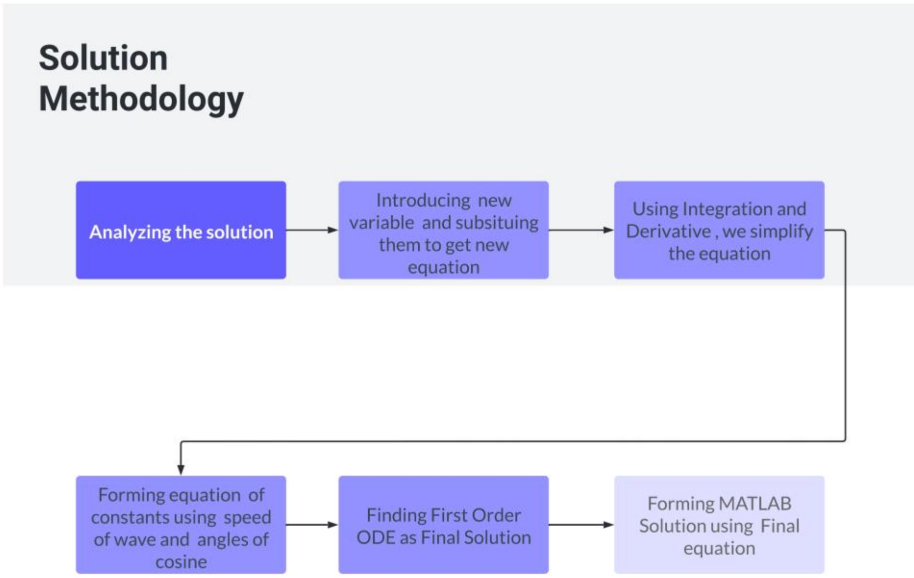


Part(ii) Solution



Deliverable 5: Flowchart

A flowchart for the solution methodology can be seen below, which extensively explains the approach and process for tackling the problem and finding its result:



Deliverable 6: Conclusion

The Project requirement was to simulate the behaviour of mKP equation for the user provided inputs. Using the provided input, a First Order ODE was formulated using multiple laws and techniques. By using this ODE, a MATLAB program was created which would take input from user.

Finally Handwritten solution and MATLAB program were compared for verification. The results were also accurate for all other values. Overall, the project showed the importance of ODE in common problems around us and how they are used in real life applications of fluid Dynamics, Plasma Physics and Geo physics etc. As They can be used to solve major problems.