Vectors and Vector Equations

x-component   y-component

$\underline{V}_1 = 2\hat{i} + 3\hat{j} \equiv \underline{V}_1 = \begin{bmatrix}2\\3\end{bmatrix} \begin{matrix}\to \hat{i}\\ \to \hat{j}\end{matrix}$

$\underline{V}_2 = \hat{i} + 2\hat{j}$

$\underline{V}_3 = \begin{bmatrix}2\\-1\end{bmatrix} \in \mathbb{R}^{\boxed{2}}$

$\underline{V}_3 = 2\hat{i} + \hat{j}$

$= \hat{i} + \hat{j} + 4\hat{k}$

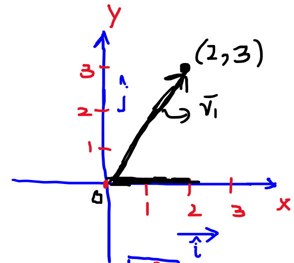
$\underline{V}_4 = \hat{i} + \hat{j} + 4\hat{k}$ $\underline{V}_4 = \begin{bmatrix}1\\1\\4\end{bmatrix} \in \mathbb{R}^{\boxed{3}}$

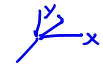
belong

$\underline{V}_1 + \underline{V}_2 = \hat{i} + 2\hat{j} + 2\hat{i} + \hat{j}$

$= (1+2)\hat{i} + (2+1)\hat{j}$

$= 3\hat{i} + 3\hat{j}$





$\mathbb{R} \to$ Set of real number

$\boxed{\mathbb{R}^2}$-space → 2D-space

$\mathbb{R}^3$-space → 3D-space

$\mathbb{R}^{\boxed{n}}$ → space — n dimensional

$\underline{V}_1 + \underline{V}_2 = \begin{bmatrix}2\\3\end{bmatrix} + \begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}2+1\\3+2\end{bmatrix} = \begin{bmatrix}3\\5\end{bmatrix}$

$\underline{V}_1 + \underline{V}_4$

$\underline{V}_1 = \underset{2\times1}{\begin{bmatrix}2\\3\end{bmatrix}} + \underset{3\times1}{\begin{bmatrix}1\\1\\4\end{bmatrix}}$ → not pos

Parallelogram Rule for Addition

If **u** and **v** in $\mathbb{R}^2$ are represented as points in the plane, then **u** + **v** corresponds to the fourth vertex of the parallelogram whose other vertices are **u**, **0**, and **v**. See Fig. 3.

$\underline{V}_1 = \begin{bmatrix}2\\3\\0\end{bmatrix}$

$\underline{V}_1 = 2\hat{i} + 3\hat{j} + 0\hat{k} \in \mathbb{R}^3$

$V_1 = 2\hat{i} + 3\hat{j} \in \mathbb{R}^2$

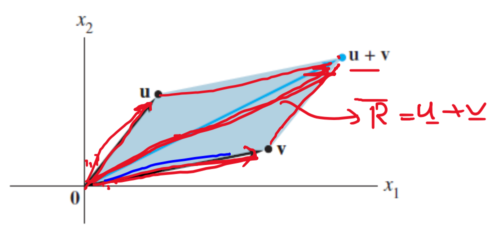


FIGURE 3 The parallelogram rule.

$u + v = \begin{bmatrix}2-6\\2+1\end{bmatrix}$

EXAMPLE 2 The vectors $\underline{\mathbf{u}} = \begin{bmatrix}2\\2\end{bmatrix}$, $\underline{\mathbf{v}} = \begin{bmatrix}-6\\1\end{bmatrix}$, and $\mathbf{u} + \mathbf{v} = \begin{bmatrix}-4\\3\end{bmatrix}$ are displayed in Fig. 4. ■

$2\underline{u} = 2\begin{bmatrix}2\\2\end{bmatrix} = \begin{bmatrix}2\times2\\2\times2\end{bmatrix}$

$-3\underline{u} = [$

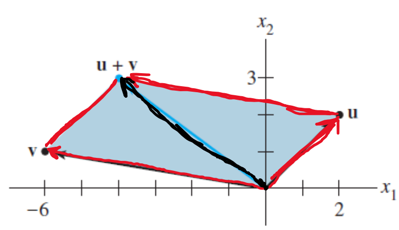
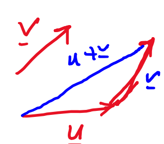


FIGURE 4

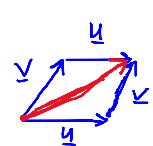
$\underline{u} + \underline{v}$

$\underline{u} - \underline{v}$

$2\underline{u}$ $\frac{1}{2}\underline{u}$



The next example illustrates the fact that the set of all scalar multiples of one fixed nonzero vector is a line through the origin, (0, 0).



Linear System in Vector Equation

In general, a linear system in n variable $x_1, x_2, \dots, x_n$ having m equations can be written as

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \vdots \qquad\qquad & \quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

① → Equation

n-variables

m-equation

A row x column

→ matrix form

$$\begin{array}{cccc} x_1 & x_2 & & x_n \\ \end{array}$$
$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

$A\,\underline{x} = \underline{b}$ $\quad i+j = [\,!\,]$

$$\boxed{\begin{aligned} x_1 - x_2 &= 5 \\ x_1 + x_2 &= 10 \end{aligned}}$$ → 3-unkn

$$\boxed{x_1}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \boxed{x_2}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$ → vector form

$$\boxed{x_1\underline{v_1} + x_2\underline{v_2} = \underline{b}}$$

Vector Equation

A vector equation

$$\boxed{x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n = \mathbf{b}}$$ → const.

has the same solution set as the linear system whose augmented matrix is

$$A_d = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_n & \vdots & \mathbf{b} \end{bmatrix} \tag{5}$$

In particular, $\mathbf{b}$ can be generated by a linear combination of $\mathbf{a}_1, \dots, \mathbf{a}_n$ if and only if there exists a solution to the linear system corresponding to the matrix (5).

3-variables
3-Equatn

$$\begin{cases} 2x_1 - 3x_2 + 4x_3 = 1 \\ x_1 + 4x_2 - x_3 = 2 \\ \qquad\; x_2 + x_3 = -1 \end{cases} \Rightarrow x_1\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + x_2\begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix} + x_3\begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$\boxed{x_1\,\boxed{\underline{a}_1} + x_2\,\boxed{\underline{a}_2} + x_3\,\underline{a}_3 = \underline{b}}$$

$$A_d = \left[\begin{array}{ccc|c} 2 & -3 & 4 & 1 \\ 1 & 4 & -1 & 2 \\ 0 & 1 & 1 & -1 \end{array}\right] \Leftarrow [\,\underline{a}_1 \quad \underline{a}_2 \quad \underline{a}_3 \mid \underline{b}\,] = A_d$$

Linearly Independent and dependent Vectors

Linear Combinations

Given vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$ in $\mathbb{R}^n$ and given scalars $c_1, c_2, \ldots, c_p$, the vector $\mathbf{y}$ defined by

$$\mathbf{y} = \boxed{c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p}$$

is called a **linear combination** of $\mathbf{v}_1, \ldots, \mathbf{v}_p$ with **weights** $c_1, \ldots, c_p$. Property (ii)

$$\underline{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \underline{v} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\boxed{3\underline{u} + 4\underline{v}} = 3\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 4\begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix} + \begin{bmatrix} -8 \\ 12 \end{bmatrix} = \begin{bmatrix} 6-8 \\ 3+12 \end{bmatrix} = \begin{bmatrix} -2 \\ 15 \end{bmatrix}$$

$\Rightarrow$ $\boxed{\begin{bmatrix} -2 \\ 15 \end{bmatrix}}$ $= 3u + 4v$ is linear combination of vectors $\boxed{\underline{u} \text{ and } \underline{v}}$ with weights 3 & 4

EXAMPLE 5 Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix}$. Determine whether $\mathbf{b}$ can be generated (or written) as a linear combination of $\mathbf{a}_1$ and $\mathbf{a}_2$. That is, determine whether weights x_1 and x_2 exist such that

$$x_1\mathbf{a}_1 + x_2\mathbf{a}_2 = \mathbf{b} \tag{1}$$

If vector equation (1) has a solution, find it.

SOLUTION Use the definitions of scalar multiplication and vector addition to rewrite the vector equation

$$x_1 \underbrace{\begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix}}_{\mathbf{a}_1} + x_2 \underbrace{\begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}}_{\mathbf{a}_2} = \underbrace{\begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix}}_{\mathbf{b}}$$

$$\underline{V}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \underline{V}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \quad \underline{V}_3 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

$$\boxed{2}\underline{V}_1 + \boxed{3}\underline{V}_2 (-4)\underline{V}_3 = 2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} - 4\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix} + \begin{bmatrix} 4 \\ -4 \\ -8 \end{bmatrix} = \begin{bmatrix} 2+0+4 \\ 4+3-4 \\ 6-3-8 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ -5 \end{bmatrix}$$

$$\underline{V}_1 + \underline{V}_2 + \underline{V}_3 =$$

$$\underline{V}_1 - \underline{V}_2 + 2\underline{V}_3 =$$

Existence of Solutions

LS →

The definition of $A\mathbf{x}$ leads directly to the following useful fact.

The equation $\boxed{A\mathbf{x} = \mathbf{b}}$ has a solution if and only if $\mathbf{b}$ is a linear combination of the columns of A.

$A\underline{x} = \boxed{b}$ $\Rightarrow$ $x_1 \underline{a}_1 + x_2 \underline{a}_2 + \cdots + x_n \underline{a}_n = b$

Rank (A) = Rank (Ab)

Consistent = Solution exist

Rank(A) ≠ Rank(Ab)

Solution doesnot exist = inconsistent

If $\underline{b}$ is linear combination of Columns of A

$$\underline{A} = \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{array}\right]$$

Rank(A) = Rank(Ab)

$\Rightarrow$ $\underline{b}$ is linear combination a_1, a_2, a_3

Linearly Independent and dependent Vectors

$V_i = \begin{bmatrix} - \\ \vdots \\ - \end{bmatrix}$

An indexed set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ in $\mathbb{R}^n$ is said to be **linearly independent** if the vector equation

$$\boxed{x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots + x_p\mathbf{v}_p = \mathbf{0}} \in R^n \qquad \underline{0} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} n\text{-th}$$

has only the trivial solution. The set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is said to be **linearly dependent** if there exist weights $c_1, \ldots, c_p$, not all zero, such that

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_p\mathbf{v}_p = \mathbf{0} \qquad (2)$$

For example:-

$\underline{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$; $\underline{v} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$\boxed{x_1 \underline{u} + x_2 \underline{v}_2 = \underline{0}}$

$x_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$Ab = \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 2 & -1 & 0 \end{array}\right] R \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & -7 & 0 \end{array}\right] R_2 - 2R_1 \qquad \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \text{homog-}$$

P.c P.c

Only trivial solution exist → $\underline{u}$, $\underline{v}$ are linearly independ

Linear Independence of Matrix Columns

Suppose that we begin with a matrix $A = [\,\mathbf{a}_1 \quad \cdots \quad \mathbf{a}_n\,]$ instead of a set of vectors. The matrix equation $A\mathbf{x} = \mathbf{0}$ can be written as

$$x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n = \mathbf{0}$$

Each linear dependence relation among the columns of A corresponds to a nontrivial solution of $A\mathbf{x} = \mathbf{0}$. Thus we have the following important fact.

> The columns of a matrix A are linearly independent if and only if the equation $A\mathbf{x} = \mathbf{0}$ has *only* the trivial solution. (3)

EXAMPLE 2 Determine if the columns of the matrix $A = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix}$ are linearly independent.

$$\begin{array}{ccc} v_1 & v_2 & v_3 \end{array}$$

$$A = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix}$$

$$A \overset{R}{=} \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 4 \\ 0 & -2 & 5 \end{bmatrix} \begin{array}{l} R_1 \leftrightarrow R_2 \\ R_3 - 5R_1 \end{array}$$

$$A \overset{R}{=} \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 4 \\ 0 & 0 & 13 \end{bmatrix} R_3 + 2R_2$$

$\Rightarrow$ Columns of A are linearly independents

1.7 EXERCISES

In Exercises 1–4, determine if the vectors are linearly independent. Justify each answer.

1. $\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -8 \end{bmatrix}$

2. $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -8 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$

3. $\begin{bmatrix} 2 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ 6 \end{bmatrix}$

4. $\begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ -9 \end{bmatrix}$

In Exercises 5–8, determine if the columns of the matrix form a linearly independent set. Justify each answer.

5. $\begin{array}{c} \begin{array}{ccc} \underline{v}_1 & \underline{v}_2 & \underline{v}_3 \end{array} \\ \begin{bmatrix} 0 & -3 & 9 \\ 2 & 1 & -7 \\ -1 & 4 & -5 \\ 1 & -4 & -2 \end{bmatrix} \end{array}$

6. $\begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 5 \\ 1 & 1 & -5 \\ 2 & 1 & -10 \end{bmatrix}$

7. $\begin{bmatrix} 1 & 4 & -3 & 0 \\ -2 & -7 & 5 & 1 \\ -4 & -5 & 7 & 5 \end{bmatrix}$

8. $\begin{array}{c} \begin{array}{cccc} \underline{v}_1 & \underline{v}_2 & \underline{v}_3 & \underline{v}_4 \end{array} \\ \begin{bmatrix} 1 & -2 & 3 & 2 \\ -2 & 4 & -6 & 2 \\ 0 & 1 & -1 & 3 \end{bmatrix} \\ \begin{array}{cccc} \text{P.C} & \text{P.C} & \text{P.C} & \end{array} \end{array}$ → $\underline{v}_1, \underline{v}_2, \underline{v}_4$ Linearly dependent

In Exercises 9 and 10, (a) for what values of h is $\mathbf{v}_3$ in Span $\{\mathbf{v}_1, \mathbf{v}_2\}$, and (b) for what values of h is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly *dependent*? Justify each answer.

Q1:- $\underline{v}_1 = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \underline{v}_2 = \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix}, \underline{v}_3 = \begin{bmatrix} 9 \\ 4 \\ -8 \end{bmatrix}$

Now

$$x_1\underline{v}_1 + x_2\underline{v}_2 + x_3\underline{v}_3 = \underline{0}$$

$$x_1\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} + x_2\begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix} + x_3\begin{bmatrix} 9 \\ 4 \\ -8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 7 & 9 \\ 0 & 2 & 4 \\ 0 & -6 & -8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad -(1)$$

If HS (1) has only trivial solution then the $\underline{v}_1, \underline{v}_2, \underline{v}_3$ are LI otherwise $\underline{v}_1, \underline{v}_2, \underline{v}_3$ are LD vector

$$A_d = \left[\begin{array}{ccc|c} 5 & 7 & 9 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & -6 & -8 & 0 \end{array}\right]$$

$$A_d \cong \left[\begin{array}{ccc|c} 5 & 7 & 9 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 0 & 4 & 0 \end{array}\right] \quad R_3 + 3R_2$$

All the column are PC so non-trivial does not exist and the vectors are LI.

9. $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 5 \\ -7 \\ h \end{bmatrix}$

10. $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \\ -5 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -3 \\ 9 \\ 15 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 2 \\ -5 \\ h \end{bmatrix}$

In Exercises 11–14, find the value(s) of h for which the vectors are $\boxed{\text{linearly } dependent}$. Justify each answer.

11. $\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ 7 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ h \end{bmatrix}$ -4

12. $\begin{bmatrix} 3 \\ -6 \\ 1 \end{bmatrix}, \begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}, \begin{bmatrix} 9 \\ h \\ 3 \end{bmatrix}$

13. $\begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}, \begin{bmatrix} -2 \\ -9 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ h \\ -9 \end{bmatrix}$

14. $\begin{bmatrix} 1 \\ -2 \\ -4 \end{bmatrix}, \begin{bmatrix} -3 \\ 7 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ h \end{bmatrix}$

Determine by inspection whether the vectors in Exercises 15–20 are linearly *independent*. Justify each answer.

15. $\begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 7 \end{bmatrix}$

16. $\begin{bmatrix} 2 \\ -4 \\ 8 \end{bmatrix}, \begin{bmatrix} -3 \\ 6 \\ -12 \end{bmatrix}$

17. $\begin{bmatrix} 5 \\ -3 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 2 \\ 4 \end{bmatrix}$

18. $\begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 7 \\ 1 \end{bmatrix}$

19. $\begin{bmatrix} -8 \\ 12 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$

20. $\begin{bmatrix} 1 \\ 4 \\ -7 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

In Exercises 21 and 22, mark each statement True or False. Justify each answer on the basis of a careful reading of the text.

21. a. The columns of a matrix A are linearly independent if the equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution.

Q11:- $\underline{V}_1 = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}, \underline{V}_2 = \begin{bmatrix} 4 \\ -6 \\ 7 \end{bmatrix}, \underline{V}_3 = \begin{bmatrix} -2 \\ 2 \\ h \end{bmatrix}$

$$\boxed{x_1 \underline{V}_1 + x_2 \underline{V}_2 + x_3 \underline{V}_3 = \underline{0}}$$

$$A_d = \left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ -2 & -6 & 2 & 0 \\ 4 & 7 & h & 0 \end{array}\right] \text{ — (1)}$$

$$A_d \overset{R}{=} \left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & -1 & h+4 & 0 \end{array}\right] \begin{matrix} R_2+R_1 \\ R_3-2R_1 \end{matrix}$$

$$A_d \overset{R}{=} \left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & -1 & h+4 & 0 \\ 0 & -2 & 0 & 0 \end{array}\right] R_2 \leftrightarrow R_3$$

$$A_d \overset{R}{=} \left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & -1 & h+4 & 0 \\ 0 & 0 & -2h-8 & 0 \end{array}\right] R_3-2R_2$$

p.c p.c p.c

$-2h-8 \neq 0$ Vectors are LI

For LD vectors we have $-2h-8=0$

$$\boxed{h=-4}$$