Types of System of Linear Equations

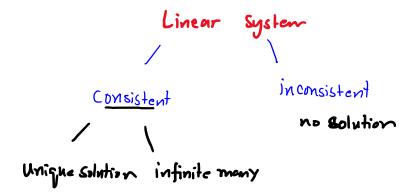
Consistent and Inconsistent System

$$\frac{2X-Y=1}{X+Y=2}$$

A system of linear equations has

- 1. no solution, or] Inconsistent
- $X = 2 \quad X = 2$ $X = 2 \quad Y = 3$ X + Y = 10

A system of linear equations is said to be **consistent** if it has either one solution or infinitely many solutions; a system is **inconsistent** if it has no solution.



Examples,

In
$$x_1 - 2x_2 = -1$$
 $-x_1 + 3x_2 = 3$

In $x_1 - 2x_2 = -1$
 $-x_1 + 2x_2 = 1$

In finite many solution

Consistent systems

 $x_1 - 2x_2 = -1$
 $-x_1 + 2x_2 = 3$

In finite many solution

 $x_1 - 2x_2 = -1$
 $-x_1 + 2x_2 = 3$
 $x_1 - 2x_2 = -1$
 $-x_1 + 2x_2 = 3$

In finite many solution

 $x_1 - 2x_2 = -1$
 $-x_1 + 2x_2 = 3$

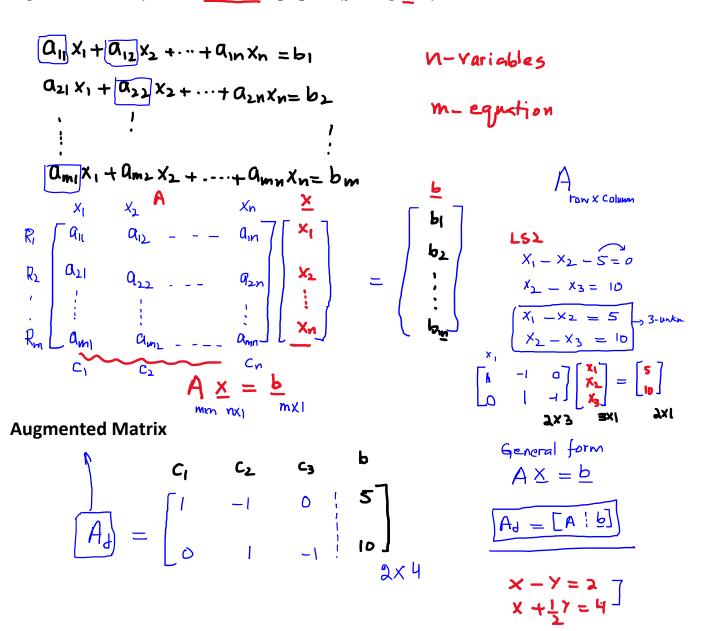
 $A_d = \begin{bmatrix} 1 & -(1 & 2) \\ 1 & 1 & 4 \end{bmatrix}$

Linear System in Matrix Notation/ Augmented Matrix

The essential information of a linear system can be recorded compactly in a rectangular array called a **matrix**. Given the system

3-egn
$$\begin{cases} x_1' - 2x_2' + x_3' = 0 \\ 0x_1 + 2x_2' - 8x_3' = 0 \\ -4x_1' + 5x_2' + 9x_3' = -9 \end{cases} \Rightarrow \begin{cases} x_1 & x_2 & x_3 \\ 0 & 2 & -8 \\ -4 & 5 & 9 \end{cases} \begin{cases} x_1 & x_2 \\ x_3 & -8 \\ x_3 & -8 \end{cases}$$
 Ve clar of term $\begin{cases} x_1 & x_2 & x_3 \\ x_3 & -8 \\ x_3 & -8 \end{cases}$

In general, a linear system in \underline{n} variable x_1, x_2, \dots, x_n having \underline{m} equations can be written as



ELEMENTARY ROW OPERATIONS

- 1. (Replacement) Replace one row by the sum of itself and a multiple of another row.²
- 2. (Interchange) Interchange two rows.
- 3. (Scaling) Multiply all entries in a row by a nonzero constant

X - Y = C

Example

$$\begin{bmatrix} 0 & 2 & -8 & 8 \\ 1 & -2 & 1 & 0 \\ -4 & 5 & 9 & 9 \end{bmatrix} \begin{array}{c} \text{Interchange} \\ R_1 \longleftrightarrow R_2 \\ -4 & 5 & 9 & 9 \\ -1 & -2 & 1 & 0 \\ \end{array}$$

$$\begin{bmatrix}
0 & 2 & -8 & 8 \\
1+26) & -2+263 & 1+6-16 \\
-4 & 5 & 9 & -9
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 2 & -8 & 8 \\
1+26) & -2+263 & 1+6-16 \\
-4 & 5 & 9 & -9
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 2 & -8 & 8 \\
R_2 + C_2 R_1 \\
R_2 + C_2 R_1
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 2 & -8 & 8 \\
R_3 + R_2 \\
R_2 + C_2 R_1
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 2 & -8 & 8 \\
R_3 + R_2 \\
R_2 + C_2 R_1
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 2 & -8 & 8 \\
R_3 + R_2 \\
R_2 + C_2 R_1
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 2 & -8 & 8 \\
R_3 + R_2 \\
R_2 + C_2 R_1
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 2 & -8 & 8 \\
R_3 + R_2 \\
R_2 + C_2 R_1
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 2 & -8 & 8 \\
R_3 + R_2 \\
R_2 + C_2 R_1
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 2 & -8 & 8 \\
R_3 + R_2 \\
R_2 + C_2 R_1
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 2 & -8 & 8 \\
R_3 + R_2 \\
R_2 + C_2 R_1
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 2 & -8 & 8 \\
R_3 + R_2 \\
R_2 + C_2 R_1
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 2 & -8 & 8 \\
R_3 + R_2 \\
R_2 + C_2 R_1
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 2 & -8 & 8 \\
R_3 + R_2 \\
R_2 + C_2 R_1
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 2 & -8 & 8 \\
R_3 + R_2 \\
R_3 + R_2 \\
R_3 + R_2 \\
R_3 + R_3 \\
R_$$

Remarks

- Row operations can be applied to any matrix, not merely to one that arises as the augmented matrix of a linear system
- The row operations are reversible. If two rows are interchanged, they can be returned to their original positions by another interchange.

Row Reduction and Echelon Form

A rectangular matrix is in **echelon form** (or **row echelon form**) if it has the following three properties:

- 1. All nonzero rows are above any rows of all zeros.
- 2. Each leading entry of a row is in a column to the right of the leading entry of the row above it.
- 3. All entries in a column below a leading entry are zeros.

If a matrix in echelon form satisfies the following additional conditions, then it is in **reduced echelon form** (or **reduced row echelon form**):

- **4.** The leading entry in each nonzero row is 1.
- **5.** Each leading 1 is the only nonzero entry in its column.