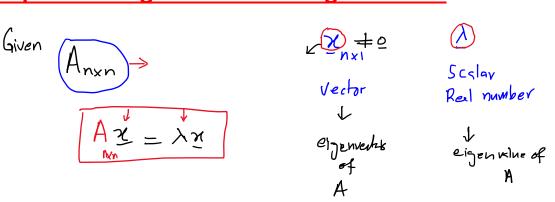
Chapter 05: Eigenvalues and Eigenvectors



An eigenvector of an $\underline{n \times n}$ matrix \underline{A} is a nonzero vector $\underline{\mathbf{x}}$ such that $\underline{A}\mathbf{x} = \lambda \mathbf{x}$ for some scalar $\underline{\lambda}$. A scalar λ is called an eigenvalue of \underline{A} if there is a nontrivial solution $\underline{\mathbf{x}}$ of $A\mathbf{x} = \lambda \mathbf{x}$; such an \mathbf{x} is called an eigenvector corresponding to $\underline{\lambda}$.

EXAMPLE 2 Let
$$A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$$
, $A = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$, and $A = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Are $A = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ -5 \end{bmatrix} = \begin{bmatrix} -24 \\ -25 \end{bmatrix} = \begin{bmatrix} -41 \\ -41 \end{bmatrix}$

Au = $\begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -9 \\ 11 \end{bmatrix} \neq \lambda \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

Au = $A = \begin{bmatrix} -9 \\ -5 \end{bmatrix}$

Au = $A = \begin{bmatrix} -9 \\ -5 \end{bmatrix}$

Au = $A = \begin{bmatrix} -9 \\ -5 \end{bmatrix}$

V is not an eigenvalue of $A = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$

Is eigenvalue of $A = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$

Au = $A = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$

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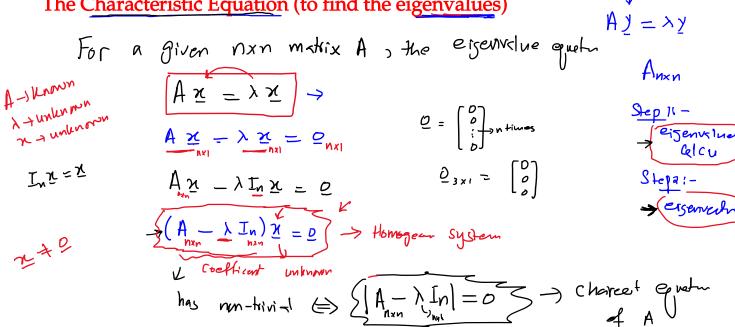
Au = $A = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$

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Au = $A = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$

The Characteristic Equation (to find the eigenvalues)



A scalar λ is an eigenvalue of an $n \times n$ matrix A if and only if λ satisfies the characteristic equation

$$\det(A - \lambda I) = 0$$

Solution:

For
$$A_{a\times a}$$
 matrix the characteristic equilibrium:

$$\begin{vmatrix} A_{a\times a} - \lambda I_{a\times a} \\ 3 - 6 \end{vmatrix} = 0$$

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$$\begin{vmatrix} A_{a\times a} - \lambda I_{a\times a} \\ 3 - 6 \end{vmatrix} = 0$$

 $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \dots + 1 = 0$ $\frac{1}{2} + \frac{1}{2} + \dots + 1 = 0$

EXAMPLE 3 Find the characteristic equation of

$$A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{\text{UXU}} \qquad [A - \lambda \bar{L}] = \begin{bmatrix} 1 - \lambda & 2 \\ 3 & \mu - \lambda \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & u \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & -1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$|A - AI_3| = \begin{vmatrix} 3 - \lambda & 1 & 2 \\ 1 & -1 - \lambda & 1 \\ 0 & 0 & 3 - \lambda \end{vmatrix}$$

$$\begin{bmatrix}
5 & -2 & 6 & -1 \\
0 & 3 & -3 & 0 \\
0 & 0 & 5 & 4 \\
0 & 0 & 0 & 1
\end{bmatrix} - \lambda \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} = 0$$

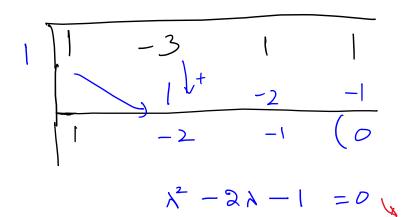
$$(5-\lambda)(3-\lambda)(5-\lambda)(1-\lambda) = 0$$

 $\lambda = 5,5,3,1$

$$\begin{bmatrix} \lambda^{3} - 3\lambda^{2} + \lambda + 1 = 0 \\ 1^{3} - 3 + 1 + 1 = 0 \\ 0 = 0 \end{bmatrix}$$

$$x^3 - 3x^2 + x + l = 0$$

$$\lambda = \pm 1$$



$$\lambda^{3} - \lambda^{2} + \lambda + 6 = 0$$

$$\lambda = 1$$

$$\lambda = 1$$

$$\lambda = -1$$

$$\lambda = -2$$

$$\lambda = 3$$

$$\lambda = -3$$

$$\lambda = -3$$

$$\lambda = -6$$

$$\lambda = -1 \times 4 + 6 = 0$$

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