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# NATIONAL UNIVERSITY OF MODERN LANGUAGES

*Faculty of Engg & Computer Science*

**BSCS-34 M-IV– Final Term Examination, Fall 2020**

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| **Paper: Linear Algebra** | **Time allowed: 2 hours** |
| **Program: BSCS (Morning)** | **Total marks :30** |

**Instructions:**

(1) Be mindful of time. Try to finish your answers within prescribed time. Upload your answers within 15 minutes after the paper time.
(2) Honestly observe all online examination protocols with your cameras on.
(3) If you upload hand-written answer scripts, write legibly so that teachers can conveniently read and grade your papers.
(4) Be precise and relevant in your answers.

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**Question#1: (i)** Find the value of "$k$" such that the matrix $A$ is non-invertiable.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & k \end{bmatrix}$$

Find the inverse of $A$, that is, $A^{-1}$ for $k = 3$.

**(ii)** For $k = 3$**,** find the cofectors of elements corresponding to $\boldsymbol{a_{21}}$ and $\boldsymbol{a_{32}}$**.**

**(iii)** For what value(s) of $k$ the columns of the matrix A are linear independent.

**(iv)** For what value(s) of $k$**,** the linear system $A\boldsymbol{x} = \boldsymbol{b}$ have unique solution for any vector $\boldsymbol{b}$.

**(v)** Using the inverse square matrix method **s**olve the linear system $A\boldsymbol{x} = \boldsymbol{b}$, for $\boldsymbol{b} = [1 \;\; 2 \;\; 3]^t$ (where $t$ represents transpose of the vector) for $k = 3$.

**(vi)** Define symmetric matrix. Is the matrix "$A$" a symmetric matrix?

**(4+2+2+2+2+2=14 Marks)**

**Question#02: (i)** Combine the methods of row reduction and cofactor expansion to compute the $\det(A)$,

$$A = \begin{bmatrix} -1 & -1 & 2 & -1 \\ 0 & 2 & -7 & 3 \\ 1 & -2 & 1 & -2 \\ 1 & 3 & 2 & 3 \end{bmatrix}$$

**(ii)** Let $A$ and $B$ be square matrices with $B$ invertible. Show that

$$det(A^{-1}BA) = \det(B)$$

**(iii)** If $det(A) = 2$, then find determinant of adjoint of the matrix A, that is, $det(adjA)$.

**(04+02+02 = 08 Marks)**

**Question#03: (i)** Show that the charteristic equation of a $3 \times 3$ metrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

for eigenvalue $\lambda$ is

$$\lambda^3 - trace(A)\lambda^2 + (a_{11}a_{22} + a_{11}a_{33} + a_{22}a_{33} - a_{12}a_{21} - a_{13}a_{31} - a_{23}a_{32})\lambda - |A| = 0.$$

**(ii)** Using the charteristic equation obtained in part (i), find the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & k \end{bmatrix}$$

for $k = 1$, and the eigenvector corresponding to $\lambda = 0$.

**(iii)** Show that for a general $2 \times 2$ matric $A$

Sum of eigenvalues = trace(A)

Product of eigenvalues = det(A)

**(4+2+2=08 Marks)**

Note: You can use the results obtained from Question 1 in Question 3 and vice versa.

**Good Luck**