Solution of Linear System by Inverse Method

The linear system involving n variables $x_1, x_2, \ldots, x_n$ and having m equations in matrices form can be written as

$$\underset{\text{Coefficient matrix}}{A}\ \underset{\text{vector of unknown}}{x} = \underset{\text{vectors}}{b} \quad \text{--- (1)}$$

where $A = [a_{ij}]_{m\times n} \rightarrow$ a coefficient matrix

$x = [x_i]_{n\times 1} \rightarrow$ unknown vector

$b = [b_i]_{m\times 1} \rightarrow$ vector of constant terms

If

$$\boxed{\text{no of unknown in the LS} = \text{no of equations}}$$

i.e. if the coefficient matrix A is square matrix & the coefficient matrix A is non-singular, then we can solve Linear System (1) as follows.

A^{-1} exist $|A| \neq 0$

$$A\,x = b \quad \text{--- (1)}$$

xing A^{-1} b/s

If A is non-singular then A^{-1} exist

$$A^{-1}A\,x = A^{-1}b$$

$$I_n\,x = A^{-1}b$$

$$\boxed{x = A^{-1}b}$$

unknown

If (i) A is square

(ii) A is non-singular

$$2 \cdot \frac{1}{2} = 1$$

$A \rightarrow n\times n$ non-s.

$A^{-1} \rightarrow n\times n \rightarrow$

$AA^{-1} = I = A^{-1}A$

$$\boxed{x = A^{-1}b}$$

Unknown

inverse of square matrix

Example:- Solve the linear System by inverse square matrix method

no of equation = 3
no of unknown = 3

$$\begin{cases} x_1 - x_2 + x_3 = 1 \\ \quad\ x_2 - 2x_3 = -1 \\ -x_1 - x_3 = 0 \end{cases}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\underset{3\times3}{A} \quad \underline{x} = \underline{b}$$

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ -1 & 0 & -1 \end{bmatrix} \qquad |A| = 1\begin{vmatrix} 1 & -2 \\ 0 & -1 \end{vmatrix} - 0\begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} - 1\begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix}$$

$$|A| = 1(-1-0) - 1(-2-1) = -1 + 3 = 2 \neq 0$$

$|A| = 0 \Leftrightarrow A$ is singular

$|A| \neq 0 \Leftrightarrow A$ is non-singular

So A is non-singular matrix

A is non-singular $\Leftrightarrow A^{-1}$ exist

A is singular $\Leftrightarrow A^{-1}$ does not exist

$$A^{-1} = \frac{\operatorname{adj} A}{|A|}$$

$$|A| = 1\begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix} + 1\begin{vmatrix} 0 & 2 \\ -1 & -1 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix}$$

$$= 1(-1-0) + 1(2) + 1(1)$$

$$|A| = -1 + 2 + 1 = 2 \neq 0$$

$$A\underline{x} = \underline{b}$$

xing A^{-1} on b/s

$$\underline{x} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$A^{-1}A\underline{x} = A^{-1}\underline{b}$$

$$I_n \underline{x} = A^{-1}\underline{b}$$

$$\underline{x} = \begin{bmatrix} \ \end{bmatrix}$$

$$\boxed{\underline{x} = A^{-1}\underline{b}}$$

we have to calculate A^{-1}

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ -1 & 0 & -1 \end{bmatrix} = A \qquad A^{-1} = ?$$

Convert into identity matrix

$$\underbrace{[A \vdots I]}_{\text{identity, seperator}} = \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\underbrace{\hphantom{xxxxxx}}_{A} \qquad \underbrace{\hphantom{xxxxxx}}_{I_3}$$

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$a_{ii} = 1 \ \forall i$

$a_{ij} = 0 \ \forall i \neq j$

Consider

$$[A_{n\times n} \mid I_{n\times n}] \xrightarrow[\text{to convert}]{\text{apply row operations}} [I_n \mid A^{-1}_{n\times n}]$$

(seperator)

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 \end{array}\right] \quad R_3 + R_1$$

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