Answer sheet # 1

NATIONALUNIVERSITYOFMODERNLANGUAGES,ISLAMABAD

Department of Computer science

End TermExamination Fall2020

RollNo:	2115	Class/ Section:	BSCS-4 -Morning
SystemID:	NUML-S19-10403	Program:	BSCS-4-Morning
EmailAddress:	Usman302@gmail.com	Teacher'sName:	Sir Rizwan
CourseTitle:	Linear Algebra	Campus:	Main
CourseCode:		Class/Section:	

Instructions:

(1) Bemindfuloftime.Trytofinishyouranswerswithinprescribedtime.Uploadyour answers within 15minutesafterthepaper time.
(2) Honestlyobserve all online examination protocols withyour cameras on.
(3) Ifyouuploadhand-writtenanswerscripts,writelegiblyon A4 sheets sothatteacherscan convenientlyreadandgradeyour papers.
(4) Be precise andrelevantinyour answers.

Q1) i) value of k such that matrix is non-invertible.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & k \end{bmatrix}$$

A matrix is non-invertible if its $|A| = 0$ or in other words, if is singular. So

A matrix is

$$|A| = 1\begin{vmatrix} 3 & 1 \\ 2 & k \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 1 & k \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix}$$

$$= (3k-2) - 2(2k-1) + (4-3) = 3k - \cancel{2} - 4k + \cancel{2} + 1$$

Scanned with CamScanner

$$= A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{7}{2} & 2 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{5}{2} & -1 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \end{array}\right] \quad \begin{matrix} R_1 + R_3 \\ R_2 - R_3 \end{matrix}$$

Thus inverse of Matrix is

ii) Part (ii)

$k = 3$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

Cofactor $a_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} = (-1)(6-2) = -4$

Cofactor $a_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1\,(1-2) = 1$

thus $\boxed{a_{21} = -4}$, $\boxed{a_{32} = 1}$

iii) Part 3 (iii)

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 1 \\ k \end{bmatrix}$$

$$x_1 v_1 + x_2 v_2 + x_3 v_3 = 0$$

$$Ad = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 3 & 1 & 0 \\ 1 & 2 & k & 0 \end{array}\right]$$

Scanned with CamScanner

Name: Roll No Page: 2

$= 3k - 4k + 1 \Rightarrow -k+1$

So As $|A| = 0$, so $-k+1 = 0$

$\boxed{1 = k}$

if value of $k = 1$, Matrix A is non-invertible.

Find inverse of A^{-1} for $k = 3$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

$= [A \,\vdots\, I]$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 2 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} R_2 - 2R_1 \\ R_3 - R_1 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -3 & 2 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 2 & -1 & 0 & 1 \end{array}\right] \quad R_1 + R_2$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -3 & 2 & 0 \\ 0 & 1 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \end{array}\right] \begin{array}{l} \sim -R_2 \\ \sim \frac{R_3}{2} \end{array}$$

Name: Roll No Page: 3

$$= A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{7}{2} & 2 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{5}{2} & -1 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \end{array}\right] \quad \begin{array}{l} R_1 + R_3 \\ R_2 - R_3 \end{array}$$

Thus inverse of Matrix is

ii) Part (ii)

$k = 3$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\text{Cofactor } a_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} = (-1)(6-2) = -4$$

$$\text{Cofactor } a_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1(1-2) = 1$$

Thus $\boxed{a_{21} = -4}$, $\boxed{a_{32} = 1}$

iii) Part 3 (iii)

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 1 \\ k \end{bmatrix}$$

$$x_1 v_1 + x_2 v_2 + x_3 v_3 = 0$$

$$Ad = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 3 & 1 & 0 \\ 1 & 2 & k & 0 \end{array}\right]$$

Part III)

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & k-1 & 0 \end{array}\right] \begin{array}{l} R_2 - R_1 \\ R_3 - R_1 \end{array}$$

vectors v_1, v_2 and v_3 are linearly independent when Matrix has trivial solution i.e no free variable

= Matrix is linearly independent ~~..~~ for all values of k where $\boxed{k \neq 1.}$

Scanned with CamScanner

Name: Roll No Page:

Question 2) i)

~~k = 3~~

$$A = \begin{bmatrix} -1 & -1 & 2 & -1 \\ 0 & 2 & -7 & 3 \\ 1 & -2 & 1 & -2 \\ 1 & -3 & 2 & 3 \end{bmatrix}$$

Row Reduction

$$= \begin{bmatrix} -1 & -1 & 2 & -1 \\ 0 & 2 & -7 & 3 \\ 0 & -3 & 3 & -3 \\ 0 & 2 & 4 & 2 \end{bmatrix} \quad \begin{matrix} R_3 + R_1 \\ R_4 + R_1 \end{matrix}$$

Cofactor

= Expanding from C_1

$$= -1 \begin{bmatrix} 2 & -7 & 3 \\ -3 & -3 & -3 \\ 2 & 4 & 2 \end{bmatrix} + 0 + 0 + 0$$

Expanding from R_1

$$= -1 \left[2 \begin{bmatrix} -3 & -3 \\ 4 & 2 \end{bmatrix} - (-7) \begin{bmatrix} -3 & -3 \\ 2 & 2 \end{bmatrix} + 3 \begin{bmatrix} -3 & 3 \\ 2 & 4 \end{bmatrix} \right]$$

$$= -1 \left[2(6+12) + 7(-6+6) + 3(-12-6) \right]$$

$$= \cancel{-1 \left[36 + -126 \right]}$$

Name: Roll No Page:

$= -1 \, [36 - 54]$

$= -1 \, [-18]$

$= \boxed{18 = |A|}$

Question 2) b) $\det(A^{-1}BA) = \det(B)$

As we know,

$$\det(A.B) = \det(A).\det(B)$$

So $\det(A^{-1}.B.A) = \det(A^{-1}) \det(B).\det(A)$

$= \det(A^{-1}) \det(A).\det(B)$

$= \det(AA^{-1}).\det(B)$

$= \det(I).\det(B)$ As $A.A^{-1} = I$

$= 1.\det(B)$

Thus $\boxed{\det(A^{-1}.BA) = \det(B)}$

Proved.

Question 2) iii)

$$Adj(A) = \begin{bmatrix} -7 & 0 & 8 & 3 \\ -38 & -18 & -26 & -12 \\ 5 & 0 & 2 & 3 \\ 37 & 18 & 22 & 15 \end{bmatrix}$$

$$\det(Adj(A)) = \begin{vmatrix} -7 & 0 & 8 & 3 \\ -38 & -18 & -26 & -12 \\ 5 & 0 & 2 & 3 \\ -1 & 0 & -4 & 3 \end{vmatrix} \quad R_4 + R_2$$

Expanding By C_2.

$$= 0 + 18 \begin{vmatrix} -7 & 8 & 3 \\ 5 & 2 & 3 \\ -1 & -4 & 3 \end{vmatrix} + 0 + 0$$

$$= 18 \left[-7(6+12) + 8(15+3) + 3(20-2) \right]$$

$$= 18 \left[+126 + 144 + 54 \right]$$

$$= \boxed{5832}$$ Ans

iv)

Part (iv)

As found in part (iii), we can clearly see that for all values of k except k=1, the system will have unique solution. ;

if there are no free variables in the system, There will be a unique solution for that system.

Name:________________ Roll No________________ Page:________

Question 3) i)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= |A - \lambda I|_{3\times 3} = 0$$

$$= \begin{vmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{vmatrix} = 0$$

$$= (a_{11}-\lambda)\left[(a_{22}-\lambda)(a_{33}-\lambda) - a_{23}a_{32}\right] - a_{21}\left[(a_{12})(a_{33}-\lambda) - (a_{13})(a_{32})\right] + (a_{31})\left[(a_{12})(a_{23}) - (a_{22}-\lambda)(a_{13})\right]$$

$$= \lambda^3 - \text{trace}(A)\lambda^2 + (a_{11}a_{22} + a_{11}a_{33} + a_{22}a_{33} - a_{12}a_{21} - a_{13}a_{31} - a_{23}a_{32})\lambda - |A|$$

Question 3) (ii)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

as A has two rows identical, $R_1 = R_2$

so $|A| = 0$. Also $A = 0$

Name: Roll No Page:

Question 3) iii)

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Scanned with CamScanner