Stochastic Model 03: Algoritmus MCMC (Přímé vzorkování)

In [1]:
```
# Instalace potřebných knihoven
#%pip install pandas
#%pip install numpy
#%pip install seaborn matplotlib
```

In [2]:
```
# Import potřebných knihoven
import pandas as pd
import numpy as np

import seaborn as sns
import matplotlib.pyplot as plt
```

Načtení přechodové matice

In [3]:
```
# Soubor je načten a přiřazen do proměnné ‚df‘
other_path = '../../data/03_StochModel/transition_matrix.csv'
df = pd.read_csv(other_path, header=0)
P = df.to_numpy()
P
```

Out[3]:
```
array([[0.00000000e+00, 9.98657124e-01, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        4.87149817e-05, 1.21787454e-03, 6.93511892e-06, 6.93511892e-05],
       [0.00000000e+00, 0.00000000e+00, 9.98657124e-01, 0.00000000e+00,
        0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        4.87149817e-05, 1.21787454e-03, 6.93511892e-06, 6.93511892e-05],
       [0.00000000e+00, 0.00000000e+00, 7.48655318e-01, 2.50000000e-01,
        0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        4.87804878e-05, 1.21951220e-03, 6.94444444e-06, 6.94444444e-05],
       [0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 4.98655318e-01,
        5.00000000e-01, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        4.87804878e-05, 1.21951220e-03, 6.94444444e-06, 6.94444444e-05],
       [0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        6.65321985e-01, 3.33333333e-01, 0.00000000e+00, 0.00000000e+00,
        4.87804878e-05, 1.21951220e-03, 6.94444444e-06, 6.94444444e-05],
       [0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 9.21732242e-01, 7.69230769e-02, 0.00000000e+00,
        4.87804878e-05, 1.21951220e-03, 6.94444444e-06, 6.94444444e-05],
       [0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 0.00000000e+00, 7.48655318e-01, 2.50000000e-01,
        4.87804878e-05, 1.21951220e-03, 6.94444444e-06, 6.94444444e-05],
       [7.69230769e-02, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 9.21732242e-01,
        4.87804878e-05, 1.21951220e-03, 6.94444444e-06, 6.94444444e-05],
       [1.66666667e-03, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        9.98333333e-01, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00],
       [8.33333333e-03, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 9.91666667e-01, 0.00000000e+00, 0.00000000e+00],
       [3.33333333e-04, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 0.00000000e+00, 9.99666667e-01, 0.00000000e+00],
       [1.00000000e-03, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
        0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 9.99000000e-01]])
```

Parametry simulace

In [4]:
```
n_states = 12

# počet iteračních kroků (sekund)
n_iter = 1_000_000

# nastavení seedu (počátečního stavu generátoru náhodných čísel)
rng = np.random.default_rng(seed=122)

samples = np.zeros(n_iter, dtype=int)

# počáteční stav S1 (index 0)
current_state = 0
samples[0] = current_state
```

Přímé vzorkování z přechodové matice

In [5]:
```
for t in range(1, n_iter):

    current_state = rng.choice(
        n_states,
        p=P[current_state]
    )
```

```
    samples[t] = current_state
```

Výsledná trajektorie stavů

In [6]:
```
# převod na stavy S1–S12
states = samples + 1
```

Empirické rozdělení

In [7]:
```
hist = np.bincount(samples, minlength=n_states)
empirical_pi = hist / np.sum(hist)

print("Empirické rozdělení:")
print(empirical_pi)
```

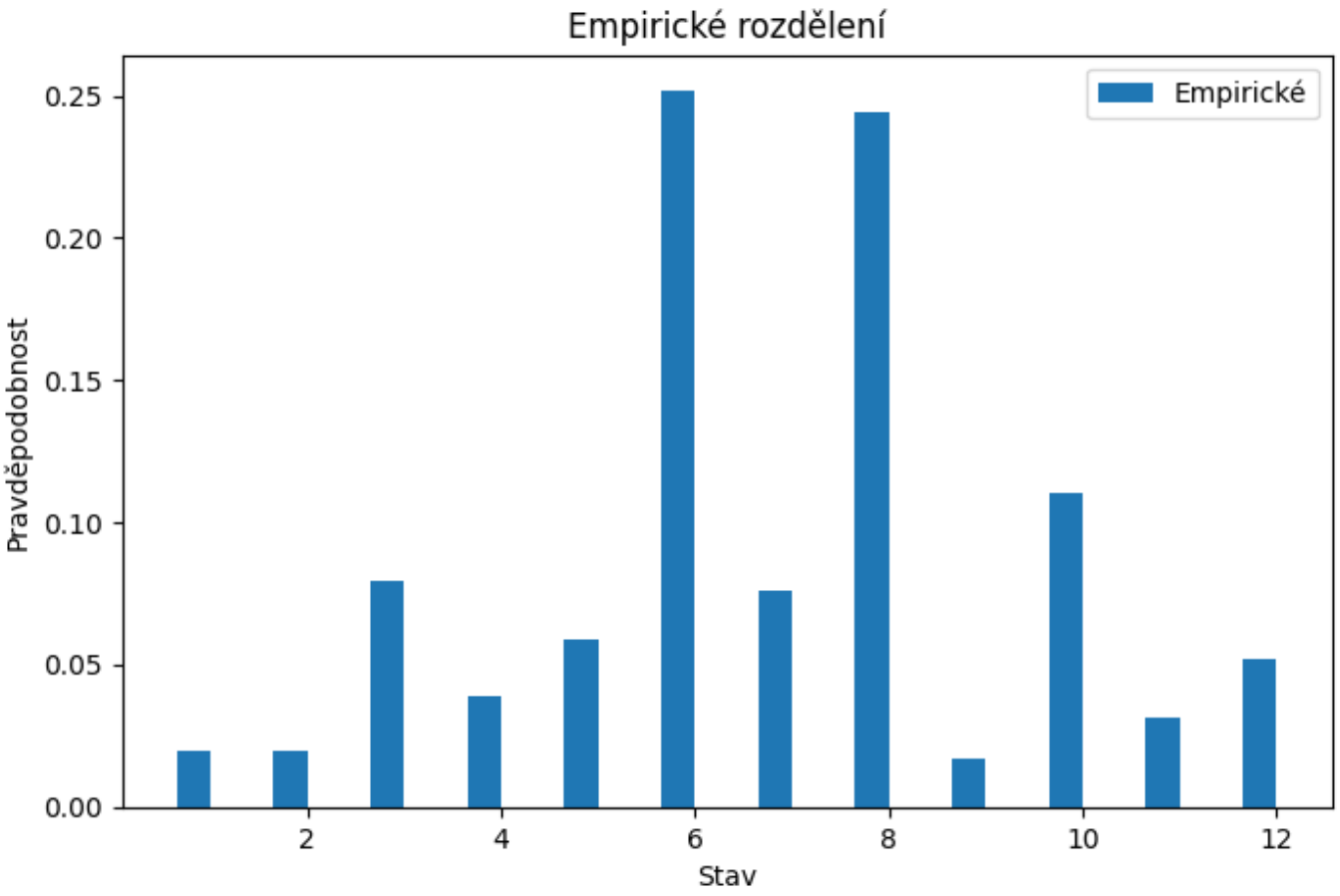
```
Empirické rozdělení:
[0.019898 0.019866 0.079479 0.039166 0.058999 0.251703 0.076208 0.244172
 0.016793 0.110242 0.031284 0.05219 ]
```

Sloupcový density graf

In [8]:
```
width = 0.35
states = np.arange(1,13)
plt.figure(figsize=(8,5))

plt.bar(states - width/2, empirical_pi, width, label='Empirické')

plt.xlabel("Stav")
plt.ylabel("Pravděpodobnost")
plt.title("Empirické rozdělení")
plt.legend()
plt.show()
```



Ilustrační graf simulace

In [9]:
```
time = np.arange(len(samples))
states = samples + 1   # převod na S1–S12
```

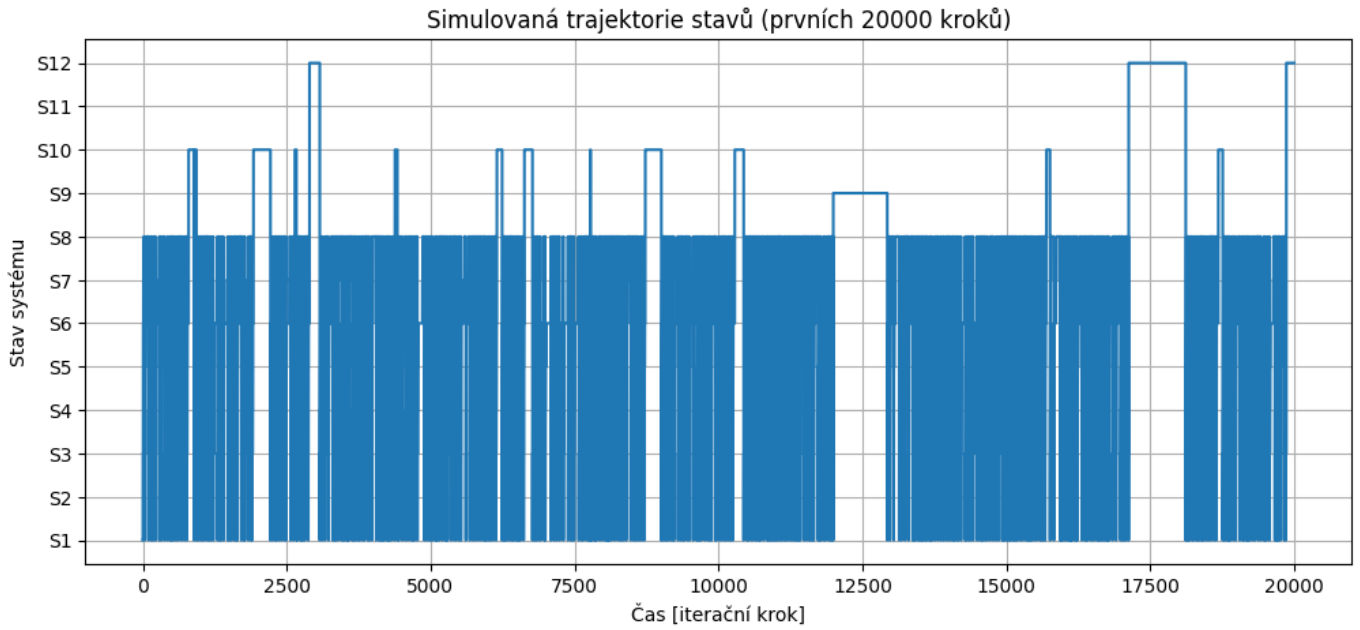
In [10]:
```
N = 20_000

plt.figure(figsize=(12,5))
plt.step(time[:N], states[:N], where='post')

plt.xlabel("Čas [iterační krok]")
plt.ylabel("Stav systému")
plt.title(f"Simulovaná trajektorie stavů (prvních {N} kroků)")

plt.yticks(np.arange(1,13), [f"S{i}" for i in range(1,13)])
plt.grid(True)

plt.show()
```

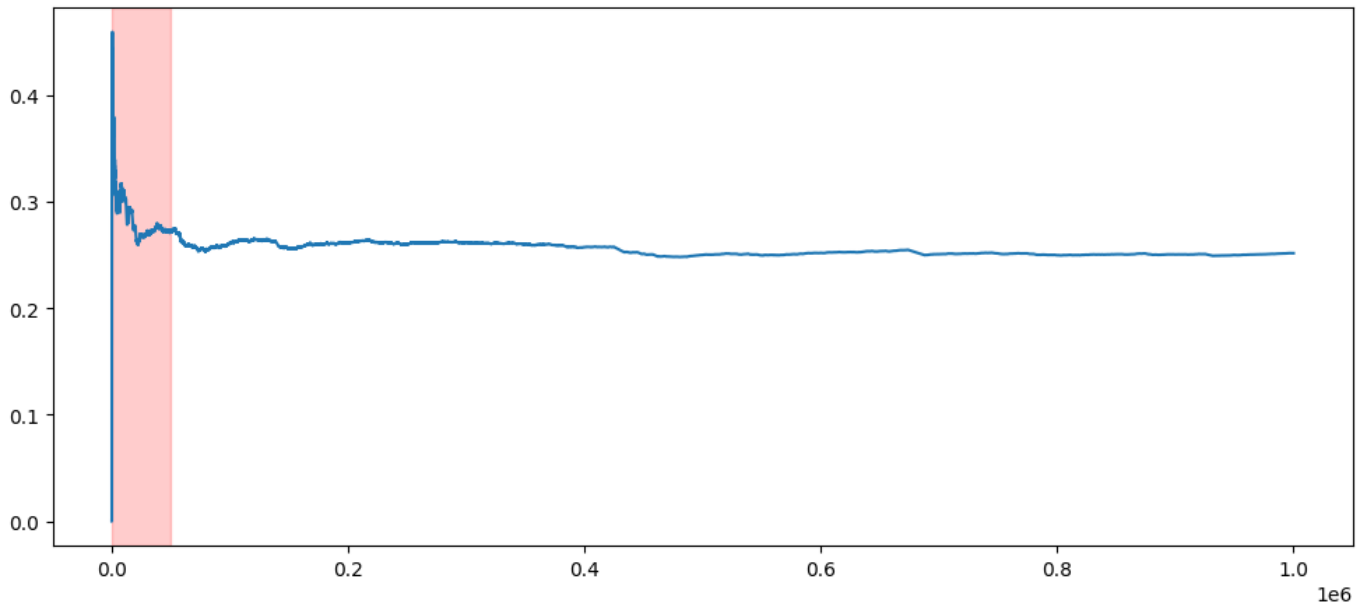


Spálení počáteční části simulace (Burn-in)

Počáteční transientní fáze simulace (burn-in) byla identifikována a odstraněna z další analýzy. Tato oblast je v grafu vyznačena červeně.

In [11]:
```
burn_in = 50_000
running_mean = np.cumsum(samples==5)/np.arange(1,len(samples)+1)
plt.figure(figsize=(12,5))
plt.axvspan(0, burn_in, color='red', alpha=0.2, label='Burn-in')
plt.plot(running_mean)
```

Out[11]:
```
[<matplotlib.lines.Line2D at 0x21e46900f10>]
```

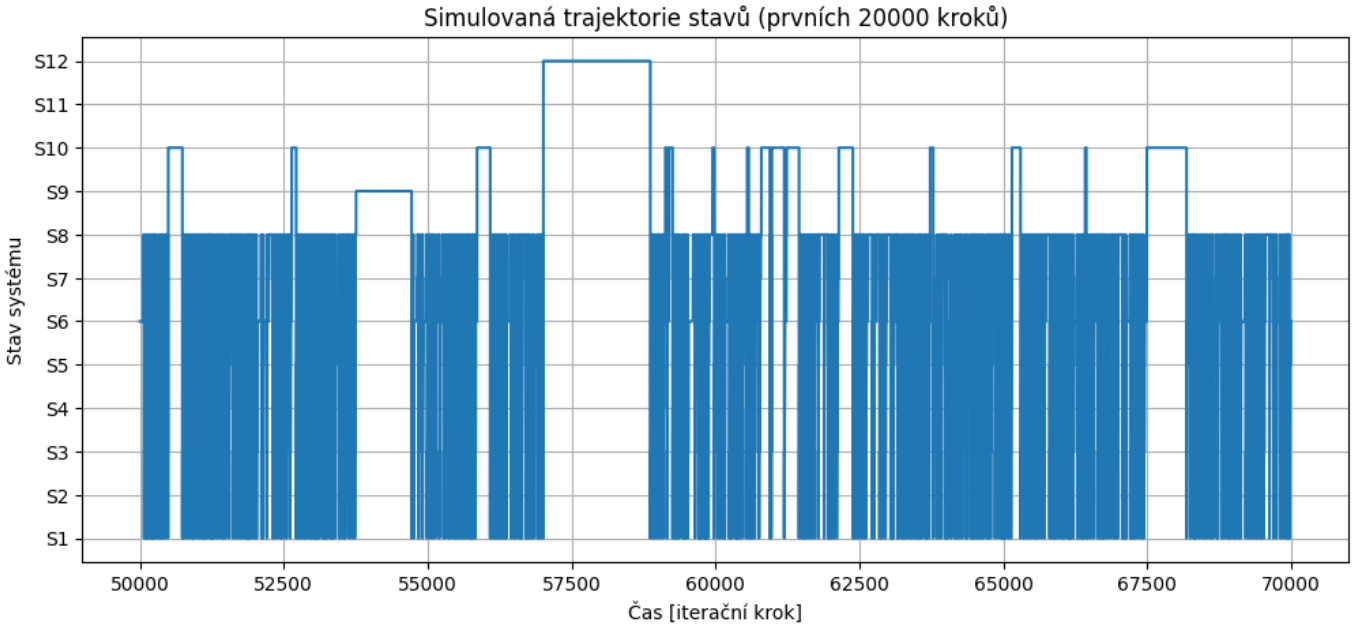


In [12]:
```
plt.figure(figsize=(12,5))
plt.step(time[burn_in:burn_in+N], states[burn_in:burn_in+N], where='post')

plt.xlabel("Čas [iterační krok]")
plt.ylabel("Stav systému")
plt.title(f"Simulovaná trajektorie stavů (prvních {N} kroků)")

plt.yticks(np.arange(1,13), [f"S{i}" for i in range(1,13)])
plt.grid(True)
```

```
plt.show()
```



Odstranění burn-in

In [13]:
```
samples_burned = samples[burn_in:]
```

Empirické rozdělení

In [14]:
```
hist = np.bincount(samples_burned, minlength=n_states)
empirical_pi = hist / np.sum(hist)

print("Empirické rozdělení:")
print(empirical_pi)
```

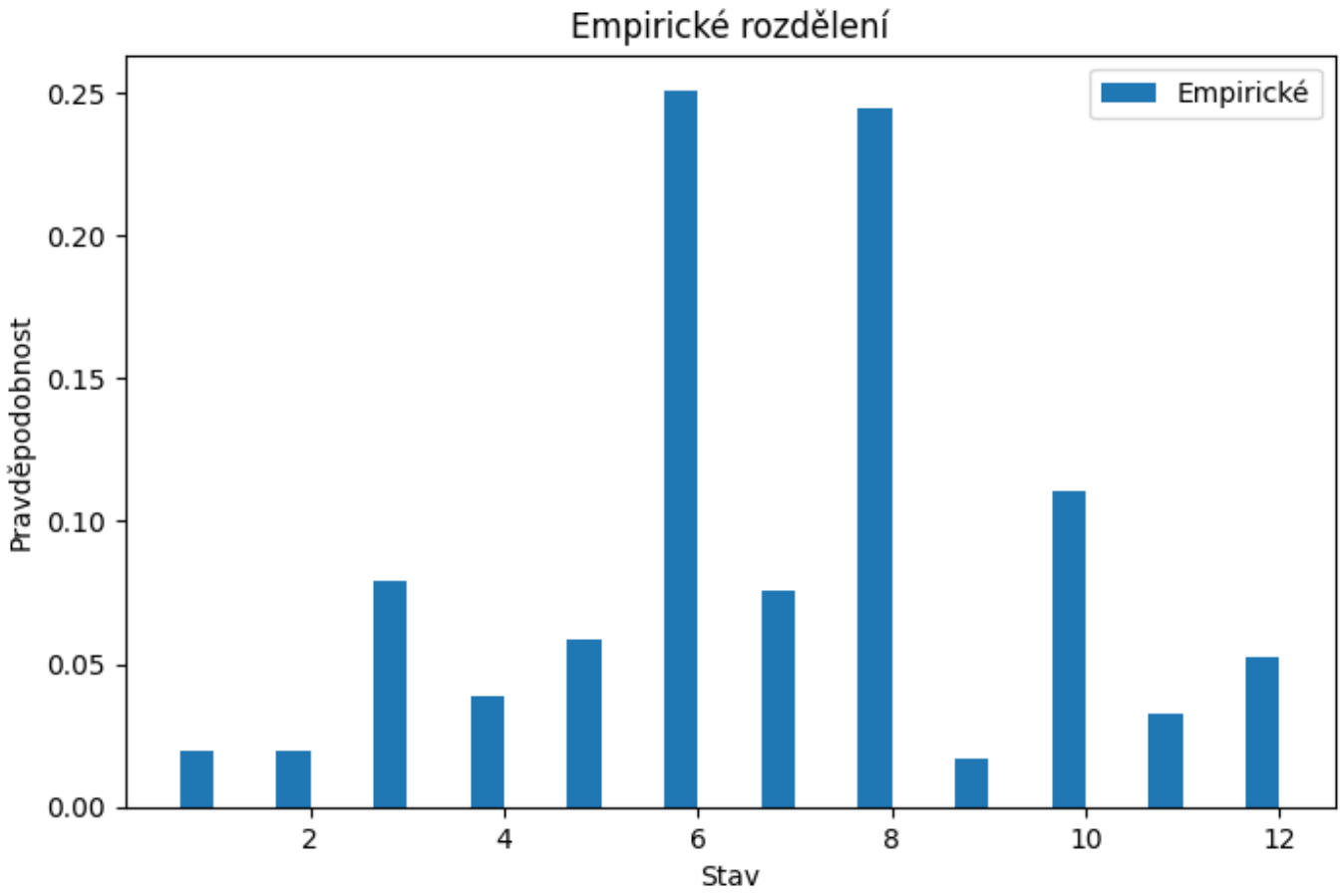
```
Empirické rozdělení:
[0.01984421 0.01981158 0.07932421 0.03907263 0.05886    0.25059579
 0.07574947 0.24442105 0.01668947 0.11052211 0.03293053 0.05217895]
```

Sloupcový density graf

In [15]:
```
width = 0.35
states = np.arange(1,13)
plt.figure(figsize=(8,5))

plt.bar(states - width/2, empirical_pi, width, label='Empirické')

plt.xlabel("Stav")
plt.ylabel("Pravděpodobnost")
plt.title("Empirické rozdělení")
plt.legend()
plt.show()
```



Export matice formátu CSV

In [16]:
```
df_sim = pd.DataFrame({
    "time": np.arange(len(samples_burned)),
    "state_index": samples_burned,
    "state": samples_burned + 1
})
```

In [17]:
```
df_sim.to_csv('../../data/03_StochModel/simulation_MCMC_samples.csv', index=False)
```

Autor / Organizace / Datum

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Přehled změn

Datum (YYYY-MM-DD)	Verze	Autor změny	Popis změny
2026-01-25	1.1	Vjačeslav Usmanov	added SM_03_MCMC_samples.ipynb
2026-02-15	1.2	Vjačeslav Usmanov	changed SM_03_MCMC_samples.ipynb