Stochastic Model 01: Matice přechodů (Transition matrix)

In [1]:
```
# Instalace potřebných knihoven
#%pip install pandas
#%pip install numpy
#%pip install seaborn matplotlib
```

In [2]:
```
# Import potřebných knihoven
import pandas as pd
import numpy as np

import seaborn as sns
import matplotlib.pyplot as plt
```

Definice modelových parametrů

In [3]:
```
# nastavení formátu výpisu pro zobrazení až 8 desetinných míst.
np.set_printoptions(precision=8, suppress=True)

# Definice průměrných dob trvání jednotlivých stavů systému a intenzit přechodu do neprovozníc
dt = 1  # časový krok (s)

# prumerne doby trvani (s)
T = {
    "S1": 2, "S2": 2, "S3": 5, "S4": 2,
    "S5": 5, "S6": 7, "S7": 3, "S8": 6,
    "S9": 60, "S10": 70, "S11": 160, "S12": 20
}


# intenzity poruch (1/s)
lambda_vals = {
    "S9": 4/8896,
    "S10": 7/8896,
    "S11": 3/8896,
    "S12": 13/8896
}
```

In [4]:
```
# Inicializace přechodové matice Markovského řetězce
states = list(T.keys())
n = len(states)

P = np.zeros((n,n))
```

Pomocne funkce

In [5]:
```
def exit_probability(Ti):
    return 1 - np.exp(-dt / Ti)

def fault_probability(lam):
    return 1 - np.exp(-lam * dt)
```

Definice prechodu

In [6]:
```
transitions = {
```

```
    "S1": ["S2", "S9", "S11"],
    "S2": ["S3", "S9", "S11"],
    "S3": ["S4", "S9", "S10", "S11"],
    "S4": ["S5", "S9", "S11"],
    "S5": ["S6", "S9", "S10", "S11", "S12"],
    "S6": ["S7", "S9", "S11"],
    "S7": ["S8", "S9", "S10", "S11"],
    "S8": ["S1", "S9", "S11"],
    "S9": ["S1"],
    "S10": ["S3"],
    "S11": ["S1"],
    "S12": ["S5"]
}
```

Sestaveni matice

In [7]:
```
for i, si in enumerate(states):

    p_exit = exit_probability(T[si])
    total_fault_prob = 0

    # poruchove prechody
    for sj in transitions[si]:
        if sj in lambda_vals:
            total_fault_prob += fault_probability(lambda_vals[sj])

    # rozdeleni pravdepodobnosti
    for sj in transitions[si]:
        j = states.index(sj)

        if sj in lambda_vals:
            P[i, j] = fault_probability(lambda_vals[sj])
        else:
            P[i, j] = p_exit

    # pravdepodobnost setrvani
    P[i, i] = 1 - P[i].sum()
```

Kontrola matice

In [8]:
```
print("Soucty radku:", P.sum(axis=1))

print("Prechodova matice P:")
print(np.round(P, 8))
```

```
Soucty radku: [1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
Prechodova matice P:
[[0.60574395 0.39346934 0.         0.         0.         0.
  0.         0.         0.00044954 0.         0.00033717 0.        ]
 [0.         0.60574395 0.39346934 0.         0.         0.
  0.         0.         0.00044954 0.         0.00033717 0.        ]
 [0.         0.         0.81715748 0.18126925 0.         0.
  0.         0.         0.00044954 0.00078656 0.00033717 0.        ]
 [0.         0.         0.         0.60574395 0.39346934 0.
  0.         0.         0.00044954 0.         0.00033717 0.        ]
 [0.         0.         0.         0.         0.81569722 0.18126925
  0.         0.         0.00044954 0.00078656 0.00033717 0.00146026]
 [0.         0.         0.         0.         0.         0.86609119
  0.1331221  0.         0.00044954 0.         0.00033717 0.        ]
 [0.         0.         0.         0.         0.         0.
  0.71495804 0.28346869 0.00044954 0.00078656 0.00033717 0.        ]
 [0.15351828 0.         0.         0.         0.         0.
  0.         0.84569501 0.00044954 0.         0.00033717 0.        ]
 [0.01652855 0.         0.         0.         0.         0.
  0.         0.         0.98347145 0.         0.         0.        ]
 [0.         0.         0.01418416 0.         0.         0.
  0.         0.         0.         0.98581584 0.         0.        ]
 [0.00623051 0.         0.         0.         0.         0.
  0.         0.         0.         0.         0.99376949 0.        ]
 [0.         0.         0.         0.         0.04877058 0.
  0.         0.         0.         0.         0.         0.95122942]]
```

Výpočet stacionárního rozdělení

In [9]:
```
# Výpočet stacionárního rozdělení pravděpodobností pomocí vlastního řešení matice.
eigvals, eigvecs = np.linalg.eig(P.T)

idx = np.argmin(np.abs(eigvals-1))

pi = eigvecs[:,idx]
pi = np.real(pi / np.sum(pi))

print("\nStacionární rozdělení pi:")
print(np.round(pi,6))
```

```
Stacionární rozdělení pi:
[0.064078 0.06395  0.139183 0.063993 0.13771  0.186414 0.08706  0.159936
 0.024541 0.020182 0.048831 0.004123]
```

Vyhodnocení dostupnosti systému

In [10]:
```
# Stanovení dlouhodobé provozní dostupnosti systému
availability = np.sum(pi[0:8])
print("\nDostupnost systému:")
print(round(availability,4))

# Stanovení podílu neprovozního času
downtime = np.sum(pi[8:12])
print("\nPodíl neprovozního času:")
print(round(downtime,4))
```

```
Dostupnost systému:
0.9023

Podíl neprovozního času:
0.0977
```

Export matice formátu CSV

In [11]:
```
states = [
    "S1","S2","S3","S4","S5","S6","S7","S8",
    "S9","S10","S11","S12"
]

df = pd.DataFrame(P, index=states, columns=states)
```

In [12]:
```
df
```

Out[12]:

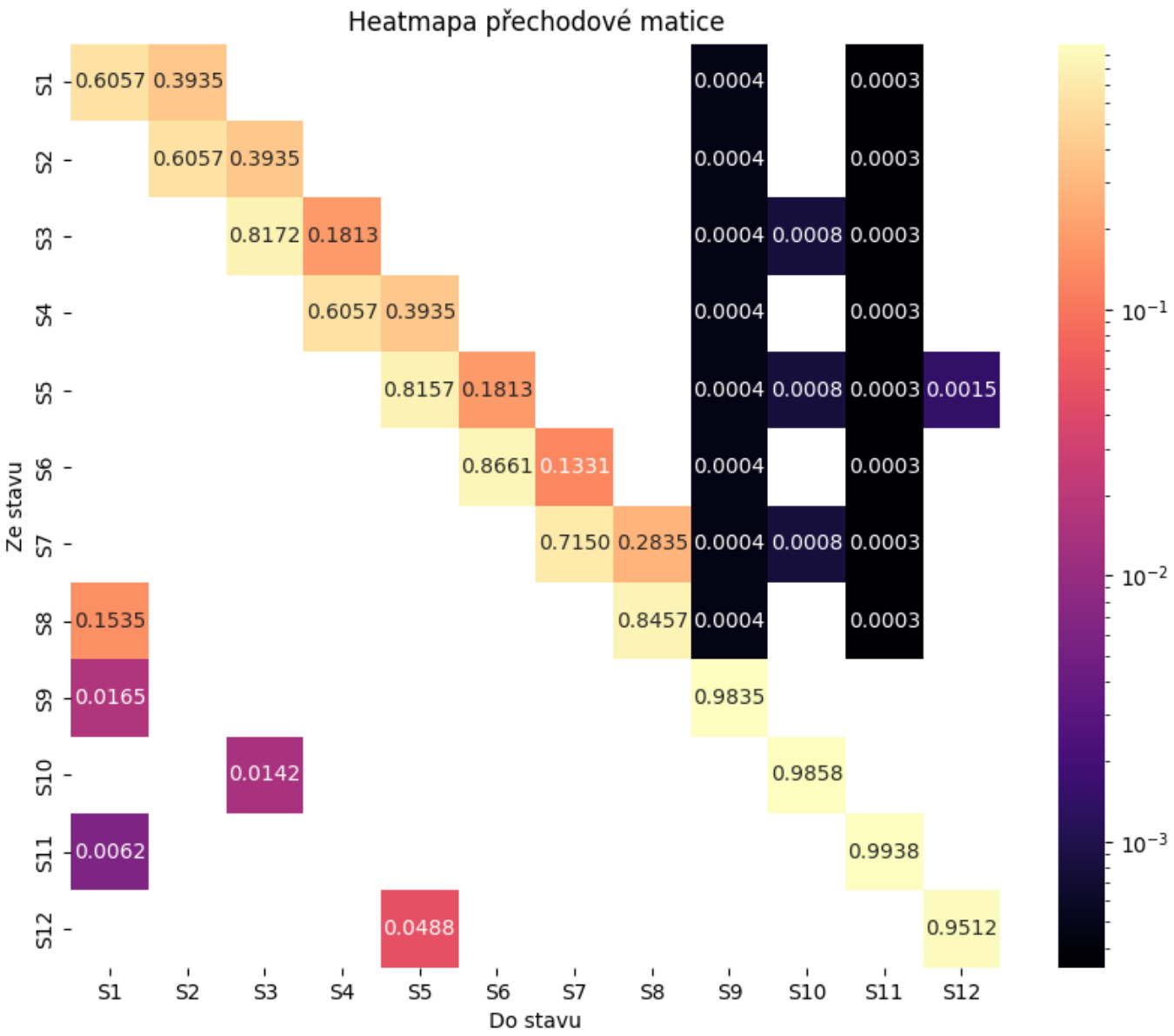
	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
S1	0.605744	0.393469	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000450	0.000000
S2	0.000000	0.605744	0.393469	0.000000	0.000000	0.000000	0.000000	0.000000	0.000450	0.000000
S3	0.000000	0.000000	0.817157	0.181269	0.000000	0.000000	0.000000	0.000000	0.000450	0.000787
S4	0.000000	0.000000	0.000000	0.605744	0.393469	0.000000	0.000000	0.000000	0.000450	0.000000
S5	0.000000	0.000000	0.000000	0.000000	0.815697	0.181269	0.000000	0.000000	0.000450	0.000787
S6	0.000000	0.000000	0.000000	0.000000	0.000000	0.866091	0.133122	0.000000	0.000450	0.000000
S7	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.714958	0.283469	0.000450	0.000787
S8	0.153518	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.845695	0.000450	0.000000
S9	0.016529	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.983471	0.000000
S10	0.000000	0.000000	0.014184	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.985816
S11	0.006231	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
S12	0.000000	0.000000	0.000000	0.000000	0.048771	0.000000	0.000000	0.000000	0.000000	0.000000

Vykreslení heatmapy

In [13]:
```
plt.figure(figsize=(10,8))

sns.heatmap(
    df,
    annot=True,
    fmt=".4f",
    cmap="magma",
    norm=plt.matplotlib.colors.LogNorm()
)

plt.title("Heatmapa přechodové matice")
plt.xlabel("Do stavu")
plt.ylabel("Ze stavu")
plt.show()
```



In [14]:
```
df.to_csv('../../data/03_StochModel/transition_matrix.csv', index=False)
```

Zdroj: vlastní implementace na základě [Stewart, 2009]

- Stewart, W.J. (2009). Probability, Markov Chains, Queues, and Simulation. Princeton University Press.
- Norris, J.R. (1998). Markov Chains. Cambridge University Press.

Autor / Organizace / Datum

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Přehled změn

Datum (YYYY-MM-DD)	Verze	Autor změny	Popis změny
2026-01-25	1.1	Vjačeslav Usmanov	added SM_01_Transition_matrix.ipynb
2026-02-15	1.2	Vjačeslav Usmanov	changed SM_01_Transition_matrix.ipynb