Homework 4

Instructions

Points: Please see the points for each problem.
Submission: Submit completed homework as a PDF file. Handwritten work or photos of handwritten work must be neat and legible.

Points Summary

Question Number	Points Possible	Points Earned
1	4	_____
2	3	_____
3	3	_____
Total	**10**	_____

Consider a Markov Model with a binary state $X_t \in \{0, 1\}$. The transition probabilities are given as follows:

X_t	X_{t+1}	$P(X_{t+1} \mid X_t)$
0	0	0.9
0	1	0.1
1	0	0.5
1	1	0.5

1. The prior belief distribution over the initial state X_0 is uniform: $P(X_0 = 1) = P(X_0 = 0) = 0.5$. After one timestep, what is the new belief distribution $P(X_1)$?

Now, we incorporate sensor readings. The sensor model is parameterized by a number $\beta \in [0, 1]$:

X_t	E_t	$P(E_t \mid X_t)$
0	0	β
0	1	$1 - \beta$
1	0	$1 - \beta$
1	1	β

2. At $t = 1$, we get the first sensor reading, $E_1 = 0$. Use your answer from Question 1 to compute $P(X_1 = 0 \mid E_1 = 0)$. Leave your answer in terms of β.

3. For what range of values of β will a sensor reading $E_1 = 0$ increase our belief that $X_1 = 0$? That is, what is the range of β for which $P(X_1 = 0 \mid E_1 = 0) > P(X_1 = 0)$?