Homework 2

Instructions

Points: Please see the points for each problem.
Submission Instructions: Please submit a PDF in Canvas.

Points

Question	Part	Points Possible	Points Earned
1. Minimax and α-β pruning	(a)	4	______
	(b)	4	______
2. MDP and Value Iteration	(a)	4	______
	(b)	4	______
	(c)	4	______
3. Policy Gradient and PPO	(a)	3	______
	(b)	3	______
Total		**26**	______

1. Minimax and α-β pruning Consider the minimax tree shown below for parts (a) and (b).

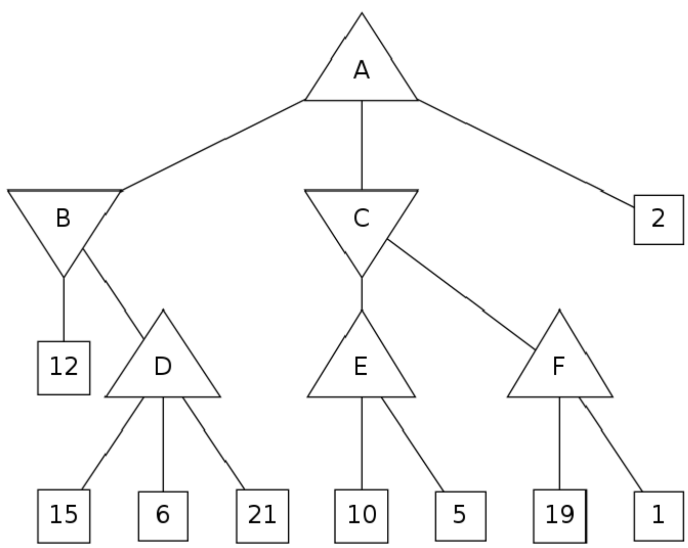

Figure 1: Minimax tree for Question 1.

(a) What value will the root node A have?

Solution: From the leaves, compute internal node values bottom-up:

$$\begin{aligned} D &= \max\{15, 6, 21\} = 21, \\ B &= \min\{12, D\} = 12, \\ E &= \max\{5, 10\} = 10, \\ F &= \max\{1, 19\} = 19, \\ C &= \min\{E, F\} = 10, \\ A &= \max\{B, C, 2\} = 12. \end{aligned}$$

Thus, the root value is $A = 12$.

(b) Cross off the nodes that are pruned by α-β pruning. Assume standard left-to-right traversal. If a non-terminal state (A, B, C, D, E, or F) is pruned, cross off the entire subtree.

Solution: With left-to-right traversal, the leaf with value 6 and the leaf with value 21 under D are pruned, and the entire subtree under F is pruned.

2. Pacman is using MDPs and Value Iteration to maximize his expected utility. He has the standard actions {North, East, South, West} unless blocked by an outer wall. There is a reward of 1 when eating a dot. The game ends when the dot is eaten.

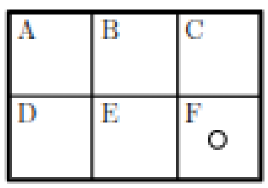

Figure 2: Grid for Question 2.

(a) Consider the grid where there is a single food pellet in the bottom-right corner (F) as shown in figure 2. The discount factor is $\gamma = 0.5$. There is no living reward. The states are the grid locations A, B, C, D, E, F. What is the optimal policy for each state?

State s	Policy $\pi(s)$
A	
B	
C	
D	
E	

Solution: An optimal policy (one of possibly several, where ties are allowed) is:

State s	Policy $\pi(s)$
A	East or South
B	East or South
C	South
D	East
E	East

(b) What is the optimal value for the upper-left corner state A?

Solution: $V^*(A) = 0.375$.

We use value iteration with the Bellman equation. The reward $R(s, a, s') = 1$ when transitioning into the terminal state F (eating the dot), and 0 otherwise. The value iteration update is:

$$V^k(s) = \max_a \left[R(s, a, s') + \gamma V^{k-1}(s')\right]$$

where s' is the state reached by taking action a from state s, and $\gamma = 0.5$.

Value Iteration Table:

k	$V^k(A)$	$V^k(B)$	$V^k(C)$	$V^k(D)$	$V^k(E)$	$V^k(F)$
0	0	0	0	0	0	0
1	0	1	0	0	1	1
2	0	0.5	1.5	0.5	1.5	1
3	0.25	0.75	1.5	0.75	1.5	1
4	0.375	0.75	1.5	0.75	1.5	1
5	0.375	0.75	1.5	0.75	1.5	1

Explanation by iteration:

Iteration $k = 0$: All values initialized to 0, including $V^0(F) = 0$.

Iteration $k = 1$: The terminal state F gets its value:

$$V^1(F) = 1 \quad \text{(terminal state: when you're in } F\text{, you've already received reward } R = 1 \text{ and the game ends)}$$

Since F is terminal, once you reach it, you get the reward of 1 and the episode ends. Therefore, $V^k(F) = 1$ for all $k \geq 1$. The value stays 1 because there are no future actions or rewards after reaching the terminal state.

States that can reach F in one step receive the reward:

$$V^1(E) = R(E, \text{East}, F) + \gamma V^0(F) = 1 + 0.5 \cdot 0 = 1$$
$$V^1(B) = R(B, \text{action}, F) + \gamma V^0(F) = 1 + 0.5 \cdot 0 = 1 \quad \text{(can reach } F \text{ via optimal path)}$$
$$V^1(A) = \max_a [0 + \gamma \cdot 0] = 0 \quad \text{(cannot reach } F \text{ in one step)}$$
$$V^1(C) = \max_a [0 + \gamma \cdot 0] = 0 \quad \text{(cannot reach } F \text{ in one step)}$$
$$V^1(D) = \max_a [0 + \gamma \cdot 0] = 0 \quad \text{(cannot reach } F \text{ in one step)}$$

Iteration $k = 2$: States update based on values from iteration 1:

$$
\begin{aligned}
V^2(F) &= 1 \quad \text{(terminal state, unchanged)} \\
V^2(E) &= R(E, \text{East}, F) + \gamma V^1(F) = 1 + 0.5 \cdot 1 = 1.5 \\
V^2(C) &= R(C, \text{South}, E?) + \gamma V^1(E) = 0 + 0.5 \cdot 1 = 0.5, \text{ or} \\
&\qquad \max\{\text{other actions}\} = 1.5 \quad \text{(optimal path gives higher value)} \\
V^2(D) &= R(D, \text{South}, F) + \gamma V^1(F) = 1 + 0.5 \cdot 1 = 1.5, \text{ or} \\
&\qquad \max\{R(D, \text{East}, E) + \gamma V^1(E)\} = 0 + 0.5 \cdot 1 = 0.5 \\
V^2(B) &= \max_a [0 + \gamma V^1(\text{next state})] = \max\{0.5 \cdot 1, 0.5 \cdot 0\} = 0.5 \\
V^2(A) &= \max_a [0 + \gamma V^1(\text{next state})] = \max\{0.5 \cdot 0, 0.5 \cdot 0\} = 0
\end{aligned}
$$

Iteration $k = 3$:

$$
\begin{aligned}
V^3(F) &= 1 \quad \text{(terminal state, unchanged)} \\
V^3(E) &= 1 + 0.5 \cdot 1 = 1.5 \quad \text{(unchanged, optimal to go East to } F\text{)} \\
V^3(C) &= 1.5 \quad \text{(unchanged, optimal path established)} \\
V^3(D) &= \max\{1 + 0.5 \cdot 1, 0 + 0.5 \cdot 1.5\} = \max\{1.5, 0.75\} = 1.5, \text{ or } 0.75 \\
V^3(B) &= \max\{0 + 0.5 \cdot 1.5, 0 + 0.5 \cdot 0.5\} = \max\{0.75, 0.25\} = 0.75 \\
V^3(A) &= \max\{0 + 0.5 \cdot 0.5, 0 + 0.5 \cdot 1.5\} = \max\{0.25, 0.75\} = 0.75, \text{ or } 0.25
\end{aligned}
$$

Iteration $k = 4$: Values update from iteration 3:

$$
\begin{aligned}
V^4(F) &= 1 \quad \text{(terminal state, unchanged)} \\
V^4(E) &= R(E, \text{East}, F) + \gamma V^3(F) = 1 + 0.5 \cdot 1 = 1.5 = V^3(E) \quad \text{(unchanged)} \\
V^4(C) &= 1.5 = V^3(C) \quad \text{(unchanged, optimal path established)} \\
V^4(D) &= \max\{R(D, \text{South}, F) + \gamma V^3(F), R(D, \text{East}, E) + \gamma V^3(E)\} \\
&= \max\{1 + 0.5 \cdot 1, 0 + 0.5 \cdot 1.5\} = \max\{1.5, 0.75\} = 0.75 = V^3(D) \\
V^4(B) &= \max\{0 + \gamma V^3(C), 0 + \gamma V^3(D)\} \\
&= \max\{0 + 0.5 \cdot 1.5, 0 + 0.5 \cdot 0.75\} = \max\{0.75, 0.375\} = 0.75 = V^3(B) \\
V^4(A) &= \max\{0 + \gamma V^3(B), 0 + \gamma V^3(C)\} \\
&= \max\{0 + 0.5 \cdot 0.75, 0 + 0.5 \cdot 1.5\} = \max\{0.375, 0.75\} = 0.375
\end{aligned}
$$

Note that $V^4(A) = 0.375$ changed from $V^3(A) = 0.25$, while other states remained unchanged from iteration 3.

Iteration $k = 5$: No change in any state value:

$$V^5(F) = 1 = V^4(F)$$
$$V^5(E) = 1.5 = V^4(E)$$
$$V^5(C) = 1.5 = V^4(C)$$
$$V^5(D) = 0.75 = V^4(D)$$
$$V^5(B) = 0.75 = V^4(B)$$
$$V^5(A) = 0.375 = V^4(A)$$

Since $V^5(s) = V^4(s)$ for all states s, the algorithm has converged. We stop value iteration when there is no change in any state value between consecutive iterations, which indicates we have reached the optimal values $V^*(s)$.

The optimal value $V^*(A) = 0.375$ is reached at iteration $k = 4$. This represents the expected discounted reward from state A following the optimal policy, where the discount factor $\gamma = 0.5$ reduces the value of future rewards.

(c) Using value iteration initialized with $V^0(\cdot) = 0$, at which iteration k does $V^k(A)$ first equal $V^*(A)$?

Solution: At iteration $k = 4$. Iteration $k = 5$ matches iteration $k = 4$.

3. Consider some of the newer RL algorithms such as Policy Gradient and Proximal Policy Optimization.

(a) What issues with RL are they designed to solve?

Solution: Policy Gradient and PPO algorithms address several key issues in RL:

- **Sample inefficiency**: Traditional value-based methods (like Q-learning) require many samples to learn good policies, especially in high-dimensional or continuous action spaces.
- **High variance**: Policy gradient methods suffer from high variance in gradient estimates, making training unstable.
- **Stability**: Vanilla policy gradients can make large, destructive policy updates that degrade performance.
- **On-policy data requirement**: Many RL algorithms require on-policy data, making sample reuse difficult and inefficient.
- **Continuous action spaces**: Value-based methods struggle with continuous actions, while policy gradients handle them naturally.

Note: Stating any 3 of these is sufficient for full credit.

(f) How do they improve on previous RL algorithms?

Solution: Policy Gradient and PPO improve on previous RL algorithms in several ways:

- **Direct policy optimization**: Policy gradients optimize the policy directly, avoiding the need to learn value functions first (unlike value-based methods like Q-learning).
- **PPO clipping**: PPO uses a clipped objective function that prevents large policy updates, maintaining training stability while allowing multiple updates from the same data.
- **Better sample efficiency**: PPO can perform multiple gradient updates on the same batch of data, improving sample efficiency compared to vanilla policy gradients.
- **Natural continuous actions**: Policy gradients parameterize policies directly, making them well-suited for continuous action spaces without discretization.
- **Reduced variance**: PPO's clipped objective and trust region approach reduce variance compared to standard policy gradient methods.
- **Stable learning**: The clipping mechanism ensures policy updates stay within a trust region, preventing performance collapse that can occur with large updates.

Note: Stating any 3 of these is sufficient for full credit.