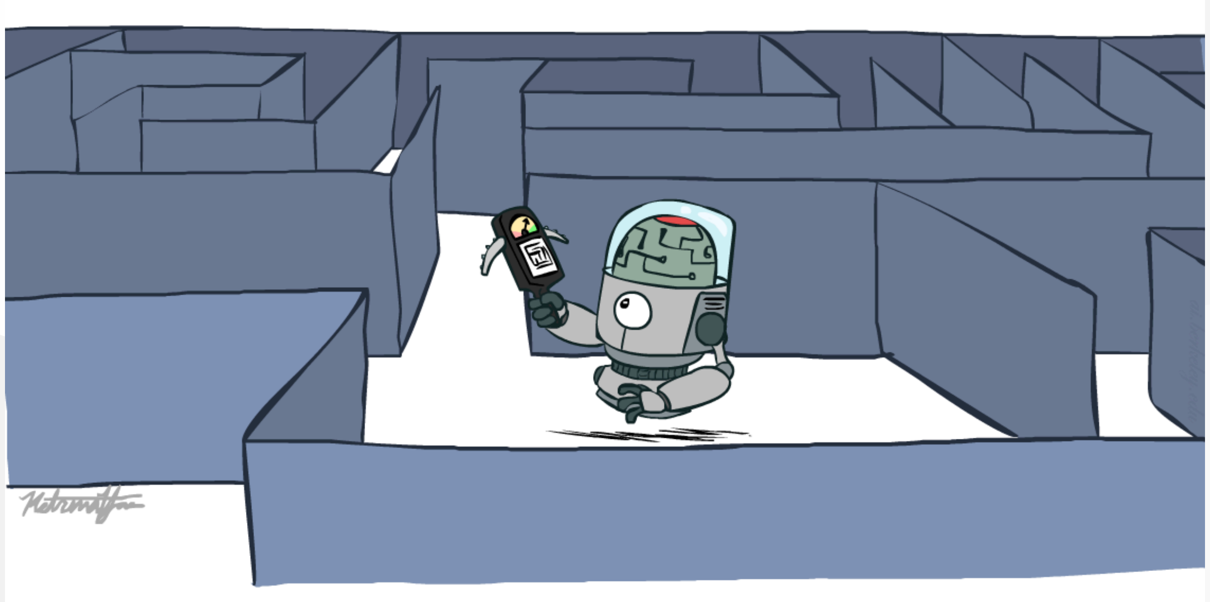
CS 6460: Artificial Intelligence

Informed Search



Instructor: George Rudolph
Utah Valley University Spring 2025

[These slides adapted from Dan Klein and Pieter Abbeel at UC Berkley]

Learning Outcomes

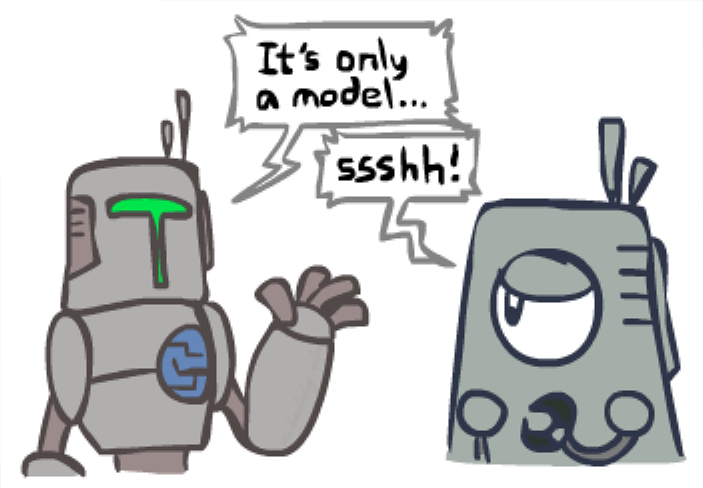
1. Solve Problems using Informed Searches

- Heuristics
- Greedy Search
- A* Search

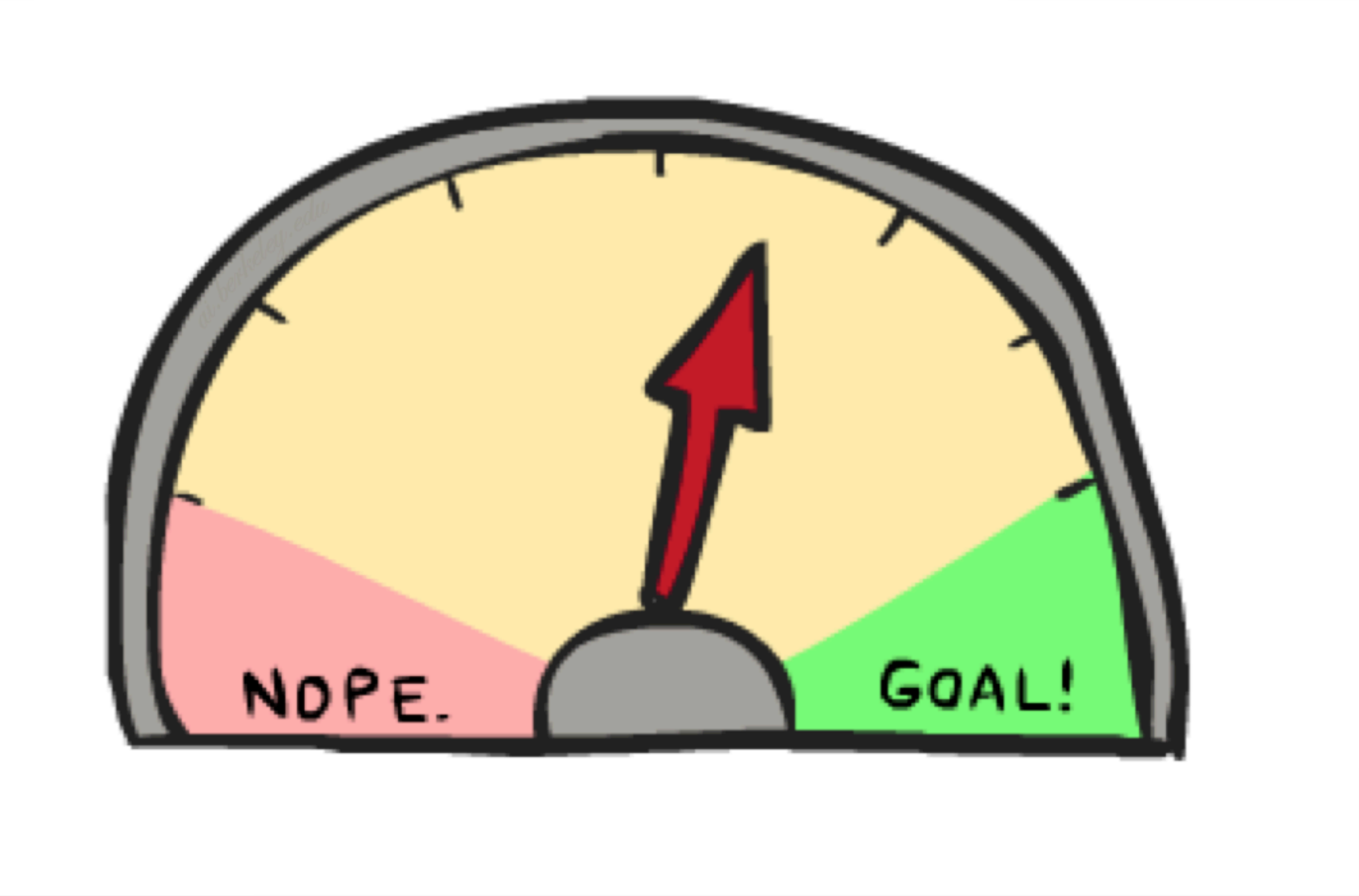
2. Model Problems as Graph Search

Search and Models

- Search operates over models of the world
- The agent doesn't actually try all the plans out in the real world!
- Planning is all in simulation
- Your search is only as good as your models…



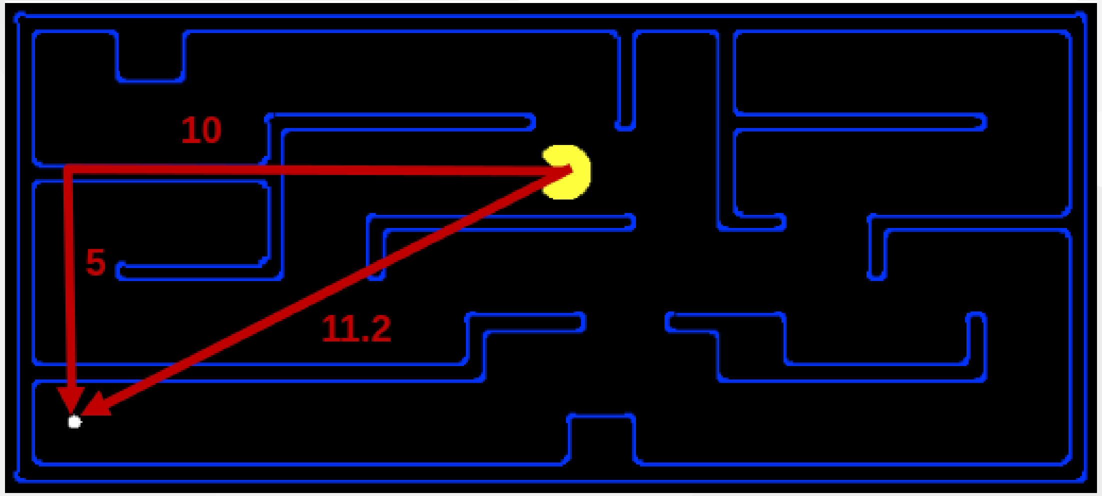
Informed Search

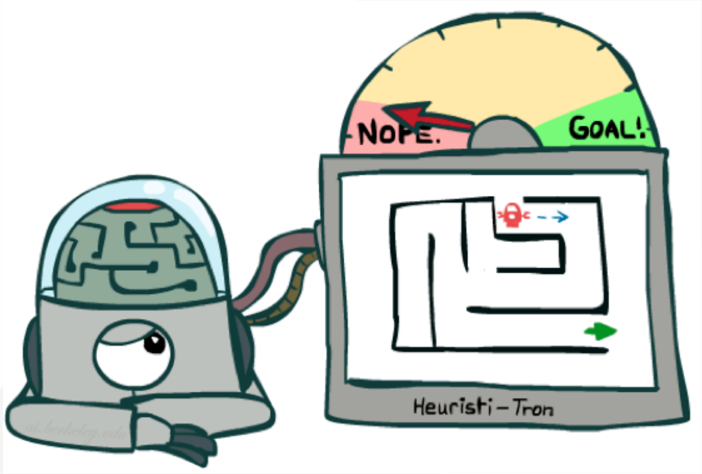


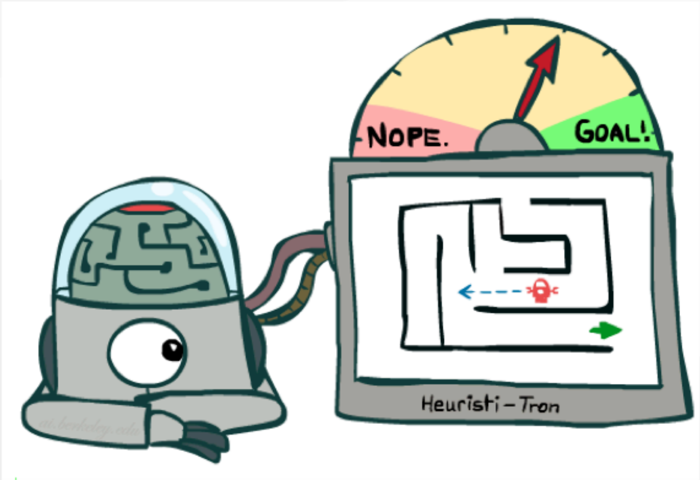
Search Heuristics

A heuristic is:

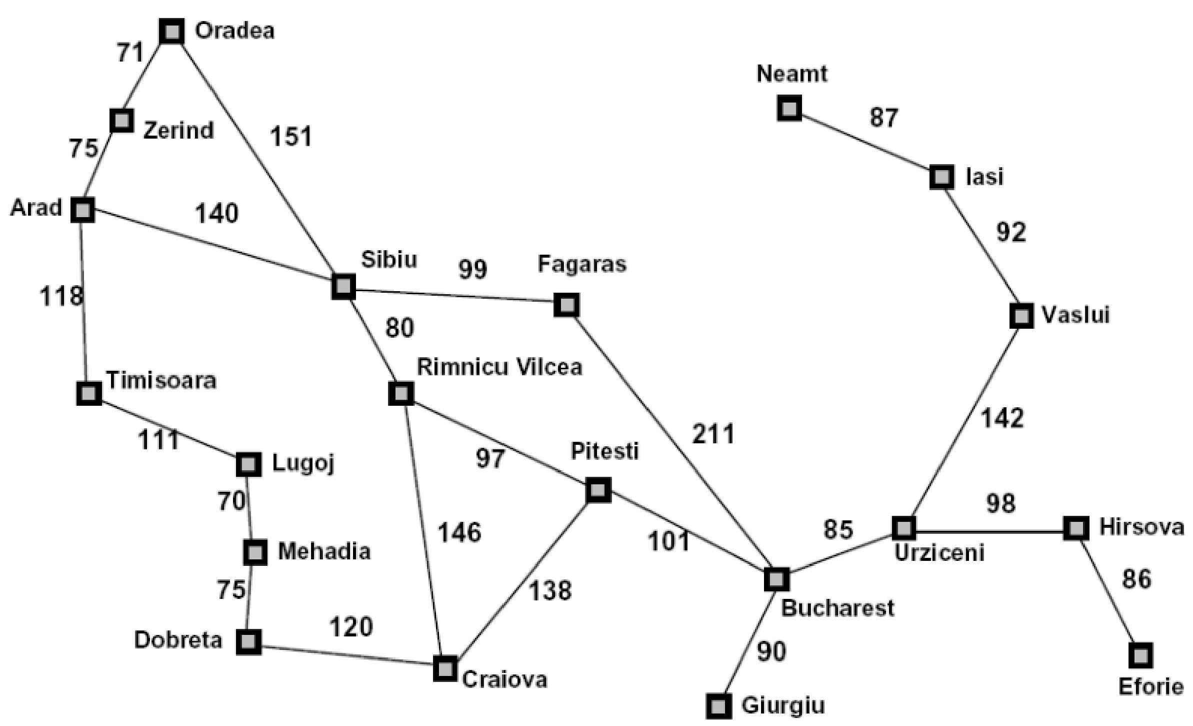
- A function that **estimates** how close a state is to a goal
- Designed for a **particular** search problem
- Examples: Manhattan distance, Euclidean distance







Example: Heuristic Function



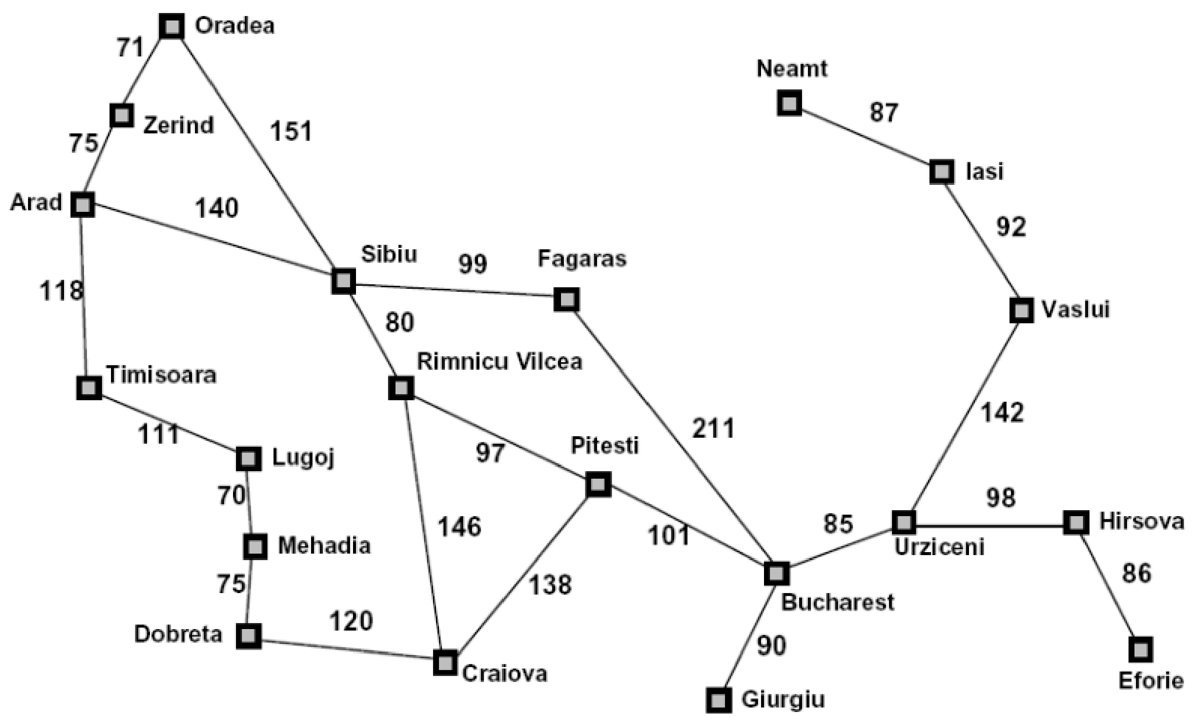
Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

h(x)

Greedy Search

Example: Greedy Heuristic Function



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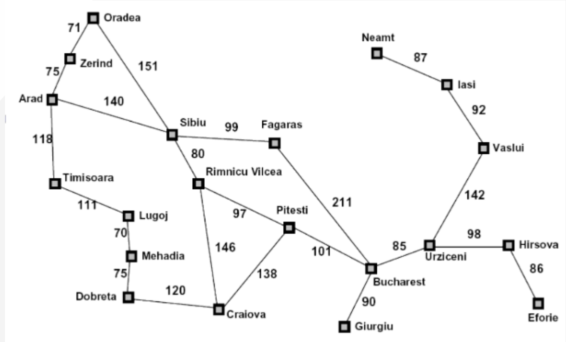
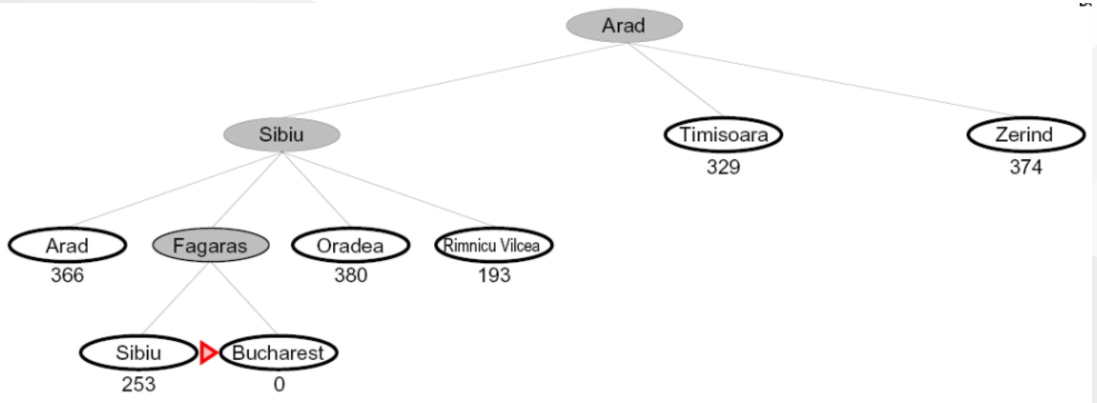
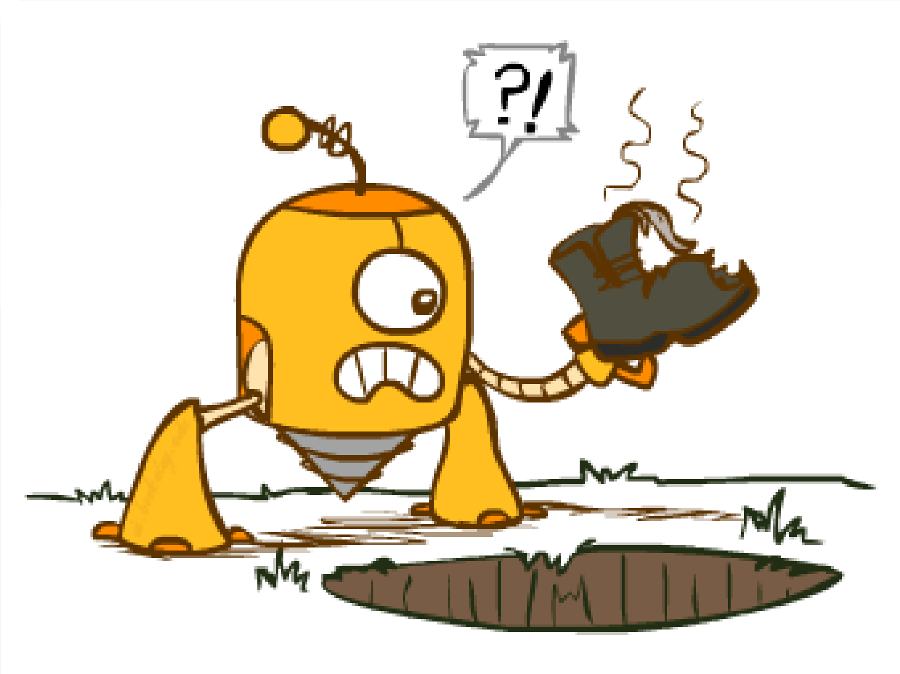
h(x)

Greedy Search



- Expand the node that seems closest…
- What can go wrong?





Greedy Search

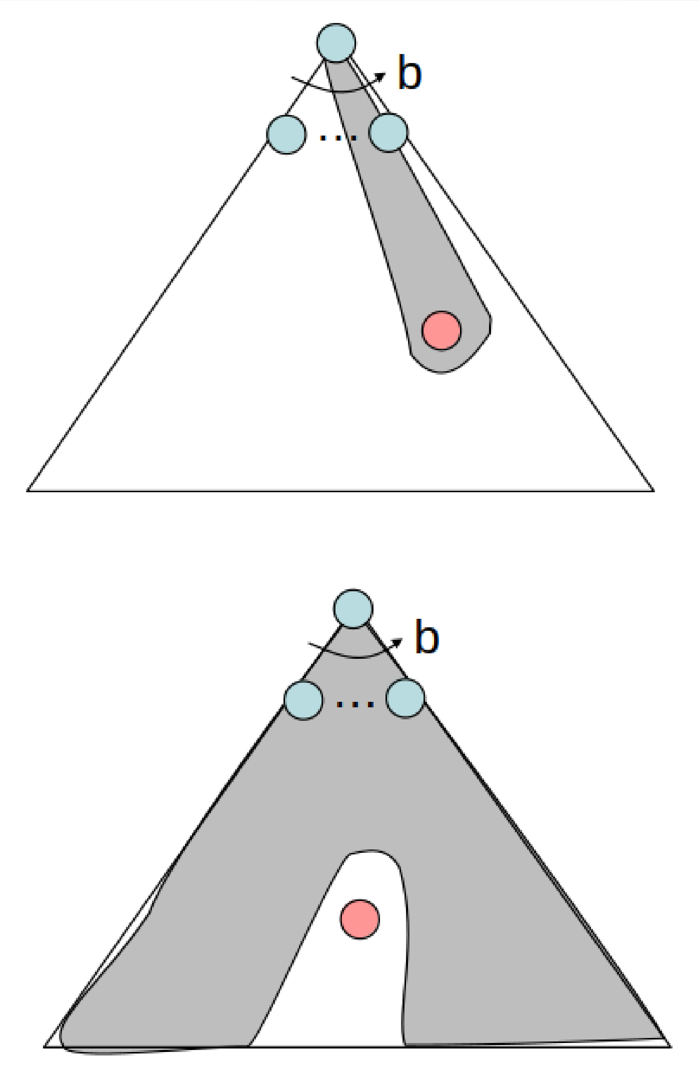
- Strategy: expand a node that you think is closest to a goal state
  - Heuristic: estimate of distance to nearest goal for each state

A common case

- Best-first takes you straight to the (wrong) goal

Worst-case

- behaves like a badly-guided DFS



Video of Demo Contours Greedy (Empty)

Video of Demo Contours Greedy (Pacman Small Maze)

A* Search

A* Search

UCS

Greedy

A*

Combining UCS and Greedy

- Uniform-cost orders by path cost, or backward cost g(n)
- Greedy orders by goal proximity, or forward cost h(n)
- A* Search orders by the sum: f(n) = g(n) + h(n)
- S
- a
- d
- b
- G
- h=5
- h=6
- h=2
- 1
- 8

When should A* terminate?

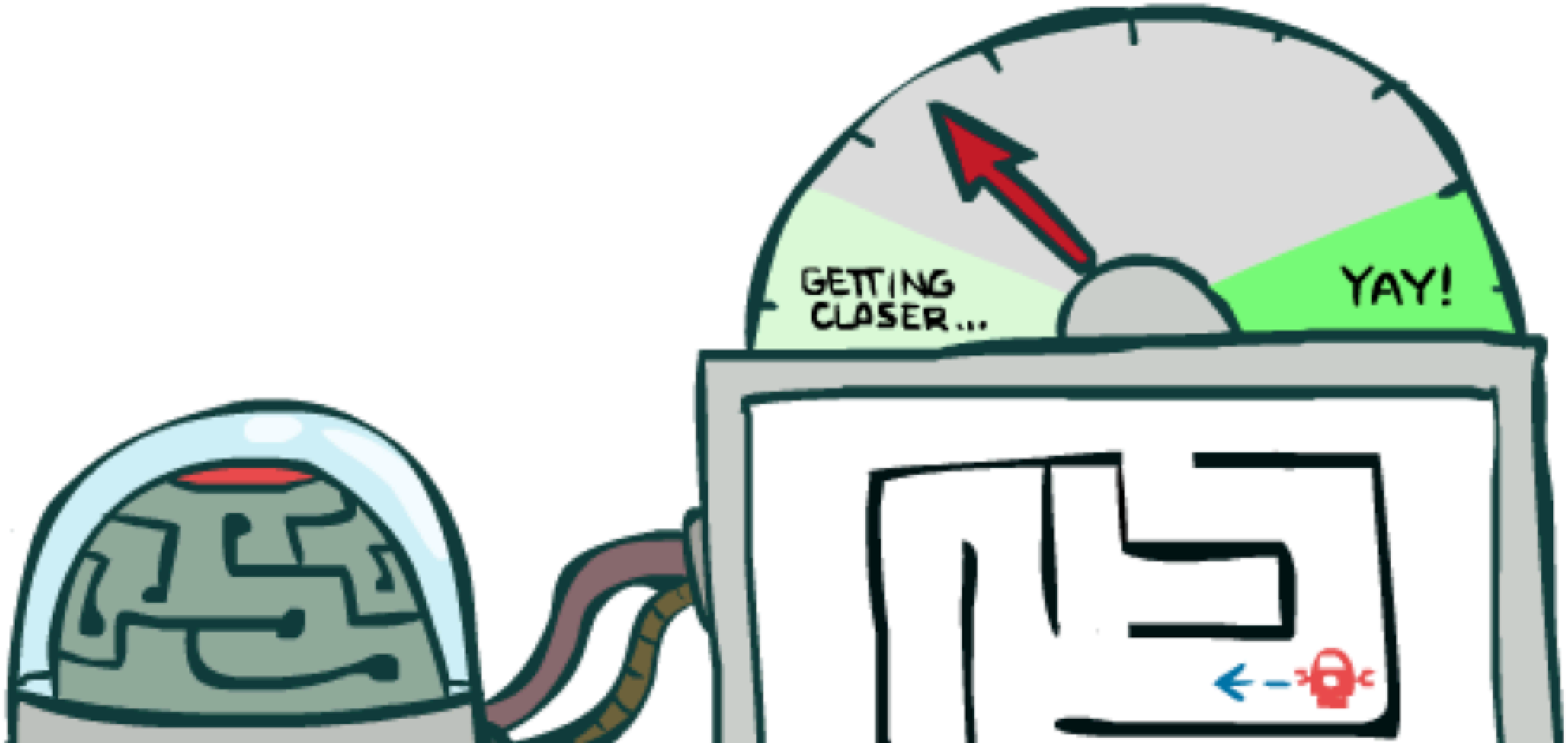
- Should we stop when we put a goal in the fringe?
- No: only stop when we pull a goal off the fringe
- S
- B
- A
- G
- 2
- 3
- 2
- 2
- h = 1
- h = 2
- h = 0

Is A* Optimal?

- What is wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!
- A
- G
- S
- 1
- 3
- h = 6
- h = 0
- 5
- h = 7

Converted shape

Admissible Heuristics



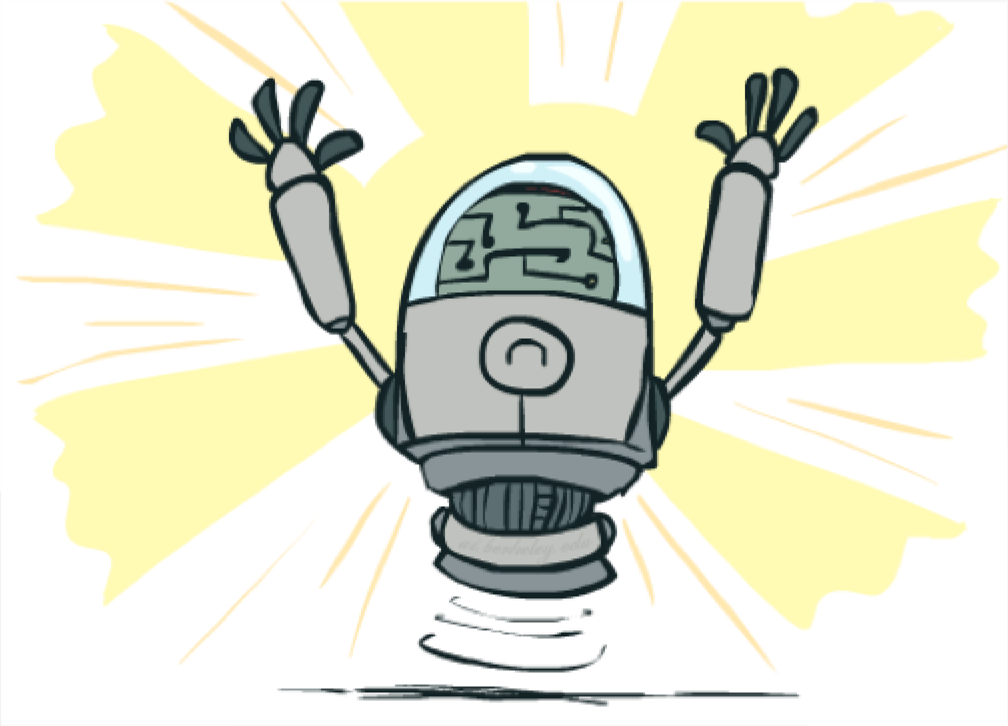
Admissible Heuristics

- A heuristic h is admissible (optimistic) if:
- where is the true cost to a nearest goal
- Examples:
- Defining admissible heuristics is the biggest effort in using A* in practice

$$0 \leq h(n) \leq h^*(n)$$

$$h^*(n)$$

Optimality of A* Tree Search



Optimality of A* Tree Search

- Assume:

Optimality of A* Tree Search: Blocking

- Proof:
- Imagine B is on the fringe
- Some ancestor n of A is on the fringe, too (maybe A!)
- Claim: n will be expanded before B
- f(n) is less or equal to f(A)
- Definition of f-cost
- Admissibility of h
- …
- h = 0 at a goal

$$f(n) = g(n) + h(n)$$

Converted shape

Optimality of A* Tree Search: Blocking

- Proof:
- Imagine B is on the fringe
- Some ancestor n of A is on the fringe, too (maybe A!)
- Claim: n will be expanded before B
- f(n) is less or equal to f(A)
- f(A) is less than f(B)
- B is suboptimal
- h = 0 at a goal
- …

$$f(A) < f(B)$$

Optimality of A* Tree Search: Blocking

- Proof:
- Imagine B is on the fringe
- Some ancestor n of A is on the fringe, too (maybe A!)
- Claim: n will be expanded before B
- f(n) is less or equal to f(A)
- f(A) is less than f(B)
- n expands before B
- All ancestors of A expand before B
- A expands before B
- A* search is optimal
- ...

$$f(n) \leq f(A) < f(B)$$

Properties of A*

Properties of A*

- ...
- b
- ...
- b
- Uniform-Cost
- A*

Converted shape

...

Converted shape

b

UCS vs A* Contours

- Uniform-cost expands equally in all “directions”
- A* expands mainly toward the goal, but does hedge its bets to ensure optimality
- Start
- Goal
- Start
- Goal

Converted shape

Start

Converted shape

Goal

Video of Demo Contours (Empty) -- UCS

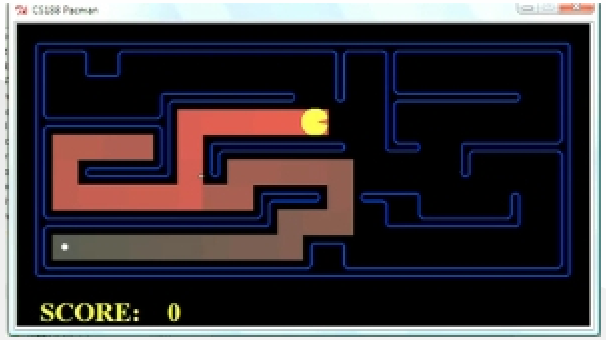
Video of Demo Contours (Empty) -- Greedy

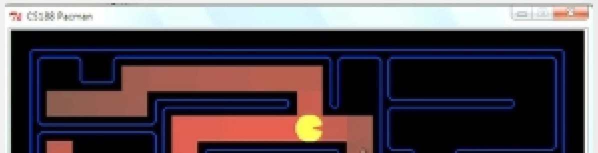
Video of Demo Contours (Empty) – A*

Video of Demo Contours (Pacman Small Maze) – A*

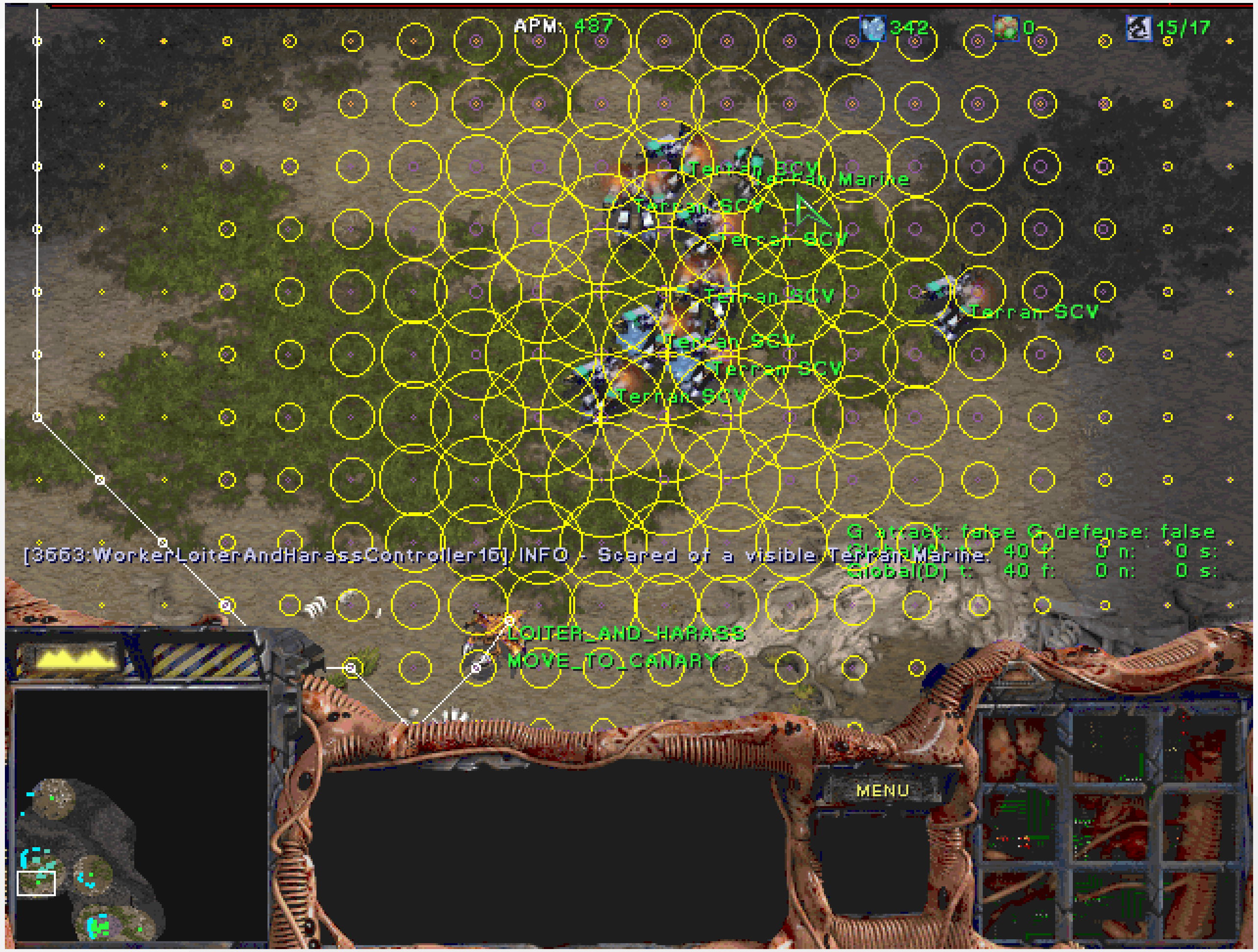
Comparison

- Greedy
- Uniform Cost
- A*





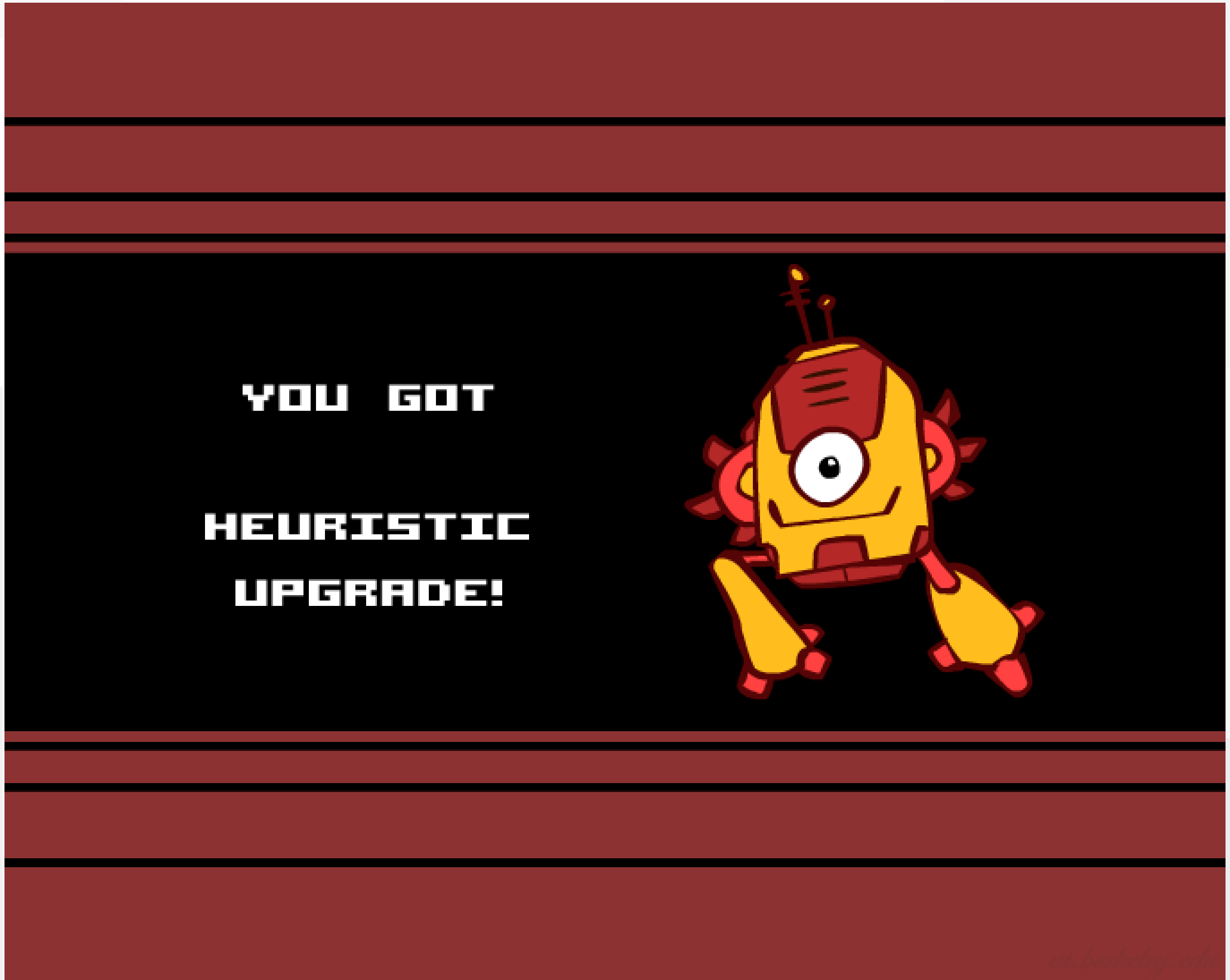
A* Applications



Video of Demo Pacman (Tiny Maze) – UCS / A*

Video of Demo Empty Water Shallow/Deep – Guess Algorithm

Creating Heuristics



Example: 8 Puzzle

- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?
- Start State
- Goal State
- Actions

Converted shape

Group of shapes

8 Puzzle I

- Heuristic: Number of tiles misplaced
- Why is it admissible?
- h(start) =
- This is a relaxed-problem heuristic
- 8
- Statistics from Andrew Moore

Converted shape

8

8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?
- Total Manhattan distance
- Why is it admissible?
- h(start) =
- 3 + 1 + 2 + … = 18

Converted shape

3 + 1 + 2 + … = 18

Converted shape

Start State Goal State

8 Puzzle III

- How about using the actual cost as a heuristic?
- Would it be admissible?
- Would we save on nodes expanded?
- What’s wrong with it?
- With A*: a trade-off between quality of estimate and work per node
- As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself

Semi-Lattice of Heuristics

Trivial Heuristics, Dominance

- Dominance: ha ≥ hc if
- Heuristics form a semi-lattice:
- Max of admissible heuristics is admissible
- Trivial heuristics
- Bottom of lattice is the zero heuristic (what does this give us?)
- Top of lattice is the exact heuristic

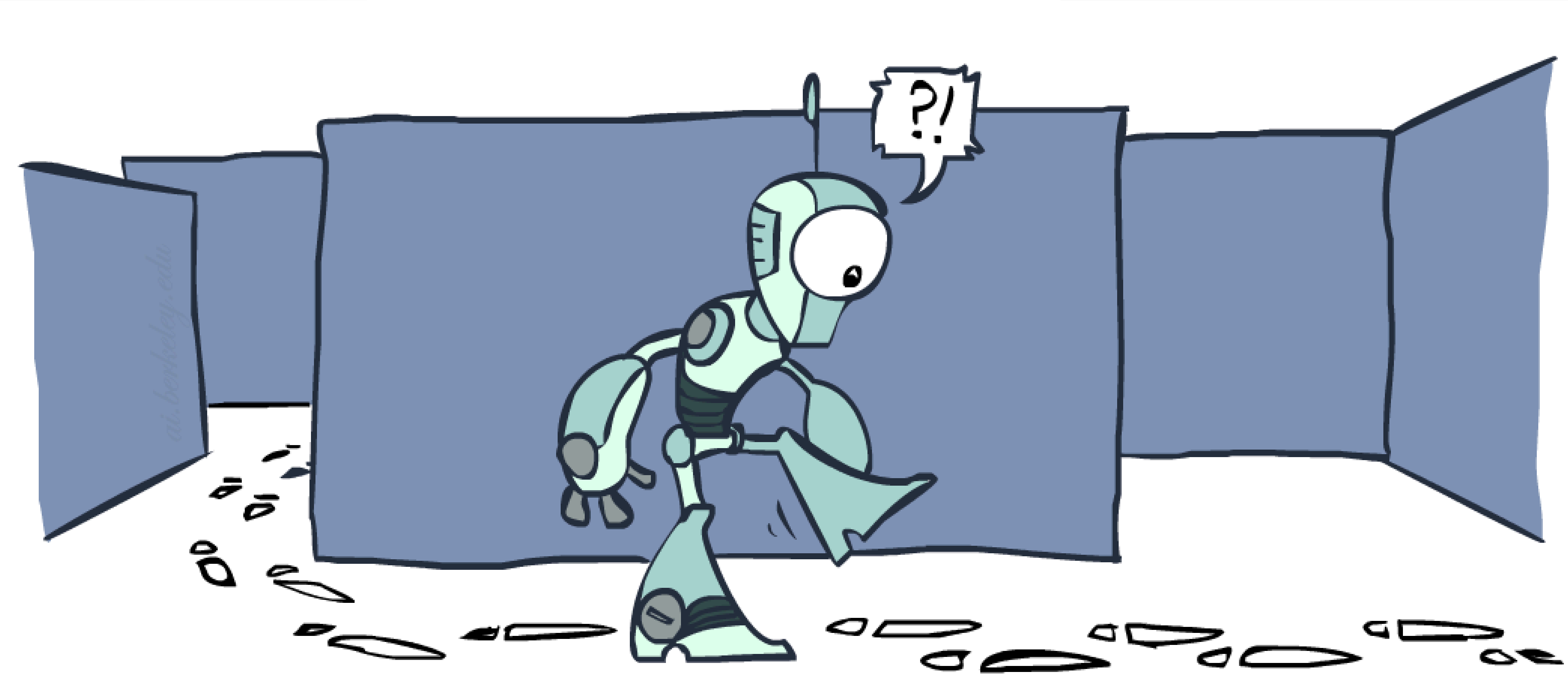
$$h(n) = max(h_a(n), h_b(n))$$

$$\forall n : h_a(n) \geq h_c(n)$$

Converted shape

Group of shapes

Graph Search



Tree Search: Extra Work!

BFS Graph Search Example

- we shouldn't bother expanding the circled nodes: WHY?

Converted shape

S a b d p a c q

Graph Search

- Idea: never expand a state twice
- How to implement:
- Tree search + set of expanded states (“closed set”)
- Expand the search tree node-by-node, but…
- Before expanding a node, check to make sure its state has never been expanded before
- If not new, skip it, if new add to closed set
- Important: store the closed set as a set, not a list
- Can graph search wreck completeness? Why/why not?
- How about optimality?

A* Graph Search Gone Wrong?

- S
- A
- B
- C
- G
- 1
- 1
- 1
- 2
- 3
- S (0+2)
- State space graph
- Search tree

Consistency of Heuristics

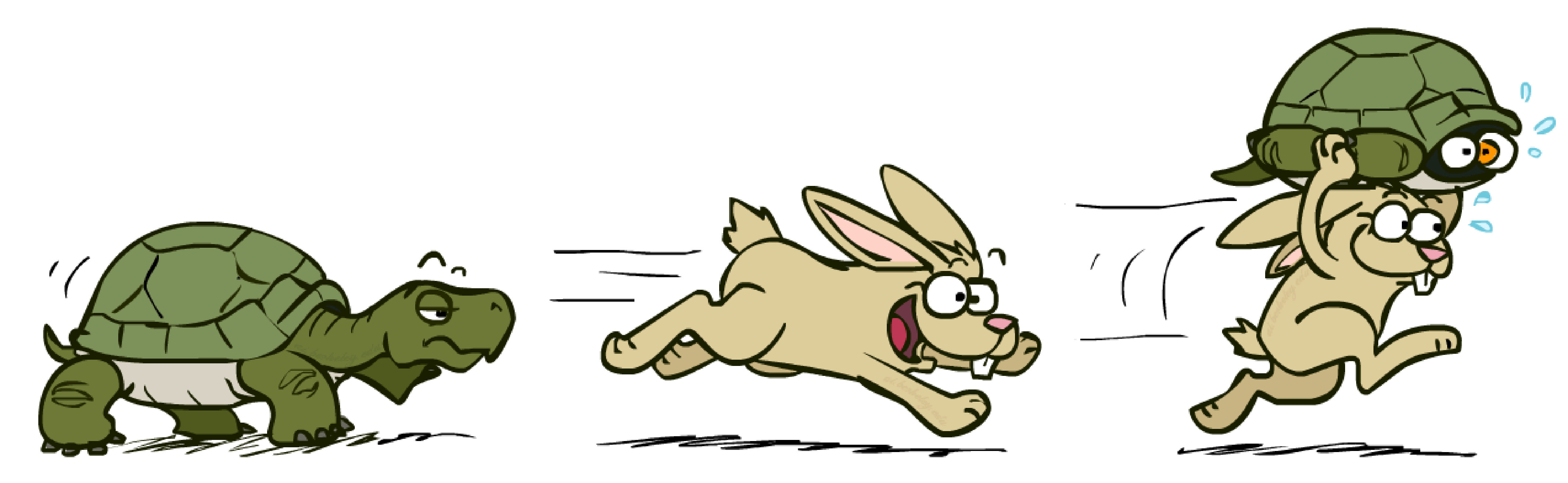
- Main idea: estimated heuristic costs ≤ actual costs
- Admissibility: heuristic cost ≤ actual cost to goal
- h(A) ≤ actual cost from A to G
- Consistency: heuristic “arc” cost ≤ actual cost for each arc
- h(A) – h(C) ≤ cost(A to C)
- Consequences of consistency:
- The f value along a path never decreases
- h(A) ≤ cost(A to C) + h(C)
- A* graph search is optimal
- 3
- A
- C
- G

Optimality

- Tree search:
- A* is optimal if heuristic is admissible
- UCS is a special case (h = 0)
- Graph search:
- A* optimal if heuristic is consistent
- UCS optimal (h = 0 is consistent)
- Consistency implies admissibility
- In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems

A*: Summary

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems



Tree Search Pseudo-Code

```
function TREE-SEARCH(problem, fringe) return a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        for child-node in EXPAND(STATE[node], problem) do
            fringe ← INSERT(child-node, fringe)
        end
    end
```

Graph Search Pseudo-Code

```
function GRAPH-SEARCH(problem, fringe) return a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
```