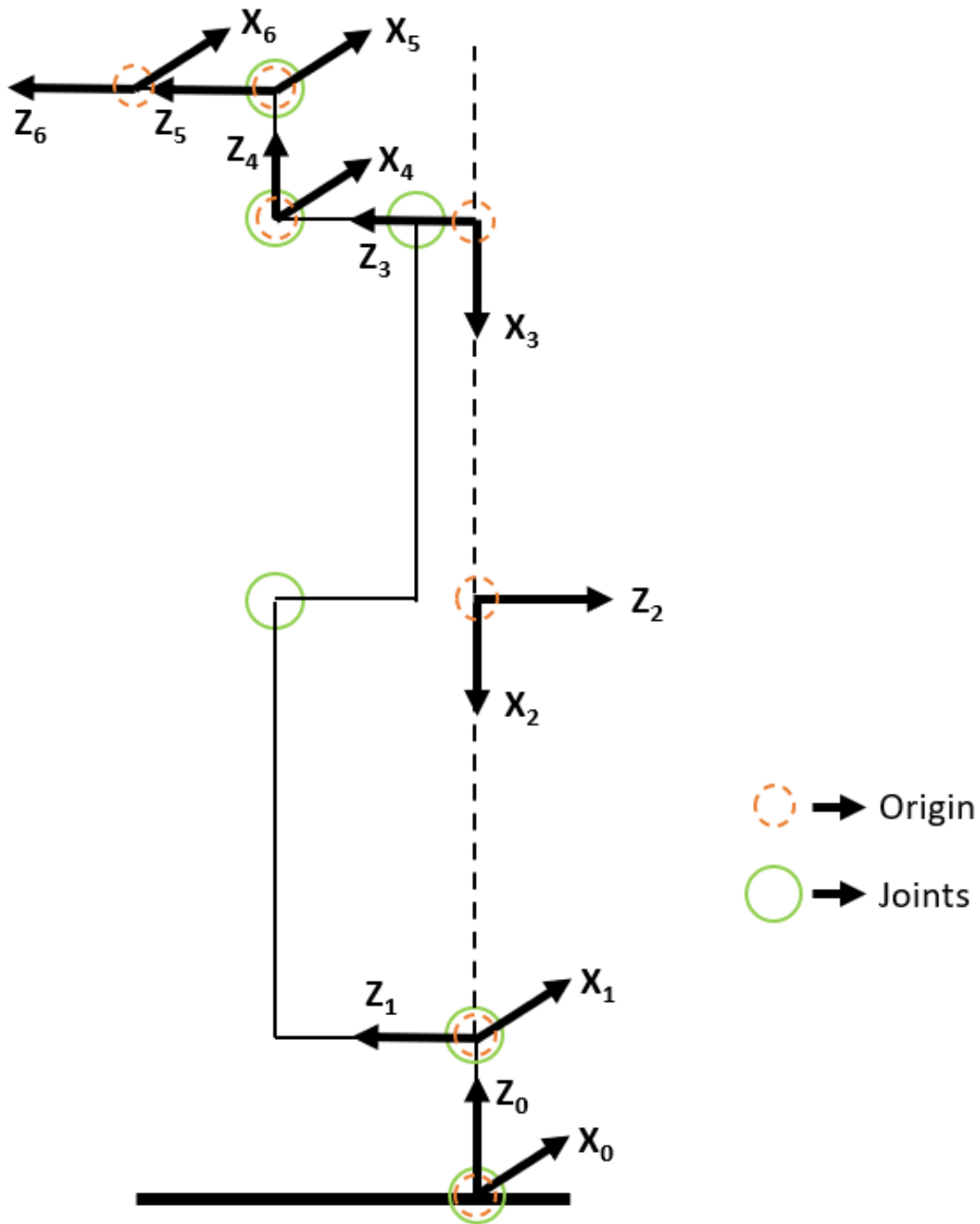


HOMEWORK 3

October 2023

1 Problem 1 - Position kinematics UR10

The DH coordinate frame for the robot is assigned as follows:



After assigning the DH coordinate frames for the robot, we construct the DH table as follows:

frame	a	α	d	θ
0 - 1	0	-90	128	θ_1
1 - 2	-612.7	180	0	$90 + \theta_2$
2 - 3	-571.6	-180	0	θ_3
3 - 4	0	90	163.9	$-90 + \theta_4$
4 - 5	0	-90	115.7	θ_5
5 - 6	0	0	92.2	θ_6

Once the DH table is constructed, we can proceed with the transformation matrix for each frame transformation.

$$T_1^0 = \begin{bmatrix} \cos \theta_1 & 0 & -\sin \theta_1 & 0 \\ \sin \theta_1 & 0 & \cos \theta_1 & 0 \\ 0 & -1 & 0 & 128 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} -\sin \theta_2 & \cos \theta_2 & 0 & 612.7 \sin \theta_2 \\ \cos \theta_2 & \sin \theta_2 & 0 & -612.7 \cos \theta_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} \cos \theta_3 & \sin \theta_3 & 0 & -571.6 \cos \theta_3 \\ \sin \theta_3 & -\cos \theta_3 & 0 & -571.6 \sin \theta_3 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^3 = \begin{bmatrix} \sin \theta_4 & 0 & -\cos \theta_4 & 0 \\ -\cos \theta_4 & 0 & -\sin \theta_4 & 0 \\ 0 & 1 & 0 & 163.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5^4 = \begin{bmatrix} \cos \theta_5 & 0 & -\sin \theta_5 & 0 \\ \sin \theta_5 & 0 & \cos \theta_5 & 0 \\ 0 & -1 & 0 & 115.7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6^5 = \begin{bmatrix} \cos \theta_6 & -\sin \theta_6 & 0 & 0 \\ \sin \theta_6 & \cos \theta_6 & 0 & 0 \\ 0 & 0 & 1 & 92.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The overall transformation matrix can be obtained by:

$$T_6^0 = T_1^0 * T_2^1 * T_3^2 * T_4^3 * T_5^4 * T_6^5$$

The robot position at 5 different configurations:

- 1) When $\theta = 0$ for all the frames.

2) When $\theta_6 = 90$ and rest of the angles are zero.

```

47 # -----
48 # #Robot at theta6 = 90 and rest of the angles are zero.
49
50 T_1 = Matrix([[1, 0, 0, 0], [0, 0, 1, 0], [0, -1, 0, 128], [0, 0, 0, 1]])
51 T_2 = Matrix([[0, 1, 0, 0], [1, 0, 0, -612.7], [0, 0, -1, 0], [0, 0, 0, 1]])
52 T_3 = Matrix([[1, 0, 0, -571.6], [0, -1, 0, 0], [0, 0, -1, 0], [0, 0, 0, 1]])
53 T_4 = Matrix([[0, 0, -1, 0], [-1, 0, 0, 0], [0, 1, 0, 163.9], [0, 0, 0, 1]])
54 T_5 = Matrix([[1, 0, 0, 0], [0, 0, 1, 0], [0, -1, 0, 115.7], [0, 0, 0, 1]])
55 T_6 = Matrix([[0, -1, 0, 0], [1, 0, 0, 0], [0, 0, 1, 92.2], [0, 0, 0, 1]])
56
57 T = T_1 * T_2 * T_3 * T_4 * T_5 * T_6
58 pprint(T)
59

```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL

```

uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$ /bin/python3 /home/uthappa/Documents/662/HW3/uthu_hw3_code.py
[[0 -1 0 0]
 [0 0 1 256.1]
 [-1 0 0 1428.0]
 [0 0 0 1]]
uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$

```

3) When $\theta_5 = 90$ and rest of the angles are zero.

```

60 # -----
61 # #Robot at theta5 = 90 and rest of the angles are zero.
62
63 T_1 = Matrix([[1, 0, 0, 0], [0, 0, 1, 0], [0, -1, 0, 128], [0, 0, 0, 1]])
64 T_2 = Matrix([[0, 1, 0, 0], [1, 0, 0, -612.7], [0, 0, -1, 0], [0, 0, 0, 1]])
65 T_3 = Matrix([[1, 0, 0, -571.6], [0, -1, 0, 0], [0, 0, -1, 0], [0, 0, 0, 1]])
66 T_4 = Matrix([[0, 0, -1, 0], [-1, 0, 0, 0], [0, 1, 0, 163.9], [0, 0, 0, 1]])
67 T_5 = Matrix([[0, 0, -1, 0], [1, 0, 0, 0], [0, -1, 0, 115.7], [0, 0, 0, 1]])
68 T_6 = Matrix([[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 92.2], [0, 0, 0, 1]])
69
70 T = T_1 * T_2 * T_3 * T_4 * T_5 * T_6
71 pprint(T)
72

```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL

```

uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$ /bin/python3 /home/uthappa/Documents/662/HW3/uthu_hw3_code.py
[[0 0 -1 -92.2]
 [1 0 0 163.9]
 [0 -1 0 1428.0]
 [0 0 0 1]]
uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$

```

4) When $\theta_5 = -90$ and rest of the angles are zero.

```

73 # -----
74 # #Robot at theta5 = -90 and rest of the angles are zero.
75
76 T_1 = Matrix([[1, 0, 0, 0], [0, 0, 1, 0], [0, -1, 0, 128], [0, 0, 0, 1]])
77 T_2 = Matrix([[0, 1, 0, 0], [1, 0, 0, -612.7], [0, 0, -1, 0], [0, 0, 0, 1]])
78 T_3 = Matrix([[1, 0, 0, -571.6], [0, -1, 0, 0], [0, 0, -1, 0], [0, 0, 0, 1]])
79 T_4 = Matrix([[0, 0, -1, 0], [-1, 0, 0, 0], [0, 1, 0, 163.9], [0, 0, 0, 1]])
80 T_5 = Matrix([[0, 0, 1, 0], [-1, 0, 0, 0], [0, -1, 0, 115.7], [0, 0, 0, 1]])
81 T_6 = Matrix([[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 92.2], [0, 0, 0, 1]])
82
83
84 T = T_1 * T_2 * T_3 * T_4 * T_5 * T_6
85 pprint(T)
86

```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL

```

uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$ /bin/python3 /home/uthappa/Documents/662/HW3/uthu_hw3_code.py
[[0 0 1 92.2]
 [-1 0 0 163.9]
 [0 -1 0 1428.0]
 [0 0 0 1]]
uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$

```

5) When $\theta_4 = 90$ and rest of the angles are zero.

```

87 # -----
88 # #Robot at theta4 = 90 and rest of the angles are zero.
89
90 T_1 = Matrix([[1, 0, 0, 0], [0, 0, 1, 0], [0, -1, 0, 128], [0, 0, 0, 1]])
91 T_2 = Matrix([[0, 1, 0, 0], [1, 0, 0, -612.7], [0, 0, -1, 0], [0, 0, 0, 1]])
92 T_3 = Matrix([[1, 0, 0, -571.6], [0, -1, 0, 0], [0, 0, -1, 0], [0, 0, 0, 1]])
93 T_4 = Matrix([[1, 0, 0, 0], [0, 0, -1, 0], [0, 1, 0, 163.9], [0, 0, 0, 1]])
94 T_5 = Matrix([[1, 0, 0, 0], [0, 0, 1, 0], [0, -1, 0, 115.7], [0, 0, 0, 1]])
95 T_6 = Matrix([[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 92.2], [0, 0, 0, 1]])
96
97
98 T = T_1 * T_2 * T_3 * T_4 * T_5 * T_6
99 pprint(T)

```

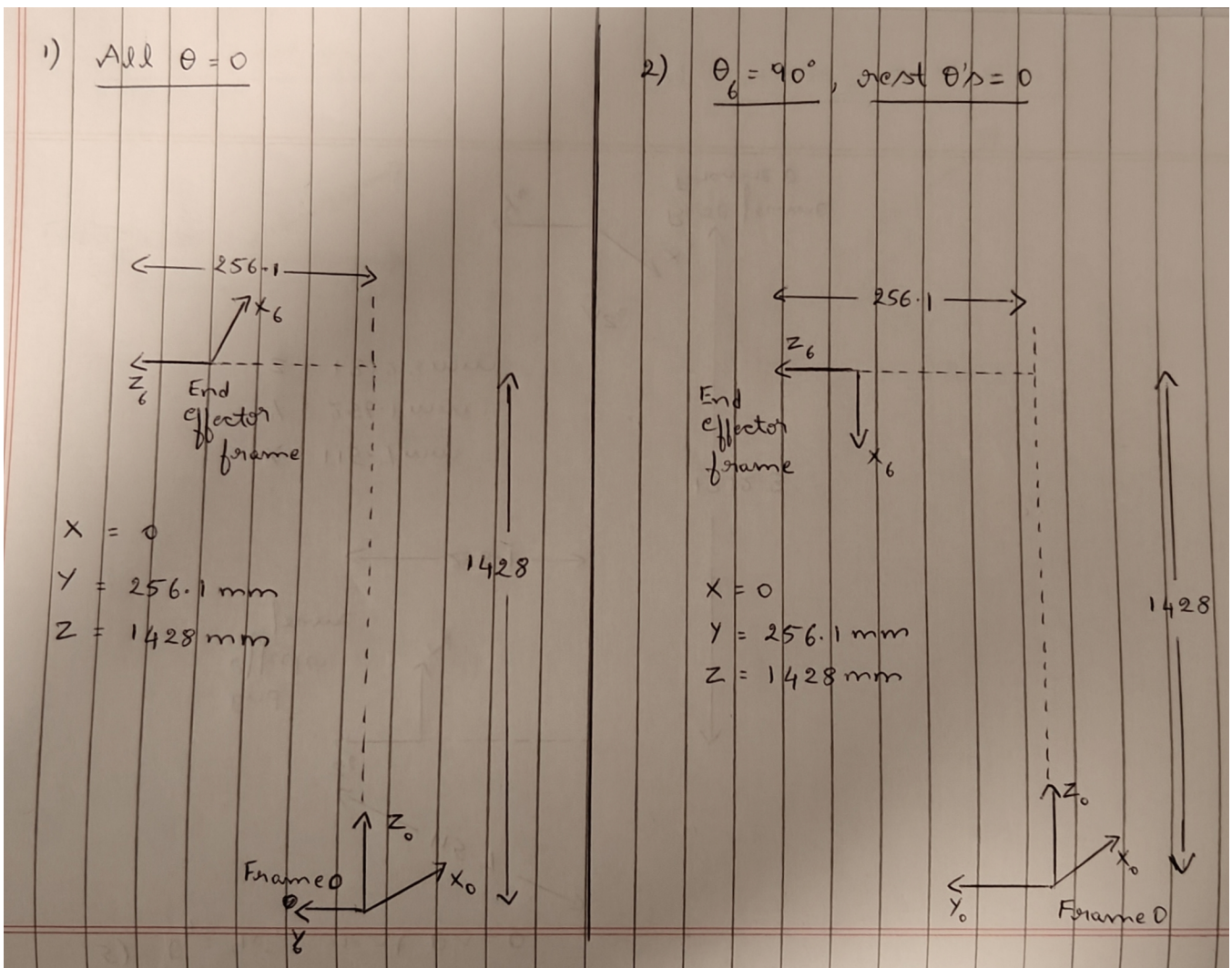
PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL

```

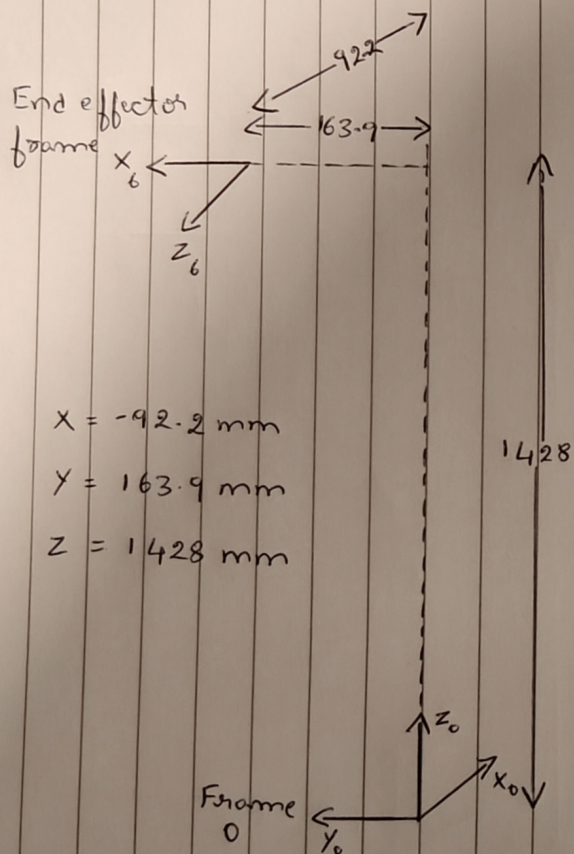
uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~/Documents/662/HW3/uthu_hw3_code.py
[0 -1 0 115.7]
[0 0 1 256.1]
[-1 0 0 1312.3]
[0 0 0 1]
uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~/Documents/662/HW3/uthu_hw3_code.py

```

The above results can be validated geometrically as follows:

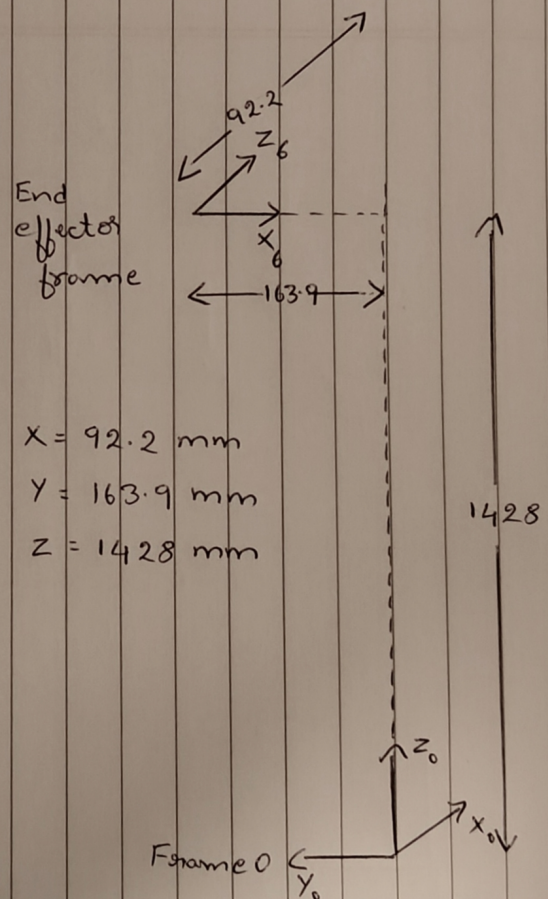


3) $\theta_5 = 90^\circ$, rest $\theta_i = 0$



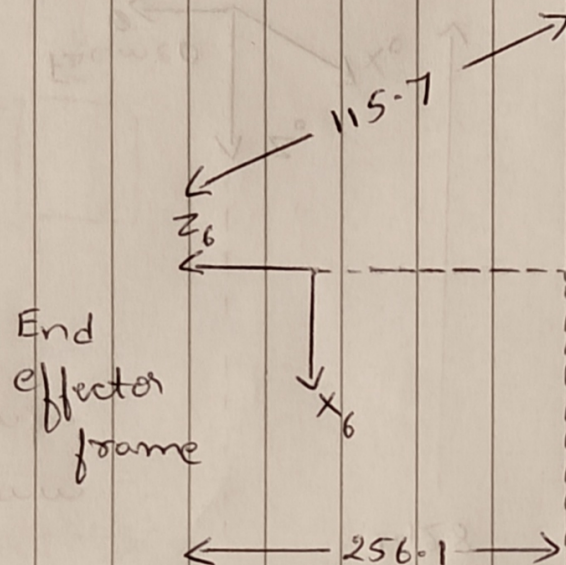
$x = -92.2 \text{ mm}$
 $y = 163.9 \text{ mm}$
 $z = 1428 \text{ mm}$

4) $\theta_5 = -90^\circ$, rest $\theta_i = 0$



$x = 92.2 \text{ mm}$
 $y = 163.9 \text{ mm}$
 $z = 1428 \text{ mm}$

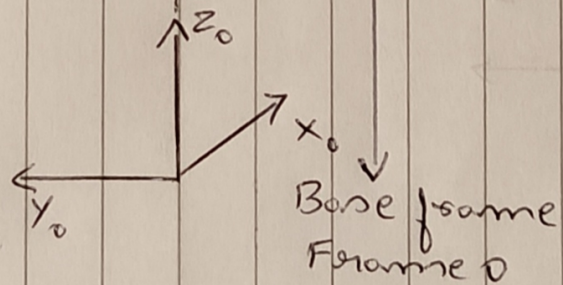
5) $\theta_4 = 90^\circ$, rest θ 's = 0



$$X = 115.7 \text{ mm}$$

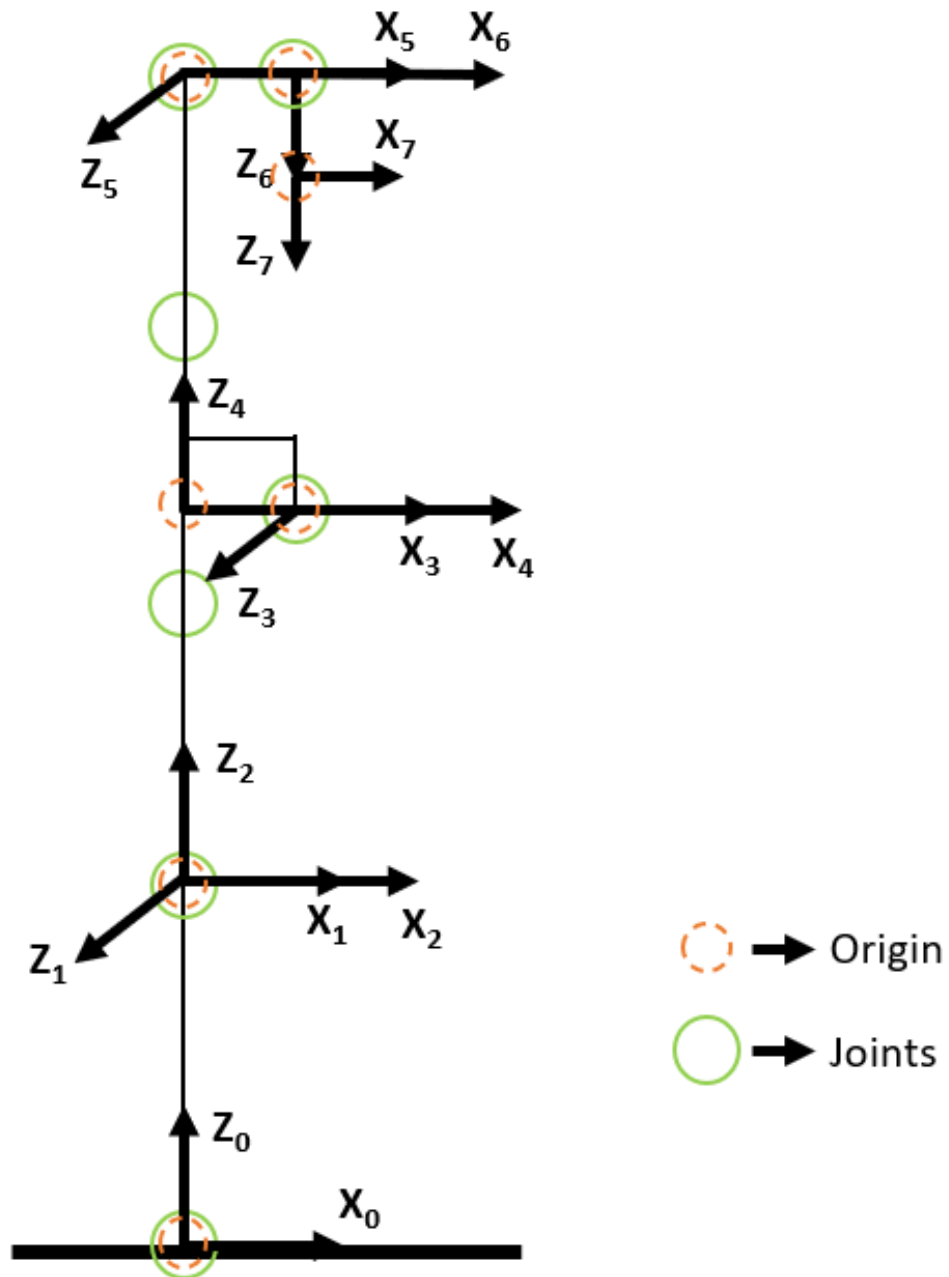
$$Y = 256.1 \text{ mm}$$

$$Z = 1312.3 \text{ mm}$$



2 Problem 2 - Position kinematics KUKA

The DH coordinate frame for the robot is assigned as follows:



After assigning the DH coordinate frames for the robot, we construct the DH table as follows:

frame	a	α	d	θ
0 - 1	0	90	d_1	θ_1
1 - 2	0	-90	0	θ_2
2 - 3	a_3	90	d_3	θ_3
3 - 4	$-a_3$	-90	0	θ_4
4 - 5	0	90	d_5	θ_5
5 - 6	a_3	90	0	θ_6
6 - 7	0	0	d_7	θ_7