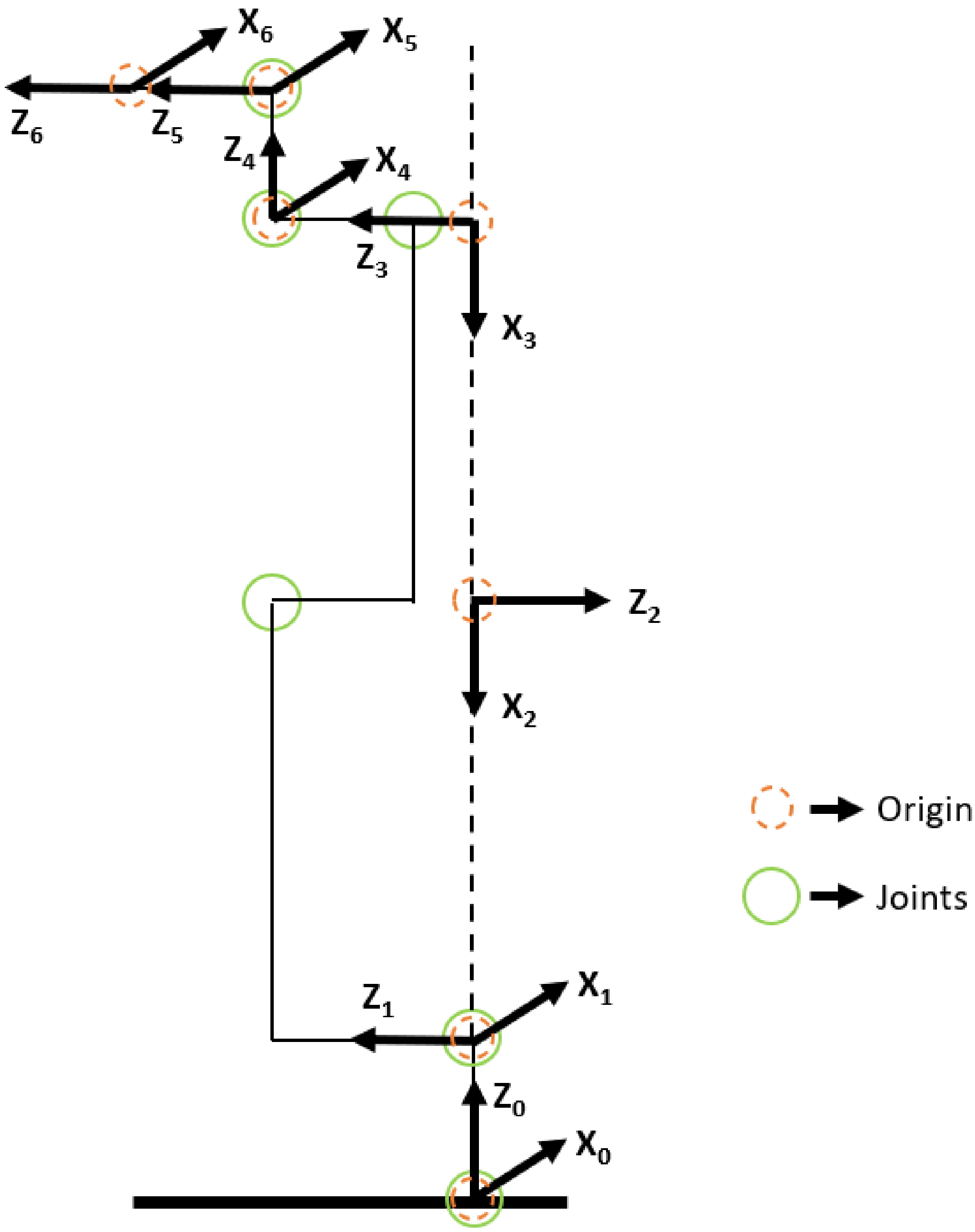
HOMEWORK 4

November 2023

1 Trajectory Generation - UR10

The DH coordinate frame for the robot is assigned as follows:

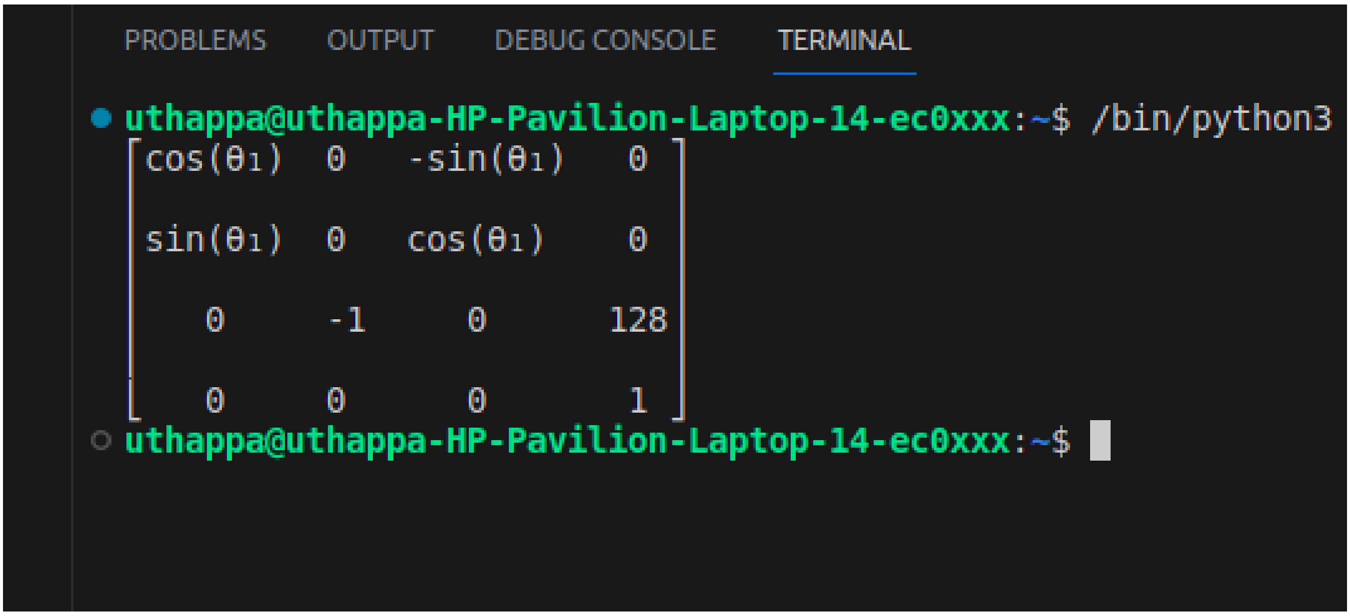


After assigning the DH coordinate frames for the robot, we construct the DH table as follows:

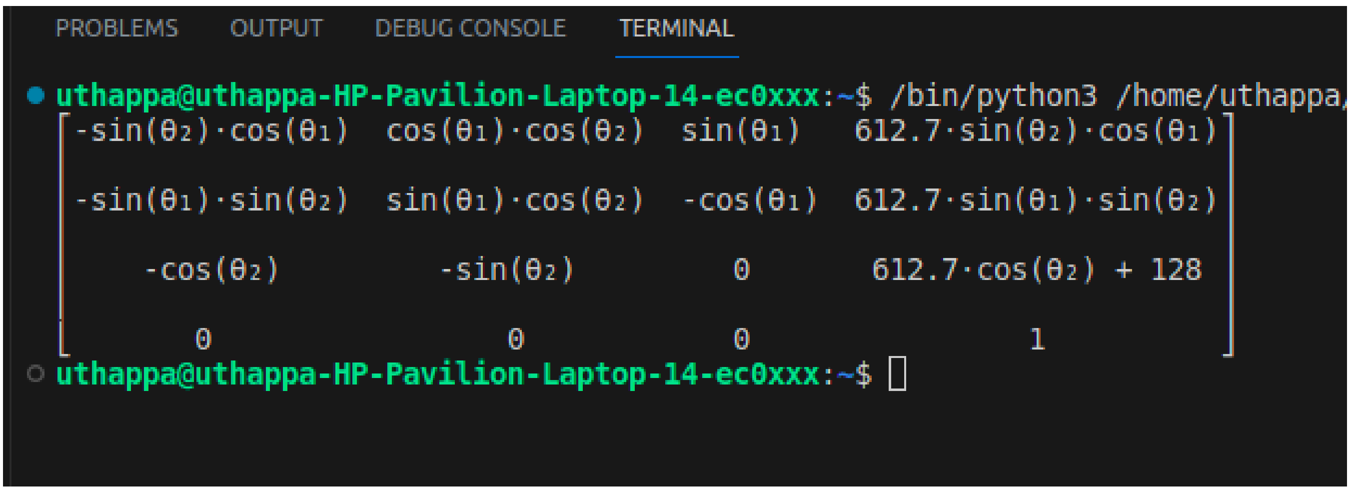
frame	a	α	d	θ
0 - 1	0	-90	128	θ_1
1 - 2	-612.7	180	0	90 + θ_2
2 - 3	-571.6	-180	0	θ_3
3 - 4	0	90	163.9	-90 + θ_4
4 - 5	0	-90	115.7	θ_5
5 - 6	0	0	192.2	θ_6

Once the DH parameters are set, the transformation matrices for each frame with respect to the base frame is noted down as follows:

T_1^0 = Transformation of frame 1 with respect to the base frame.



T_2^0 = Transformation of frame 2 with respect to the base frame.



T_3^0 = Transformation of frame 3 with respect to the base frame.

```
PROBLEMS    OUTPUT    DEBUG CONSOLE    TERMINAL                                                    Python

uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$ /bin/python3 /home/uthappa/Documents/662/HW4/uthu_hw3_code.py
⎡-sin(θ₂)⋅cos(θ₁)⋅cos(θ₃) + sin(θ₃)⋅cos(θ₁)⋅cos(θ₂)  -sin(θ₂)⋅sin(θ₃)⋅cos(θ₁) - cos(θ₁)⋅cos(θ₂)⋅cos(θ₃)  -sin(θ₁)  571.6⋅sin(θ₂)⋅cos(θ₁)⋅cos(θ₃)
 + 612.7⋅sin(θ₂)⋅cos(θ₁) - 571.6⋅sin(θ₃)⋅cos(θ₁)⋅cos(θ₂)⎤
⎢                                                    ⎥
⎢-sin(θ₁)⋅sin(θ₂)⋅cos(θ₃) + sin(θ₁)⋅sin(θ₃)⋅cos(θ₂)  -sin(θ₁)⋅sin(θ₂)⋅sin(θ₃) - sin(θ₁)⋅cos(θ₂)⋅cos(θ₃)  cos(θ₁)   571.6⋅sin(θ₁)⋅sin(θ₂)⋅cos(θ₃)
 + 612.7⋅sin(θ₁)⋅sin(θ₂) - 571.6⋅sin(θ₁)⋅sin(θ₃)⋅cos(θ₂)⎥
⎢                                                    ⎥
⎢       -sin(θ₂)⋅sin(θ₃) - cos(θ₂)⋅cos(θ₃)                sin(θ₂)⋅cos(θ₃) - sin(θ₃)⋅cos(θ₂)            0             571.6⋅sin(θ₂)⋅sin(θ₃
) + 571.6⋅cos(θ₂)⋅cos(θ₃) + 612.7⋅cos(θ₂) + 128        ⎥
⎢                                                    ⎥
⎣                    0                                              0                                  0
               1                                     ⎦
uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$ 
```

T_4^0 = Transformation of frame 4 with respect to the base frame.

```
PROBLEMS    OUTPUT    DEBUG CONSOLE    TERMINAL                                                    Python
uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$ /bin/python3 /home/uthappa/Documents/662/HW4/uthu_hw3_code.py
⎡-(-sin(θ₂)⋅sin(θ₃)⋅cos(θ₁) - cos(θ₁)⋅cos(θ₂)⋅cos(θ₃))⋅cos(θ₄) + (-sin(θ₂)⋅cos(θ₁)⋅cos(θ₃) + sin(θ₃)⋅cos(θ₁)⋅cos(θ₂))⋅sin(θ₄)  -sin(θ₁)  -(-si
⎢
⎢-(-sin(θ₁)⋅sin(θ₂)⋅sin(θ₃) - sin(θ₁)⋅cos(θ₂)⋅cos(θ₃))⋅cos(θ₄) + (-sin(θ₁)⋅sin(θ₂)⋅cos(θ₃) + sin(θ₁)⋅sin(θ₃)⋅cos(θ₂))⋅sin(θ₄)  cos(θ₁)   -(-si
⎢
⎢                 (-sin(θ₂)⋅sin(θ₃) - cos(θ₂)⋅cos(θ₃))⋅sin(θ₄) - (sin(θ₂)⋅cos(θ₃) - sin(θ₃)⋅cos(θ₂))⋅cos(θ₄)                    0
⎢
⎣                                                            0                                                              0

n(θ₂)⋅sin(θ₃)⋅cos(θ₁) - cos(θ₁)⋅cos(θ₂)⋅cos(θ₃))⋅sin(θ₄) - (-sin(θ₂)⋅cos(θ₁)⋅cos(θ₃) + sin(θ₃)⋅cos(θ₁)⋅cos(θ₂))⋅cos(θ₄)  -163.9⋅sin(θ₁) + 571.

n(θ₁)⋅sin(θ₂)⋅sin(θ₃) - sin(θ₁)⋅cos(θ₂)⋅cos(θ₃))⋅sin(θ₄) - (-sin(θ₁)⋅sin(θ₂)⋅cos(θ₃) + sin(θ₁)⋅sin(θ₃)⋅cos(θ₂))⋅cos(θ₄)  571.6⋅sin(θ₁)⋅sin(θ₂)

          -(-sin(θ₂)⋅sin(θ₃) - cos(θ₂)⋅cos(θ₃))⋅cos(θ₄) - (sin(θ₂)⋅cos(θ₃) - sin(θ₃)⋅cos(θ₂))⋅sin(θ₄)                         571.

                                                        0

6⋅sin(θ₂)⋅cos(θ₁)⋅cos(θ₃) + 612.7⋅sin(θ₂)⋅cos(θ₁) - 571.6⋅sin(θ₃)⋅cos(θ₁)⋅cos(θ₂)⎤
                                                                                 ⎥
⋅cos(θ₃) + 612.7⋅sin(θ₁)⋅sin(θ₂) - 571.6⋅sin(θ₁)⋅sin(θ₃)⋅cos(θ₂) + 163.9⋅cos(θ₁) ⎥
                                                                                 ⎥
6⋅sin(θ₂)⋅sin(θ₃) + 571.6⋅cos(θ₂)⋅cos(θ₃) + 612.7⋅cos(θ₂) + 128                  ⎥
                                                                                 ⎥
                            1                                                    ⎦
uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$ 
```

T_5^0 = Transformation of frame 5 with respect to the base frame.

```
PROBLEMS    OUTPUT    DEBUG CONSOLE    TERMINAL                                                        Python
θ₁)·sin(θ₅)  (-sin(θ₂)·sin(θ₃)·cos(θ₁) - cos(θ₁)·cos(θ₂)·cos(θ₃))·sin(θ₄) + (-sin(θ₂)·cos(θ₁)·cos(θ₃) + sin(θ₃)·cos(θ₁)·cos(θ₂))·cos(θ₄)  -(-(
θ₅)·cos(θ₁)  (-sin(θ₁)·sin(θ₂)·sin(θ₃) - sin(θ₁)·cos(θ₂)·cos(θ₃))·sin(θ₄) + (-sin(θ₁)·sin(θ₂)·cos(θ₃) + sin(θ₁)·sin(θ₃)·cos(θ₂))·cos(θ₄)  -(-(
                         (-sin(θ₂)·sin(θ₃) - cos(θ₂)·cos(θ₃))·cos(θ₄) + (sin(θ₂)·cos(θ₃) - sin(θ₃)·cos(θ₂))·sin(θ₄)
                                                                         0

-sin(θ₂)·sin(θ₃)·cos(θ₁) - cos(θ₁)·cos(θ₂)·cos(θ₃))·cos(θ₄) + (-sin(θ₂)·cos(θ₁)·cos(θ₃) + sin(θ₃)·cos(θ₁)·cos(θ₂))·sin(θ₄))·sin(θ₅) - sin(θ₁)·
-sin(θ₁)·sin(θ₂)·sin(θ₃) - sin(θ₁)·cos(θ₂)·cos(θ₃))·cos(θ₄) + (-sin(θ₁)·sin(θ₂)·cos(θ₃) + sin(θ₁)·sin(θ₃)·cos(θ₂))·sin(θ₄))·sin(θ₅) + cos(θ₁)·
                  -((-sin(θ₂)·sin(θ₃) - cos(θ₂)·cos(θ₃))·sin(θ₄) - (sin(θ₂)·cos(θ₃) - sin(θ₃)·cos(θ₂))·cos(θ₄))·sin(θ₅)
                                                                       0

cos(θ₅)  -115.7·(-sin(θ₂)·sin(θ₃)·cos(θ₁) - cos(θ₁)·cos(θ₂)·cos(θ₃))·sin(θ₄) - 115.7·(-sin(θ₂)·cos(θ₁)·cos(θ₃) + sin(θ₃)·cos(θ₁)·cos(θ₂))·cos(
cos(θ₅)  -115.7·(-sin(θ₁)·sin(θ₂)·sin(θ₃) - sin(θ₁)·cos(θ₂)·cos(θ₃))·sin(θ₄) - 115.7·(-sin(θ₁)·sin(θ₂)·cos(θ₃) + sin(θ₁)·sin(θ₃)·cos(θ₂))·cos(
                                   -115.7·(-sin(θ₂)·sin(θ₃) - cos(θ₂)·cos(θ₃))·cos(θ₄) - 115.7·(sin(θ₂)·cos(θ₃) - sin(θ₃)·cos(θ₂))·sin(
                                                                                                                    1

θ₄) - 163.9·sin(θ₁) + 571.6·sin(θ₂)·cos(θ₁)·cos(θ₃) + 612.7·sin(θ₂)·cos(θ₁) - 571.6·sin(θ₃)·cos(θ₁)·cos(θ₂)]
θ₄) + 571.6·sin(θ₁)·sin(θ₂)·cos(θ₃) + 612.7·sin(θ₁)·sin(θ₂) - 571.6·sin(θ₁)·sin(θ₃)·cos(θ₂) + 163.9·cos(θ₁)|
θ₄) + 571.6·sin(θ₂)·sin(θ₃) + 571.6·cos(θ₂)·cos(θ₃) + 612.7·cos(θ₂) + 128                                   |
```

T_6^0 = Transformation of frame 6 with respect to the base frame.

```
PROBLEMS    OUTPUT    DEBUG CONSOLE    TERMINAL                                                        Python
• uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$ /bin/python3 /home/uthappa/Documents/662/HW4/uthu_hw3_code.py
⎡((-(-sin(θ₂)·sin(θ₃)·cos(θ₁) - cos(θ₁)·cos(θ₂)·cos(θ₃))·cos(θ₄) + (-sin(θ₂)·cos(θ₁)·cos(θ₃) + sin(θ₃)·cos(θ₁)·cos(θ₂))·sin(θ₄))·cos(θ₅) - sin(θ₁)·sin(θ₅))·cos(θ₆) + ((-sin(θ₂)·sin(θ₃)·cos(θ₁) - cos(θ₁)·cos(θ₂)·cos(θ₃))
⎢((-(-sin(θ₁)·sin(θ₂)·sin(θ₃) - sin(θ₁)·cos(θ₂)·cos(θ₃))·cos(θ₄) + (-sin(θ₁)·sin(θ₂)·cos(θ₃) + sin(θ₁)·sin(θ₃)·cos(θ₂))·sin(θ₄))·cos(θ₅) + sin(θ₅)·cos(θ₁))·cos(θ₆) + ((-sin(θ₁)·sin(θ₂)·sin(θ₃) - sin(θ₁)·cos(θ₂)·cos(θ₃))
⎢                                    ((-sin(θ₂)·sin(θ₃) - cos(θ₂)·cos(θ₃))·sin(θ₄) - (sin(θ₂)·cos(θ₃) - sin(θ₃)·cos(θ₂))·cos(θ₄))·cos(θ₅)·cos(θ₆) + ((-sin(θ₂)·sin(θ₃) - cos(θ₂)·cos(θ₃))·cos(θ₄) + (sin(θ₂)·cos(θ₃)
⎣                                                                                                            0

·sin(θ₄) + (-sin(θ₂)·cos(θ₁)·cos(θ₃) + sin(θ₃)·cos(θ₁)·cos(θ₂))·cos(θ₄))·sin(θ₆)  -((-(-sin(θ₂)·sin(θ₃)·cos(θ₁) - cos(θ₁)·cos(θ₂)·cos(θ₃))·cos(θ₄) + (-sin(θ₂)·cos(θ₁)·cos(θ₃) + sin(θ₃)·cos(θ₁)·cos(θ₂))·sin(θ₄))·cos(θ₅)
·sin(θ₄) + (-sin(θ₁)·sin(θ₂)·cos(θ₃) + sin(θ₁)·sin(θ₃)·cos(θ₂))·cos(θ₄))·sin(θ₆)  -((-(-sin(θ₁)·sin(θ₂)·sin(θ₃) - sin(θ₁)·cos(θ₂)·cos(θ₃))·cos(θ₄) + (-sin(θ₁)·sin(θ₂)·cos(θ₃) + sin(θ₁)·sin(θ₃)·cos(θ₂))·sin(θ₄))·cos(θ₅)
 - sin(θ₃)·cos(θ₂))·sin(θ₄))·sin(θ₆)                                                        -((-sin(θ₂)·sin(θ₃) - cos(θ₂)·cos(θ₃))·sin(θ₄) - (sin(θ₂)·cos(θ₃) - sin(θ₃)·cos(θ₂))·cos(θ₄))·

 - sin(θ₁)·sin(θ₅))·sin(θ₆) + ((-sin(θ₂)·sin(θ₃)·cos(θ₁) - cos(θ₁)·cos(θ₂)·cos(θ₃))·sin(θ₄) + (-sin(θ₂)·cos(θ₁)·cos(θ₃) + sin(θ₃)·cos(θ₁)·cos(θ₂))·cos(θ₄))·cos(θ₆)  -(-(-sin(θ₂)·sin(θ₃)·cos(θ₁) - cos(θ₁)·cos(θ₂)·cos(θ₃))
 + sin(θ₅)·cos(θ₁))·sin(θ₆) + ((-sin(θ₁)·sin(θ₂)·sin(θ₃) - sin(θ₁)·cos(θ₂)·cos(θ₃))·sin(θ₄) + (-sin(θ₁)·sin(θ₂)·cos(θ₃) + sin(θ₁)·sin(θ₃)·cos(θ₂))·cos(θ₄))·cos(θ₆)  -(-(-sin(θ₁)·sin(θ₂)·sin(θ₃) - sin(θ₁)·cos(θ₂)·cos(θ₃))
sin(θ₆)·cos(θ₅) + ((-sin(θ₂)·sin(θ₃) - cos(θ₂)·cos(θ₃))·cos(θ₄) + (sin(θ₂)·cos(θ₃) - sin(θ₃)·cos(θ₂))·sin(θ₄))·cos(θ₆)                                                     -((-sin(θ₂)·sin(θ₃) - cos(θ₂)
         0

·cos(θ₄) + (-sin(θ₂)·cos(θ₁)·cos(θ₃) + sin(θ₃)·cos(θ₁)·cos(θ₂))·sin(θ₄))·sin(θ₅) - sin(θ₁)·cos(θ₅)  -192.2·(-(-sin(θ₂)·sin(θ₃)·cos(θ₁) - cos(θ₁)·cos(θ₂)·cos(θ₃))·cos(θ₄) + (-sin(θ₂)·cos(θ₁)·cos(θ₃) + sin(θ₃)·cos(θ₁)·cos
·cos(θ₄) + (-sin(θ₁)·sin(θ₂)·cos(θ₃) + sin(θ₁)·sin(θ₃)·cos(θ₂))·sin(θ₄))·sin(θ₅) + cos(θ₁)·cos(θ₅)  -192.2·(-(-sin(θ₁)·sin(θ₂)·sin(θ₃) - sin(θ₁)·cos(θ₂)·cos(θ₃))·cos(θ₄) + (-sin(θ₁)·sin(θ₂)·cos(θ₃) + sin(θ₁)·sin(θ₃)·cos
·cos(θ₃))·sin(θ₄) - (sin(θ₂)·cos(θ₃) - sin(θ₃)·cos(θ₂))·cos(θ₄))·sin(θ₅)                                                               -192.2·((-sin(θ₂)·sin(θ₃) - cos(θ₂)·cos(θ₃))·sin(θ₄) - (s
                 0

(θ₂))·sin(θ₄))·sin(θ₅) - 115.7·(-sin(θ₂)·sin(θ₃)·cos(θ₁) - cos(θ₁)·cos(θ₂)·cos(θ₃))·sin(θ₄) - 115.7·(-sin(θ₂)·cos(θ₁)·cos(θ₃) + sin(θ₃)·cos(θ₁)·cos(θ₂))·cos(θ₄) - 192.2·sin(θ₁)·cos(θ₅) - 163.9·sin(θ₁) + 571.6·sin(θ₂)·co
(θ₂))·sin(θ₄))·sin(θ₅) - 115.7·(-sin(θ₁)·sin(θ₂)·sin(θ₃) - sin(θ₁)·cos(θ₂)·cos(θ₃))·sin(θ₄) - 115.7·(-sin(θ₁)·sin(θ₂)·cos(θ₃) + sin(θ₁)·sin(θ₃)·cos(θ₂))·cos(θ₄) + 571.6·sin(θ₁)·sin(θ₂)·cos(θ₃) + 612.7·sin(θ₁)·sin(θ₂) -
in(θ₂)·cos(θ₃) - sin(θ₃)·cos(θ₂))·cos(θ₄))·sin(θ₅) - 115.7·(-sin(θ₂)·sin(θ₃) - cos(θ₂)·cos(θ₃))·cos(θ₄) - 115.7·(sin(θ₂)·cos(θ₃) - sin(θ₃)·cos(θ₂))·sin(θ₄) + 571.6·sin(θ₂)·sin(θ₃) + 571.6·cos(θ₂)·cos(θ₃) + 612.7·cos(θ₂)
                                                              1

s(θ₁)·cos(θ₃) + 612.7·sin(θ₂)·cos(θ₁) - 571.6·sin(θ₃)·cos(θ₁)·cos(θ₂)⎤
571.6·sin(θ₁)·sin(θ₃)·cos(θ₂) + 192.2·cos(θ₁)·cos(θ₅) + 163.9·cos(θ₁)⎥
 + 128                                                               ⎥
                                                                     ⎦
○ uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$ 
```

Once the transformation matrices are established we proceed with the calculation of the jacobian.
Using method 2 for the calculation of the jacobian.

$$J = \begin{bmatrix} \frac{\partial P}{\partial q_1} & \frac{\partial P}{\partial q_2} & \frac{\partial P}{\partial q_3} & \frac{\partial P}{\partial q_4} & \frac{\partial P}{\partial q_5} & \frac{\partial P}{\partial q_6} \\ Z_0 & Z_1 & Z_2 & Z_3 & Z_4 & Z_5 \end{bmatrix}$$

We will first write down the Z_i part.

$$J_\omega = \begin{bmatrix} Z_0 & Z_1 & Z_2 & Z_3 & Z_4 & Z_5 \end{bmatrix}$$

Which is given by:

```
PROBLEMS   OUTPUT   DEBUG CONSOLE   TERMINAL                                        Python

uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$ /bin/python3 /home/uthappa/Documents/662/HW4/uthu_hw3_code.py
⎡0  -sin(θ₁)  sin(θ₁)   -sin(θ₁)  -(-sin(θ₂)⋅sin(θ₃)⋅cos(θ₁) - cos(θ₁)⋅cos(θ₂)⋅cos(θ₃))⋅sin(θ₄) - (-sin(θ₂)⋅cos(θ₁)⋅cos(θ₃) + sin(θ₃)⋅cos(θ₁)⋅co
s(θ₂))⋅cos(θ₄)  -(-(-sin(θ₂)⋅sin(θ₃)⋅cos(θ₁) - cos(θ₁)⋅cos(θ₂)⋅cos(θ₃))⋅cos
⎢

⎢0  cos(θ₁)   -cos(θ₁)  cos(θ₁)   -(-sin(θ₁)⋅sin(θ₂)⋅sin(θ₃) - sin(θ₁)⋅cos(θ₂)⋅cos(θ₃))⋅sin(θ₄) - (-sin(θ₁)⋅sin(θ₂)⋅cos(θ₃) + sin(θ₁)⋅sin(θ₃)⋅co
s(θ₂))⋅cos(θ₄)  -(-(-sin(θ₁)⋅sin(θ₂)⋅sin(θ₃) - sin(θ₁)⋅cos(θ₂)⋅cos(θ₃))⋅cos
⎢

⎣1     0         0         0                      -(-sin(θ₂)⋅sin(θ₃) - cos(θ₂)⋅cos(θ₃))⋅cos(θ₄) - (sin(θ₂)⋅cos(θ₃) - sin(θ₃)⋅cos(θ₂))⋅sin(θ₄)
                                                -((-sin(θ₂)⋅sin(θ₃) - cos(θ₂)⋅cos

(θ₄) + (-sin(θ₂)⋅cos(θ₁)⋅cos(θ₃) + sin(θ₃)⋅cos(θ₁)⋅cos(θ₂))⋅sin(θ₄))⋅sin(θ₅) - sin(θ₁)⋅cos(θ₅)⎤
                                                                                              ⎥
(θ₄) + (-sin(θ₁)⋅sin(θ₂)⋅cos(θ₃) + sin(θ₁)⋅sin(θ₃)⋅cos(θ₂))⋅sin(θ₄))⋅sin(θ₅) + cos(θ₁)⋅cos(θ₅)⎥
                                                                                              ⎥
(θ₃))⋅sin(θ₄) - (sin(θ₂)⋅cos(θ₃) - sin(θ₃)⋅cos(θ₂))⋅cos(θ₄))⋅sin(θ₅)                          ⎦
```

We proceed with the calculation of J_v

$$J_v = \begin{bmatrix} \frac{\partial P}{\partial q_1} & \frac{\partial P}{\partial q_2} & \frac{\partial P}{\partial q_3} & \frac{\partial P}{\partial q_4} & \frac{\partial P}{\partial q_5} & \frac{\partial P}{\partial q_6} \end{bmatrix}$$

The function P is given by the end effector coordinates of the final transformation matrix. $P = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$

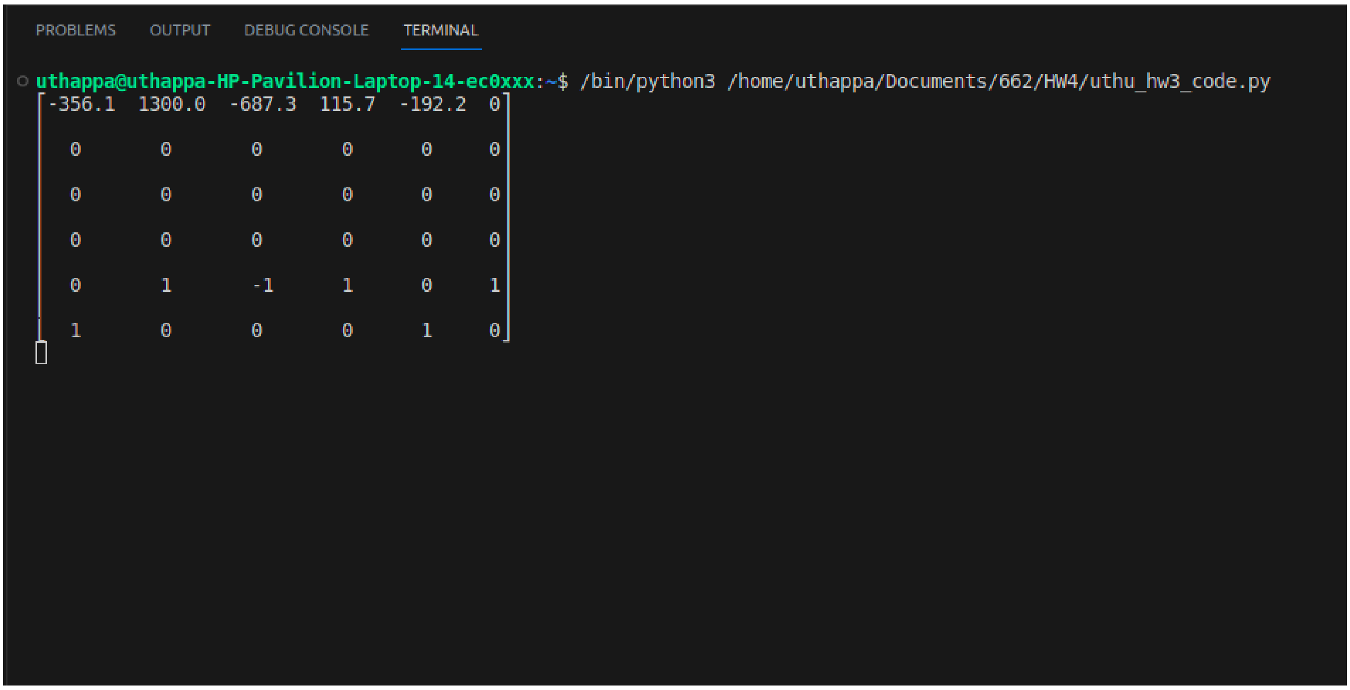
P_x, P_y and P_z is as follows:

```
PROBLEMS   OUTPUT   DEBUG CONSOLE   TERMINAL                                        Python

uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$ /bin/python3 /home/uthappa/Documents/662/HW4/uthu_hw3_code.py

P_x is as follows

-192.2⋅(-(-sin(θ₂)⋅sin(θ₃)⋅cos(θ₁) - cos(θ₁)⋅cos(θ₂)⋅cos(θ₃))⋅cos(θ₄) + (-sin(θ₂)⋅cos(θ₁)⋅cos(θ₃) + sin(θ₃)⋅cos(θ₁)⋅cos(θ₂))⋅sin(θ₄))⋅sin(θ₅) -
115.7⋅(-sin(θ₂)⋅sin(θ₃)⋅cos(θ₁) - cos(θ₁)⋅cos(θ₂)⋅cos(θ₃))⋅sin(θ₄) - 115.7⋅
(-sin(θ₂)⋅cos(θ₁)⋅cos(θ₃) + sin(θ₃)⋅cos(θ₁)⋅cos(θ₂))⋅cos(θ₄) - 192.2⋅sin(θ₁)⋅cos(θ₅) - 163.9⋅sin(θ₁) + 571.6⋅sin(θ₂)⋅cos(θ₁)⋅cos(θ₃) + 612.7⋅sin
(θ₂)⋅cos(θ₁) - 571.6⋅sin(θ₃)⋅cos(θ₁)⋅cos(θ₂)

P_y is as follows

-192.2⋅(-(-sin(θ₁)⋅sin(θ₂)⋅sin(θ₃) - sin(θ₁)⋅cos(θ₂)⋅cos(θ₃))⋅cos(θ₄) + (-sin(θ₁)⋅sin(θ₂)⋅cos(θ₃) + sin(θ₁)⋅sin(θ₃)⋅cos(θ₂))⋅sin(θ₄))⋅sin(θ₅) -
115.7⋅(-sin(θ₁)⋅sin(θ₂)⋅sin(θ₃) - sin(θ₁)⋅cos(θ₂)⋅cos(θ₃))⋅sin(θ₄) - 115.7⋅
(-sin(θ₁)⋅sin(θ₂)⋅cos(θ₃) + sin(θ₁)⋅sin(θ₃)⋅cos(θ₂))⋅cos(θ₄) + 571.6⋅sin(θ₁)⋅sin(θ₂)⋅cos(θ₃) + 612.7⋅sin(θ₁)⋅sin(θ₂) - 571.6⋅sin(θ₁)⋅sin(θ₃)⋅cos
(θ₂) + 192.2⋅cos(θ₁)⋅cos(θ₅) + 163.9⋅cos(θ₁)

P_z is as follows

-192.2⋅((-sin(θ₂)⋅sin(θ₃) - cos(θ₂)⋅cos(θ₃))⋅sin(θ₄) - (sin(θ₂)⋅cos(θ₃) - sin(θ₃)⋅cos(θ₂))⋅cos(θ₄))⋅sin(θ₅) - 115.7⋅(-sin(θ₂)⋅sin(θ₃) - cos(θ₂)⋅
cos(θ₃))⋅cos(θ₄) - 115.7⋅(sin(θ₂)⋅cos(θ₃) - sin(θ₃)⋅cos(θ₂))⋅sin(θ₄) + 571.
6⋅sin(θ₂)⋅sin(θ₃) + 571.6⋅cos(θ₂)⋅cos(θ₃) + 612.7⋅cos(θ₂) + 128
uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$ 
```

These P_x, P_y and P_z are differentiated with respect to $theta1$, $theta2$, $theta3$, $theta4$, $theta5$ and $theta6$ respectively and then setup to form J_v.
Finally the jacobian matrix is setup as:

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$$

Since the Jacobian matrix is huge it is not included in the report and is present in the code. The jacobian matrix at home position of robot is obtained as follows:



Calculation involved in the trajectory planning of a circle.

We setup the parametric equation of a circle as follows:
$x = r * sin(\theta)$
$z = r * cos(\theta)$
where θ is the angle subtended by the circle and r is the radius of the circle. Here r = 100mm.

Also the circle needs to be drawn in 20s, hence angular velocity $\omega = 2\pi/20$.

From the parametric equation we obtain the velocity $\dot{x}$ and $\dot{y}$.
$\dot{x} = r * cos(\theta) * \dot{\theta}$
$\dot{z} = -r * sin(\theta) * \dot{\theta}$

This forms the end effector velocity matrix as follows:

$$\dot{X} = \begin{bmatrix} \dot{x} \\ 0 \\ \dot{z} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

In order to find out the joint velocities we make use of the equation:
$\dot{q} = J^{-1} * \dot{X}$

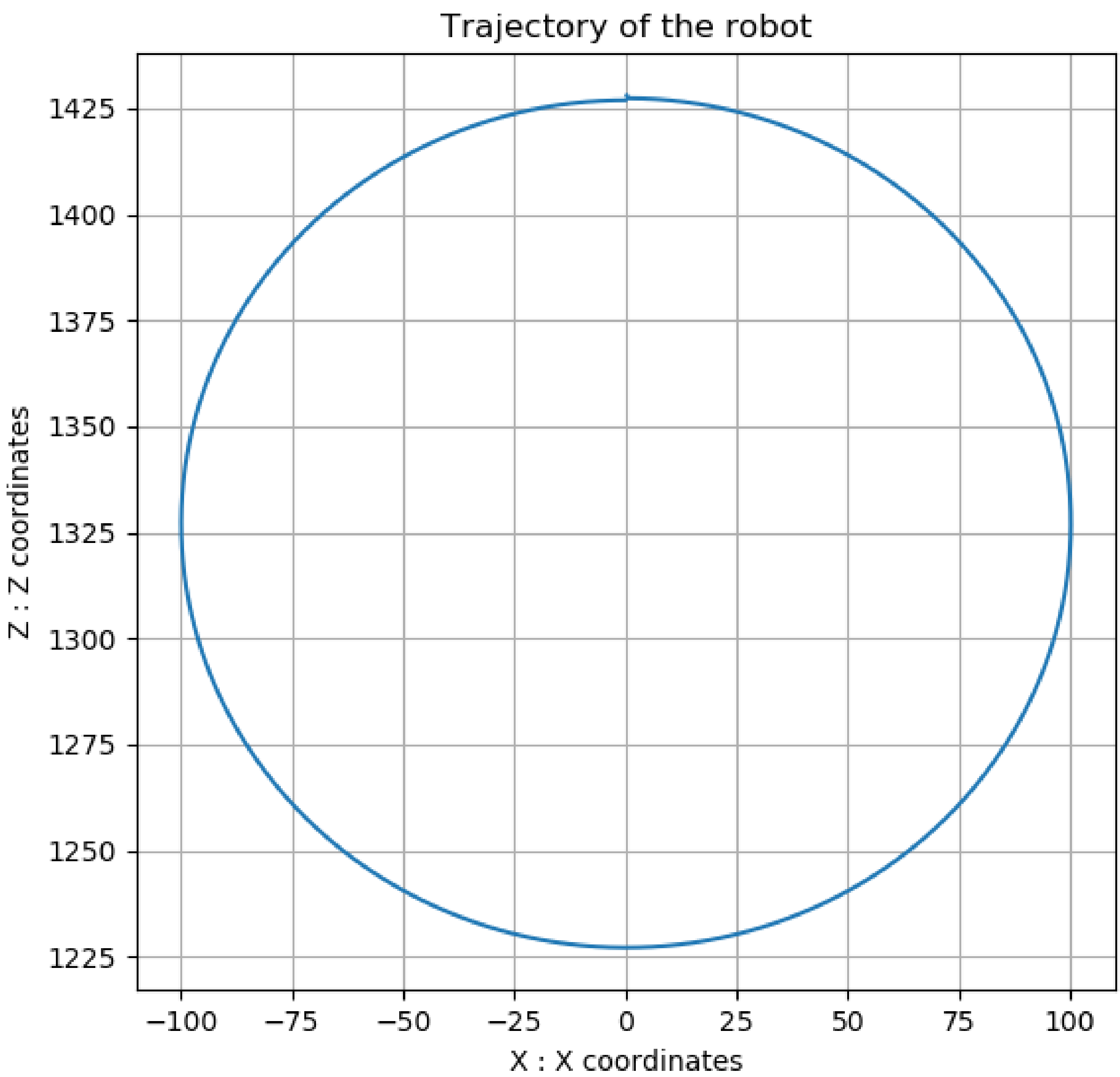
The Jacobian matrix is not invertible and hence we find the sudo inverse of it.

Once we obtain $\dot{q}$ we proceed with the numerical integration as follows:
$q = q + \dot{q} * dt$

The joint angles thus obtained are then plugged into the final transformation matrix to obtain the end effector position.
This process is repeated to get the trajectory of a circle.
All the calculations are present in the code.

The obtained plot is as follows:

The plot in 2D:



The plot in 3D: