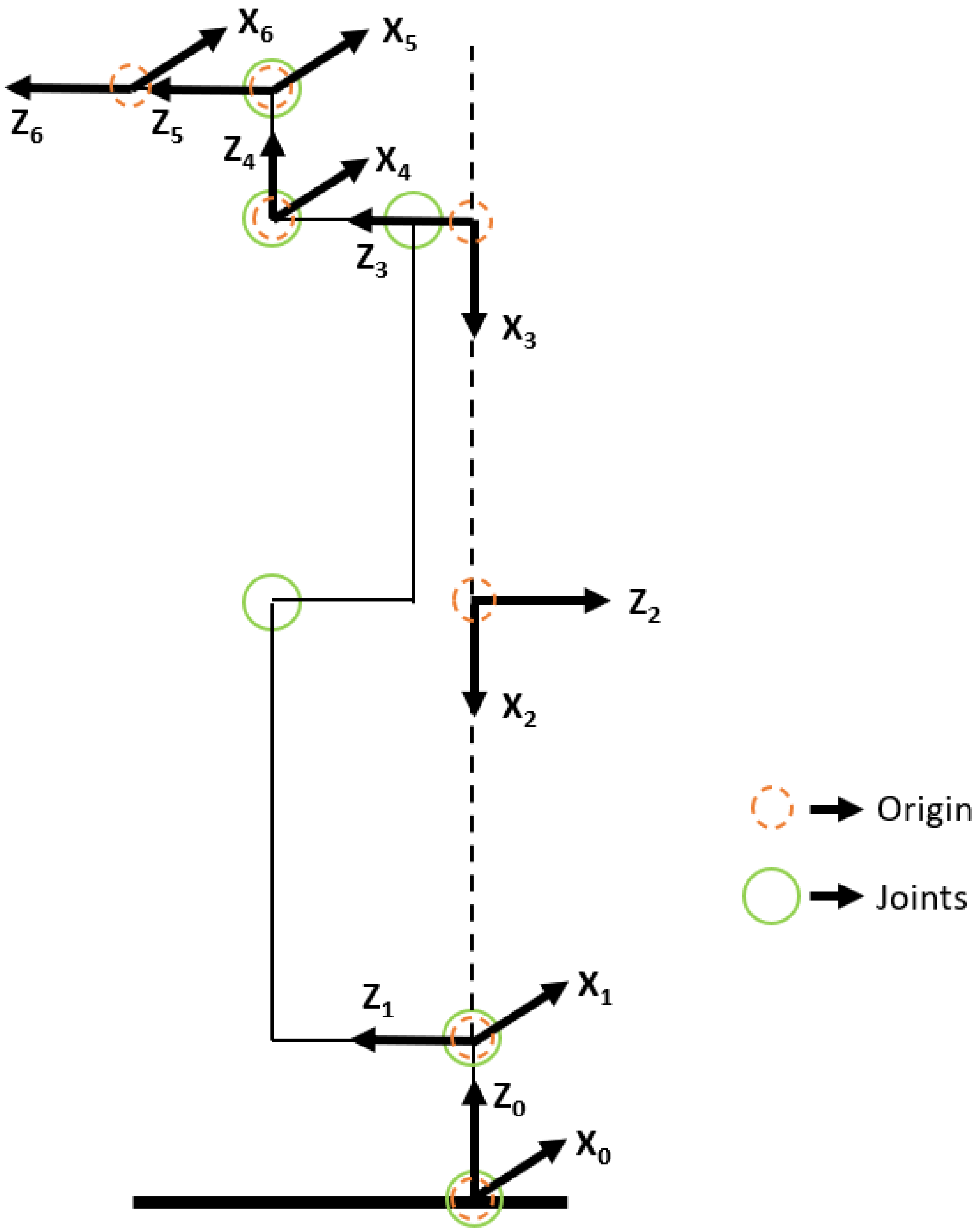
HOMEWORK 3

October 2023

1 Problem 1 - Position kinematics UR10

The DH coordinate frame for the robot is assigned as follows:



After assigning the DH coordinate frames for the robot, we construct the DH table as follows:

frame	a	α	d	θ
0 - 1	0	-90	128	θ_1
1 - 2	-612.7	180	0	90 + θ_2
2 - 3	-571.6	-180	0	θ_3
3 - 4	0	90	163.9	-90 + θ_4
4 - 5	0	-90	115.7	θ_5
5 - 6	0	0	92.2	θ_6

Once the DH table is constructed, we can proceed with the transformation matrix for each frame transformation.

$$T_1^0 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & 128 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} -\sin\theta_2 & \cos\theta_2 & 0 & 612.7\sin\theta_2 \\ \cos\theta_2 & \sin\theta_2 & 0 & -612.7\cos\theta_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} \cos\theta_3 & \sin\theta_3 & 0 & -571.6\cos\theta_3 \\ \sin\theta_3 & -\cos\theta_3 & 0 & -571.6\sin\theta_3 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

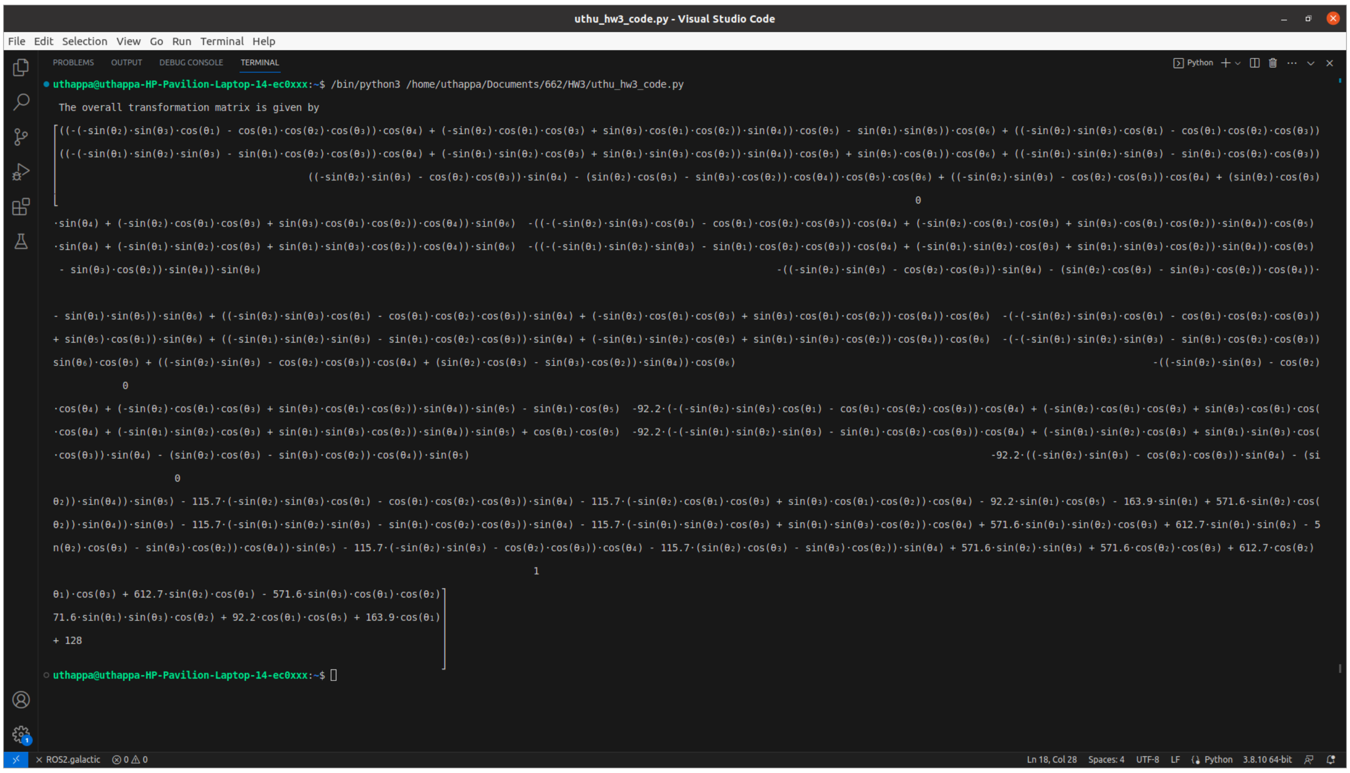
$$T_4^3 = \begin{bmatrix} \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ -\cos\theta_4 & 0 & -\sin\theta_4 & 0 \\ 0 & 1 & 0 & 163.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5^4 = \begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 115.7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6^5 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 92.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

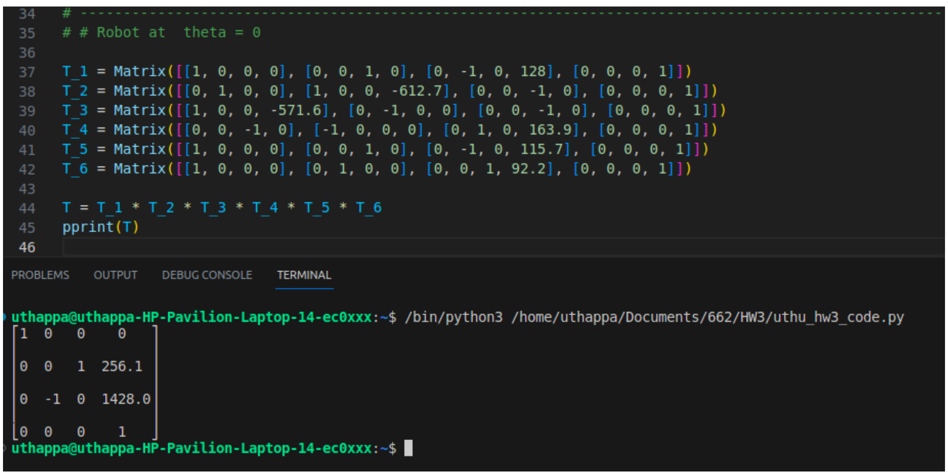
The overall transformation matrix can be obtained by:

$$T_6^0 = T_1^0 * T_2^1 * T_3^2 * T_4^3 * T_5^4 * T_6^5$$



The robot position at 5 different configurations:

1) When $\theta = 0$ for all the frames.



2) When θ_6 = 90 and rest of the angles are zero.

```python
# ------------------------------------------------------------------------------------------------
# #Robot at theta6 = 90 and rest of the angles are zero.

T_1 = Matrix([[1, 0, 0, 0], [0, 0, 1, 0], [0, -1, 0, 128], [0, 0, 0, 1]])
T_2 = Matrix([[0, 1, 0, 0], [1, 0, 0, -612.7], [0, 0, -1, 0], [0, 0, 0, 1]])
T_3 = Matrix([[1, 0, 0, -571.6], [0, -1, 0, 0], [0, 0, -1, 0], [0, 0, 0, 1]])
T_4 = Matrix([[0, 0, -1, 0], [-1, 0, 0, 0], [0, 1, 0, 163.9], [0, 0, 0, 1]])
T_5 = Matrix([[1, 0, 0, 0], [0, 0, 1, 0], [0, -1, 0, 115.7], [0, 0, 0, 1]])
T_6 = Matrix([[0, -1, 0, 0], [1, 0, 0, 0], [0, 0, 1, 92.2], [0, 0, 0, 1]])

T = T_1 * T_2 * T_3 * T_4 * T_5 * T_6
pprint(T)
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL

```
uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$ /bin/python3 /home/uthappa/Documents/662/HW3/uthu_hw3_code.py
⎡0   -1  0    0  ⎤
⎢                ⎥
⎢0   0   1  256.1⎥
⎢                ⎥
⎢-1  0   0  1428.0⎥
⎢                ⎥
⎣0   0   0    1  ⎦
uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$
```

3) When θ_5 = 90 and rest of the angles are zero.

```python
# ------------------------------------------------------------------------------------------------
# #Robot at theta5 = 90 and rest of the angles are zero.

T_1 = Matrix([[1, 0, 0, 0], [0, 0, 1, 0], [0, -1, 0, 128], [0, 0, 0, 1]])
T_2 = Matrix([[0, 1, 0, 0], [1, 0, 0, -612.7], [0, 0, -1, 0], [0, 0, 0, 1]])
T_3 = Matrix([[1, 0, 0, -571.6], [0, -1, 0, 0], [0, 0, -1, 0], [0, 0, 0, 1]])
T_4 = Matrix([[0, 0, -1, 0], [-1, 0, 0, 0], [0, 1, 0, 163.9], [0, 0, 0, 1]])
T_5 = Matrix([[0, 0, -1, 0], [1, 0, 0, 0], [0, -1, 0, 115.7], [0, 0, 0, 1]])
T_6 = Matrix([[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 92.2], [0, 0, 0, 1]])

T = T_1 * T_2 * T_3 * T_4 * T_5 * T_6
pprint(T)
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL

```
uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$ /bin/python3 /home/uthappa/Documents/662/HW3/uthu_hw3_code.py
⎡0  0   -1  -92.2 ⎤
⎢                 ⎥
⎢1  0   0   163.9 ⎥
⎢                 ⎥
⎢0  -1  0   1428.0⎥
⎢                 ⎥
⎣0  0   0     1   ⎦
uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$
```

4) When θ_5 = -90 and rest of the angles are zero.

```python
# ------------------------------------------------------------------------------------------------
# #Robot at theta5 = -90 and rest of the angles are zero.

T_1 = Matrix([[1, 0, 0, 0], [0, 0, 1, 0], [0, -1, 0, 128], [0, 0, 0, 1]])
T_2 = Matrix([[0, 1, 0, 0], [1, 0, 0, -612.7], [0, 0, -1, 0], [0, 0, 0, 1]])
T_3 = Matrix([[1, 0, 0, -571.6], [0, -1, 0, 0], [0, 0, -1, 0], [0, 0, 0, 1]])
T_4 = Matrix([[0, 0, -1, 0], [-1, 0, 0, 0], [0, 1, 0, 163.9], [0, 0, 0, 1]])
T_5 = Matrix([[0, 0, 1, 0], [-1, 0, 0, 0], [0, -1, 0, 115.7], [0, 0, 0, 1]])
T_6 = Matrix([[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 92.2], [0, 0, 0, 1]])


T = T_1 * T_2 * T_3 * T_4 * T_5 * T_6
pprint(T)
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL

```
uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$ /bin/python3 /home/uthappa/Documents/662/HW3/uthu_hw3_code.py
⎡0   0   1   92.2 ⎤
⎢                 ⎥
⎢-1  0   0   163.9⎥
⎢                 ⎥
⎢0   -1  0  1428.0⎥
⎢                 ⎥
⎣0   0   0    1   ⎦
uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$
```

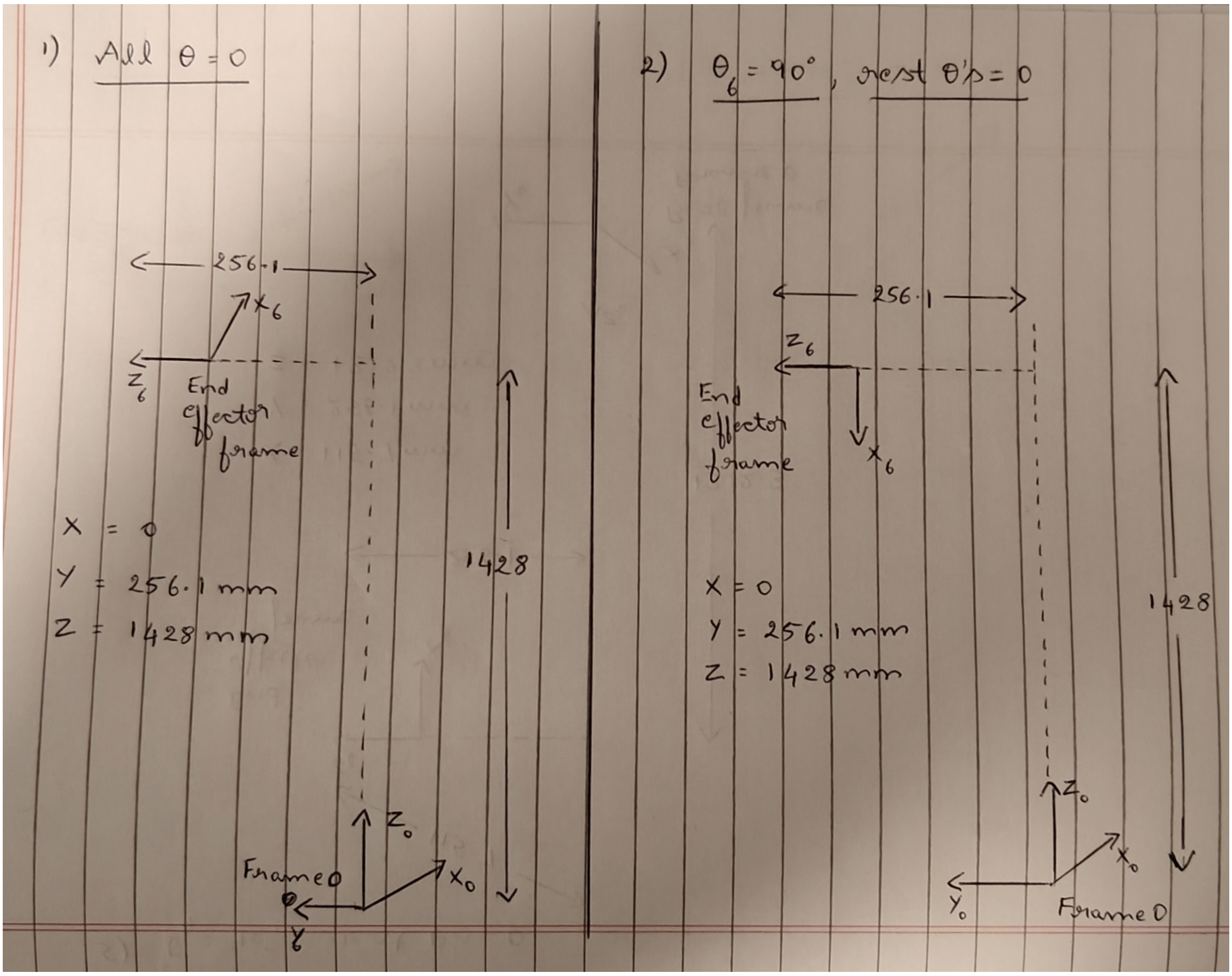
5) When $\theta_4 = 90$ and rest of the angles are zero.

```
# -----------------------------------------------------------------------------------------------
# #Robot at theta4 = 90 and rest of the angles are zero.

T_1 = Matrix([[1, 0, 0, 0], [0, 0, 1, 0], [0, -1, 0, 128], [0, 0, 0, 1]])
T_2 = Matrix([[0, 1, 0, 0], [1, 0, 0, -612.7], [0, 0, -1, 0], [0, 0, 0, 1]])
T_3 = Matrix([[1, 0, 0, -571.6], [0, -1, 0, 0], [0, 0, -1, 0], [0, 0, 0, 1]])
T_4 = Matrix([[1, 0, 0, 0], [0, 0, -1, 0], [0, 1, 0, 163.9], [0, 0, 0, 1]])
T_5 = Matrix([[1, 0, 0, 0], [0, 0, 1, 0], [0, -1, 0, 115.7], [0, 0, 0, 1]])
T_6 = Matrix([[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 92.2], [0, 0, 0, 1]])


T = T_1 * T_2 * T_3 * T_4 * T_5 * T_6
pprint(T)
```

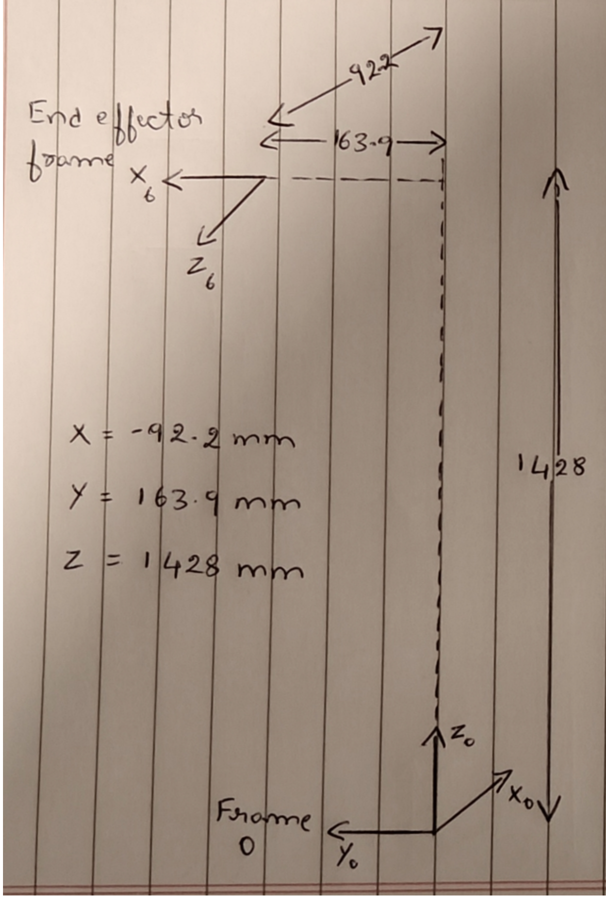
PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL

```
uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$ /bin/python3 /home/uthappa/Documents/662/HW3/uthu_hw3_code.py
⎡0   -1  0  115.7 ⎤
⎢0   0   1  256.1 ⎥
⎢-1  0   0  1312.3⎥
⎣0   0   0    1   ⎦
uthappa@uthappa-HP-Pavilion-Laptop-14-ec0xxx:~$
```

The above results can be validated geometrically as follows:



3) $\theta_5 = 90°$, rest θ's = 0

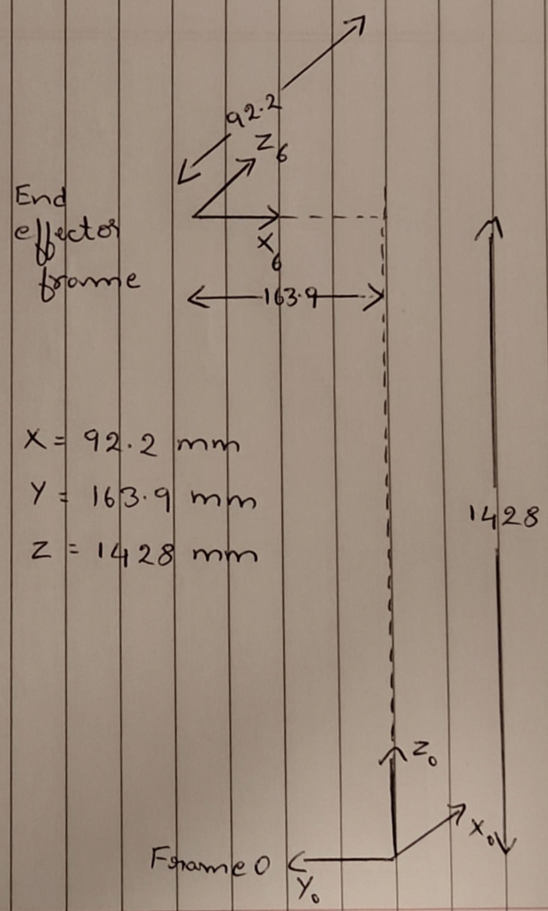


X = -92.2 mm

Y = 163.9 mm

Z = 1428 mm

4) $\theta_5 = -90°$, rest θ's = 0

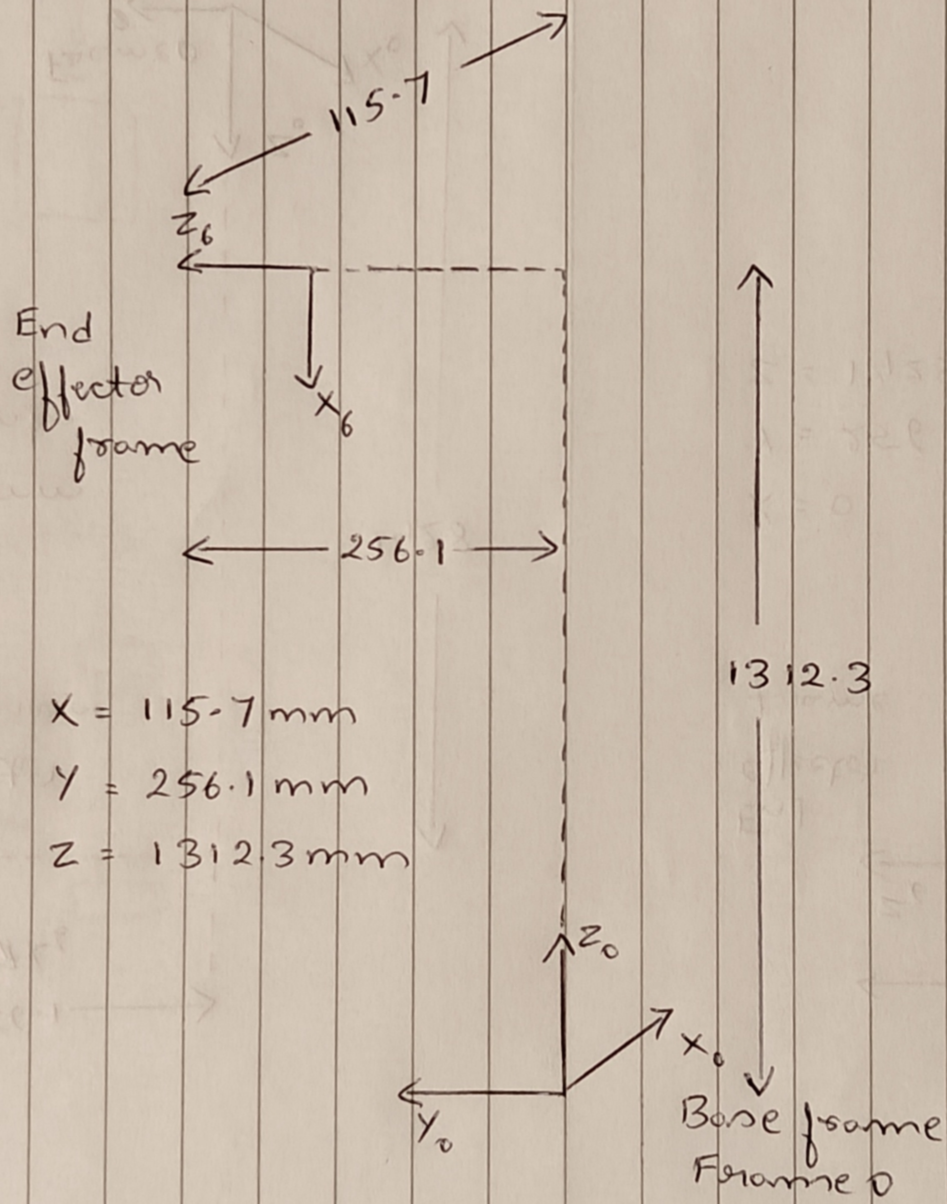


X = 92.2 mm

Y = 163.9 mm

Z = 1428 mm

5) $\theta_4 = 90°$, rest θ's = 0



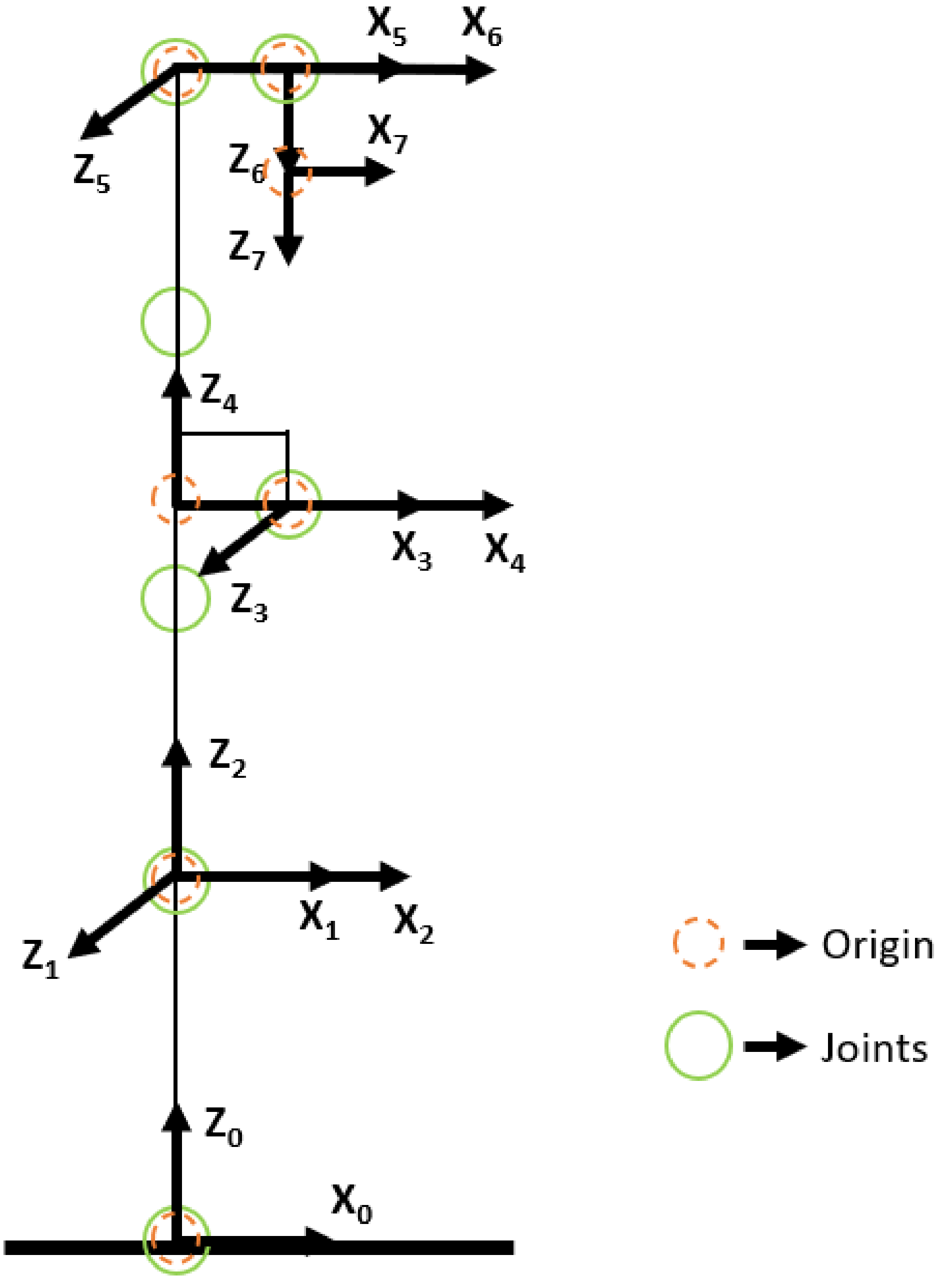
$X = 115.7\,mm$

$Y = 256.1\,mm$

$Z = 1312.3\,mm$

2 Problem 2 - Position kinematics KUKA

The DH coordinate frame for the robot is assigned as follows:



After assigning the DH coordinate frames for the robot, we construct the DH table as follows:

frame	a	α	d	θ
0 - 1	0	90	d_1	θ_1
1 - 2	0	-90	0	θ_2
2 - 3	a_3	90	d_3	θ_3
3 - 4	$-a_3$	-90	0	θ_4
4 - 5	0	90	d_5	θ_5
5 - 6	a_3	90	0	θ_6
6 - 7	0	0	d_7	θ_7