Bilinear control theory

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Contents

1 The Definitions

2 The Theorems

m denotes the number of control fields, n denotes the number of modes of the system. The control functions are usually either unspecified or constrained to $u \in \mathbb{R}$. By symmetric controls I mean that the set of controls parameters is equal to its negative.

2.1 Normal Accessibility

$m = m$, $n = n$, any controls. Necessary and sufficient condition for control on a Lie group. p154 JurdjevicGeo or my pdf.

2.2 Recursivity theorems

2.2.1 Neutrality

$m = m$, $n = n$, any controls. Sufficient condition for control on a Lie group. Elliott p97.

- Elliott contains other, equivalent definitions of neutrality.
- I have a necessary and sufficient condition for symplectic neutrality.

2.2.2 Genoni2012 recursivity

$m = m$, $n = n$, any controls. It is sufficient for Lie group controllability. Note this is not as broad as the neutrality theorem above. Genoni2012.

2.2.3 Wu non-empty set

$m = 1$, $n = 1$, symmetric controls. Necessary and sufficient condition for control on a Lie group. Wu2007 and also p117 of Elliott seems to look very similar.

2.3 Single input unbounded

2.3.1 Jurdjevic and Kupka sufficient condition

$m = 1$, $n = n$, unbounded controls. Sufficient condition for control on a semi-simple? Lie group. Theorem 3.11 in Elliott. Also found in Jurdjevic I think.

- This seems like it must be broader than neutrality after fixing the control parameters. If this is true then neutrality is not equivalent to controllability.

2.3.2 El-Assoudi sufficient condition

$m = 1$, $n = n$, unbounded controls. Sufficient condition for control on a semi-simple Lie group. Presumably broader than above; pretty much takes all properties of the symplectic group. ElAssoudi2014.

2.4 Huang's theorem

$m = m$, $n = n$, unbounded controls. Controllability on a homogeneous space of the Lie group in infinite dimensional Hilbert space, intersected with the set of analytic vectors. Huang1983, Wu2005.

- It is the equality in the theorem that is most confusing. Why can it not be higher than the dimension of the manifold?

2.5 Trivial theorems

2.5.1 Blow away the drift field

$m > 1$, $n = n$, unbounded controls. Can blow away the drift field and generate the algebra with the control fields, thus making LARC necessary and sufficient. Necessary and sufficient condition for control on a Lie group. JurdjevicGeo.

2.5.2 Generic Generation

$m > 1$, $n = n$, unbounded controls. Can blow away the drift field. Generators are dense in the algebra and so LARC not required. JurdjevicGeo.

3 The map of Gaussian states

This map of bilinear control theory is applied to my map of Gaussian states. Symplectic group manifold of dimension $n(2n + 1)$, symplectic vector space, metaplectic group manifold, acting on the Hilbert space with a sub-Kahler manifold: the unit sphere or projective Hilbert space in which is a sub-manifold of pure Gaussian states diffeomorphic to the open convex cone of covariance matrices in dimension $n(n + 1)$.