EPSRC Centre for Doctoral Training in Delivering Quantum Technologies

PHASQ01 – Atoms and photons

2014-2015, Problem Set 1

1. Assuming an electric dipole coupling, provide a rough estimate of the mode volume needed to couple a single photon of one mode of the electromagnetic field to a $2P \to 1S$ electronic transition in hydrogen (the "mode volume" would correspond to the volume of an optical cavity where the coupling takes place). Recall that the energy E_N of the N^{th} level of hydrogen is given by

$$E_N = \frac{E_0}{N^2} \,, \quad \text{with} \quad E_0 = -\frac{e^2}{8\pi\epsilon_0 a_0}$$

and that the Bohr radius is $a_0 \simeq 0.5\ 10^{-10} m$, and assume the photon to be at resonance with the transition.

Warning: don't try too hard on this one. It's not worth it, really. **[6 marks]**

2. A coherent state $|\alpha\rangle$ is the eigenstate of the annihilation operator a with eigenvalue $\alpha \in \mathbb{C}$. Knowing that a acts on a state $|n\rangle$ of the Fock basis according to

$$a|n\rangle = \sqrt{n}|n-1\rangle \quad \text{for} \quad n \in [0, 1, 2, 3, \ldots, \infty] \,,$$

determine the expression of $|\alpha\rangle$ in the Fock basis. **[10 marks]**

3. A "beam splitter" operation describes the action of a semi-reflective mirror that mixes two input modes of light into two combined output modes. Its action on the vector $\hat{\mathbf{r}}$ of canonical operators is represented by the matrix

$$B = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & -\sin\theta & 0 & \cos\theta \end{pmatrix} \,,$$

where $\cos^2\theta$ can be interpreted with the transmissivity of the mirror (or reflectivity, depending on the identification of the output modes). Show that B is a symplectic transformation and find a Hamiltonian matrix H that generates B. Also show that B preserves the free Hamiltonian of the system (the Hamiltonian matrix corresponding to the free Hamiltonain is just the identity matrix). **[14 marks]**