NUMERICAL METHODS LABORATORY(MA29202) & NUMERICAL TECHNIQUES LABORATORY(MA39110)

Linear and non-linear Boundary-Value Problems [1]

1. Solve the following boundary-value problem

$$\begin{cases} y'' + e^x y' - xy = (-x^2 + 2x - 3)e^{-x} - x + 2, & 0 \le x \le 1, \\ y(0) = -1,\ y(1) = 0, \end{cases}$$

using central finite difference method with $h = 0.1$. Compare the results with the exact solution, $y(x) = (x-1)\,e^{-x}$.

2. Apply central finite difference method to solve the boundary-value problem

$$\begin{cases} y'' + x^2 y' - 4xy = 0, & 0 \le x \le 1, \\ y(0) = 0,\ y(1) = 5, \end{cases}$$

with $h = 0.1$. Compare the results with the exact solution, $y(x) = x^4 + 4x$.

3. Solve the boundary-value problem

$$\begin{cases} y'' - \dfrac{3}{x}y' + \dfrac{3}{x^2}y = 2x^2 e^x, & 1 \le x \le 2, \\ y(1) = 0,\ y(2) = 4e^2, \end{cases}$$

using the shooting method with $h = 0.1$. Compare the results with the exact solution $y(x) = 2xe^x(x-1)$.

4. Use the non-linear shooting method to approximate the solution of the following boundary-value problem

$$\begin{cases} y'' = \dfrac{1 - y'^2}{y}, & 0 \le x \le 2, \\ y(0) = 1,\ y(2) = 2, \end{cases}$$

with $h = 0.4$. Use $TOL = 10^{-4}$.

Central Finite-Difference Method:

$$y'(x_i) \approx \frac{1}{2h}\left[y(x_{i+1}) - y(x_{i-1})\right] \text{ and } y''(x_i) \approx \frac{1}{h^2}\left[y(x_{i+1}) - 2y(x_i) + y(x_{i-1})\right].$$