

MATHEMATICAL FORMULATION AND TYPE CURVES FOR INDUCED POLARIZATION*

HAROLD O. SEIGEL†

ABSTRACT

A basic mathematical formulation is developed for overvoltage and other induced polarization phenomena. Starting from the fundamental representation of a volume dipolar distribution, one is led to the concept of a change in apparent conductivity due to polarization effects. The mathematical solution of induced polarization phenomena, therefore, reduces to the appropriate solution of Laplace's equation for the same geometry and conductivity distribution ignoring polarization, followed by partial differentiation of the apparent resistivity function so determined. The dielectric constants of the media are not involved in the solution.

As examples of the use of the representation, the response of a polarizable sphere and of a polarizable lower layer in a typical two-layer case are presented. Actual field results are shown illustrating the use of the latter solution.

INTRODUCTION

The early workers in the field of induced polarization, such as Schlumberger (1930) and Dakhnov (1941) have given no published indication of any attempt at mathematical representation of the phenomena they were investigating. Bleil (1953), who has been the first in North America to publish on overvoltage or any other induced polarization phenomenon, also presented no mathematical formulation.

The writer's first attempt at a representation for a volume polarization distribution was presented in a thesis for the National Research Council of Canada (1948) and was expanded in his Ph.D. thesis on overvoltage (1949). In the latter thesis the experimental proof of the successful application to the detection of sulphide mineralization *in situ* was presented.

The field procedure involved the application of a constant potential between two points of a medium under investigation for a few seconds; the abrupt interruption of the applied voltage and the measurement of transient voltages existing in the medium from a few milliseconds after the interception.

A second phenomenon (or group of phenomena), termed the "normal" or background effect, was recognized in the course of the field investigations as being present in all rocks, consolidated or otherwise, even when totally lacking in metallic sulphide minerals. It was found that the "normal" effect generally lay within relatively narrow limits for most rocks and could, therefore, be resolved from overvoltage effects. In the thesis (p. 63-71) it was suggested that the "normal" effect in consolidated rocks might be due to a number of phenomena of which certain electrokinetic effects (electro-osmosis plus "streaming potential") might predominate. The more recent investigations of Vacquier (1957) add

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† Harold O. Seigel & Associates, Limited, Toronto 1, Ontario, Canada.

electrodialysis of clay across semi-permeable partitions formed by sand grains in unconsolidated sediments.

The need of a mathematical model of the overvoltage and "normal" effect became great when it was apparent that the overvoltage phenomenon could be employed in exploration for sulphide mineralization. The model could be used in the interpretation of field data in terms of dimensions and physical properties of the various geological factors involved.

In the Ph.D. thesis it was shown that the overvoltage effect associated with a single metallic conducting sphere in an electrolyte is the same as that due to a dipolar source whose moment is proportional and antiparallel to the average current density vector in the vicinity. A small volume containing a number of such particles would then be expected to act at an external point as a dipolar source whose moment is the combined effect (including interaction) of the particles. In the thesis treatment it was assumed that a dipolar source acted at a distance in accordance with electrostatic laws rather than electrodynamic laws. Simple, idealized geologic examples were then solved mathematically, e.g., a uniform mineral dissemination through a homogeneous semi-infinite earth. An extension of this case to the homogeneous infinite earth, i.e., wherein all electrodes are buried in the earth at a depth much greater than their separation, reveals that the polarization potentials for the same current flow remain unchanged, but the primary voltage drops to half its value. In other words, the ratio of the secondary (transient) voltage to primary voltage (steady state) should be dependent on the position of the electrodes relative to the boundaries of a mass of homogeneous polarizable material.

Using a large box containing a homogeneous mixture of crushed pyrite of a certain particle size range, crushed rock and saline electrolyte, this deduction was tested and found to be incorrect. It was quickly found that the ratio of secondary (transient) voltage over primary voltage depends only on the material, not on its shape, nor on the configuration of electrodes used or their location within the mass. This took place in January, 1950. About the same time it became apparent that the "normal" ratio displayed the same disregard for electrode configuration or surface topography, even when the latter was extremely dissected, as at Jerome, Arizona, and was approximately the same for electrodes on surface or in mine workings far underground.

It became apparent that a different mathematical model was required to predict the dependence of the observed transient voltages on the physical properties of the materials involved. This formulation, which is reproduced in detail below, has been tested experimentally in the laboratory and on field examples and leads to agreement with the results obtained. Starting with the fundamental approach of a volume distribution of dipolar sources, it leads directly to the concept of change of apparent resistivity with time, due to either normal induced effects or to the effect of a dissemination of sulphide particles.

Of the mathematical representations which have been published since this

development was made, it is interesting to note that the mathematical formulation of Frische and von Buttlar (1957) employs the electrostatic model in conjunction with induced polarization in unconsolidated sediments. A recent representation, by Henkel (1958), leads to the same end product as the writer's present formulation, but it is less fundamental in approach, since it ignores the basic concept of a volume dipolar distribution.

MATHEMATICAL FORMULATION OF THE DYNAMIC DIPOLAR REPRESENTATION

All dipolar charge effects will be included in the following discussion, whether due to dielectric absorption, electroanalysis, overvoltage, or other sources. The experimental conditions assumed are that a step function of voltage is instantaneously applied to two electrodes immersed in a medium, and, after a selected interval, it is interrupted instantaneously. The voltage existing between two non-polarizing electrodes in the medium is observed after the interruption. These residual voltages may be due to any combination of the various phenomena mentioned above.

The earlier representation of the dipolar distribution set up by an electric field in an ionic conductor was a static one, i.e., we considered only the electrostatic field of such dipoles as causing the observed voltage gradients after the interruption of the primary field. Such a representation ignores the effect of conductivity changes in the medium on the field of the secondary dipoles in favor of the effect of changes in the dielectric constant, and hence is not valid for slowly varying fields.

We now propose to introduce a dynamic representation or model for polarization effects.

The secondary response due to these phenomena is represented as that due to a volume density of dipolar sources, the current from which must obey the normal steady current flow boundary conditions at discontinuities in conductivity.

Our postulate is: The action of the primary field is to create a volume distribution of current dipoles antiparallel to the field at each point in the medium, and with volume current moment strength $\vec{M} = -mj$, where j is the primary current density. The factor m is assumed to be a constant, called the "chargeability." It is a property of the medium and includes all polarization effects, but it depends on the duration interval of the applied voltage.

The potential at a point, P, at a distance r from the volume dipole element of current strength $\vec{M}dv$ in a medium of conductivity σ is

$$d\Phi = \frac{1}{4\pi\sigma} \vec{M} \cdot \nabla \left(\frac{1}{r} \right) dv$$

when the gradient is taken at the dipole (see Figure 1).

The total potential at the field point, P, due to a volume distribution of such dipoles, is then

$$\Phi = \frac{1}{4\pi} \iiint \frac{\vec{M}}{\sigma} \cdot \nabla \left(\frac{1}{r} \right) dv.$$

However, by use of the identity

$$\operatorname{div} \left(\frac{\vec{M}}{\sigma r} \right) = \frac{1}{r} \operatorname{div} \left(\frac{\vec{M}}{\sigma} \right) + \frac{\vec{M}}{\sigma} \cdot \nabla \left(\frac{1}{r} \right),$$

we obtain

$$\Phi = \frac{1}{4\pi} \iiint \left[\operatorname{div} \left(\frac{\vec{M}}{\sigma r} \right) - \frac{1}{r} \operatorname{div} \left(\frac{\vec{M}}{\sigma} \right) \right] dv.$$

By Gauss' theorem we may write this as

$$\Phi = \frac{1}{4\pi} \iint_S \frac{M_n}{\sigma \cdot r} dS - \frac{1}{4\pi} \iiint_V \frac{1}{r} \operatorname{div} \left(\frac{\vec{M}}{\sigma} \right) dv$$

where S is the surface bounding the volume V .

Thus, the volume distribution of current dipoles is mathematically equivalent to a volume distribution of current sources of density equal to $-\operatorname{div} \vec{M}$ (σ constant within V , understood) and a surface distribution of strength M_n (normal component of \vec{M} out of the surface S).

Now if \vec{j} is the current density vector at any point in the medium, and i is the volume density of free current sources (i.e., due to primary current electrodes) then by the law of conservation of charge:

$$\operatorname{div} \vec{j} = i.$$

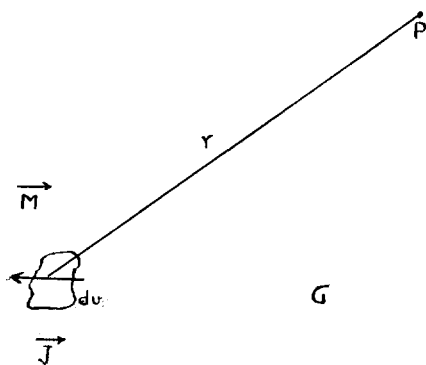


FIG. 1. Volume dipolar element.

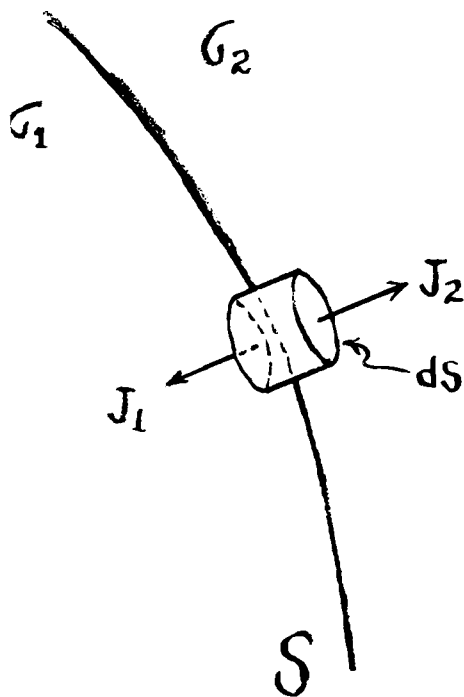


FIG. 2. Boundary conditions.

To these free sources i we must now add the effective source density $-\text{div } \vec{M}$,

$$\text{i.e., } \text{div } \vec{j} = i - \text{div } \vec{M},$$

$$\text{or, } \text{div } (\vec{j} + \vec{M}) = i. \quad (1)$$

At a discontinuity in M , σ , or both, we may determine our boundary conditions by considering the normal current flow out of the faces of a small disc with sides normal to the interface and faces dS parallel to it (see Figure 2).

Equating current outflow to current inflow within the disc, we find, in the absence of free sources on the interface, that

$$-j_1 + j_2 = M_1 - M_2,$$

$$\text{or, } (j + M)_1 = (j + M)_2 \quad (2)$$

Thus, the vector $\vec{j} + \vec{M}$ or $\vec{j}(1 - m)$ is everywhere solenoidal, except in the presence of free sources or sinks, and its normal component is continuous across discontinuities in σ and m .

If the impressed electric field due to externally supplied sources is \vec{E} , then

the original impressed current density vector is $\vec{j} = \sigma \vec{E}$. The vector $\vec{j}(1-m)$ in the presence of the dipolar distribution plays the role of \vec{j} in the absence of such dipoles. Thus the total current density in the presence of the dipoles is $\sigma(1-m)\vec{E}$. The net effect is to reduce σ by the factor $(1-m)$.

Thus, whenever we can obtain the current flow steady state solution for any problem involving conductors whose conductivities σ_i are known, we have also obtained the solution for the same potential problem taking into account the secondary dipolar distributions. The one step required is to change σ_i to $\sigma_i(1-m_i)$ in each instance. The difference between the two potential functions will then give us the peak secondary voltages to be experienced from polarization effects.

The question of form of decay curve is another matter entirely and depends on determination of the relative amounts contributed to the peak secondary by the dipoles in the various media involved, assuming the decay form for each medium is known.

SPECIAL CASES

(1) *Homogeneous Medium*

If we have a mass of homogeneous material of any size and shape and induce current I into it at two or more points (sources and sinks), then the primary potential drop between any two electrodes (serving as potential electrodes) will be, in the absence of volume polarization effects,

$$\Phi = \frac{I}{\sigma} F$$

where σ is the uniform conductivity of the medium, F is a function of the shape of the body and positions of all electrodes in it.

In the presence of the dipoles, the primary potential drop is altered to

$$\Phi' = \frac{I}{\sigma(1-m)} F$$

where m is the uniform chargeability of the medium. The peak secondary voltage is

$$\Phi' - \Phi = \left(\frac{IF}{\sigma} \right) \left(\frac{m}{1-m} \right)$$

and the ratio of the peak secondary to the primary voltage is

$$(\Phi' - \Phi)/\Phi' = m. \quad (3)$$

Thus, in any homogeneous isotropic medium, for an arbitrary electrode configuration, the ratio of peak secondary to primary voltage is equal to the charge-

ability m of the medium, provided that by the "primary" voltage we mean the voltage between the potential electrodes at the instant of the interruption of the primary field.

The importance of this is manifold. Firstly, in uniform isotropic media, all electrode configurations will yield the same ratio of peak secondary to primary voltages. Secondly, the ratio will be independent of shape of surface or boundaries, i.e., topography will not affect the ratio. This has been one of the puzzling, but welcome, observations in areas of great ruggedness, e.g., Arizona, Peru, and British Columbia. The ratios are expected to be unaltered in the transition from surface to underground operations. Thirdly, the calculation of polarization characteristics of rock samples from measurements in the laboratory is greatly simplified. Assuming that there will be negligible interaction between electrodes due to overvoltage at the current electrodes, the ratios of primary and secondary voltages will, for homogeneous isotropic samples, be independent of the electrode positions, and specimen size and shape.

(2) *Heterogeneous Media*

If the medium under consideration is composed of n different materials of conductivities σ_i , $i=1 \cdots n$, then the primary voltage in the absence of dipolar action is generally expressible as $\Phi=\Phi(\sigma_i)$ for fixed dimensions and electrode positions.

In the presence of the dipoles $\Phi'=\Phi(\sigma'_i)$ where $\sigma'_i=\sigma_i(1-m_i)$ and where m_i ($i=1, 2, \cdots, n$) are the chargeabilities of the various media

Now

$$\Phi' = \Phi(\sigma'_i) = \Phi(\sigma_i) + \sum_i (\sigma'_i - \sigma_i) \Phi_i,$$

where $\Phi_i = \partial\Phi/\partial\sigma_i$. After differentiation σ_i is replaced by $\sigma_i(1-\theta_i m_i)$, where $0 \leq \theta_i \leq 1$. But $\sigma'_i - \sigma_i = -\sigma_i m_i$. Therefore,

$$\Phi' = \Phi - \sum_i \sigma_i m_i \Phi_i. \quad (4)$$

Thus, whenever an increase of conductivity of one of the components of the system will result in a decrease of primary voltage (which is true except in certain special configurations) the effect of the presence of the dipoles is to increase the primary voltage.

The apparent chargeability of the medium is given by

$$m_a = (\Phi' - \Phi)/\Phi' = \frac{-1}{\Phi'} \sum_i \sigma_i m_i \Phi_i = + \sum_i m_i \rho_i \partial\Phi/\partial\rho_i, \quad (5)$$

where $\rho_i = 1/\sigma_i$ is the resistivity of the i th medium.

Generally, a function ρ_a is defined which is merely a constant geometric factor times the primary voltage.

Then

$$m_a = \sum_i m_i \rho_i \frac{\partial \rho_a}{\partial \rho_i} / \rho_a' \quad (6)$$

(accent indicating inclusive of dipolar action)

$$\frac{\partial \rho_a}{\partial \rho_i}$$

is taken at

$$\rho_i/1 - \theta_i m_i.$$

For the most types of polarization $m \ll 1$ and is often of the order of 10^{-2} . Then $\theta_i m_i \ll 1$, and $\rho_a' \approx \rho_a$.

Therefore, we may write

$$m_a \approx \sum_i m_i \partial \log \rho_a / \partial \log \rho_i \quad (6')$$

Now ρ_a is homogeneous of the first order in the ρ_i , i.e.,

$$\rho_a(t\rho_i) = t\rho_a(\rho_i)$$

This follows from the expression for the primary voltage between two points, P_1 and P_2 , as $V = \int_{P_1}^{P_2} E_s dS$, i.e., as the line integral of the tangential component of the field over an arbitrary path joining the points. We may write $E_s = j_s \rho$, where j_s is the tangential component of the current vector, and ρ is the resistivity at the point of integration.

If all resistivities are multiplied by the factor t , then the primary voltage and hence apparent resistivity is altered proportionally as well, since the current distribution for constant total current will remain unaltered.

For homogeneous functions of the first order, the following theorem holds:

$$\rho_a = \sum_i \rho_i \partial \rho_a / \partial \rho_i. \quad (7)$$

This may be formally derived simply from the Taylor expansion

$$\begin{aligned} \rho_a(0) &= \rho_a(t\rho_i) - t \left(\sum_i \rho_i \partial \rho_a(t\rho_i) / \partial \rho_i \right) \\ &\quad + \frac{t^2}{2!} \left(\sum_i \rho_i \partial^2 \rho_a(t\rho_i) / \partial \rho_i^2 \right) \cdot \dots \end{aligned}$$

Since $\rho_a(0) = 0$ and $\rho_a(t\rho_i) = t\rho_a(\rho_i)$ we obtain, after equating to zero the coefficient of t in the identity on the right side

$$\rho_a = \sum_i \rho_i \partial \rho_a / \partial \rho_i.$$

This may be otherwise expressed as

$$\sum_i \partial \log \rho_a / \partial \log \rho_i = 1. \quad (8)$$

Thus we may write

$$m_a = \left[\sum_i m_i \partial \log \rho_a / \partial \log \rho_i \right] / \sum_i \partial \log \rho_a / \partial \log \rho_i. \quad (9)$$

In general $\partial \log \rho_a / \partial \log \rho_i$ is positive, and m_a will lie intermediate to the extreme m_i . If all m_i are equal, say to m_0 , then regardless of the resistivity distribution $m_a = m_0$ also.

m_a can lie externally to the extreme m_i only when $\partial \log \rho_a / \partial \log \rho_i$ is negative for some i . Actual examples might be a thin bed lying entirely between a current and potential electrode; or measurements down a drill hole wherein there is a marked resistivity contrast between the drill fluid and the wall rock. An increase in resistivity of the thin bed or of the drill hole fluid may cause a decrease in apparent resistivity. Other possibilities may present themselves.

When we are dealing with only two media, formula 6' becomes

$$m_a = m_1 \frac{\partial \log \rho_a}{\partial \log \rho_1} + m_2 \frac{\partial \log \rho_a}{\partial \log \rho_2} = m_1 + (m_2 - m_1) \frac{\partial \log \rho_a}{\partial \log \rho_2}$$

or

$$\frac{m_a - m_1}{m_2 - m_1} = \frac{\partial \log \rho_a}{\partial \log \rho_2} \quad (6'')$$

This is a useful formula in dealing with two media, of which only the second is abnormally polarizable.

The mathematical representation above applies to peak polarization voltages, and formulas 6, 6', 9, etc., apply to the resultant secondary voltage observed immediately after the interruption of the primary field. Assuming little dipolar interaction, however, the above formulas apply to any phase of the voltage decay curve and to any type of measurement, e.g., integration over a specified time interval.

To the approximation that $m_i \ll 1$, for all i the procedure for determining the net polarization response of a heterogeneous medium reduces to the following two steps.

1. Obtain the mathematical or experimental (model) apparent resistivity solution of the problem as a function of the individual media resistivity values using the electrode configuration desired.

2. Differentiate the logarithm of the apparent resistivity curve, mathematically or graphically, with respect to the resistivity of each component.

The polarization factor, or "chargeability" m_i of the i th medium is then weighted in proportion to (2).

This is a direct procedure and remarkably simple in relation to the dictates of the older dielectric representation which involved elaborate integration of volume dipolar effects.

Significance of Conductivity Contrast

As formula 6'' indicates, the polarization effect of an anomalous body is related both to its polarization contrast and to the effect which a change in its resistivity would have on the overall apparent resistivity picture.

In a variety of examples where precise mathematical solutions are possible, it is found that when the ratio of ρ_2/ρ_1 becomes either very large or very small, a small percentage change in ρ_2 will no longer affect the voltage distribution (and apparent resistivity) in the outside medium. This might be termed a "saturation" phenomenon. In either saturation region, therefore, the weighting factor

$$\frac{\partial \log \rho_a}{\partial \log \rho_2}$$

will be very small and likewise the polarization effect for given m_2 and m_1 .

As a practical consequence, the overvoltage response of a concentrated sulphide body of relatively high conductivity would be less than one might have expected from its percentage of sulphide mineralization. For this reason and because interconnected sulphide particles give overvoltage effects only at their points of current entry and exit—the overvoltage method is rather weak in the realm of concentrated sulphides (25 percent or greater). Its best field of application is in the realm of investigation of truly disseminated sulphide bodies, as, for example, most "porphyry" copper-type deposits.

As an example of the saturation effect, one may refer to Figure 8 of Bleil's paper (1953), which shows that the overvoltage effect of the Gossan Lead (pyrrhotite) body decreases as the more massive portion of the body, is approached.

Electrode Arrays

An electrode array for resistivity or induced polarization surveys generally consists of four earthed contacts, two for current passage and two for voltage measurements. Three different arrays that have been employed are as follows

1. *Wenner or Four-Electrode Array*. This is the classic array consisting of four collinear, equally-spaced electrodes with common separation designated a . When surveying with this type of array, all four electrodes are moved in unison along a line. The spacing a is selected on the basis of the desired depth of penetration, i.e., probable depth to the target desired. Measurements are plotted against the mid-point of the spread.

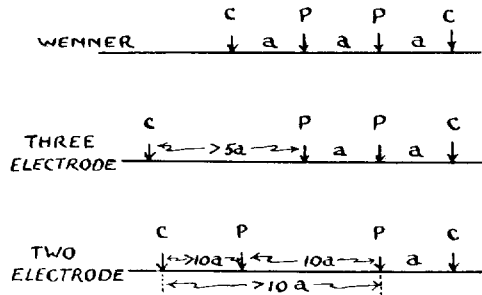


FIG. 3. Typical electrode arrays.

2. *Three-Electrode Array.* When one of the current electrodes from the Wenner array is moved to a point a great distance, such as $5a$, from all the other electrodes, its contribution to the voltage picture will become so small (usually) that it may be ignored in calculations. Now only the remaining three equi-spaced electrodes need be moved in unison during a survey. Measurements are plotted against the mid-point of the current and first potential electrode.
3. *Two-Electrode Array.* When one of the current electrodes and its adjacent potential electrode of the Wenner array are moved to locations a large distance, such as $10a$, from each other and from the two remaining electrodes we are left with the two-electrode array. Only the two remaining electrodes need now be moved along lines to carry out a survey. The far electrode need not be collinear with the near electrode. Measurements are plotted against the position of the mid-point of the current and near potential electrode (see Figure 3).

Each of these various electrode arrays has its own advantages and disadvantages. For example, it is not surprising that the removal of one or more electrodes out of the field of influence of the other increases the effective depth of penetration of the array. This is illustrated in the Example 1, the Response of a Sphere, which follows below.

Example 1. The Response of a Sphere. The effect of a sphere on the voltage distribution due to current flow from a point current source may, to a good approximation, be represented by the effect of the same sphere in a uniform field equal in magnitude and direction to that of the point source at its center (Stratton, 1941).

Consider a sphere of volume V , resistivity ρ_2 , chargeability m_2 , in a medium of resistivity ρ_1 , chargeability m_1 in the presence of a uniform electric field of strength E , inclined at an angle θ to the axis of a spherical co-ordinate system (r, θ, ϕ) with the origin of the system at the center of the sphere (see Figure 4). The modification to the potential distribution in the external medium due to the presence of the sphere may be expressed as (Stratton, 1941)

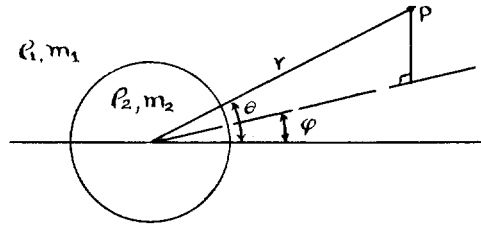


FIG. 4. Co-ordinate system for response of a sphere.

$$\Delta\Phi = E \frac{3V}{2\pi} \frac{\cos \theta}{r^2} \left(\frac{\rho_2 - \rho_1}{\rho_1 + 2\rho_2} \right). \quad (10)$$

The value of E , for a current electrode C on the surface of the earth is $I/2\pi R^2$, where I is the current and R the distance from the electrode to the center of the sphere (see Figure 5).

The voltages and apparent resistivities measured in the presence of the sphere will consist of two parts, the first due only to the external medium and the second to the disturbing effect of the sphere of anomalous material. Both will be functions of the array selected and the geometry of the situation.

On the assumption that the primary effect on the apparent resistivity is small, we can, to the first approximation, separate the geometric factor from the physical property factor. It can be shown that we may write

$$\frac{m_a - m_1}{m_2 - m_1} = F \frac{9V}{2\pi d^3} \rho_1 \rho_2 / (\rho_1 + 2\rho_2)^2. \quad (11)$$

Where F is a form factor depending on the array employed, the electrode spacing relative to the depth " d " of the center of the sphere below the array, and the lateral displacement of the line of traverse relative to the vertical axis through

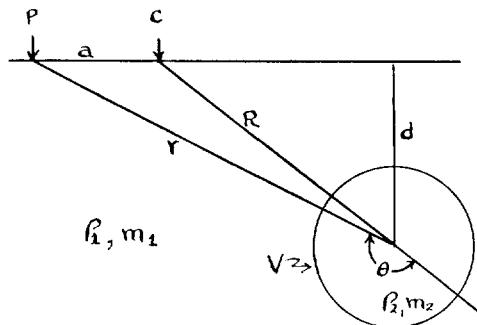


FIG. 5. The buried sphere parameters.

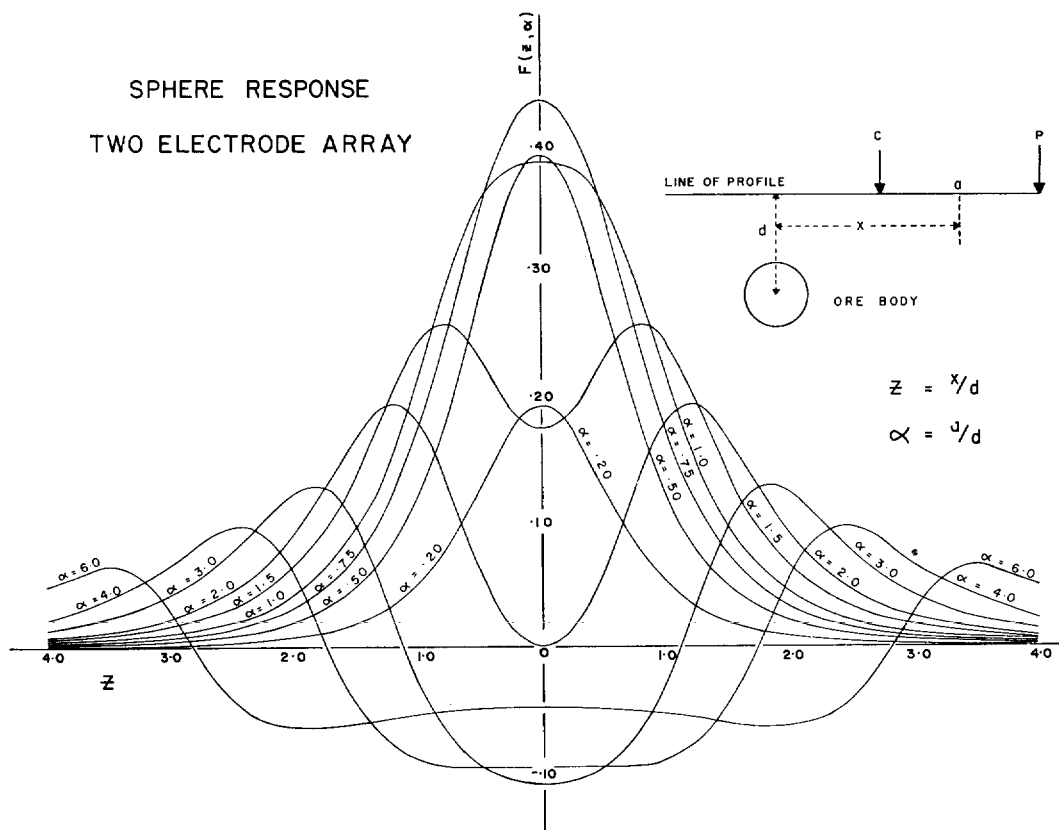


FIG. 6. Sphere response: two-electrode arrays.

the center of the sphere. Incidentally, the same form factor applies for the shape of the corresponding resistivity anomaly curve. That portion of the right hand side of (11), which entails only the resistivities, is of interest. It is actually dimensionless and depends only on the ratio ρ_2/ρ_1 . For very large or very small values of this ratio, this resistivity factor approaches zero. This is the "saturation effect" referred to earlier. The factor is a maximum when $\rho_2/\rho_1 = 0.5$, i.e., when the sphere is just twice as conductive as its surroundings. This illustrates the optimum efficiency of the overvoltage method on detection of moderate disseminations of sulphide mineralization.

The form factor F has been plotted in Figure 6 for the two-electrode array and in Figure 7 for the three-electrode array. The following points are worthy of note:

1. The curve form depends on the electrode array and the value of α , the ratio of the spacing to the minimum distance to the center of the sphere.

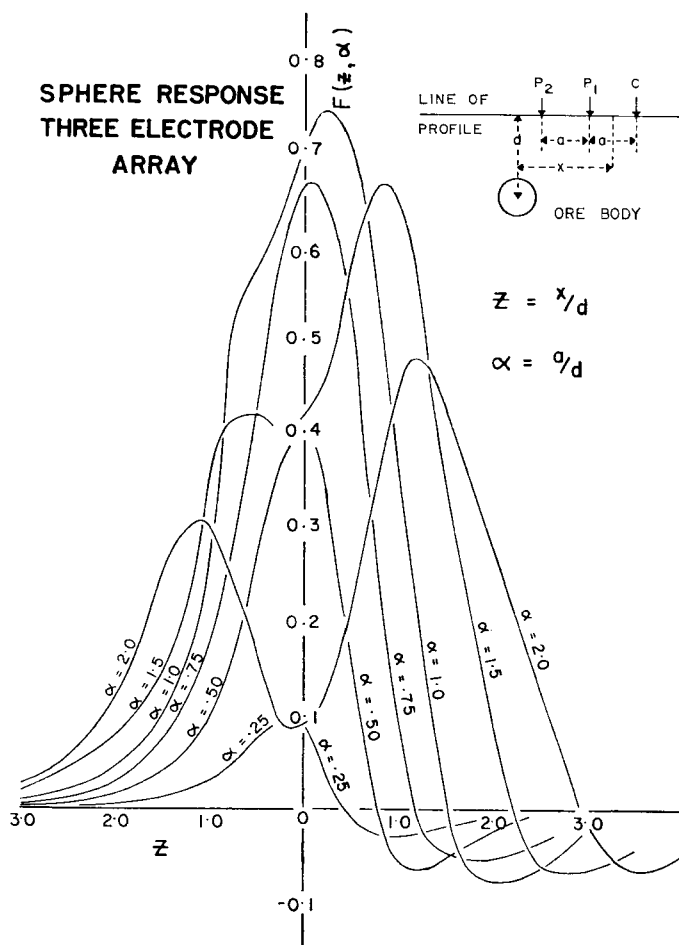


FIG. 7. Sphere response: three-electrode arrays.

2. For each array the response increases with α until a certain value is obtained, then decreases. The optimum value of α has been found to be
 - 0.70 for the two-electrode,
 - 1.1 for the three-electrode,
 - 1.2 for the Wenner array.

There is thus some meaning to the term "depth of penetration" and it may be said that for a given electrode separation, the two-electrode, three-electrode, and Wenner arrays stand in order of decreasing depth penetration.

3. The maximum anomalous indication for any spacing on the various arrays stands in the ratio 2/1. 2/1 with the three-electrode array leading and the Wenner lagging all.

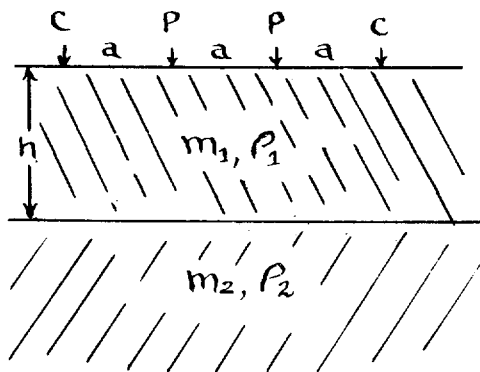


FIG. 8. The two-layer parameters.

The above are some of the factors which may influence the selection of the array to use for a particular problem.

Example 2. The Two-Layer Case, Wenner Array. The mathematical resistivity solution of the two-layer case has been variously presented (e.g., Roman, 1934).

It is assumed that the upper medium is characterized by resistivity ρ_1 and chargeability m_1 and the lower medium by parameters ρ_2 and m_2 . The electrode spacing is a and the thickness of the upper layer is h , both in arbitrary units. The array employed is the Wenner (see Figure 8).

As the solution of the resistivity problem we may write as apparent resistivity

$$\rho_a = \rho_1 \left(1 + 4 \sum_{n=1}^{\infty} \frac{k^n}{(1 + 4n^2/\alpha^2)^{1/2}} - 2 \sum_{n=1}^{\infty} \frac{k^n}{(1 + n^2/\alpha^2)^{1/2}} \right), \quad (12)$$

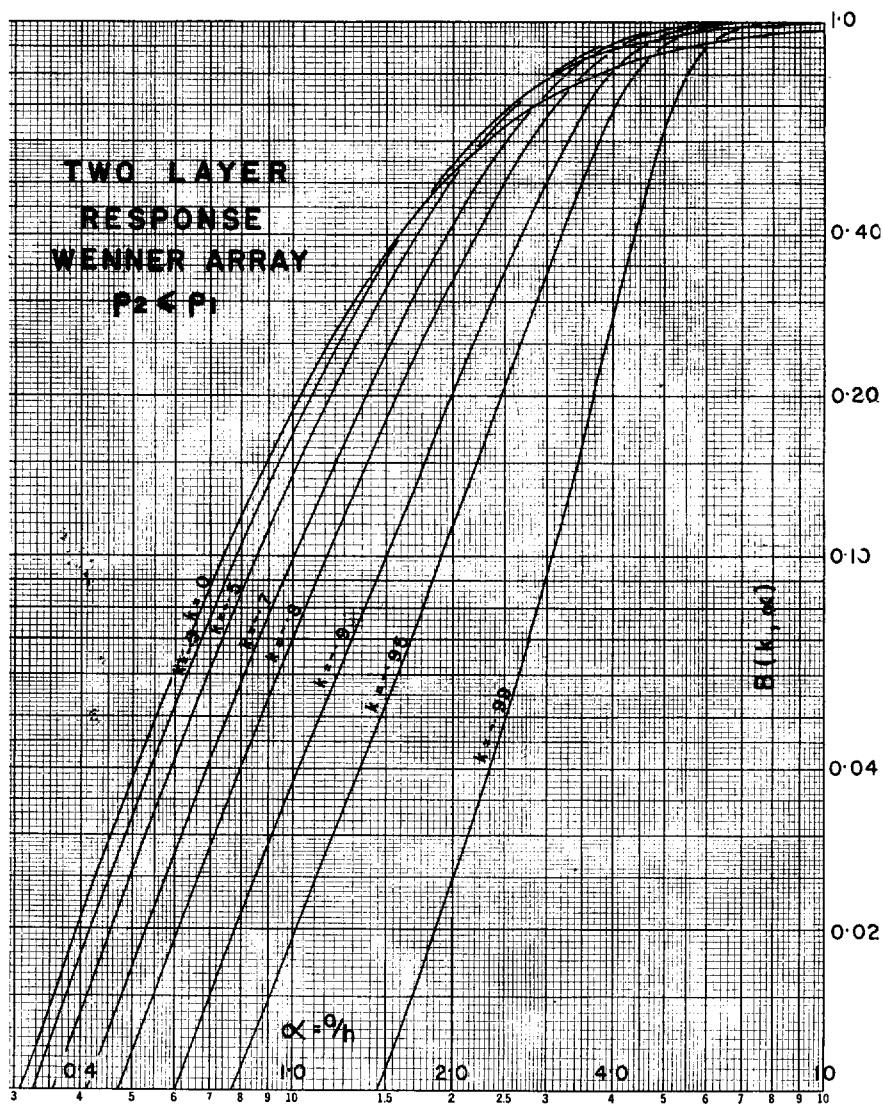
where

$$k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}, \quad \text{and} \quad \alpha = a/h.$$

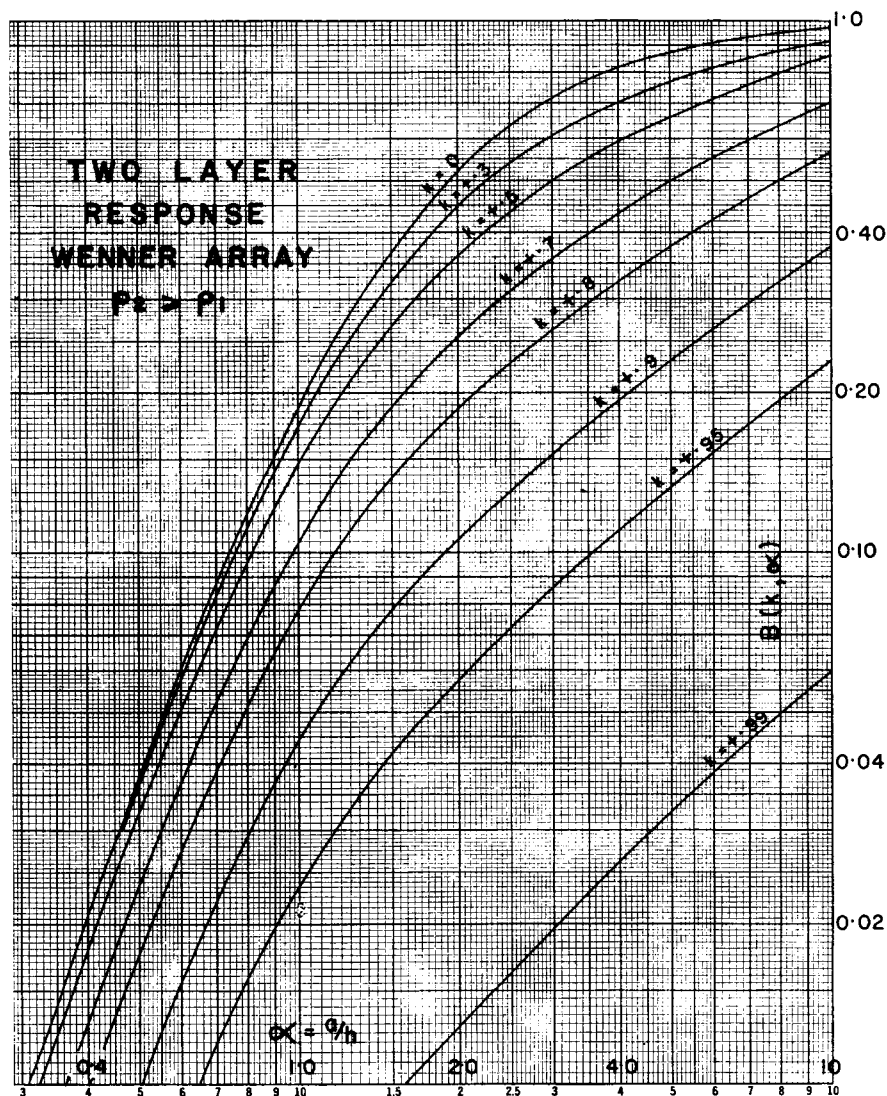
A plot of ρ_a/ρ_1 versus α may be seen, for example, in the publication of Mooney and Wetzel (1956).

$$\begin{aligned} \frac{\partial \log \rho_a}{\partial \log \rho_2} &= \frac{m_a - m_1}{m_2 - m_1} \\ &= \frac{\rho_1}{\rho_a} (1 - k^2) \left[2 \sum_{n=1}^{\infty} \frac{nk^{n-1}}{(1 + 4n^2/\alpha^2)^{1/2}} - 2 \sum_{n=1}^{\infty} \frac{nk^{n-1}}{(1 + n^2/\alpha^2)^{1/2}} \right] \end{aligned} \quad (13)$$

The function on the right, called for simplicity $B(k, \alpha)$ is plotted on the accompanying charts, on a log-log basis as a function of α for negative values of k in Figure 9 (i.e., $\rho_2 < \rho_1$) and for positive values of k in Figure 10.

FIG. 9. Two-layer response, k negative.

To illustrate the application of these curves to actual field examples, two pairs of field curves are presented. The first (Figure 11) was obtained in Custer County, Colorado, where disseminated pyritic mineralization is widespread in an acid intrusive stock, but where there has been complete oxidation of sulphides down to the present water table (about 20'-30'). On the log-log chart are shown the


 FIG. 10. Two-layer response, k positive.

theoretical curves which fit the experimental resistivity and chargeability curves. The experimental data are well fitted by the following parameters:

$\rho_1 = 135$ ohm meters, $\rho_2 = 30$ ohm meters ($k = -0.6$);

$h = 24$ ft, $m_1 = 10$, $m_2 = 72.0 \times 10^{-3}$ seconds.

The second example (Figure 12) is from Cuajone, Peru, where disseminated

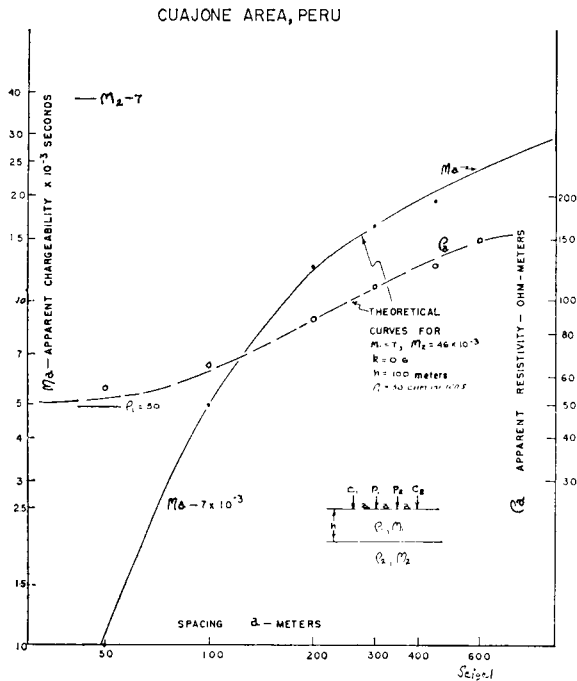


FIG. 12. Field example: Cuajone, Peru.

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