Lab Assigment

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18EX20030

For horizontal reflection,

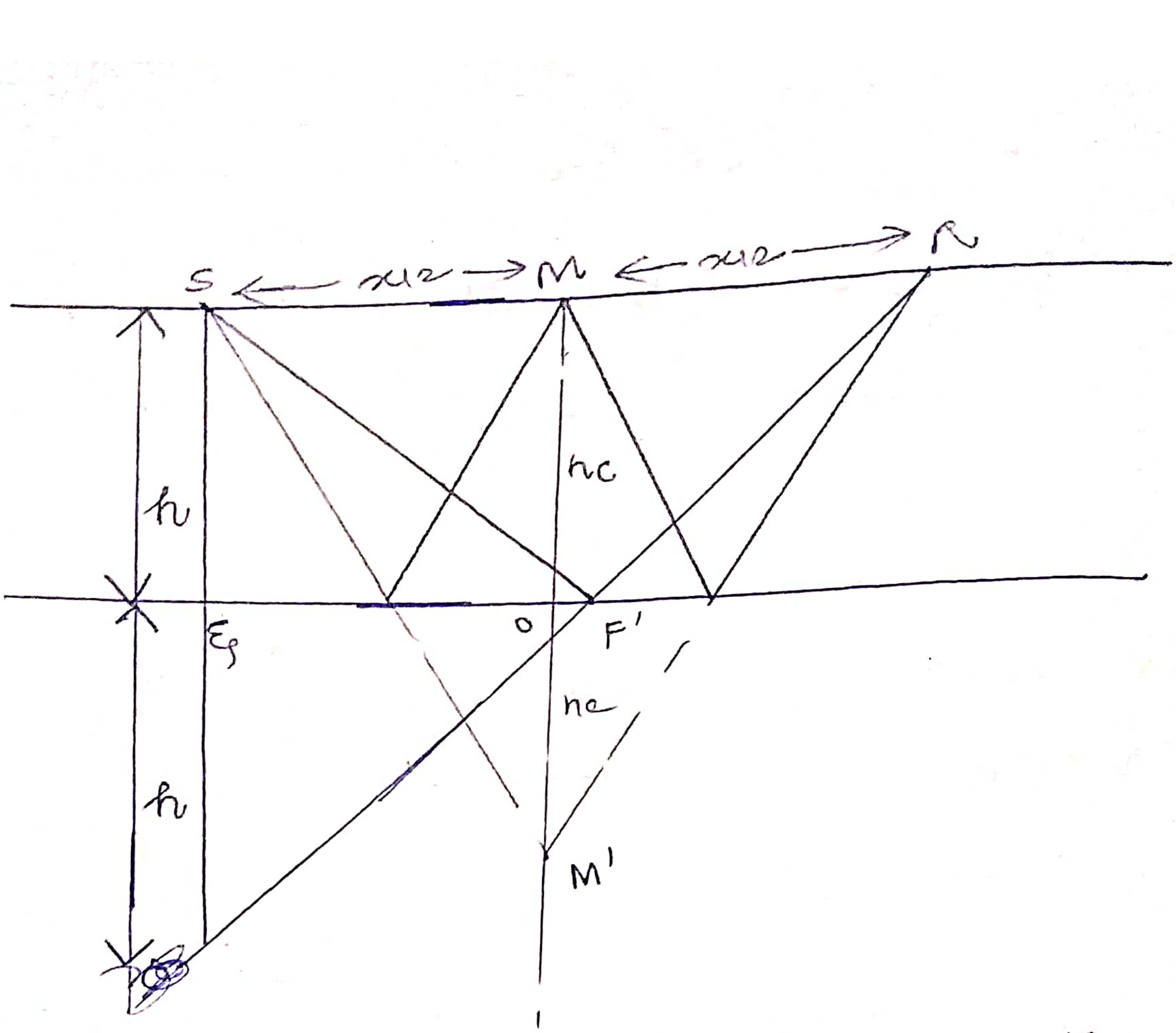
$$V^2 tA^2 = x^2 + 4h^2$$

$$\Rightarrow tA^2 = \frac{x^2}{V^2} + \frac{4h^2}{V^2}$$

From this equation we ~~we~~ get slope of plot tA^2 vs $x^2 = \frac{1}{V^2}$.

dipping reflection,

$$V^2 tB^2 = x^2 + 4h^2 - 4hx \sin\xi$$



Substituting $h = h_c + \frac{x}{2}\sin\xi$ we get

$$V^2 t_B^2 = x^2 + 4\left(h_c + \frac{x}{2}\sin\xi\right)^2 - 4x\sin\xi \times \left(h_c + \frac{x}{2}\sin\xi\right)$$

$$\Rightarrow \quad V^2 t_B^2 = x^2\cos^2\xi + 4h_c^2$$

$$\Rightarrow \quad t_B^2 = \frac{(x\cos\xi)^2}{V^2} + \frac{4h_c^2}{V^2}$$

$\therefore$ slope of the plot t_B^2 vs $(x\cos\xi)^2 = \frac{1}{V^2}$

Table

x^2	$x^2\cos^2\theta$	t_A^2	t_B^2
0	0	0.731	0.820
0.01	0.009	0.733	0.814
0.04	0.038	0.736	0.806
0.09	0.087	0.746	0.806
0.16	0.155	0.753	0.808
0.25	0.242	0.764	0.814
0.36	0.349	0.772	0.815
0.49	0.475	0.796	0.826
0.64	0.620	0.817	0.839
0.81	0.785	0.821	0.850
1.00	0.970	0.865	0.869
1.21	1.173	0.893	0.889
1.44	1.396	0.902	0.902
1.69	1.639	0.958	0.931
1.96	1.901	1.010	0.955
2.25	2.182	1.034	0.982
2.56	2.483	1.075	1.008
2.89	2.803	1.14	1.038
3.24	3.142	1.168	1.075
3.16	3.501	1.221	1.119
4.00	3.82	1.25	1.136
4.41	4.277	1.325	1.172
4.84	4.694	1.359	1.214
5.29	5.131	1.447	1.257
5.76	5.587	1.53	1.270
6.25	6.062	1.575	1.340
6.76	6.557	1.646	1.385

x^2	$(x\cos\xi)^2$	t_A^2	t_B^2
7.29	7.071	1.70	1.428
7.84	7.604	1.769	1.445
8.41	8.157	1.850	1.523
9.00	8.730	1.97	1.570
9.61	9.321	2.05	1.618
10.24	9.932	2.172	1.679
10.89	10.563	2.21	1.780
11.56	11.213	2.289	1.779
12.25	11.882	2.396	1.839
12.96	12.571	2.496	1.896
13.69	13.279	2.596	1.980
14.44	14.006	2.719	2.002
15.22	14.753	2.802	2.068
16.00	15.520	2.917	2.129

From graph of t_A^2 vs x^2 for horizontal reflection,

slope $= \frac{1}{v^2} = \frac{0.55}{4.125}$

$\Rightarrow$ $v^2 = 7.5\ km^2/s^2$

$\therefore$ $v = 2.738\ km/s$

From graph ② t_B^2 vs $(x\cos\xi)^2$ for dipping reflection,

Slope $= \frac{1}{v^2} = \frac{0.2}{2.25}$

$\Rightarrow$ $v^2 = 11.25\ km^2/s^2$

$\therefore$ $v = 3.35\ km/s.$

VRMS is defined as,

$$V^2_{rms \cdot n} = \frac{\sum_{i=1}^{n} V_i^2 \Delta t_i}{\sum_{i=1}^{n} \Delta t_i} \quad \text{---} (1)$$

again, $$V^2_{rms \cdot n-1} = \frac{\sum_{i=1}^{n-1} V_i^2 \Delta t_i}{\sum_{i=1}^{n-1} \Delta t_i} \quad \text{---} (2)$$

where V_i is the interval velocity of layer.

subtracting eq (2) from eq (1) we get,

$$V^2_{rms\, n} \cdot \sum_{i=1}^{n} \Delta t_i - V^2_{rms, n-1} \cdot \sum_{i=1}^{n-1} \Delta t_i = \sum_{i=1}^{n} V_i^2 \Delta t_i - \sum_{i=1}^{n-1} V_i^2 \Delta t_i$$

$$\Rightarrow V_n^2 \Delta t_n = V^2_{rms \cdot n} \cdot \Delta t_n - V^2_{rms, n-1} \Delta t_{n-1}$$

$$\Rightarrow V_n = \sqrt{\frac{V^2_{rms \cdot n} \Delta t_n - V^2_{rms\, n-1} \Delta t_{n-1}}{\Delta t_n}}$$

$$\Rightarrow V_n = \sqrt{\frac{V^2_L t_L - V^2_n t_n}{t_L - t_n}}$$

Given,

i	Z(Km)	t_i (s)	$V_{rms, i}$ (km/s)
1	1.20	1.100	2.18
2	2.50	1.786	2.80
3	3.10	1.935	3.20
4	4.10	2.250	3.64

For $i=1$, $V_1 = 2.18$ km/s.

For $i=2$, $$V_2 = \sqrt{\frac{V_2^2 t_2 - V_1^2 t_1}{t_2 - t_1}}$$

$$= \sqrt{\frac{(2.80)^2 \times 1.786 - (2.18)^2 \times 1.100}{1.786 - 1.100}} = 3.5\cancel{67}6 \text{ km/s}$$

For $i=3$, $V_3 = \sqrt{\dfrac{V_3^2 t_3 - V_2^2 t_2}{t_3 - t_2}}$

$$= \sqrt{\frac{(3.2)^2 \times 1.935 - (2.80)^2 \times 1.786}{1.935 - 1.786}} = 6.2456 \text{ km/s}$$

For $i=4$, $V_4 = \sqrt{\dfrac{V_4^2 t_4 - V_3^2 t_3}{t_4 - t_3}}$

$$= \sqrt{\frac{(3.64)^2 \times 2.250 - (3.20)^2 \times 1.935}{2.250 - 1.935}}$$

$$= 5.633 \text{ km/s}$$

(30) Thickness of each layer = 300m.

Velocity of each layer → $V_1 = 1.5$ km/s, $V_2 = 1.8$ km/s, $V_3 = 2.1$ km/s, $V_4 = 2.4$ km/s, $V_5 = 2.7$ km/s, $V_6 = 3.0$ km/s.

For normal incident,

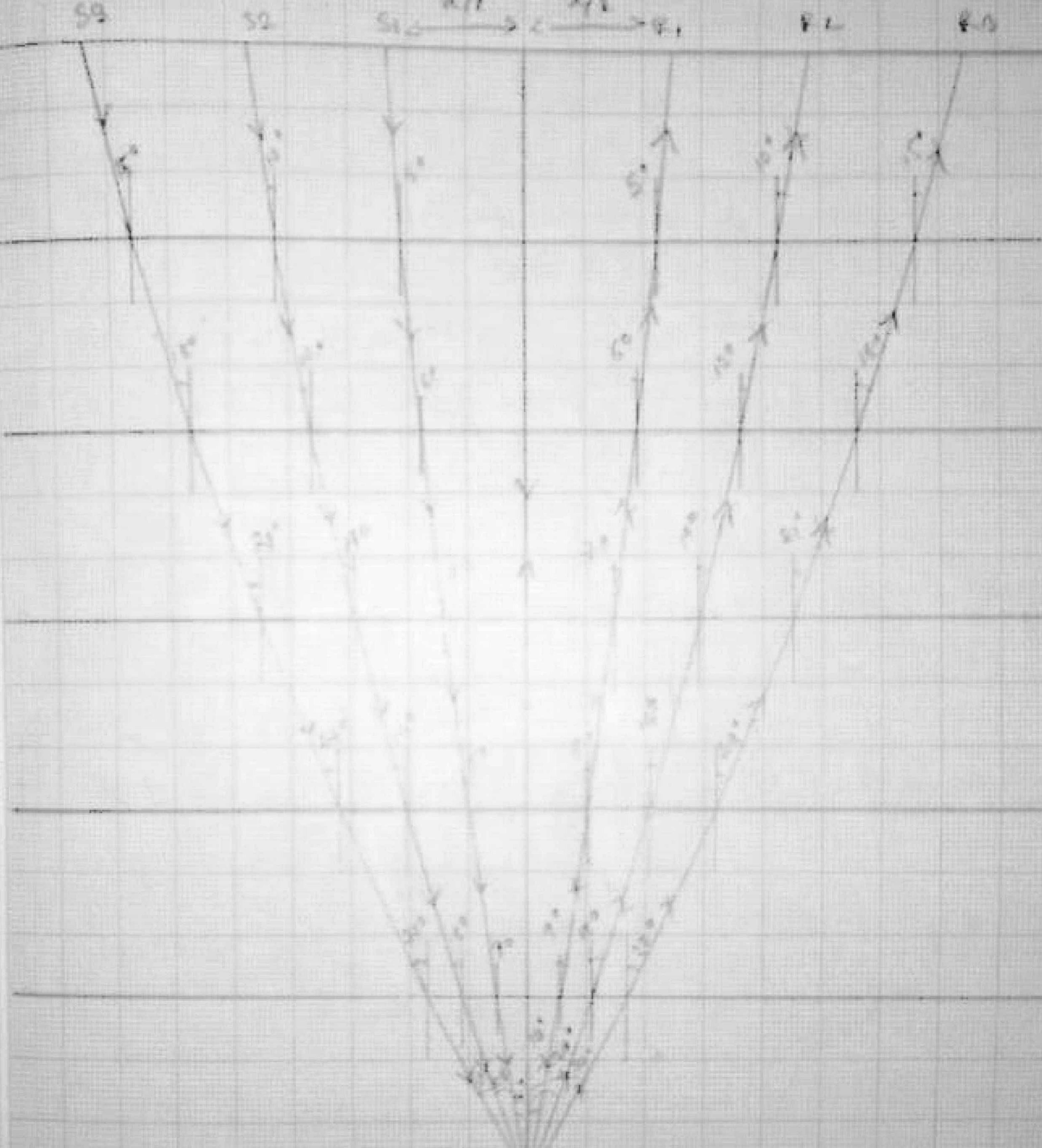
$$t_0 = 2\left[\frac{300}{1.5} + \frac{300}{1.8} + \frac{300}{2.1} + \frac{300}{2.4} + \frac{300}{2.7} + \frac{300}{3.0}\right]$$

$$\Rightarrow \quad t_0 = 1.691 \text{ s}.$$

Case 1 :- For $\theta = 10^\circ$

$$\frac{x}{2} = 300\left[\tan 10^\circ + \tan 3^\circ + \tan 8^\circ + \tan 7^\circ + \tan 6^\circ + \tan 5^\circ\right]$$

$$x = 474.37 \text{ m}.$$



S3
S2
S1
x/2
C
x/2
R1
R2
R3
A

$$a_1 = 2\left[\frac{300}{\cos 10^\circ} + \frac{300}{\cos 9^\circ} + \frac{300}{\cos 8^\circ} + \frac{300}{\cos 7^\circ} + \frac{300}{\cos 6^\circ} + \frac{300}{\cos 5^\circ}\right]$$

$$\Rightarrow \quad a_1 = 3632.73\,m.$$

$$t_1 = 2\left[\frac{300}{\cos 10^\circ} \times \frac{1}{3.0} + \frac{300}{\cos 9^\circ} \times \frac{1}{2.7} + \frac{300}{\cos 8^\circ} \times \frac{1}{2.4} + \frac{300}{\cos 7^\circ} \times \frac{1}{2.1} + \frac{300}{\cos 6^\circ} \times \frac{1}{1.8} + \frac{300}{\cos 5^\circ} \times \frac{1}{1.5}\right]$$

$$\Rightarrow \quad t_1 = 1.7058\,s$$

$$V_{\text{stack}} = \sqrt{\frac{x_2}{t_1^2 - t_0^2}} = \sqrt{\frac{(474.37)^2}{(1.7098)^2 - 1.691)^2}} = 2.175\,Km/s$$

$$V_{rms} = \sqrt{\frac{\Sigma V_i^2 \Delta t_i}{\Sigma \Delta t_i}}$$

$$= \left[\left\{ (1.5)^2 \times \frac{300}{\cos 5^\circ \times 1.5} + (1.8)^2 \times \frac{300}{\cos 6^\circ \times 1.8} + (2.1)^2 \times \frac{300}{\cos 7^\circ \times 2.1} + (2.4)^2 \times \frac{300}{\cos 8^\circ \times 2.4} + (2.7)^2 \times \frac{300}{\cos 9^\circ \times 2.7} + (3.0)^2 \times \frac{300}{\cos 10^\circ \times 3.0} \right\}\right.$$

$$\left. \div \left\{ \frac{300}{\cos 5^\circ \times 1.5} + \frac{300}{\cos 6^\circ \times 1.8} + \frac{300}{\cos 7^\circ \times 2.1} + \frac{300}{\cos 8^\circ \times 2.4} + \frac{300}{\cos 9^\circ \times 2.7} + \frac{300}{\cos 10^\circ \times 3.0} \right\} \right]^{1/2} = 2.190\,Km/s.$$

$$\Rightarrow \quad V_{rms} = 2.190\,Km/s.$$

$$V_{avg} = \frac{\Sigma z_i}{\Sigma t_i} = \frac{\left(\frac{300}{\cos 5^\circ} + \frac{300}{\cos 6^\circ} + \frac{300}{\cos 7^\circ} + \frac{300}{\cos 8^\circ} + \frac{300}{\cos 9^\circ} + \frac{300}{\cos 10^\circ}\right)}{\frac{Z_1}{1.5} + \frac{Z_2}{1.8} + \frac{Z_3}{2.1} + \frac{Z_4}{2.4} + \frac{Z_5}{2.7} + \frac{Z_6}{3.0}}$$

$$= 2.130\,Km/s$$

$$\Rightarrow \quad V_{avg} = 2.130\,Km/s.$$

Case 2 :- For $\theta = 20^\circ$

$$\frac{x}{2} = 300\left[\tan 20^\circ + \tan 18^\circ + \tan 16^\circ + \tan 14^\circ + \tan 12^\circ + \tan 10^\circ\right]$$

$$= 961\text{m}$$

$$t_2 = \left[\frac{300}{\cos 20^\circ \times 3.0} + \frac{300}{\cos 18^\circ \times 2.7} + \frac{300}{\cos 16^\circ \times 2.4} + \frac{300}{\cos 14^\circ \times 2.1} + \frac{300}{\cos 12^\circ \times 1.8} + \frac{300}{\cos 10^\circ \times 1.5}\right]$$

$$= 1.747\text{s.}$$

$$V_{stack} = \sqrt{\frac{(961)^2}{(1.747)^2 - (1.691)^2}} = 2.19\text{km/s.}$$

$$V_{rms} = \sqrt{\frac{\Sigma V_i^2 \Delta t_i}{\Sigma \Delta t_i}} = 2.197\text{km/s.}$$

$$V_{Avg} = \frac{\Sigma z_i}{\Sigma \Delta t_i} = \frac{\left(\frac{300}{\cos 20^\circ} + \frac{300}{\cos 18^\circ} + \frac{300}{\cos 16^\circ} + \frac{300}{\cos 14^\circ} + \frac{300}{\cos 12^\circ} + \frac{300}{\cos 10^\circ}\right)}{\left(\frac{300}{\cos 20^\circ \times 3.0} + \frac{300}{\cos 18^\circ \times 2.7} + \frac{300}{\cos 16^\circ \times 2.4} + \frac{300}{\cos 14^\circ \times 2.1} + \frac{300}{\cos 12^\circ \times 1.8} + \frac{300}{\cos 10^\circ \times 1.5}\right)}$$

$$= 2.137\text{km/s.}$$

Case 3 :- For $\theta = 30^\circ$

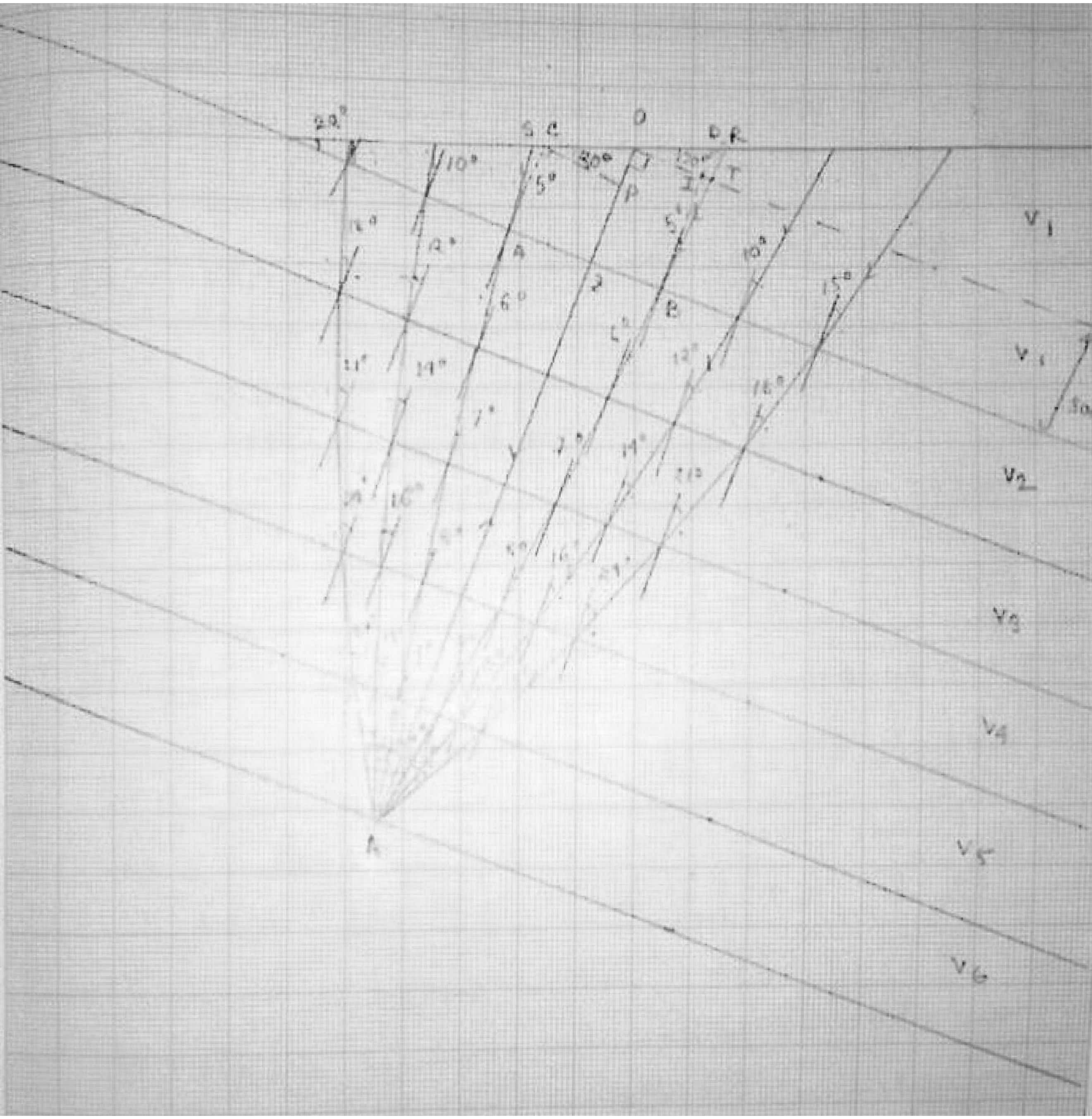
$$\frac{x}{2} = 300\left[\tan 30^\circ + \tan 27^\circ + \tan 24^\circ + \tan 21^\circ + \tan 18^\circ + \tan 15^\circ\right] = 1475\text{m.}$$

$$t_3 = 2\left[\frac{300}{\cos 30^\circ \times 3.0} + \frac{300}{\cos 27^\circ \times 2.7} + \frac{300}{\cos 24^\circ \times 2.4} + \frac{300}{\cos 21^\circ \times 2.1} + \frac{300}{\cos 18^\circ \times 1.8} + \frac{300}{\cos 15^\circ \times 1.5}\right] = 1.820\text{s}$$

$$V_{stack} = \sqrt{\frac{(1478)^2}{(1.820)^2 - (1.691)^2}} = 2.196\text{km/s.}$$

$$V_{RMS} = \sqrt{\frac{\Sigma V_i^2 \Delta t_i}{\Sigma \Delta t_i}} = 2.207\text{km/s}$$

$$V_{Avg} = \frac{\Sigma z_i}{\Delta t_i} = 2.147\text{km/s.}$$



S.C
O
D.R
V1
V2
V3
V4
V5
V6

(36) For normal incidence, $t_0 = 1.6913$

Case 1 :- For $\theta = 10^\circ$

$ZB = 300 \times [\tan 10^\circ + \tan 9^\circ + \tan 8^\circ + \tan 7^\circ + \tan 6^\circ]$

$= 210.94m$

$BI = 300m$

$BT = \frac{300}{\cos 5^\circ} = 301.14m$

$DI = OI \times \tan 20^\circ = 110$

In $\triangle RDB$, $\frac{\sin 5^\circ}{RD} = \frac{\sin 65^\circ}{DI + IB}$ $\left[\begin{array}{l}\angle DBR = 45^\circ, \angle RDB = 110^\circ \\ \angle DRB = 180^\circ - 110^\circ - 5^\circ = 65^\circ\end{array}\right]$

$\therefore RD = \frac{\sin 5^\circ}{\sin 65^\circ} \times (76.77 + 300) = 36.23m$

$OI = OD \cos 20^\circ$

$\Rightarrow OD = \frac{210.94}{\cos 20^\circ} = 224.47m$

$\therefore OR = OD + DR = 260.7m$

So on the receiver end,

Offset $= 260.7m$.

Again, $AZ = 300 [\tan 10^\circ + \tan 9^\circ + \tan 8^\circ + \tan 7^\circ + \tan 6^\circ]$

$= 210.94m$.

$OP = CP \tan 20^\circ = 210.94 \times \tan 20^\circ = 76.77m$.

Case 2 :- For $\theta = 20^\circ$

$ZB = 300 \times [\tan 20^\circ + \tan 18^\circ + \tan 16^\circ + \tan 14^\circ + \tan 12^\circ]$

$= 428.46m$.

$BI = 300m$

$BT = \frac{300}{\cos 10^\circ} = 304.44m$

$DI = OI \times \tan 20^\circ = 428.46 \times \tan 20^\circ = 155.94m$

$$RD = \frac{\sin 10^\circ}{\sin 60^\circ} \times (300 + 155.94) = 89.43\,m$$

$$OD = \frac{OI}{\cos 20^\circ} = 455.96\,m$$

$$OR = (455.96 + 89.43)\,m = 545.39\,m.$$

Again on the source side.

$$OP = 428.46 \times \tan 20^\circ = 155.95\,m$$

$$OC = \frac{428.46}{\cos 20^\circ} = 455.96\,m$$

$$OS = 455.96 + (300 - 155.95) \times \frac{\sin 10^\circ}{\sin 100^\circ} = 480.87\,m$$

Source to receiver $= OS + OR = 1026.26\,m.$

$$t_2 = 2\left[\frac{300}{\cos 20 \times 3.0} + \frac{300}{\cos 18^\circ \times 2.7} + \frac{300}{\cos 16^\circ \times 2.4} + \frac{300}{\cos 14^\circ \times 2.1} + \frac{300}{\cos 12^\circ \times 1.8}\right] + \frac{\sin 70^\circ}{\sin 10^\circ} + \frac{24.192}{1.5} + \frac{\sin 110^\circ}{\sin 10^\circ} \times \frac{89.43}{1.5} = 1.759\,s.$$

$$V_{stack} = \sqrt{\frac{(1026.26)^2}{(1.759)^2 - (1.691)^2}} = 2.118\,km/s.$$

$$V_{rms} = \sqrt{\frac{\Sigma V_i^2 \Delta t_i}{\Sigma \Delta t}} = \sqrt{\frac{\frac{2 \times 3 \times 500}{\cos 20^\circ} + \ldots (1.5)^2 [627.28]}{1.759}}$$

$$= 2.252\,km/s$$

$$V_{avg} = \frac{\Sigma z_i}{\Sigma t_i} = \frac{1489.20 \times 2}{1.759} = 2.132\,km/s.$$

Case 3 :- For $\theta = 30^\circ$

$ZB = 300 \times [\tan 30^\circ + \tan 27^\circ + \tan 24^\circ + \tan 21^\circ + \tan 18^\circ]$

$= 661.91 m$.

$BI = 300 m$.

$BT = \dfrac{300}{\cos 15^\circ} = 309.87 m$.

Also, $DI = OI \times \tan 20^\circ = 661.91 \times \tan 20^\circ = 240.91 m$.

$\angle RDB = 110^\circ$, $RD = \dfrac{\sin 15^\circ}{\sin 60^\circ} \times 540.91 = 164.33 m$.

$OD = \dfrac{OI}{\cos 20^\circ} = 704.38 m$.

$OR = OD + OR = 868.71 m$.

On the source side,

$AZ = 300 [\tan 30^\circ + \text{---} \tan 18^\circ] = 661.91 m$.

$OP = CP \tan 20^\circ = 240.91 m$

$OC = \dfrac{CP}{\cos 20^\circ} = \dfrac{661.91}{\cos 20^\circ} = 704.38 m$

$OS = 704.38 + (300 - 240.91) \times \dfrac{\sin 15^\circ}{\sin 45^\circ} = 719.31 m$

$SR = OS + OR = (719.31 + 868.71) m = 1588.02 m$.

$t_3 = 2 \left[\dfrac{300}{\cos 30^\circ \times 3.0} + \text{———} \dfrac{300}{\cos 18^\circ \times 1.8} \right]$

$+ \dfrac{\sin 70^\circ}{\sin 15^\circ} \times \dfrac{14.498}{1.5} + \dfrac{\sin 110^\circ}{\sin 15^\circ} \times \dfrac{64.33}{1.5}$

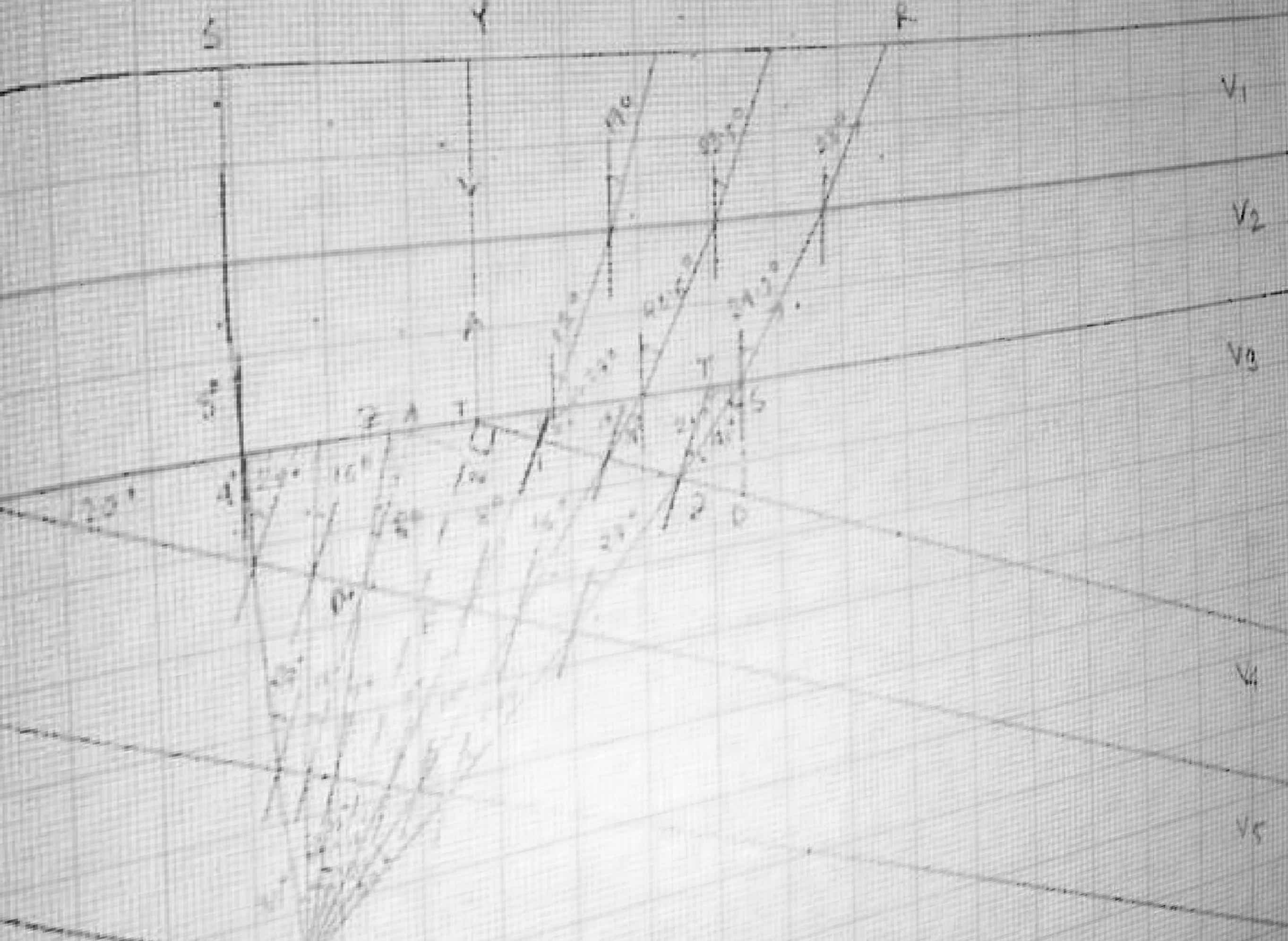
$= 1.855 s$

$V_{stack} = \sqrt{\dfrac{(1588.02)^2}{(1.855)^2 - (.697)^2}} = 2.082 km/s$.

$V_{rms} = \sqrt{\dfrac{\dfrac{2 \times 3 \times 300}{\cos 30^\circ} + \cdots (1.5)^2 \times (6.728)}{1.855}}$

$= 2.257 km/s$

$V_{avg} = \dfrac{\Sigma z_i}{\Sigma t_i} = \cancel{3.185} \; 2.135 km/s$.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(3C) case 1, $\theta_i = 10^\circ$

Angle of incidence at $C = 10^\circ$.

$OZ = 300 \times (\tan 10^\circ + \tan 9^\circ + \tan 8^\circ] = 142.57\,m.$

$\angle STZ = 110^\circ$

$\angle TSZ = 180^\circ - 110^\circ - 7^\circ = 63^\circ.$

In ΔSTZ, $\dfrac{\sin 7^\circ}{ST} = \dfrac{\sin 63^\circ}{ZT}$

$\Rightarrow ST = \dfrac{\sin 7^\circ}{\sin 63^\circ} \times [142.57 \times \tan 20^\circ] = 7.095\,m.$

Thus the ray is incident at 27° to $V_2 - V_3$ interference.

$OS = OT + TS = \dfrac{OZ}{\cos 20^\circ} + TS = \dfrac{142.57}{\cos 20^\circ} + 7.095$

$\therefore OS = 158.81\,m.$

Again, $\dfrac{\sin \theta_2}{\sin 27^\circ} = \dfrac{V_2}{V_3} \Rightarrow \theta_2 = \sin^{-1}\left(\dfrac{1.8}{2.1} \times \sin 27^\circ\right)$

$\Rightarrow \theta_2 = 23^\circ$

Also, $\dfrac{\sin \theta_1}{\sin 23^\circ} = \dfrac{1.5}{1.8} \Rightarrow \theta_1 = \sin^{-1}\left(\dfrac{1.5}{1.8} \times \sin 23^\circ\right) = 18.92^\circ$

$\therefore TR = OS + 300 \tan 23^\circ + 300 \times \tan 19^\circ = 388.364\,m.$

From source scale,

$O'F = 300 \times [\tan 10^\circ + \tan 40^\circ]$

$\Rightarrow O'F = AW = 100.41\,m,$

$FT = 300\,m.$

$\tan 20^\circ = \dfrac{TW}{AM} \Rightarrow TW = 36.54\,m$

$\therefore FW = 300 - TW = 263.46\,m = AO'$

In $\triangle ZAO'$

$$\frac{ZA}{\sin 8^\circ} = \frac{263.46}{\sin 102^\circ} \rightarrow ZA = 37.48m$$

Also,

$$\frac{AW}{AT} = \cos 20^\circ \rightarrow AT = \frac{100.41}{\cos 20^\circ} = 106.85m$$

$ZT = 144.33m.$

At $V_2 - V_4$ interference, Angle of incidence $= 12^\circ$

$$\frac{\sin\theta_2}{\sin 12^\circ} = \frac{V_2}{V_4} \Rightarrow \theta_2 = \sin^{-1}\left[\sin 12^\circ \times \frac{1.8}{2.4}\right] = 9^\circ$$

Also,

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{V_1}{V_2} \Rightarrow \theta_1 = \sin^{-1}\left(\sin 9^\circ \times \frac{1.5}{1.8}\right) = 7.46^\circ$$

$\therefore$ $SY = ZT + 300 \times \tan 9^\circ + 300 \times \tan 7.46^\circ = 230.96m.$

$\therefore$ Total source to receiver offset $= SY + YR$

$= (230.96 + 388.36)m = 619.32m.$

$$V_{stack} = \sqrt{\frac{x_2}{t_1^2 - t_0^2}}$$

$$t_1 = 2\left[\frac{300}{3 \times \cos 10^\circ} + \frac{300}{2.7 \times \cos 9^\circ}\right] + \frac{300}{2.4 \times \cos 8^\circ} +$$

$$\frac{300}{1.8 \times \cos 23^\circ} + \frac{300}{1.5 \times \cos 19^\circ} + \frac{300}{1.8 \times \cos 9^\circ} +$$

$$\frac{300}{1.5 \times \cos 7.46^\circ} + \frac{37.48}{\sin 8^\circ} + \frac{\sin 9'}{2.4} + \frac{7.098}{\sin 7^\circ} \times \frac{\sin 110^\circ}{2.1}$$

$t_1 = 1.4483.$

$$t_0 = 2\left(\frac{300}{3.0} + \frac{300}{2.7} + \frac{300}{2.4}\right) + 2\left(\frac{300}{1.8} + \frac{300}{1.5}\right)$$

$= 1.9053.$

$$V_{stack} = \sqrt{\frac{(619.32)^2}{(1.448)^2 - (1.105)^2}} = 1.780km/s.$$

$$V_{rms} = \sqrt{\frac{\Sigma V_i^2 t_i}{\Sigma t_i}} = 2.195 \text{ Km/s}.$$

Case 2 :- $\theta_i = 20°$

Angle of incidence at C $= 20°$

$OZ = [\tan 20° + \tan 17.9° + \tan 18.9°] \times 300$

$= 291.54 m.$

$\angle STZ = 110°$

$\angle TSZ = 180° - 110' - 13.9° = 56.1°.$

In ΔSTZ, $\frac{\sin 13.9°}{ST} = \frac{\sin 56.1°}{ZT}$

$\Rightarrow ST = \frac{\sin 13.9°}{\sin 56.1°} \times [291.54 \times \tan 20°] = 30.71 m$

$\therefore$ Thus the ray is incident at 33.9° to $V_2 - V_3$ ~~interface~~ interface.

$OS = OT + TS = \frac{OZ}{\cos 20°} + TS = \left(\frac{29.59}{\cos 20°} + 30.711\right) m$

$= 340.96 m$

In ΔSTZ,

$\frac{\sin 20.5°}{ST} = \frac{\sin(44.5°)}{ZT}$

$\Rightarrow ST = \frac{\sin 20.5°}{\sin 44.5°} \times [455.150 \times \tan 30°] = 121.02 m.$

Thus the ray is incident at 40.5° at $V_2 - V_3$ interface

$OS = OT + TS = \frac{OZ}{\cos 20°} + TS = \left(\frac{456.156}{\cos 20°} + 121.02\right) m$

$\therefore OS = 605.38 m.$

$$\frac{\sin\theta_2}{\sin 40.5} = \frac{V_2}{V_3} \Rightarrow \theta_2 = 33.82^\circ.$$

and
$$\frac{\sin\theta_1}{\sin 33.8^\circ} = \frac{V_1}{V_2} \Rightarrow \theta_1 = 27.63^\circ.$$

$$\therefore YR = 605.38 + 300[\tan 27.63^\circ + \tan 33.82^\circ] = 963.4m.$$

$$\frac{\sin\theta_2}{\sin 33.9^\circ} = \frac{V_2}{V_3} \rightarrow \theta_2 = 28.55^\circ.$$

Also, $\theta_1 = \sin^{-1}\left(\frac{1.5}{1.8} \times \sin 28.55^\circ\right) = 23.47^\circ.$

$$\therefore YR = OS + 300[\tan 28.55^\circ + \tan 23.47^\circ] = 634.44m.$$

From source side,

$$OF = 300 \times [\tan 20^\circ + \tan 17.9^\circ] = 206.088\,m.$$

$$OF = AW = 206.088m.$$

Also, $FT = 300m.$

$$\therefore TW = AW(\tan 20^\circ) = 206.088 \times \tan 20^\circ = 75m.$$

$$FW = (300 - 75)m = 225m = AO.$$

In ΔZAO,

$$ZA = \frac{\sin(15.9^\circ)}{\sin(94.1^\circ)} \times 225 = 61.798m.$$

Also, $AT = \frac{AW}{\cos 20^\circ} = \frac{206.088}{\cos 20^\circ} = 219.31m.$

$$ZT = 281.108m.$$

At $V_2 - V_3$ interference angle of incidence $= 4.1^\circ.$

$$\therefore \theta_2 = \sin^{-1}\left(\cancel{4.1}\,\sin 4.1^\circ \times \frac{1.8}{2.4}\right) = 3.07^\circ.$$

$$\theta_1 = \sin^{-1}\left(\sin 3.07^\circ \times \frac{1.5}{1.8}\right) = 2.55^\circ.$$

$$\therefore SX = ZT + 300[\tan 2.55^\circ + \tan 4.1^\circ] = 315.97m.$$

$\therefore$ Total source to receiver offset $= SY + YR$

$= 450.41 m.$

$$t_2 = 2\left[\frac{300}{V_6 \cos 20^\circ} + \frac{300}{V_5 \cos 17.9^\circ}\right] + \frac{300}{V_4 \cos 25.55^\circ} +$$

$$\frac{300}{V_2 \cos 23.47^\circ} + \frac{300}{V_2 \cos 89.6^\circ} + \frac{300}{V_1 \cos 7.46^\circ} +$$

$$\frac{\frac{61.798 \times \sin 70^\circ}{\sin 15.9^\circ}}{V_4} + \frac{\frac{30.72 \times \sin 60^\circ}{\sin(13.9)}}{V_3}$$

$\therefore$ $t_2 = 1.485 s.$

$t_0 = 1.4 s.$

$\therefore$ $V_{stack} = \sqrt{\frac{(950.41)^2}{(1.485)^2 - (1.40)^2}} = 1.976 km/s.$

$V_{RMS} = \sqrt{\frac{\Sigma v_i^2 t_i}{\Sigma t_i}} = 2.915 km/s.$

$V_{avg} = \frac{\Sigma z_i}{\Sigma t_i} = \frac{5592.08}{1.976} = 2.832 km/s.$

Case 3 :— $\theta_i = 30^\circ$

Angle of incidence $= 30^\circ.$

$OZ = 300[\tan 30^\circ + \tan 26.7^\circ + \tan 23.6^\circ]$

$= 455.156 m.$

$\angle STZ = 110^\circ.$

$\angle TSZ = 180^\circ - 110^\circ - 20.5 = 49.5^\circ.$

From source side,

$OF = 300 \times [\tan 30^\circ + \tan 26.7^\circ] = 324.08 m.$

$\therefore$ $OF = 1AW - 324.08 m.$

Also, $FT = 300\,m$

$TW = AW \tan 20° = 117.955m.$

$\therefore FW = 182.04m$

$$AT = \frac{AW}{\cos 20°} = \frac{324.06}{\cos 20°} = 344.878m.$$

$$ZT = ZA + AT = \frac{\sin 23.6°}{\sin 86.4°} \times 182.04 + 344.878$$

$$= 417.90m$$

At $V_2 - V_3$ interface, Angle of incidence $= 3.6°.$

$$\frac{\sin \theta_2}{\sin 3.6°} = \frac{V_2}{V_4} \Rightarrow \theta_2 = 2.699°.$$

Also,
$$\frac{\sin \theta_1}{\sin 2.7°} = \frac{V_1}{V_2} \Rightarrow \theta_1 = 2.25°.$$

$\therefore SY = ZT + 300(\tan 2.7) + 300 \tan 82.25° = 443.82m.$

$\therefore$ Total source to receiver offset $= 1407.22m.$

$$t_2 = 2\left[\frac{300}{V_6 \cos 30°} + \frac{300}{V_5 \cos 26.7°}\right] + \frac{300}{V_4 \cos 23.6°} + \frac{300}{V_2 \cos 35.5°}$$

$$+ \frac{300}{V_1 \cos 27.63°} + \frac{300}{V_2 \cos 2.7°} + \frac{300}{V_1 \cos 2.25°} +$$

$$\frac{\dfrac{ZA \times \sin 4.9}{\sin(15.9)}}{V_4} + \frac{\dfrac{ST \times \sin 110°}{\sin 15.9°}}{V_3} = 1.739s.$$

$$V_{stack} = \sqrt{\frac{(1.40722)^2}{(1.739)^2 - (1.405)^2}} = 1.37 Km/s.$$

$$V_{RMS} = \sqrt{\frac{\Sigma V_i^2 t_i}{\Sigma t_i}} = 1.92 Km/s.$$

$$V_{avg} = \frac{\Sigma z_i}{\Sigma t_i} = \cancel{30780} \; \frac{3.07803}{1.739}$$

$$= 1.77 \text{ km/s.}$$