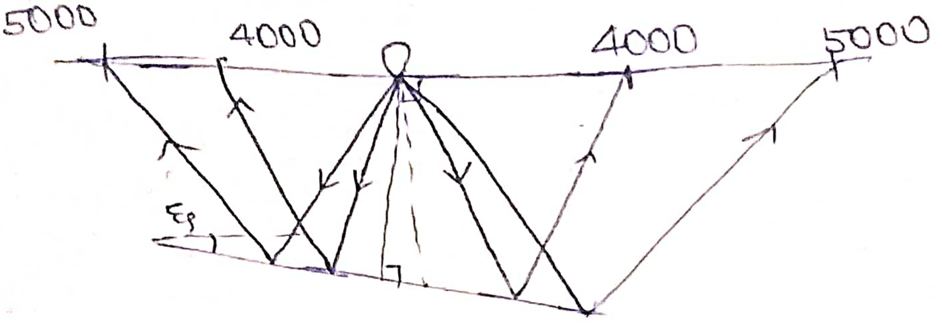
Utkarsh Jaiswal 18EX200[illegible]

Lab Assignment 1



Given :- (i) $V = 7500\,ft/s$

(ii) $\Delta t_R = 0.64\,s$

(iii) $\Delta t_L' = 0.045\,s$.

Solution :- Right side

$$v^2 t^2_{4000} = x^2 + 4h^2 - 4hx\cos\left(\frac{\pi}{2} + \xi_g\right)$$

$$\Rightarrow v^2 t^2_{4000} = x^2 + 4h^2 + 2hx\sin\xi_g \quad \left(\because \cos\left(\tfrac{\pi}{2}+\theta\right) = -\sin\theta\right)$$

$$\Rightarrow t^2_{4000} = \frac{x^2 + 4h^2 + 2hx\sin\xi_g}{v^2}$$

$$t_{4000} = \frac{2h}{V}\left(1 + \frac{x^2 + 4hx\sin\xi_g}{8h^2}\right)$$

$$t_{4000} = \frac{2h}{7500}\left(\frac{8h^2 + x^2 + 4hx\sin\xi_g}{8h^2}\right)$$

$$t_{4000} = \frac{1}{30000h}\left(8h^2 + x^2 + 4hx\sin\xi_g\right) \quad \text{—— (1)}$$

$$t_{5000} = \frac{1}{30000h}\left(8h^2 + x^2 - 4hx\sin\xi_g\right) \quad \text{—— (2)}$$

From eq ①,

$$t_{4000} = \frac{1}{30000h}\left(8h^2 + (4000)^2 + 16000\,h\sin\xi\right) \quad \text{③}$$

From eq ②

$$t_{5000} = \frac{1}{30000\,h}\left(8h^2 + (5000)^2 + 20\,000\,h\sin\xi\right) \quad \text{④}$$

eq ④ − eq ③ we get,

$$t_{5000} - t_{4000} = \frac{1}{30000\,h}\left[9000000 + 4000h\sin\xi\right]$$

Given $t_{5000} - t_{4000} = 0.064$

$$\Rightarrow \quad 0.064 = \frac{300}{h} + \frac{4}{30}\sin\xi \quad \text{⑤}$$

Left side

$$0.045 = \frac{300}{h} - \frac{4}{30}\sin\xi \quad \text{⑥}$$

Adding ⑤ and ⑥ we get,

$$0.109 = \frac{600}{h}$$

$$\Rightarrow \quad h = 5504.58\text{ ft.}$$

From eq ⑤,

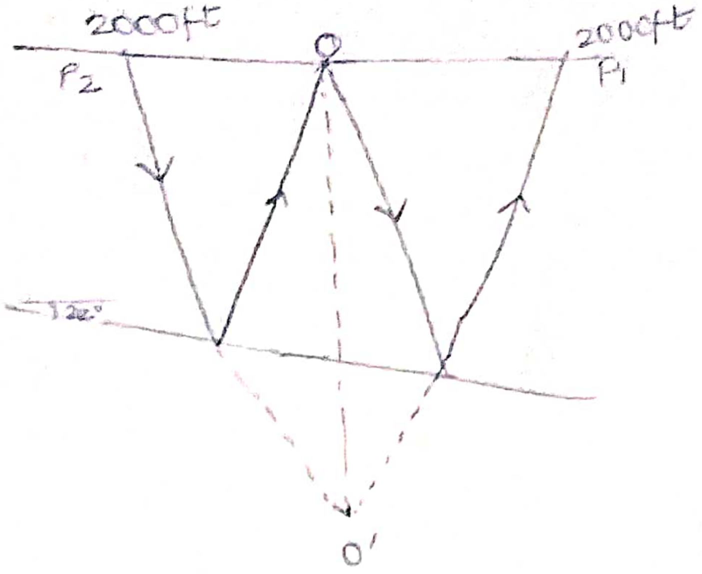
$$0.064 = \frac{300}{5504.58} + \frac{4}{30}\sin\xi$$

$$\Rightarrow \quad 0.064 = 0.0545 + 0.133\sin\xi$$

$$\Rightarrow \quad \sin\xi = \frac{9.5\times10^{-3}}{0.133}$$

$$\xi = 4.096^\circ$$

(2)



$\xi = 22°$, $\Rightarrow$ $\sin\xi = \sin 22° = 0.375$

$h' = 7200ft$, $V = 8000ft/s$

$$\sin\xi = \frac{V}{2}\left(\frac{\Delta t}{\Delta x}\right)$$

$$\Rightarrow \frac{\Delta t}{\Delta x} = \frac{2\sin\xi}{V}$$

$$\Rightarrow \frac{\Delta t}{\Delta x} = \frac{2 \times 0.375}{8000} = 9.375 \times 10^{-5} s/ft$$

$x = 2000ft$, $h = 7200 \times \cos 22° = 6675.72ft$

$$t_{P_1}^2 = \frac{x^2 + 4h^2 + 4hx\sin\xi}{V^2} = 3.1607s^2$$

$$t_{P_1} = 1.778s.$$

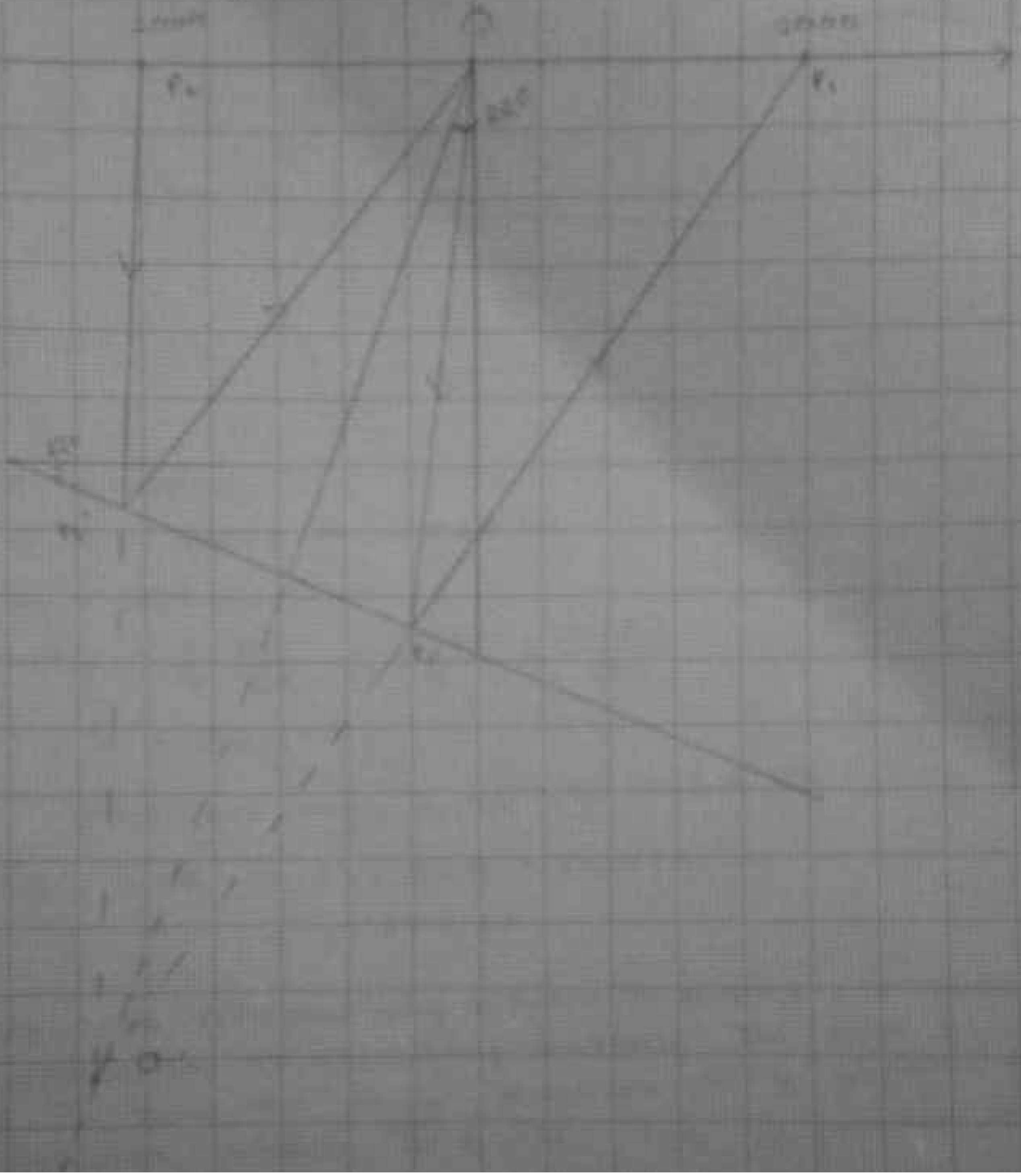
$$t_{P_2}^2 = \frac{x^2 + 4h^2 - 4hx\sin\xi}{V^2} = 2.535s^2$$

$$t_{P_2} = 1.592s$$

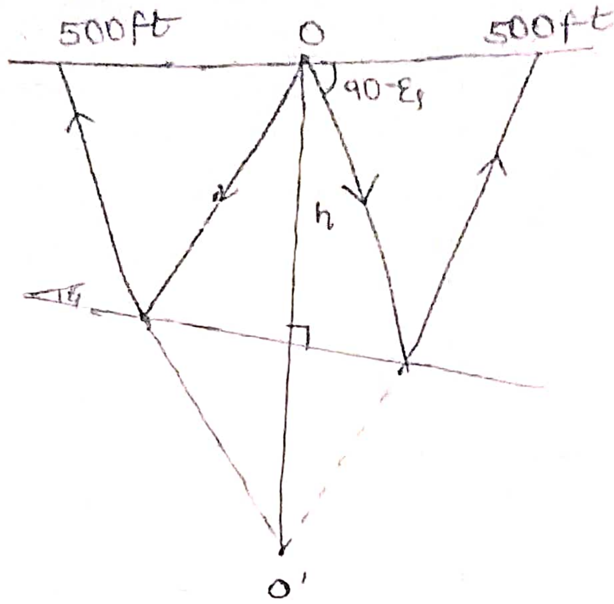
$$\Delta t = t_{P_1} - t_{P_2} = 0.186s.$$

$$\Delta x = 2000ft$$

$$\frac{\Delta t}{\Delta x} = \frac{0.186}{2000} = 9.3 \times 10^{-5} s/ft$$



(3)



(a) $V = 8000 ft/s$.

Δt for 1000ft is 0.047 s.

$t_0 = 1.669 s$

$$\sin\xi = \frac{V}{2}\frac{\Delta t}{\Delta x} = \frac{8000}{2} \times \frac{0.047}{500}$$

$\therefore$ $\xi = 22.1^\circ$

$\therefore$ Emergent angle $= 90^\circ - \xi = 67.9^\circ$

(b) $$OO' = 2h = \frac{2 \times 1.669 \times 8000}{2} ft$$

$= 13352 ft$

(c) We observe that the reflected ray returns to point O, so we can conclude that it must be perpendicular to the reflecting surface, since the image must be at 'h' distance from surface, therefore "2h/2" is the mid-point (reflection point).

(d) Perpendicular bisector of OO' is perpendicular to mid-point hence it is a reflecting bed.

④ $x_1 = 2000ft$, $t_1 = 1.377s$

$x_2 = 3000ft$, $t_2 = 1.420s$

$x_3 = 5000ft$, $t_3 = 1.537s$

$x_4 = 6000ft$, $t_4 = 1.609s$

$V^2t^2 = x^2 + 4h^2 + 4hx\sin\xi$

we equate to get,

$V^2(1.377)^2 = (2000)^2 + 4h^2 + 8000h\sin\xi$ —— ①

$V^2(1.42)^2 = (3000)^2 + 4h^2 + 12000h\sin\xi$ —— ②

$V^2(1.537)^2 = (5000)^2 + 4h^2 + 20000h\sin\xi$ —— ③

eq ② − eq ① we get,

$V^2[(1.42)^2 - (1.377)^2] = [(3000)^2 - (2000)^2] + 4000h\sin\xi$ —— ④

eq ③ − eq ② we get,

$V^2[(1.537)^2 - (1.42)^2] = [(5000)^2 - (3000)^2] + 8000h\sin\xi$ —— ⑤

eq ④ × 2 we get

$2V^2[(1.42)^2 - (1.377)^2] = 2[(3000)^2 - (2000)^2] + 8000h\sin\xi$ —— ⑥

eq ⑥ − eq ⑤ we get,

$2\times[(3000)^2 - (2000)^2] - [(5000)^2 - (3000)^2] = V^2[(2\ 61.42)^2 - (1.377)^2] - [(1.537)^2 - (1.42)^2]$

$\Rightarrow V^2[0.2405 - 0.346] = (10^7 - 1.6\times10^7)$

$\Rightarrow V = 7541.36\ ft/s$

substituting value of 'V' in eq ④ we get.

$(7541.36)^2[(1.42)^2 - (1.377)^2] = [(3000)^2 - (2000)^2] + 4000h\sin\xi$

$\Rightarrow h\sin\xi = \dfrac{(7541.36)^2[(1.42)^2 - (1.377)^2]}{[(3000)^2 - (2000)^2]\times 4000}$

$\Rightarrow h\sin\xi = 460$

Substituting value in eq ① we get,

$V^2 (1.377)^2 = (2000)^2 + 4h^2 + 8000h \sin\xi$

$\Rightarrow (7541.36 \times 1.377)^2 = (2000)^2 + 4h^2 + 8000 \times 460.$

$\Rightarrow 4h^2 = (7541.36 \times 1.377)^2 - \{(2000)^2 + 8000 \times 460)\}$

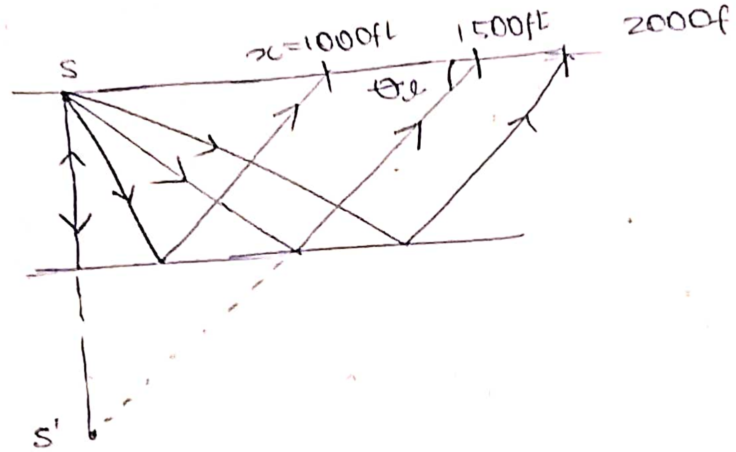
$\Rightarrow h = 5184.5 ft$

$\because h \sin\xi = 460.$

$\Rightarrow \sin\xi = \frac{460}{5184.5}$

$\xi = 5.09°$

⑤



$t_{1000} = 1.912s.$

$V = 9570 ft/s.$

$\Delta x = 1000ft$

$t_{2000} = \frac{x^2}{V^2} + \frac{4h^2}{V^2}.$

$= \frac{2000^2 + 4 \times (9135)^2}{(9570)^2} = 3.68833^2.$

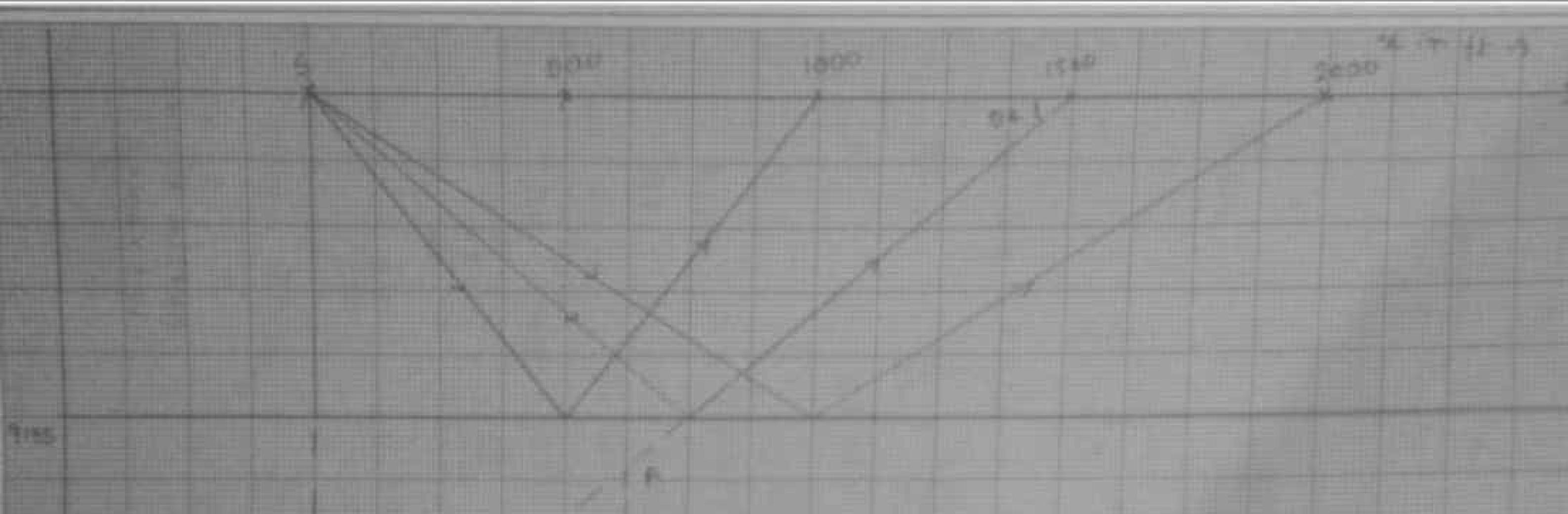
$\therefore t_{2000} = 1.9203 s.$

$\Delta t = t_{2000} - t_{1000} = 8.4 \times 10^3 s = 8.4 ms$

Let emergence angle at $x = 1500ft$ be θe

$\therefore \tan\theta e = 2h/x = 12.18.$

$\therefore \theta e = 85.30°.$



AC = 52 × 112·? = 5846.4 ft

BC = 34 × 12.5 = 425 ft

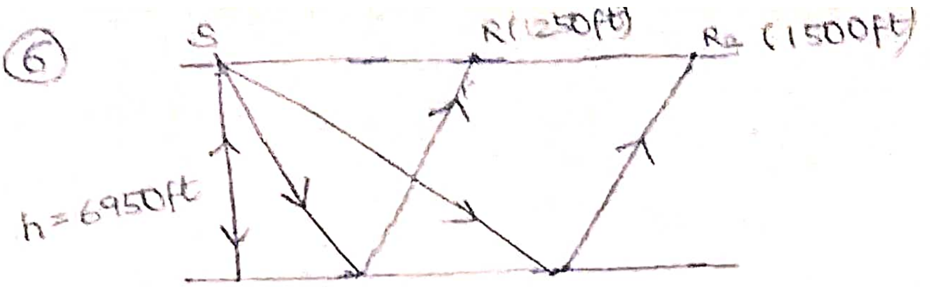
$\tan\theta = \frac{AC}{BC} = 12.108$

$\theta = 85.28°$

Direction dip along x-axis → 12.5 ft

Direction dip along y-axis → 182.3 ft

⑥



$$t^2_{1250} = \frac{x^2}{V^2} + \frac{4h^2}{V^2}$$

$$= \frac{(1250)^2}{(8700)^2} + \frac{4 \times (6950)^2}{(8700)^2}$$

$$t_{1250} = 1.60414s.$$

$$t^2_{1500} = \frac{x^2}{V^2} + \frac{4h^2}{V^2}$$

$$= \frac{(1500)^2 + 4 \times (6950)^2}{(8700)^2}$$

$$\therefore \quad t_{1500} = 1.6069s.$$

$$\Delta t = t_{1500} - t_{1250} = 2.84ms.$$

$$\Delta x = (1500 - 1250)ft = 250ft$$

$$\frac{\Delta t}{\Delta x} = \frac{2.84}{\cancel{300}\ 250} = 0.01136\ ms/ft$$

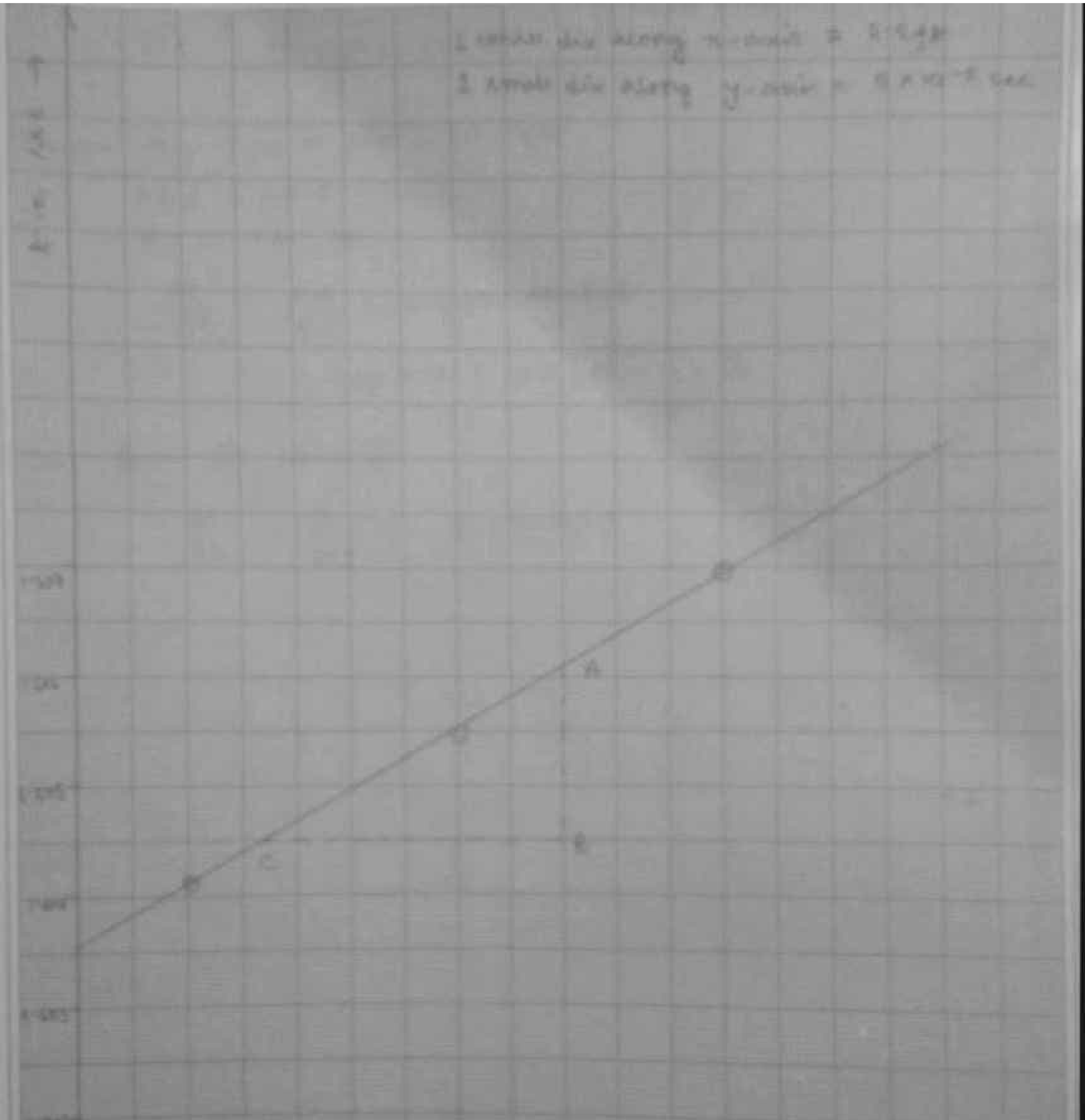
$$t^2_{1375} = \frac{x^2}{V^2} + \frac{4h^2}{V^2}$$

$$= \frac{(1375)^2 + 4 \times (6950)^2}{(8700)^2}$$

$$\therefore \quad t_{1375} = 1.60549s.$$

For $x = 1375f.$

$$\frac{x}{V^2 t} = \frac{1375}{(8700)^2 \times 1.6055} = 1.1315 \times 10^{-5} s/ft \quad ①$$


A
B
C

GRAPH

From the graph, t vs x plot,

$VAB = 1.6 \times 10^{-3}$ s.

$BC = 140$ ft

$$\frac{dt}{dx} = \frac{VAB}{BC} = \cancel{1.142 \times 10^{-5}} \frac{1.6 \times 10^{-3} \text{ s/ft}}{140}$$

$$\Rightarrow \frac{dt}{dx} = 1.142 \times 10^{-5} \text{ s/ft} \quad \text{——} (2)$$

From eq (1) and (2)

$$\frac{dt}{dx} = \frac{x}{vet}$$ (Hence Proved)