Measurement of higher moments of transverse momentum of particles produced in proton-proton collisions

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A review of "Skewness of mean transverse momentum fluctuations in heavy-ion collisions" [1] for p-p collisions at centre-of-mass energy 13 TeV is presented. Corresponding transverse momentum and mean transverse momentum distributions of particles produced have been plotted for 6 multiplicity classes. Mean transverse momentum distributions are compared against a Gaussian distribution and corresponding intrinsic and standardised skewness plotted against multiplicities using the STAR method. Positive skew is observed for mean transverse momentum distributions as concluded in the paper.

I. INTRODUCTION

Event generator	:	Pythia 8 Monte Carlo event generator.
Number of events	:	2 million
Collisions System	:	p + p at centre-of-mass energy 13 TeV.

The following variables will be used throughout the text.

p_i : Transverse momentum of ith particle
N_{ch} : The number of particles in an even
$\langle p_T \rangle$: The mean transverse momentum of emitted particles for a given collision centrality
$\langle\langle p_T \rangle\rangle$: Average of $\langle p_T \rangle$ over events in a centrality class
$\langle \Delta p_i \Delta p_j \rangle$: Variance of dynamical p_T fluctuations
$\langle \Delta p_i \Delta p_j \Delta p_k \rangle$: The skewness or the third centered moment

p-p collisions produce various particles depending on the energy of colliding particles. Following is a geometrical analysis of a typical particle collision:

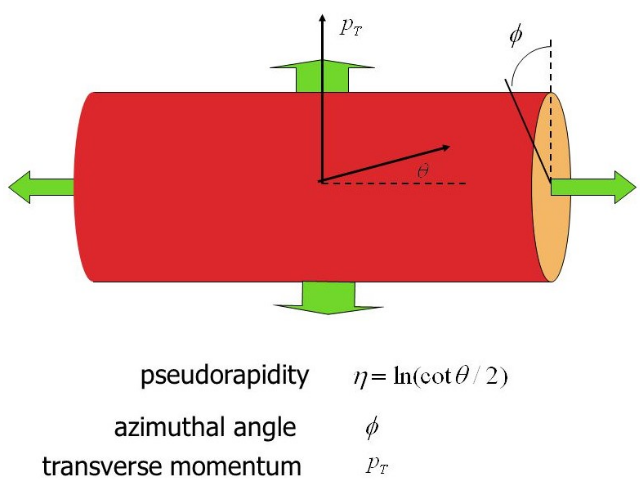

FIG. 1: Collision geometry

The paper proposes presence of positive skew for the $\langle pT \rangle$ distributions for heavy ion collisions. We provide an analysis for the same for p-p collisions. We study the pT and $\langle pT \rangle$ distributions where $\langle p_T \rangle$ is computed as,

$$\langle p_T \rangle = \sum_{i=1}^{N_{ch}} \frac{p_i}{N_{ch}} \tag{1}$$

and the STAR definition of $\langle\langle p_T \rangle\rangle$ is used as:

$$\langle\langle p_T \rangle\rangle = \left\langle \sum_{i=1}^{N_{ch}} \frac{p_i}{N_{ch}} \right\rangle \tag{2}$$

Skewness depends on standardised variance. Hence, to derive an expression for the same, the following expression is used:

$$\langle \Delta p_i \Delta p_j \rangle_{STAR} = \left\langle \frac{\sum_{i,j,i\neq j}(p_i - \langle\langle p_T \rangle\rangle)(p_j - \langle\langle p_T \rangle\rangle)}{N_{ch}(N_{ch}-1)} \right\rangle \tag{3}$$

standardised variance then becomes

$$\frac{\sqrt{\langle \Delta p_i \Delta p_j \rangle}}{\langle p_T \rangle} \tag{4}$$

Taking:

$$Q_n = \sum_{i=1}^{N_{ch}} (p_i)^n \tag{5}$$

To compute the intensive (γ_{p_t}) and standardised (Γ_{p_t}) skewness the expressions for the same are:

$$\gamma_{p_t} = \frac{\langle \Delta p_i \Delta p_j \Delta p_k \rangle}{\langle \Delta p_i \Delta p_j \rangle^{3/2}} \tag{6}$$

$$\Gamma_{p_t} = \frac{\langle \Delta p_i \Delta p_j \Delta p_k \rangle \langle\langle p_T \rangle\rangle}{\langle \Delta p_i \Delta p_j \rangle^{2}} \tag{7}$$

where

$$\langle \Delta p_i \Delta p_j \rangle_{STAR} = \left\langle \frac{Q_1^2 - Q_2}{N_{ch}(N_{ch}-1)} \right\rangle - \left\langle \frac{Q_1}{N_{ch}} \right\rangle^2 \tag{8}$$

and

$$\langle \Delta p_i \Delta p_j \Delta p_k \rangle_{STAR} = \left\langle \frac{Q_1^3 - 3Q_2 Q_1 + 2Q_3}{N_{ch}(N_{ch}-1)(N_{ch}-2)} \right\rangle - 3\left\langle \frac{Q_1^2 - Q_2}{N_{ch}(N_{ch}-1)} \right\rangle \left\langle \frac{Q_1}{N_{ch}} \right\rangle + 2\left\langle \frac{Q_1}{N_{ch}} \right\rangle^3 \tag{9}$$

II. EXPERIMENTAL OBSERVATIONS

A. p_T distributions

The following are plots of distribution of transverse momentum in logarithmic scale and have been fitted against an Exponential plot (in red). The transverse momentum values used were in GeV/c.

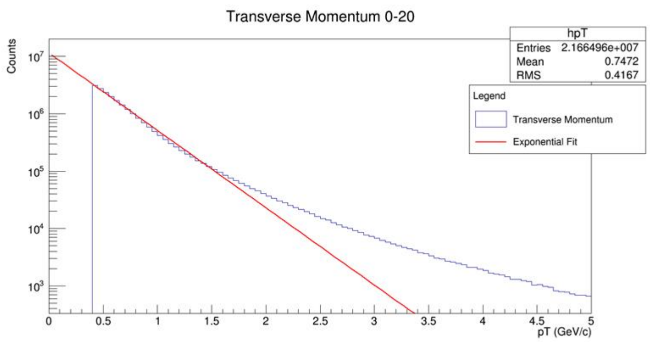

FIG. 2: Distribution of p_T for Multiplicity Class 0-20

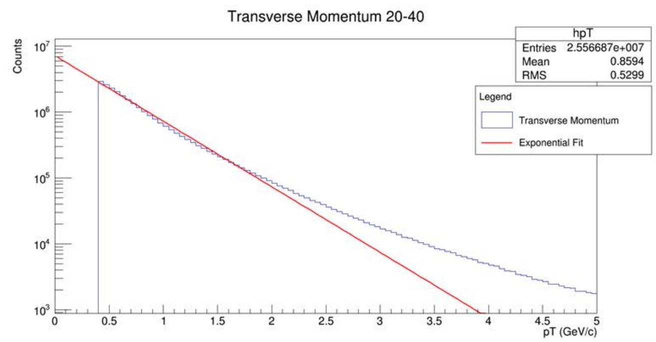

FIG. 3: Distribution of p_T for Multiplicity Class 20-40

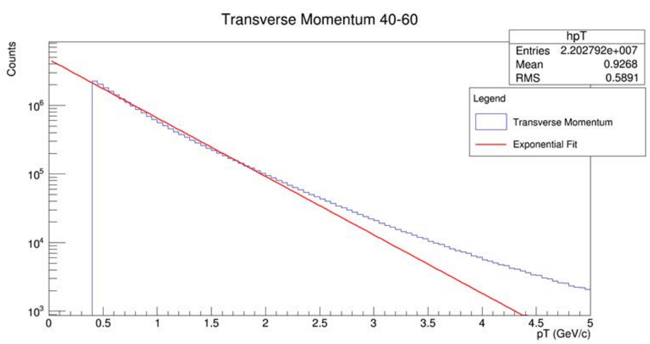

FIG. 4: Distribution of p_T for Multiplicity Class 40-60

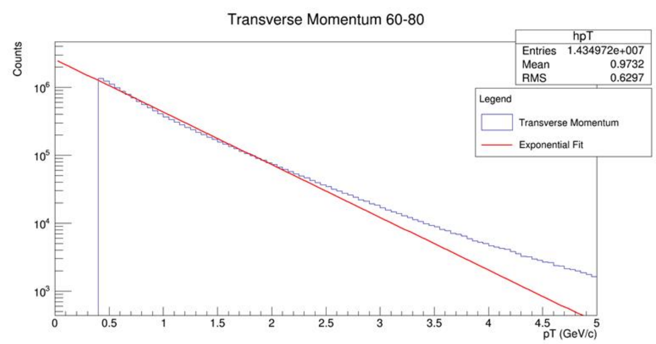

FIG. 5: Distribution of p_T for Multiplicity Class 60-80

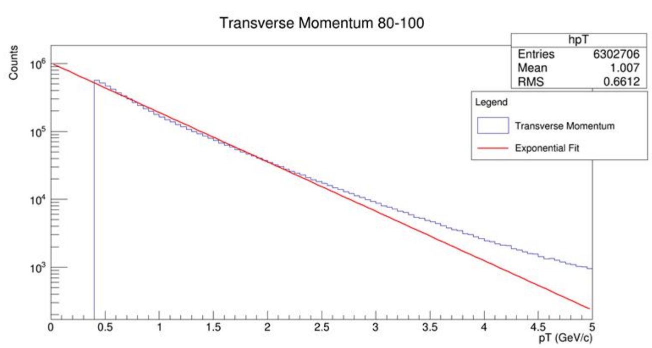

FIG. 6: Distribution of p_T for Multiplicity Class 80-100

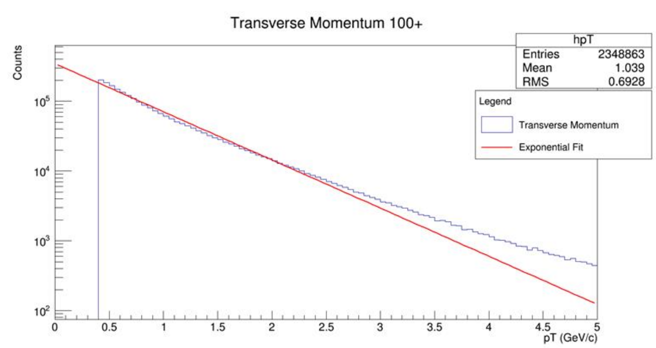

FIG. 7: Distribution of p_T for Multiplicity Class 100+

B. $\langle p_T \rangle$ distributions

The following are plots of averaged transverse momentum in logarithmic scale and have been fitted against a Gaussian curve (in red). The mean transverse momentum values used were in GeV/c.

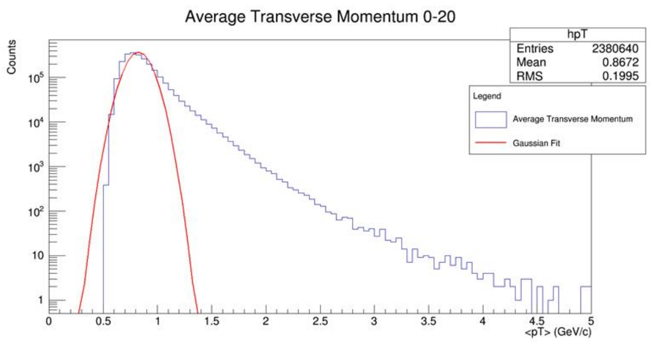

FIG. 8: Distribution of $\langle p_T \rangle$ for Multiplicity Class 0-20

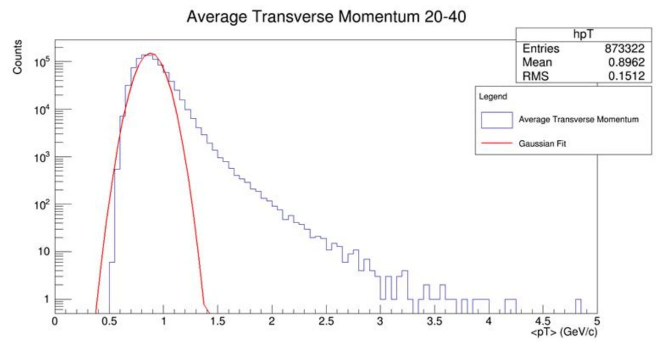

FIG. 9: Distribution of $\langle p_T \rangle$ for Multiplicity Class 20-40

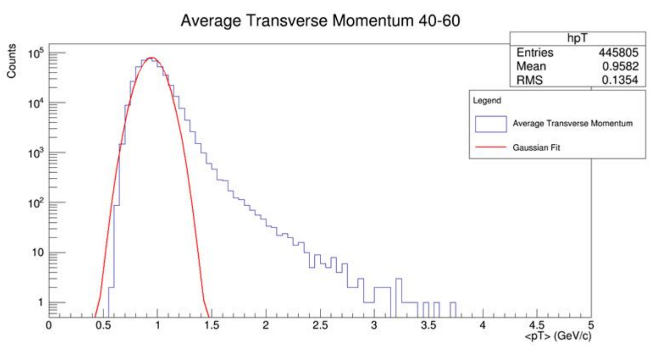

FIG. 10: Distribution of $\langle p_T \rangle$ for Multiplicity Class 40-60

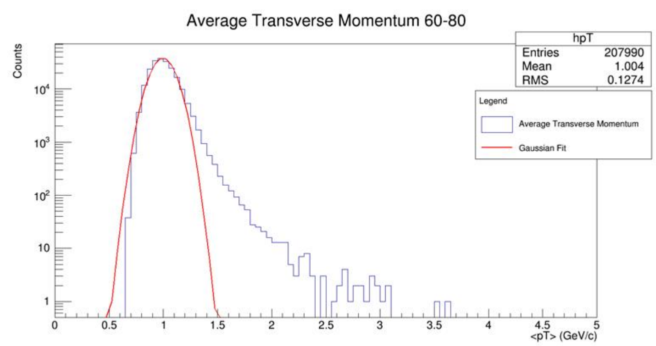

FIG. 11: Distribution of $\langle p_T \rangle$ for Multiplicity Class 60-80

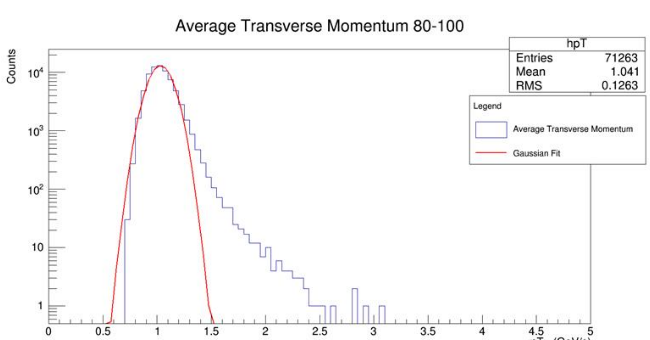

FIG. 12: Distribution of $\langle p_T \rangle$ for Multiplicity Class 80-100

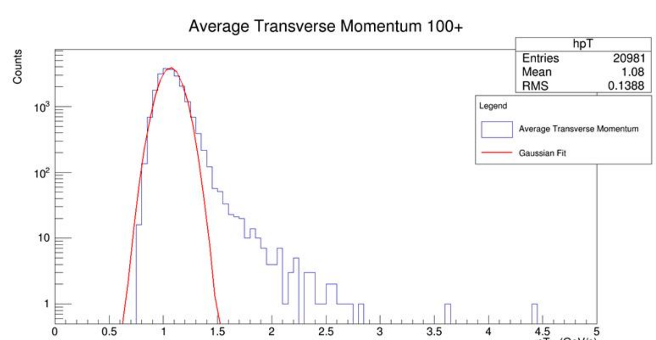

FIG. 13: Distribution of $\langle p_T \rangle$ for Multiplicity Class 100+

C. Plot for $\langle\langle p_T \rangle\rangle$

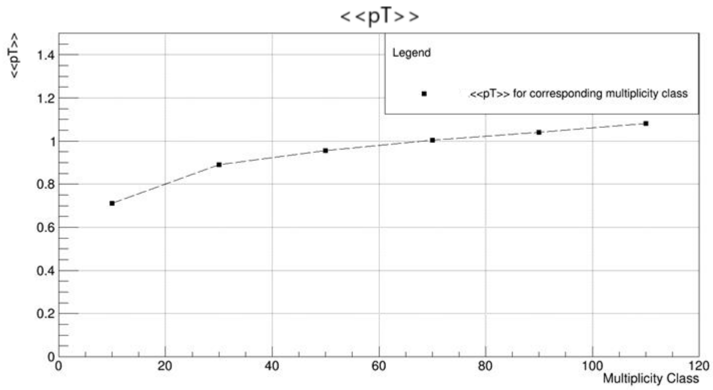

FIG. 14: Variation in $\langle\langle p_T \rangle\rangle$ across different Multiplicity Classes

D. Central Moments of $\langle p_T \rangle$

Using Eq (4), (6), (7) the graphs of Standardised Skewness, Intensive Skewness and Standardised Variance are obtained with $|\eta \leq 2.5|$.

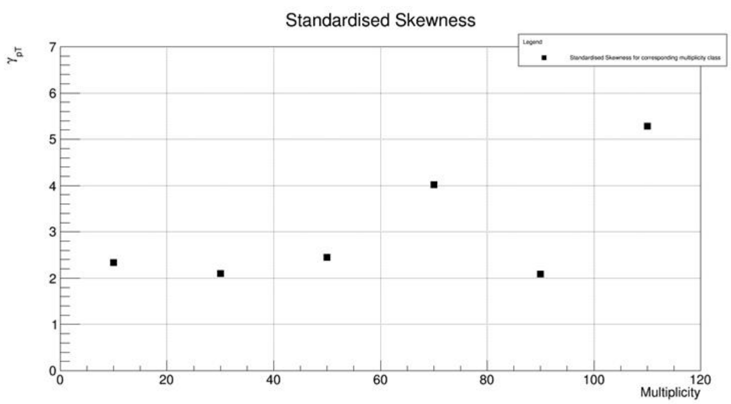

FIG. 15: Variation in Standardised Skewness across different Multiplicity Classes

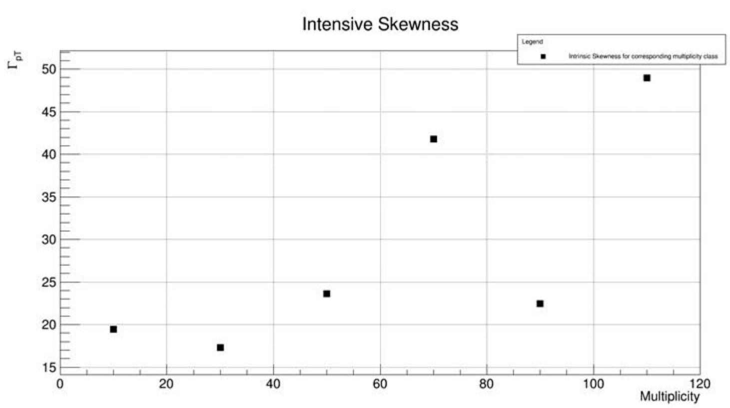

FIG. 16: Variation in Intensive Skewness across different Multiplicity Classes

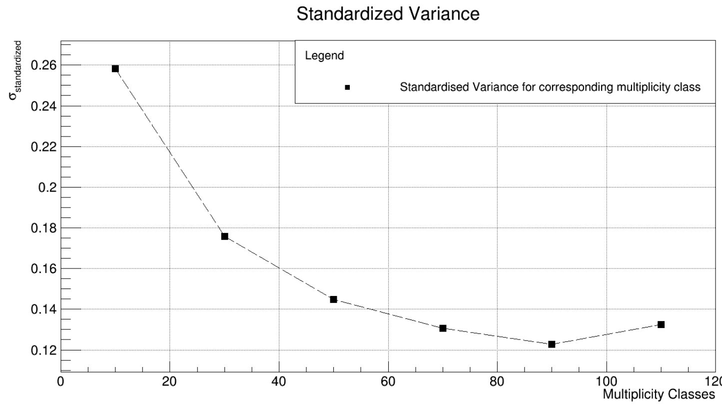

FIG. 17: Variation in Standardised Variance across different Multiplicity Classes

III. SUMMARY

ROOT Data Analysis Framework by CERN and partner institutions is a great tool for handling and visualising large data files. We started with a study and development of plots ranging from 3-D histograms to line plots; transforming huge lines of data to beautiful results. ROOT Forum and ROOT manual were our constant sources of reference.

1. We understood the p-p collision model and corresponding data storage in TTree files and applied ROOT to extract the data to plots.
2. We produced pT and $\langle pT \rangle$ distributions for multiplicity classes of bin size 20 and range 0 to 100 and fit them against Exponential and Gaussian distributions.
3. As the paper proposed observation of positive skewness in $\langle pT \rangle$ distributions for heavy-ion collisions; we observed similar results for p-p collision data generated with Pythia 8 Monte Carlo event generator in ROOT.
4. We further plot standardised variance v/s multiplicity classes and observed a decrease with an increase in the range of N_{ch} depicting an inverse relation to the same. The decrease of variance is a result of Central Limit Theorem.
5. Plot for $\langle\langle pT \rangle\rangle$ v/s multiplicity classes had a steep rise.
6. Standardized and intrinsic skewness plots present a more random picture for the data.

IV. LINK TO REPOSITORY

PH-219 ROOT Project Github Repository

REFERENCES

[1] Giuliano Giacalone **andothers**. *Skewness of mean transverse momentum fluctuations in heavy-ion collisions*. 2020. arXiv: 2004.09799 [nucl-th].