EKLAVYA.

Linear Algebra. (3Blue 1Brown).

Clip 1: Vectors

$a\hat{i} + b\hat{j} = \begin{bmatrix} a \\ b \end{bmatrix}$ - Representing 2d plane's points as vectors.
- Representing points in 3d plane as vectors.

- Addition of vectors (Parallelogram law)

$$\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} a+b \\ c+d \end{bmatrix}$$

- Scalar multiplication of vectors

Clip 2: Span.

- Representation of any vector/point by addition of scaled basis vectors (Linear combination)

Notes: ① Span of two vectors (set of all their linear combinations)
② Span of most 2-d vectors is all 2d vectors.
③ If two vectors line up then their span is all points on that line
④ Similar for 3-d vectors.

Clip 3: Linear ~~rep~~ transformation of vectors

- Visualization of linear transformation.
- Understanding ~~matrix multiplication~~ linear transformation and its importance visually
- If $\hat{i} \xrightarrow{\text{goes to}} \begin{bmatrix} a \\ b \end{bmatrix}$ and $\hat{j} \rightarrow \begin{bmatrix} c \\ d \end{bmatrix}$ then $\bar{v} = x\hat{i} + y\hat{j}$ in new plane

is same as $\bar{v} = x\begin{bmatrix} a \\ b \end{bmatrix}\hat{i} + y\begin{bmatrix} c \\ d \end{bmatrix}\hat{j}$ in old plane.

Chp 4: ~~Mat~~ Matrix multiplication as composition.

- Understanding how and why matrix multiplication works in 2 and 3-d spaces.
- Developing instincts of basic matrix properties such as → Associative $A(BC) = (AB)C$
  → Non-Commutative $AB \neq BA$
  and so on.

Chp 5: 3-d linear transformations.

- Same as 2d, 3×3 matrices.
- Visualization, cube, volume.

Chp 6: Determinant: $|A|$.

- Is the factor by which a unit square or unit volume is scaled in the transformed 2 or 3-d space.
- Negative determinant indicates flipping of space.

Chp 7: Inverse, Rank and Null Space.

- Use of matrices to solve linear sys of eqs, $A\bar{x} = \bar{v}$ (Visualization).
- Visualization of inverse (doing transformation in reverse).
- Hence understanding $AA^{-1} = I$ → does nothing
  $\therefore A\bar{x} = \bar{v}$ means if A is your transformation, you are searching for a vector $\bar{x}$ such that transforming $\bar{x}$ you land on $\bar{v}$. (Hence, $\bar{x} = A^{-1}\bar{v}$)
  This is possible only if it is not squished into a
  ↳ (Inverse method)

lower dimension (i.e. |A| = 0)
- Rank. (∴ If rank goes from 3 → 2 better than from 3 → 1, 0 → Point)
If rank reduces, possible sol exists but same.
- Column space → Span (set of) columns (→ where basis vectors land) of your matrix.

$\underbrace{\begin{bmatrix} \vdots & \vdots & \vdots \end{bmatrix}}_{\text{Column space}}$

∴ Rank better def → No of dimensions in column space.
Null space → Span of (set of) all vectors that result in 0 vector, after squishing (to the origin)

Clip 8: Non-square matrices:
- If dimensions are squished, results in non-square matrices.

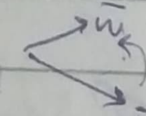
Clip 9: Dot Product and Duality

- Visualization of how projection is basically very similar to linear transformation of say a 2d vector onto a point.
- And this leading understanding similarities between dot product and its geometric interpretation.
- Projection matrix of u : $[u_x \ u_y]$
- Thus, wherever we do dot to 1d linear transf there exists a vector v such that doing that linear transf is same as doing dot product with that vector
- Duality : That vector $\vec{v}$ is dual of that linear transf

Chp 10: Cross Product (Standard)

$\bar{v} \times \bar{w}$ is area of parallelogram (+ve)

$\bar{w} \times \bar{v}$ (−ve area)



Right hand rule.

$\bar{v} \times \bar{w} = \det[\bar{v}\ \bar{w}]$

Properties: $3\bar{v} \times \bar{w} = 3[\bar{v} \times \bar{w}]$..

~~But this is fake cross product.~~

But remember, cross product is ~~not~~ just value of the area, but the vector in that dir.

Chp 11: Understanding Duality of cross product

If $\bar{p}$ is the dual of our linear transform that signifies cross product

$$\bar{v} \times \bar{w} = \vec{p} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \text{(Length of projection)} \times \text{(Length of } \bar{p}\text{)}.$$

$$\det \begin{bmatrix} x & v_1 & w_1 \\ y & v_2 & w_2 \\ z & v_3 & w_3 \end{bmatrix}$$

where $\bar{p} = [\bar{v}\ \bar{w}]$.

$$= \underbrace{\text{(Area of parallelogram)}}_{\text{Magnitude}} \times \underbrace{\left(\text{Component of } \begin{bmatrix} x \\ y \\ z \end{bmatrix} \perp \text{ to } \bar{v}\ \&\ \bar{w}\right)}_{\text{Direction}}$$

Chp 12: Cramer's rule → geometric

Chp 13: Change of basis.

- $\hat{i}$ & $\hat{j}$ are basis vectors of standard coordinate system.
- Can use a seperate coord sys with diff basis vector $\vec{b_1}$ and $\vec{b_2}$
- Origin of all systems should line up, but angle of basis ~~spe~~ vectors and grid spacing can change

Suppose $\vec{b_1} = 2\hat{i} + \hat{j}$; $\vec{b_2} = -\hat{i} + \hat{j}$.

Change of basis matrix $\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$ → Multiplying by this Matrix that by linear transformation transforms grid from $(b_1 b_2)$ system to $(\hat{i}\hat{j})$ system.

$b_1 b_2$ sys → (Scalars) ∴ $\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ → Our sys (Scalars)

∴ $2x - y = 3$ $x + y = 2$ $x = 5/3$, $y = 1/3$

Chp 14: Eigenvectors Eigenvalues

- Only in rare cases, vectors dont get knocked off their span during transformation. → Called eigenvectors
  → Scalar by which all values on span of eigenvectors → called eigenvalues

- $\underbrace{A}_{\text{linear transform matrix}} \bar{v} = \underbrace{\lambda}_{\text{Eigenvalue}} \bar{v}$ → Equivalent for eigenvector $\bar{v}$

∴ $(A - \lambda I)\, \bar{v} = \bar{0}$