ME 212: Solid Mechanics-I

Investigation of Compressive Vertical Columns

And Their Feasibility as Space Elevator Support Structures

Utkarsh Chaturvedi
190107080
Chemical Engineering

Introduction:

Space elevators are the next 'big thing' in human space exploration. By utilising a motorised lift platform that can lift people, vehicles and supplies from the surface to Earth orbit, there can potentially be a **massive reduction in costs for the space industry**, and a **fast-tracked expansion of humanity** from Earth to the rest of the solar system and beyond.

Most concepts for space elevators use a **tensile** model, as the compressive model of erecting a building thousands of kilometers high was deemed too unrealistic and impractical even with the strongest of materials available today. **This project will attempt to demonstrate why the compressive space elevator model was rejected.**

References:

The code associated with this project can be accessed on:

Google Colaboratory: https://bit.ly/37hW8mC (executable and downloadable)

GitHub: https://bit.ly/3flsf2C OR https://bit.ly/2VgguqE (only downloadable)

The notebook may be downloaded and executed on local machines, but the dependencies may need to be installed first. The list of used libraries can be viewed in the relevant section of the Python notebook. This report contains references, screenshots, LaTeX snippets and even verbatim passages from the notebook.

All Python code and LaTeX rendering in the notebook and in this report is my own.

Objective:

The aim of this project (and the associated notebook) is to analyse various column configurations and cross-sections to find out ***the maximum height that can be achieved by a vertical column while being loaded to the maximum allowed stress for the material***.

Additionally, to evaluate the feasibility of compressively-supported space elevators for currently available materials, under the already considered assumptions.

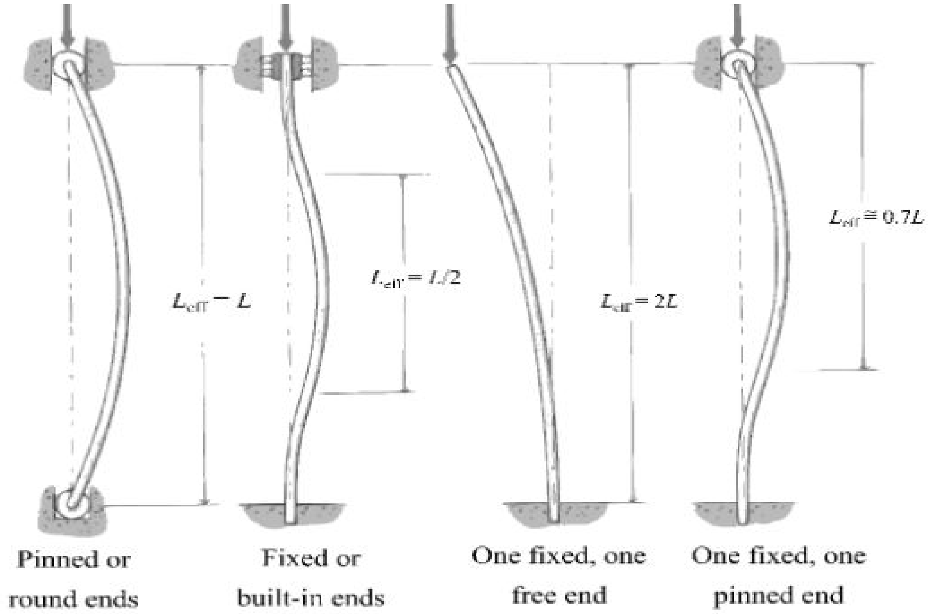
Assumptions and Methodology:

The key assumptions of the Euler Buckling Theory are:
1. The column is **massless**.
2. The material and cross-section of the column is **homogeneous** and **uniform**.
3. The column is a **slender** column, that is:

$$\frac{L_{\text{eff}}}{r} \geq 12$$

(where 'r' is a cross-sectional dimension)

Note that because we are considering lengths for which *buckling stress = yielding stress*, the slender column assumption may not hold every time. Thus, without a suitable factor of safety (FoS), the formulation serves to demonstrate an *upper limit* to the length of the column.



In the above image (*taken from [Lecture 21, Slide 6]*), the concepts of L_{eff} and column configurations are clarified. Note that **pinned** here has the same meaning as **hinged**, and it refers to the end which is not firmly bolted to the floor or ceiling. It is different from a **free** end, in that it isn't free to translate.

Apart from this, the shape of the cross-section also needs to be defined. For the purposes of this investigation, 3 types of cross-sections have been considered.

I. A **rectangular** cross-section, with defined base (**b**) and width (**h**).

$$I_{\text{rect}} = \frac{bh^3}{12};\ A_{\text{rect}} = bh;\ (\text{where } b \geq h)$$

II. An **annular** cross-section, with an outer radius (**R**) and inner radius (**r**).

$$I_{\text{tube}} = \frac{\pi}{4}(R^4 - r^4);\ A_{\text{tube}} = \pi(R^2 - r^2);\ (\text{where } R > r)$$

III. A **symmetrical I-beam** made of three **equal-area pieces** (2 flanges and 1 web). Thus, the relevant dimensions are flange (or web) length (**h**) and flange (or web) thickness (**t**).

$$I_{\text{I-beam}} = \frac{ht(t^2 + 2h^2)}{12};\ A_{\text{I-beam}} = 3ht;\ (\text{where } h \geq t)$$

These are reflected in the code in the self-adjusting parameters **a** and **b**.

Engineering safety considerations for buildings require a **FoS** of 2.0, although that is with multiple redundant members. For a single vertical column, we certainly do not have that luxury. Hence, the **FoS** was chosen as 4.0 for this section of the analysis.

Thus, the flow of the formulation loosely follows:

1. Defining the scope of the problem, and the engineering safety considerations (this yields the **FoS**)
2. Defining the physical configuration of the column; how it is likely to be used in an engineering application (this yields K)
3. Defining the geometrical properties of the column; the shape of the cross-section and its dimensions (this yields the shape, as well as parameters **a** and **b**)
4. Defining the material characteristics of the column (this yields the constants E and σ_{yield})
5. Arranging everything into easy-to-use functions that can be called again and again without much repetition of code
6. Implementing user-friendly interactive “widgets” for easy usage of the multiple functions.

Executing the Pipeline

The following cell shows the interactible widget where the parameters of the length expression can be tweaked. To see the result after tweaking the values, the below 2 cells should be executed in succession. Please note that the cross-section dimensions are in `mm`, and the material properties chosen as defaults are those of Aluminium.

In [9]:
```
# 4. calculating length of column
display(ui_final)
```

Beam Configuration: fixed at BOTH ends | Cross-Section: rectangular

Parameter 'a' in mm: 1000 | Parameter 'b'in mm: 1000

Young's Modulus (GPa): 70 | Yield Strength (MPa): 275

n [10]:
```
K = l_eff_factor(config = k_eff.children[0].value)
a = a_int.children[0].value
b = b_int.children[0].value
R, A, I = cross_sec(a, b, sec = shape_code.children[0].value)
E = E_int.children[0].value
S = S_int.children[0].value

l_trial = max_length(K, R, E, S)
print('The maximum fully-loaded length of the column is ~', '{:.3f}'.format(l_trial), 'metres.')
print('The supported load is ', S*A, ' N, which is equivalent to ', S*A/9.81, 'kilograms at the Earth\'s surface.')
print('The slenderness ratio is ', K*l_trial/np.sqrt(R*(10**-6)))
```

```
The maximum fully-loaded length of the column is ~ 57.876 metres.
The supported load is  275000000.0  N, which is equivalent to  28032619.77573904 kilograms at the Earth's surface.
The slenderness ratio is  100.24495874524781
```

Thus it is clear that even a solid Aluminium column with a square `1m` x `1m` cross-section barely reaches to `58` metres in height before buckling at maximum allowed stress for the material and safety considerations.

This interactive dashboard allows users to experiment with the function and the parameters, but an analysis of the feasibility of compressive structures for space elevators requires real-world data. To that end, 2 materials were considered.

1. Steel ($E = 200\ \text{GPa},\ S = 152\ \text{MPa}$)
2. Diamond ($E = 1220\ \text{GPa},\ S = 60,000\ \text{MPa}$)

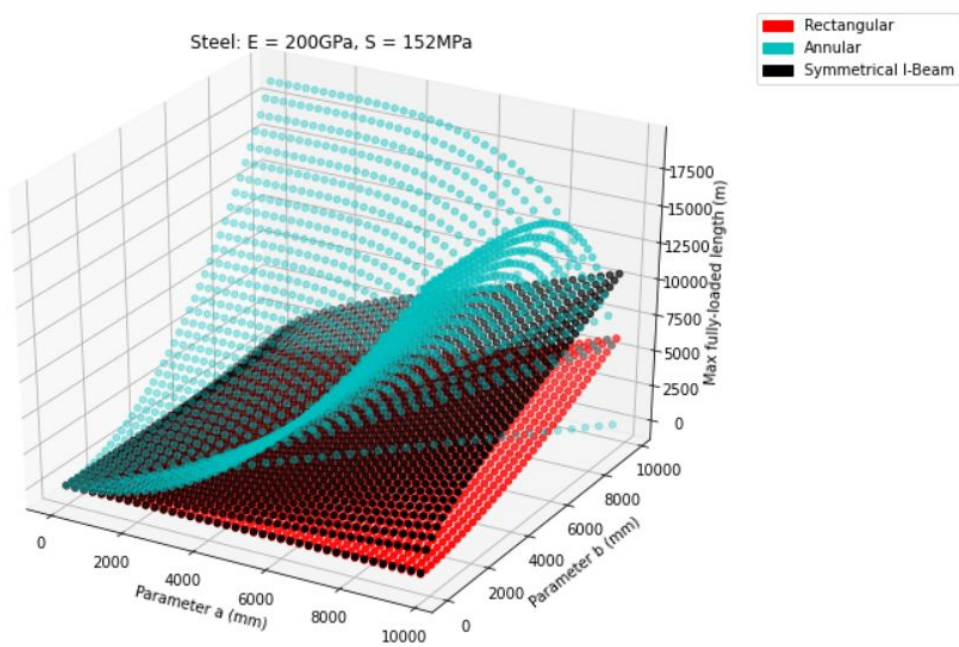
At this juncture, 2 more assumptions were made:

1. The vertical column would certainly be fixed at its bottom end, however it would be **free at its upper end**.
2. At the end of the structure would be a space station/cargo platform weighing **1000 tonnes**, or 2.5 times that of the International Space Station

The variation of gravity due to altitude isn't considered, although it would be quite unnecessary, as we shall see in the results.

$$\therefore L = \frac{\pi}{2}\sqrt{\frac{EI}{P_{\mathrm{cr}}}} \approx 5.015 \times 10^{-4}\sqrt{EI}$$

A plotting function was created to render a **3D scatter-plot** of L (as calculated above), and parameters **a** and **b**. The parameters were varied from **1** to **10000** with steps of **250**, leading to approximately **1600 data-points for each of the 3 cross-section shapes**.



Results:

The results are in accordance with earlier knowledge - compressive structures are unsuitable for constructing space elevator support columns. Additionally:

1. An annular cross-section gives the best value of maximum length for a fixed load.
2. The longest steel columns for the given range of parameters are **~18 km** long while the longest diamond columns are **~45 km**. The Kármán Line, a generally agreed definition of the boundary of space, begins at **100 km** above the surface.

3. *The dimensions of the cross-section roughly vary as the square of the length of the column*. To reach a sufficient altitude, the cross-section would need to be massive.
4. *All the values are idealisations based on the assumption that the column is entirely massless*. If the mass of the column was considered, the maximum length would be drastically lesser.

We compared two materials - one which is one of the most widely used materials for beams, and the other which is the hardest and one of the toughest materials. Both of them had glaring faults, despite massive assumptions and oversimplifications.

Thus, even after considering the absolute upper bound for the obtained values, we can conclude with certainty that compressively-loaded support columns for space elevators are impractical as space elevator support beams.

-x-x-x-x-x-x-x-x-x-x-x-x-