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OPTIMAL CONTROL STRATEGY TO DISTRIBUTE WATER THROUGH LOOP-LIKE PLANAR NETWORKS

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ABSTRACT

In this article, loop like planar networks formed by circular cross sectioned conduits with possibly different geometric measurements are studied to supply the required amount of isothermal water within the optimal time and through the shortest path. The flow optimization procedure is controlled by time varying pressures at nodes throughout the network for given specifications about pressure value at multiple demanding and single supply nodes. The flow governing equation is solved analytically to correlate transient flow rate and pressure and then studied using analogous electrical circuit. For each possible path between source and demand node, minimum equivalent flow impedance criterion is considered to pick the optimum path. This sets a multi-objective dynamic flow optimization algorithm and the same is executed under the assumption of fully developed and laminar flow. The optimum flow impedance can further be used to measure the pumping power as the cost of flow of a particular path. The algorithm can be extended to reduce the water wastages by controlling pressures efficiently.

Keywords: Flow impedance, lump parameter model, optimum flow, water distribution network.

NOMENCLATURE

The nomenclature followed in the presented work is given as follows:

m	Number of nodes in the Water Distribution Network.
n	Number of sinks to be catered in the Water Distribution Network.
k	Number of intermediate nodes in a path
A	n x n network matrix, such that a_{ij} is 0 if they are not connected with a pipe and 1 if a connection exists between them.
D	n x n diameter matrix, such that d_{ij} denotes the diameter of the pipe connecting node i and node j.
L	n x n length matrix, such that l_{ij} denotes the length of the pipe connecting node i and node j.
Z	n x n cost matrix, such that z_{ij} denotes the impedance experienced by the flow as a cost of travelling from node i to node j.
t	m x 1 demand time vector, where t_m denotes the required time in which demand has to be fulfilled at the m^{th} sink.
v	m x 1 demand volume vector, where v_m denotes the required volume at the m^{th} sink that has to be fulfilled.
f	Volume Flow Rate i.e. current analogy for lumped parameter circuit model
f_{ij}	Volume Flow rate between node i and node j.
P_{max}	Maximum pressure limit up to which pressure can be modulated at the nodes, and can be withstood by the pipes.
P_{min}	Minimum pressure limit.
V_i	Voltage at the i^{th} node i.e. pressure analogy for lumped parameter circuit model.
V_{max}, V_{min}	Lumped parameter analogy to P_{max} and P_{min}.
R	Flow resistance.
R_{ij}	Flow Resistance between node i and node j.
B_i	Component submatrices.

1. INTRODUCTION

From the last several decades, mathematical modelling and simulation of water distribution has always been a primary concern. The basic purpose of water distribution network (WDN) is to supply the required amount of water from source end to demand end(s). A general WDN consists of conduits, nodes (or junctions of conduit intersection), pumps, reservoirs and valves. The size of the network is decided by number of nodes and arrangement of conduits decides its structure. In order to supply the water, the primary aim is to minimize the pumping cost which maximizes the transportation efficiency. The pumping cost is directly proportional to the impedance of the path connecting source and demanding node. The primary aim of this article is to give a strategy to optimize the pumping cost by calculating flow impedance for distribution of water in loop-like planar networks.

Planar networks have been a topic of interest in context to material distribution networks with a varied usage towards urban, rural and agricultural water demands (Sherali and Smith [1], 1997) as well as in oil and gas pipelines. The same have also been studied for the transmission of electricity through conductors (Cross [2], 1936).

The goal related to the WDN is to distribute water throughout the network for given set of pressures at nodes or evaluating the pressures at nodes for given amount of flow rate in each pipe. The Hardy Cross method is the first systematic method developed to study this problem which uses an iterative scheme to find the flow distribution based on given pressures at source and sink side (Cross [2], 1936). This algorithm starts with initial assumption on pressure values at junction and accordingly check the flow rate in each conduit of the path. The algorithm terminates when the desired set of pressure values achieved to make flow to happen. This algorithm suits to find desired pressure values at junction points either locally or for small network. For big and/or complex network, it requires lot of computations and also the convergence is not guaranteed. To resolve this issue, a global optimization method is developed for network having the identical geometry of each conduit (Sherali and Smith [1], 1997).

Various design parameters like geometrical properties of pipes and elevation heads were studied to present a globally optimized network design based on a non-linear convex cost minimization (Sherali and Smith [1], 1997). The concept of energy minimization was also used where objective function was linked to the operating expenses to develop objective function (Sarbu [3], 2010). The optimization was in accordance with the demand variation. All these types of networks are fixed once designed and executed and are very efficient in catering the demands. But, they are unable to satisfy dynamic demand patterns and demand nodes.

Further the concept of energy minimization was extended to control flow in large WDN. The method was based on energy minimum principle and importance was given to the computation time as well. The problem of water loss was also addressed which occur due to leakages, such that, an optimal flow can be achieved and controlled (Miyaoka and Funabash [4], 1982). An efficient optimization process is developed for an open channel irrigation networks by fulfilling time and volume demands on the demand nodes based on a Multi Integer Linear Programming (MILP) Algorithm (Hong et al. [5], 2014). The flow routing process, network structure and properties, and volume available at source were some of the constraints considered to model the objective function. In the process, water loss was minimized. Both of the works addressed the problem of optimization with fixed network structure constraints. The problem of unknown flow directions was treated as an optimization variable along with the design parameters (Caballero and Ravagnani [6], 2019). Thus, a mixed integer non-linear programming (MINLP) model was developed. Although, the optimum design is fixed but flexibility in flow directions may help in addressing failures and change in demand.

In real life, there can also be situations when the demand pattern as well as the demand nodes change with periods of time. For an example, water is demanded at its maximum for a certain time period of a day, and if the same demand is supplied for the whole day, a lot of water can go to wastage. Such kind of problem is addressed in this article.

In this paper, loop-like planar networks are analysed in a way such that they make a WDN. Head loss effects are neglected in the process due to planarity and hence no effects of gravity are considered. The network is modelled in the form of a graph with a single supply node, the source, and multiple demanding nodes, the sinks. The speciality in the considered networks is the deployment of pressure check and control valves at all the nodes which can be potential sinks in the process. Finally, the optimization is defined as a path traversal problem with the costs formulated based on the network's geometrical characteristics and flow parameters. The flow path from the source to all the sinks is chosen based on a modified shortest path algorithm considering the net steady flow impedance calculated with the help of the lumped parameter, namely the flow resistance. For the given demand volumes for all the sinks, pressure along the path was obtained with the algorithm and can be executed with the help of the control valves. A multi-objective dynamic flow optimization problem is formulated. Isothermal, fully-developed and laminar flow assumptions are considered during the process and algorithms are developed for steady case. The source pressure was maintained at the maximum limit and the sink pressures were optimized with the help of the lumped parameter equivalent circuit.

The paper is organized as follows: Section 2 describes the optimization problem formulation and model development methodology for steady case. To test the developed algorithm, results of simple steady cases are considered in Section 3. This section describes the use-cases and optimized results based on certain volume demands at the sinks. Section 4 gives formulation for a possible transient case using electrical circuit analogy. The paper finally concludes in Section 5.

2. METHODOLOGY

For a given network with fixed geometrical properties, water has to be distributed from a single source to multiple sinks based on the dynamic demand volume and fulfilling time. The main goal is to determine the flow path with minimum impedance and modulate the pressure along the path such that the decided path is followed. The pressure control algorithm hence developed should also capture transient cases along with the static ones.

Let us consider a looped network with *n* number of nodes, out of which one is the source. There are *m* number of demand nodes or sinks. The notations that define the network characteristics are already mentioned in the nomenclature as **A, D, L** and **Z**. As the network is now fully defined, the flow properties and requirements are denoted as **t, v,** f_{ij}, P_{max} and P_{min}.

The planar network considered is given in figure 1 with *n* nodes, such that it is served by a single source node. The overall

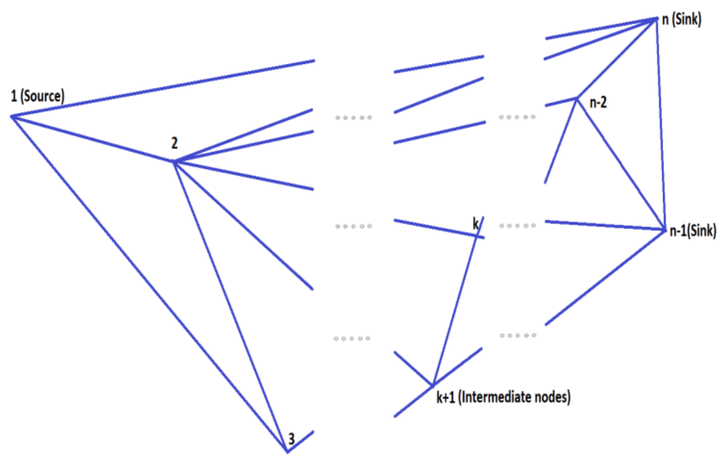

FIGURE 1: LOOPED NETWORK STRUCTURE WITH *N* NODES

demand is the cumulative of the volume needs at the multiple sinks. For every given network, matrix **A**, **D** and **L** are known, values being the geometrical properties. Also, let us assume that for any given period of time the demand time and volume vectors, **t** and **v** respectively, are provided.

For the steady case, the cost matrix **Z** will be nothing but the net flow resistance experienced by the flow from one node to another. In the lumped parameter model, resistance for a single pipe for a fully-developed, laminar and isothermal flow is calculated according to equation (1).

$$R_{ij} = \frac{8\,\eta\, l_{ij}}{\pi \left({d_{ij}}/{2} \right)^4} \tag{1}$$

For the initial case, the pressures are maintained at the same value at all the nodes, such that the magnitude is equal to the maximum allowable pressure, P_{max}. But, for the path with the minimum net impedance, pressure at all the constituting nodes are modulated to satisfy the demand in the required time. Although the source is still at the maximum, the other intermediate nodes continue to receive volume from their immediate neighbor (not on path) nodes as a result of a negative pressure gradient between them. The pressure at the remaining sink node is required to be optimized. As, all the neighboring nodes of the intermediate nodes will inflow water into the path and hence, the net impedance of the path will also incorporate the impedances due to them.

The methodology consists of only steady state case while the mathematical formulation for the unsteady case is derived in section 4. A looped planar network is constructed and the geometry was defined in terms of the constituting pipe diameters and lengths. The upper and lower bounds of the admissible pressure values at nodes are determined by the maximum pressure that the different pipes can withstand. Also, the time and volume demands are defined based on a priority scheme. Finally, the results are given as the paths from the source to the different sinks as well as the required pressure distribution along the chosen paths such that the desired flow can be achieved.

Single Source – Single Sink Case

The considered network is assumed to have the simplicity of taking conduits of equal diameter and lengths. Now the path from the source to the sink with the least possible impedance is chosen with the help of the modified shortest path algorithm (Algorithm 1) given below. Moreover, in the process, minimization of energy is achieved too.

ALGORITHM 1: FLOW IMPEDANCE MINIMIZATION

```
Function (Graph, Source, Destination)

        Convert graph to edge list

        Run Dijkstra Shortest Path Algorithm
        (using dynamic costs as per cost function)

        Choose Least cost path from source to
        destination

End


Cost Function (node I, node J) // I, J are immediate nodes

        If I == Source
                Cost = sum(1/Z)
                //Z is the cost from all neighboring
and source nodes of J

        Else if J == Destination
                Cost = Previous cost + R (I, J)

        Else
                Z1 = R (I, J) + Previous Cost
                Cost = 1/(sum(1/Z) + 1/(Z1))
        End
End
```

For the same optimized path, in order to compute the pressures at all the participating nodes, the lumped parameter model is used. Also, the consideration that the immediate nodes of the participating nodes also contribute to the total supply volume and impedance, the problem becomes complex. Let us consider a generalized path shown in the figure 2, with all nodes fitted with a pressure manipulating valve.

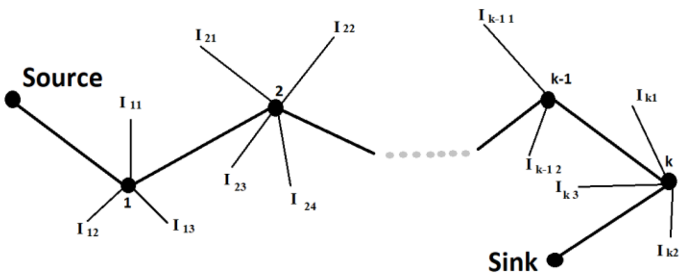

FIGURE 2: A SAMPLE PATH FROM SOURCE TO SINK WITH K INTERMEDIATE NODES, EACH WITH A SPECIFIC NUMBER OF IMMEDIATE NEIGHBOR NODES (OUTSIDE THE PATH)

Finally, the electrical circuit (lumped parameter model) can be developed from the path shown in figure 2 to the electrical circuit shown below in figure 3.

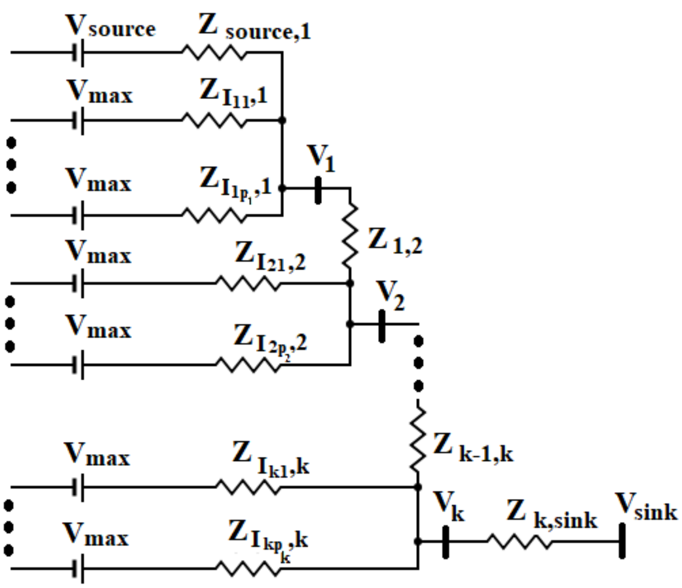

FIGURE 3: ELECTRICAL CIRCUIT ANALOGY OF FIGURE 2 PATH

In figure 3, V has been used as an analogy for the pressure values at the various nodes. With our assumption of maintaining the pressure at source to the maximum admissible pressure limit.

$$\mathrm{V}_{\max} = V_{source} \tag{2}$$

$$\mathrm{V}_{\mathrm{i}} = Pressure_{@node\ i} \quad i \in \{1, \dots, k\}$$

Thus, all the occurrences of V_{max} can be replaced with V_{source} and a simplified version of the circuit in figure 3 is shown in figure 4. The equivalent resistance of the parallel segments of the circuit are simplified to get a definite pattern. This pattern was then used to obtain the voltages and currents at all the segments of the circuit shown in figure 4. Also the net impedance was obtained in this manner.

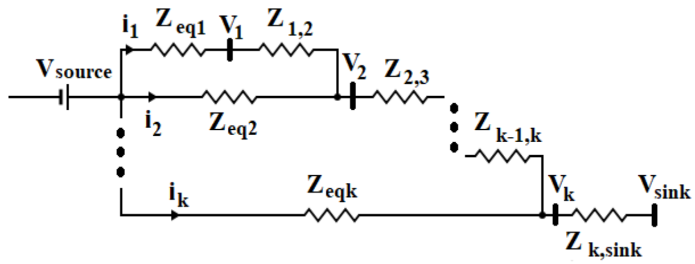

FIGURE 4: SIMPLIFIED VERSION OF THE ELECTRICAL CIRCUIT IN FIGURE 3

Now, from basic Kirchhoff's law of junction and voltage, voltages and currents at all the nodes can be obtained as follows:

$$\begin{bmatrix} B_{1\,k\,x\,k} & B_{2\,k\,x\,k} \\ B_{3\,k\,x\,k} & -B_{4\,k\,x\,k} \end{bmatrix}_{2k\,x\,2k} \begin{bmatrix} V_{k\,x\,1} \\ i_{k\,x\,1} \end{bmatrix}_{2k\,x\,1} = \begin{bmatrix} (V_{source})_{k\,x\,1} \\ (V_{sink})_{k\,x\,1} \end{bmatrix}_{2k\,x\,1}$$

where, (3)

$$B_{1\,k\,x\,k} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{k\,x\,k}$$

$$B_{2\,k\,x\,k} = \begin{bmatrix} R_{eq\,1} & 0 & 0 & 0 \\ 0 & R_{eq\,2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & R_{eq\,k} \end{bmatrix}_{k\,x\,k}$$

$$B_{3\,k\,x\,k} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & \ddots & 0 \\ 0 & 0 & \ddots & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{k\,x\,k}$$

$$B_{4\,k\,x\,k} = \begin{bmatrix} R_{1,2} & 0 & 0 & 0 \\ R_{2,3} & R_{2,3} & \dots & 0 \\ \vdots & \vdots & \dots & 0 \\ R_{k,sink} & R_{k,sink} & \dots & R_{k,sink} \end{bmatrix}_{k\,x\,k}$$

$$V_{k\,x\,1} = \begin{bmatrix} V_1 \\ \vdots \\ V_k \end{bmatrix}_{k\,x\,1} \qquad i_{k\,x\,1} = \begin{bmatrix} i_1 \\ \vdots \\ i_k \end{bmatrix}_{k\,x\,1}$$

$$(V_{source})_{k\,x\,1} = \begin{bmatrix} V_{source} \\ V_{source} \\ \vdots \\ V_{source} \end{bmatrix}_{k\,x\,1} \qquad (V_{sink})_{k\,x\,1} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ V_{sink} \end{bmatrix}_{k\,x\,1}$$

Now, we have an optimized path, the lumped parameter model of the path, hence, the next thing is to achieve the appropriate pressure distribution along the chosen paths in order to meet the time and volume demands at the sinks. Thus, a genetic algorithm is used to optimize the pressure at the sinks. Let the pressure at the sinks is given by V_{sink}. Thus,

$$volume\ flow\ rate, f = \frac{V_{source} - V_{sink}}{R} \tag{4}$$

$$R = R_{equivalent}\ for\ path\ corresponding\ to\ sink$$

where, R can be calculated from the circuit given in figure 4 corresponding to the path from the source to the sink. From the volume flow rate of the path, the time to deliver the required volume is obtained as

$$total\ time\ taken, t_{calculated} = \frac{Volume\ Needed}{f} \tag{5}$$

and finally the objective function is formulated as the sum of the squared deviations of the time taken from the desired timing at the sink i.e.

$$Objective\ function = \ minimize\,\{(t_{desired}\ -\ t_{calculated}\ )^2\} \tag{6}$$

Hence, satisfying this objective function aims at caterıng the exact demand needs in the nearest possible time.

Single Source – Double Sink Case

With the assumption of same geometry of all the conduits, this case serves as an extended and more real definition of the water distribution network. Now, we have 2 sinks being served by a single source, the node structure of which has been shown in figure 5.

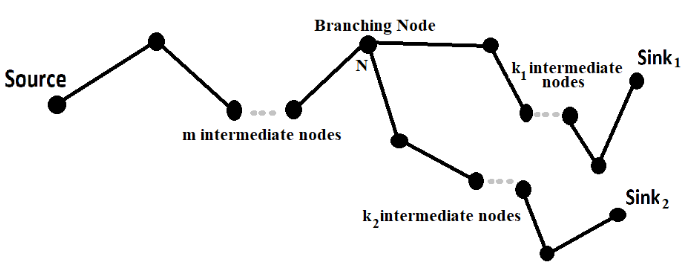

FIGURE 5: A SAMPLE PATH FROM SOURCE TO 2 SINKS WITH RESPECTIVE INTERMEDIATE NODES AND A BRANCHING NODE, EACH WITH A SPECIFIC NUMBER OF IMMEDIATE NEIGHBOR NODES (SAME AS FIGURE 2, NOT SHOWN TO AVOID COMPLEXITY)

There is a common node where the branching takes place for the different sinks. In case, the path to the two sinks are totally different with no common nodes except the source, the branching node N becomes nothing but the source node itself. Moreover, it has to be kept in mind that for every segment of path, there exists a lumped parameter model as shown in figure 4. The model has been compressed to an equivalent impedance block and the final circuit analogous is being depicted in the figure 6.

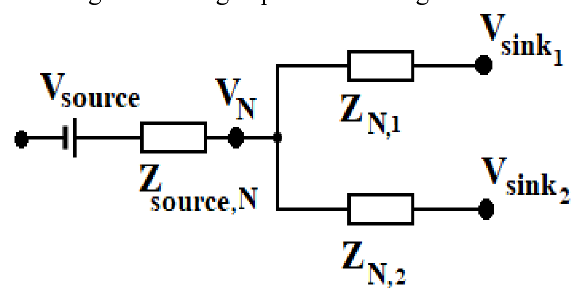

FIGURE 6: LUMPED CIRCUIT FOR SINGLE SOURCE AND 2 SINKS WITH RESPECTIVE EQUIVALENT IMPEDANCE BLOCKS, THE BLOCKS WITH $Z_{M,N}$ REPRESENT THE CIRCUIT IN FIGURE 4 BETWEEN NODES *M* AND *N*, INSTEAD OF SOURCE AND SINK

For such scenarios, it is quite difficult to come with an overall equivalent impedance of the network. The voltages or pressures at sinks may be different, but it has been considered that the current or the flowing water will divide themselves as directly proportional to the reciprocal of their path impedance. Furthermore, resistance is the only element contributing to the impedance and it is linear in nature. So, the problem can be divided into a number of single source- single sink cases. Like, V_{source} to V_N , then V_N to V_{sink1} and so on for the next sink as well.

It has to be made clear that the method of superposition is not a good measure to use in such a case, as the superposition cannot be done at the same instants of the flow. Moreover, it has to be kept in mind that the common node to the two different paths i.e. the branching node should have the same pressure in both the paths. An overall superposition may prove misleading results for the desired pressure at the branching node in such a case.

The problem is now divided into two segments, first segment optimizes the pressure at the branching node, N, for a certain value of time that the water takes to reach from source to node N. This particular time, say t_0, interval is again optimized over the total problem. Now, with the node N pressure known, pressure at both the sinks are optimized and hence the time, say t_1, for sink 1 was calculated. In order to satisfy the volume demands in the required time, both t_1 and t_2 were optimized.

3. RESULTS

The above algorithm is implemented through a fixed water distribution network. Commonly, WDNs are composed of either radial links or rectangular loops, where angle between two conduits play significant role. In this study, for the sake of simplicity, angles between conduits are not considered. Hence any WDN of the form of figure 7 can be visualized into a graphical node structure network as in figure 8.

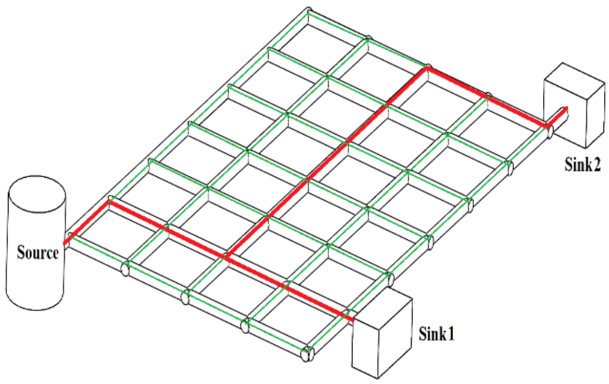

FIGURE 7: A SAMPLE PATH FROM SOURCE TO 2 SINKS WITH RESPECTIVE INTERMEDIATE

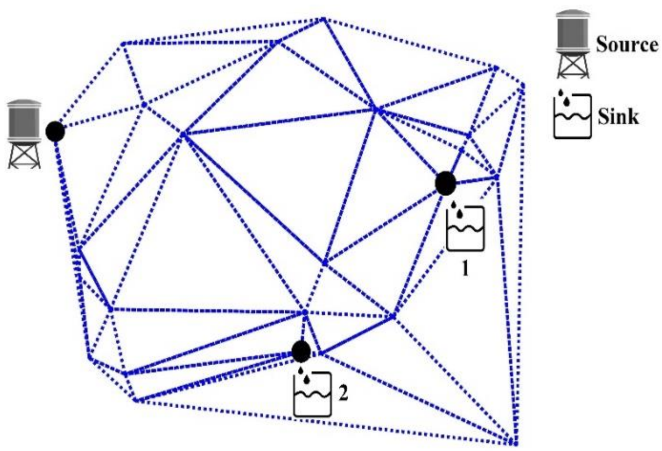

FIGURE 8: A SAMPLE PATH FROM SOURCE TO 2 SINKS WITH RESPECTIVE INTERMEDIATE

For such a transformed network, two specific steady flow cases between single source-single sink and single source-double sinks are discussed below.

Case 1: Single Source – Single Sink (Steady Flow)

The network is composed of conduits of fixed diameter and length, and thus the cost of diverting the flow through them i.e. loss due to flow resistance is already known. Suppose, there is a sink which requires 180 liters of water in a time frame of 2 minutes. The aim is to find the least impedance path and a pressure distribution along that path to cater the time and volume demands.

The least impedance desired path was obtained through the flow impedance minimization algorithm discussed in the methodology above. The path between the source and the sink is shown in the figure 9 below.

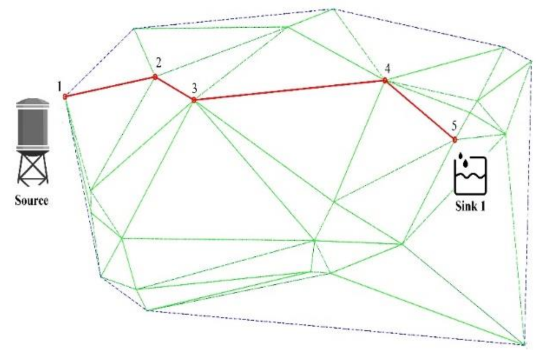

FIGURE 9: A SAMPLE PATH FROM SOURCE TO 2 SINKS WITH RESPECTIVE INTERMEDIATE

The next step is to decide the pressure distribution along the path through nodes 1 (Source) to 5 (Sink). The pressure at the source is already set to maximum allowable pressure, say 20 psi for the study. Now, the formulation was performed according to the methodology to get the equivalent lumped parameter model as shown in figure 4. Equation (3) was used to finally obtain a suitable pressure distribution as shown in Table I.

TABLE I: PRESSURE DISTRIBUTION FOR FIGURE 9

Node	Pressure (in psi)
1 (Source)	20.0000
2	19.9437
3	19.7390
4	19.6050
5 (Sink)	19.4689

Case 2: Single Source – Double Sink (Steady Flow)

Similar to the first case, suppose there are two sinks where the water has to be supplied from the source. The time and volume requirements are given as follows in table II.

TABLE II: DEMAND FOR CASE 2

Sink	Volume Needed	Time Constraint
1	200 liters	3 minutes
2	180 liters	2 minutes

In such a situation, the optimization is done in two phases, first of all, the path is obtained which has the least impedance for the two sinks collectively. If the path composed of two separate segments with no common nodes except the source, the solution would be similar to the first case. The obtained path, as shown in figure 10, has a common node (Node 3) after which the bifurcation has to be made.

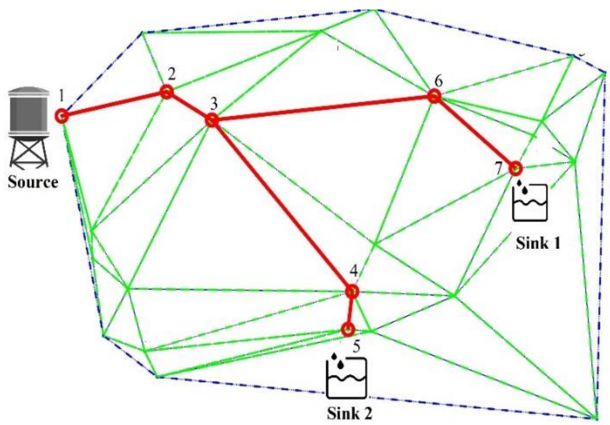

FIGURE 10: A SAMPLE PATH FROM SOURCE TO 2 SINKS WITH RESPECTIVE INTERMEDIATE

The algorithm first treats the node 3 as the sink and satisfies the demand volume as a cumulative demand of both the sinks. The time which the flow takes to reach node 3, say t_0, has to be optimized. For the next phase, the source is now transformed to the common node, i.e. node 3, and the sinks are now solved individually as in case 1. The final time required to fill the sinks with the required volume is taken to be t_1 and t_2 respectively for sink 1 and sink 2. Finally, t_0, t_1 and t_2 are optimized with the constraints in equation to give the final pressure distribution given in table III.

$$
\begin{aligned}
t_0 + t_1 &= t_{sink1}^{required} = 3\ minutes \\
t_0 + t_2 &= t_{sink2}^{required} = 2\ minutes
\end{aligned}
\tag{7}
$$

TABLE III: PRESSURE DISTRIBUTION FOR FIGURE 10

Node	Pressure (in psi)
1 (Source)	20.000
2	19.5591
3	19.1320
4	18.8738
5 (Sink)	18.3489
6	18.9675
7 (Sink)	18.7894

4. DISCUSSIONS

The study considers steady case situations where time does not play a vital role but only acts as a constraint. Also, the fact that the WDN is always filled with water cannot be considered here. The immediate neighbors of each node on the path contribute to the flow and hence add to the net impedance but the amount does not contribute to the steady state case of the problem. However, same contributes significantly for a transient case and results of which can be a possible improvement to the study.

Finally, a synopsis of the method given as follows establishes the novelty accompanying it. First, the method takes into account an already developed WDN and optimizes its functioning. The method does not optimize the WDN geometry like many other papers in the domain [4][5].

Secondly, the method brings into account a dynamic cost shortest path problem which is solved by modifying Dijkstra's algorithm for finding the least impedance path. The method takes into account various concepts of graphs and nodes instead of other iterative methods [1][2][6]. Methods like Hardy Cross are based on an iterative scheme and can be visualized as a method for obtaining the equivalent impedance of the complete network. As the method takes into account the overall network, the computations can increase for a large network. The method concerned in this paper used the modified shortest path algorithm to constrain the flow in the network in the least impedance path.

When the path for least impedance was considered the flow in the main path along with contribution from the immediate neighboring conduits of constituting nodes were also considered due to the initial pressure setup as discussed. This consideration is a new approach towards solving problems with WDNs according to the author's knowledge and thus leads to a modified analogous electrical network for the path - figure 3. Thus, this can be well seen that the cost of flowing water through a conduit not only depends on the conduit but also on the path previously traversed. The equivalent electrical circuit is finally solved by the dynamic cost path algorithm developed in the study.

Worst Case Computation Complexity of the method

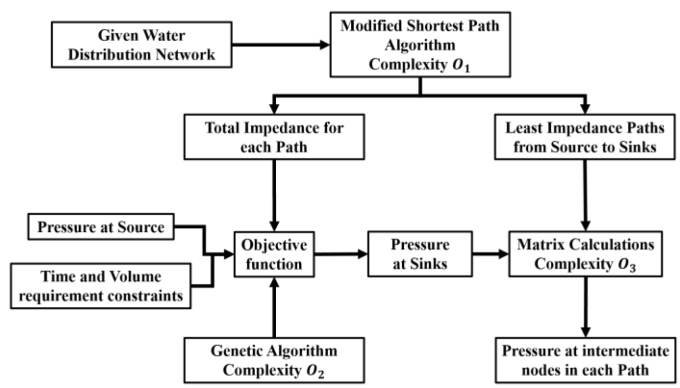

FIGURE 11: THE COMPLETE ALGORITHM OF THE METHOD DISCUSSED IN THE PRESENTED WORK

The complexities of the major components of the algorithm in the above figure 11 are denoted as O_1, O_2, O_3 for the modified shortest path algorithm, genetic algorithm and the matrix calculation (according to equation (3)) respectively.

For the modified shortest path algorithm, the complexity is $O(v^2 * (w-1))$, where v is the number of nodes of the given WDN and w be the worst case number of immediate neighbors to each of the nodes. The process runs for each sink.

The choices for the genetic algorithm being the point mutation, one-point crossover and roulette wheel selection, the

Genetic algorithm, with g number of generations, n population size and m the size of the individual (equal to number of sinks), has a complexity of the order of $O(g * m * n)$.

Fir the matrix calculation component, let the path of the least impedance path be x $(= 8$ in this case). The calculation is divided into two steps, the matrix inversion and the matrix multiplication steps with individual complexities as $O(8 * y^3)$ and $O(4 * y^2)$ respectively, where $y = x - 2$ i.e. the total number of intermediate nodes removing the source and sink nodes. This process also runs for each sink.

Thus, the final complexities for each component are as follows:

$$O_1 = O(v^2 * (w - 1)) \tag{8}$$

$$O_2 = O(g * m * n) \tag{9}$$

$$O_3 = O(8 * y^3) + O(4 * y^2) \tag{10}$$

And the final complexity, O, for the overall algorithm given in figure 11 as computed from equations (8), (9) and (10) is,

$$O = m * O_1 + O_2 + m * O_3 \tag{11}$$

where m is also the number of sinks to be catered.

5. CONCLUSION

The main problem was to find a path from source to sink(s) for the efficient flow in a water distribution network. An algorithm is developed based on the electrical circuit analogy which minimizes the overall flow resistance in choosing a suitable path. The process indirectly contributes to an energy minimization scheme. The algorithm is capable of modulating pressures at nodes in a way that can be used for periodic and dynamic volume supply demands in a constrained time situation. Finally, a controller can be used to execute the pressure distribution at the path nodes with the help of the pressure control valves.

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