Problem 3

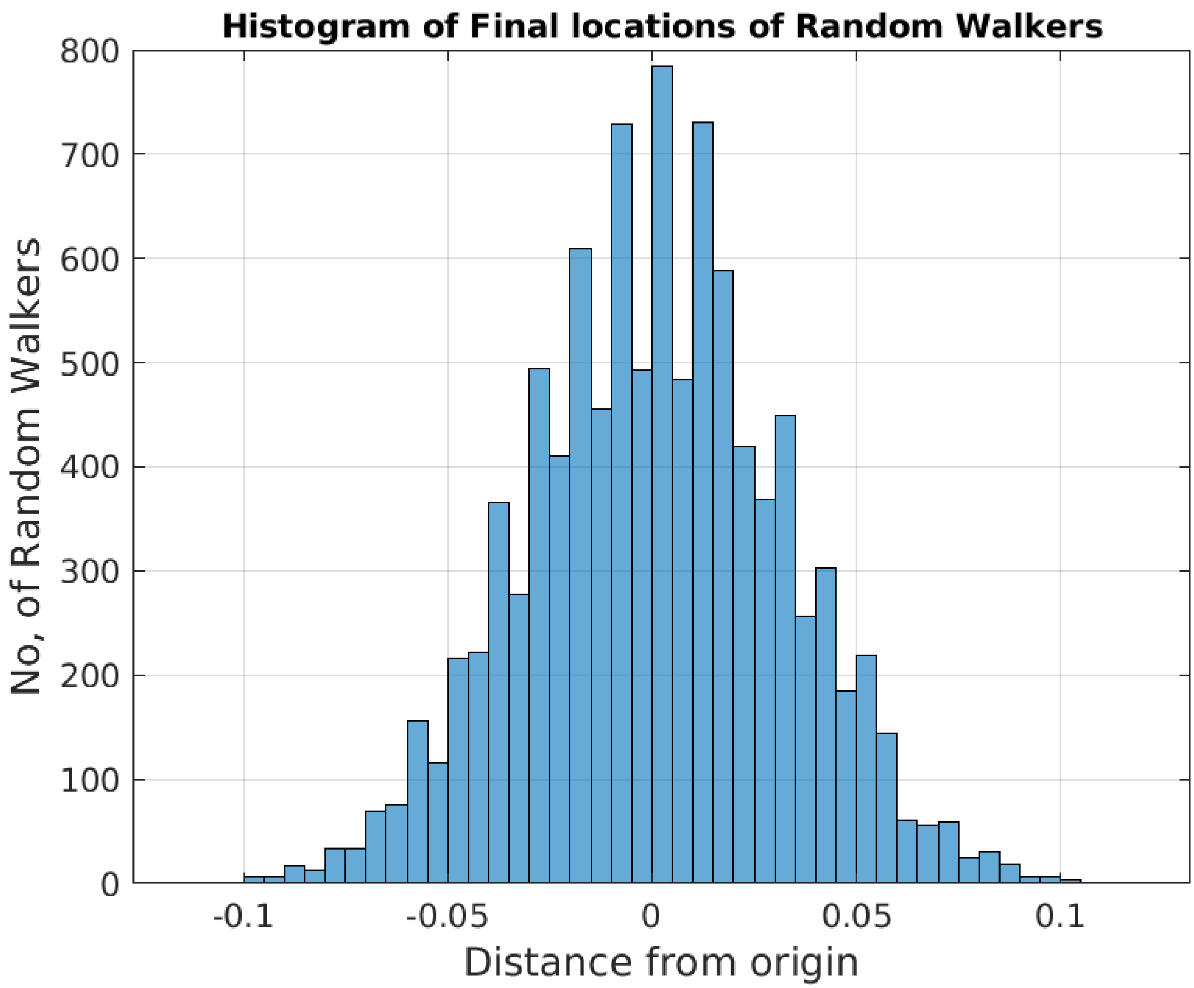
How to run code

- My submissions are live-scripts of matlab
- They are in zip format, don't unzip them
- Open matlab go to the folder of that zip and open the *.mlx files from matlab
- They would appear as jupyter notebook with markdown, codeblock and outputs.
- Press ctrl + enter to run each code block

Problem 3.1

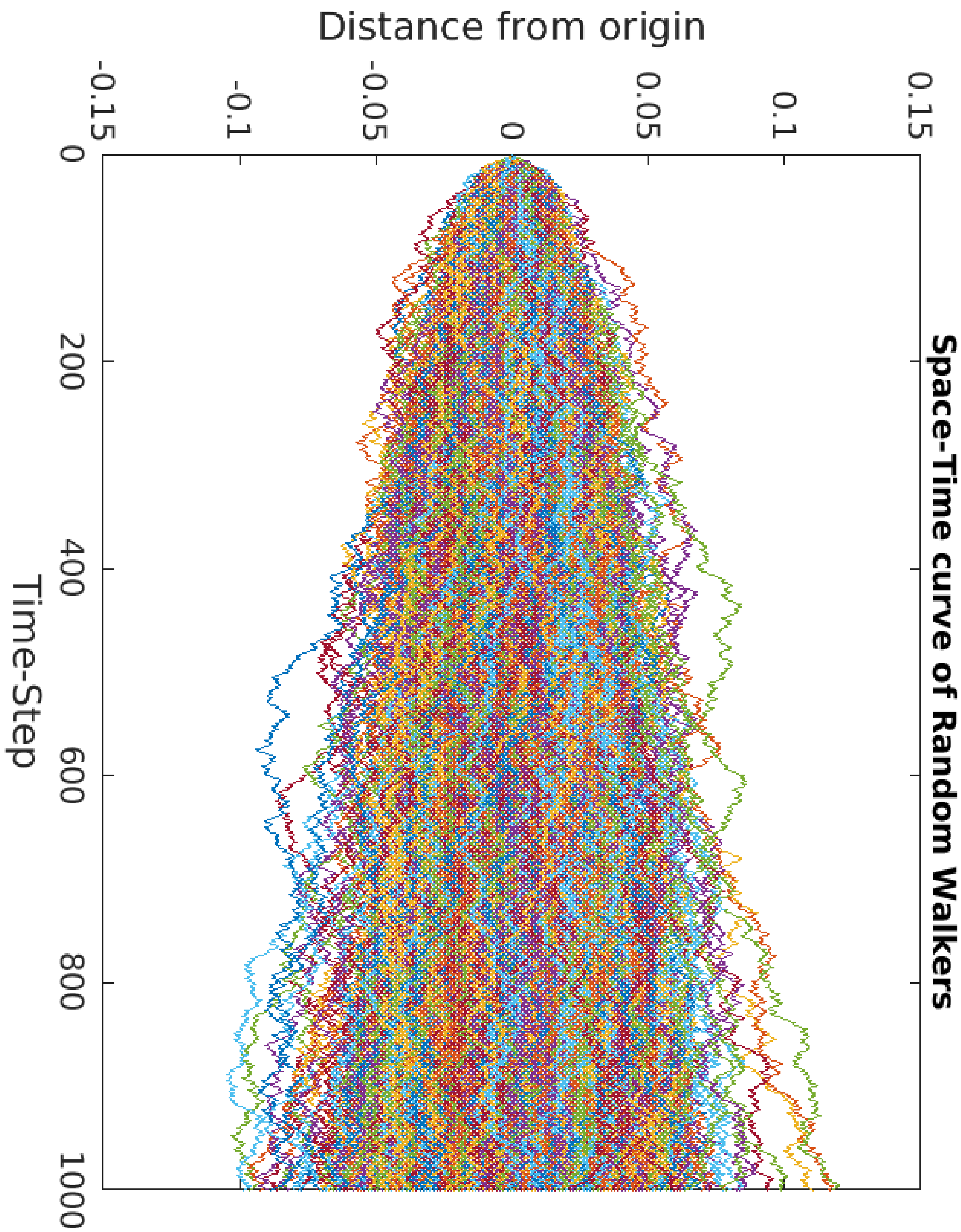
Part 1

- We have N = 10^4 random walkers.
- Further suppose within each timeframe Δt, walkers take either a $\Delta z = 10^{-3}$ step right with probability p (say 0.5 for simplicity), OR a Δz step left with probability q := 1 - p (= 0.5)
- If in an event a walker took 'x' steps right and 'n-x' steps left
  then random walker's final location is at z = $\Delta z(2x - n)$
- Where variable x can be obtained using binornd() in matlab for various random walkers $N_i \in N$, because x is the result of repeated Bernoulli trial (Number of success(here taking right step) in n trials with success probability p) and hence has a binomial distribution.
- Now we can find final location and plot the histogram



Part 2

- A 2D matrix is made to contain location of each random walker after each step 1,2,3....n
- This was done by running a loop and then adding to the previous row, 10^3 Bernoulli random variables for left and right steps, to obtain the next row.



Part 3

[code]

Problem 3.2

a) Law of large numbers:- For all $\epsilon > 0$, as $N \to \infty$, $P(|\hat{M} - E[X]| \geq \epsilon) \to 0$

By Markov inequality, we write

$$P(|\hat{M} - E[x]| \geq \epsilon) \leq \frac{E(|\hat{M} - E[x]|)}{\epsilon}$$

So we have

$$E(|\hat{M} - E[X]|) \to 0 \text{ as } N \to \infty$$

So $\hat{M} \doteq E[X]$ as $N \to \infty$

b) Replacing X_i by X_i^2 in the above proof, we have

$$\frac{X_1^2 + X_2^2 + \dots + X_N^2}{N2} \doteq E[X^2] \text{ as } N \to \infty$$

Now,

$$\hat{V} = \sum_{i=1}^{N} \frac{(x_i - \hat{M})^2}{N} = \frac{\sum_{i=1}^{N} x_i^2}{N} - \frac{2\hat{M}\sum_{i=1}^{N} x_i}{N} + \frac{\hat{M}^2 \cdot N}{N}$$

$$= \frac{\sum_{i=1}^{N} x_i^2}{N} - \hat{M}^2$$

The first ~~tends~~ tends to $E[X^2]$ and second tends to $E[X]^2$ as $N \to \infty$ by part A.

So,

$$\hat{V} \to E[X^2] - E[X^2] \text{ as } N \to \infty$$

or $\hat{V} \doteq Var(X)$ as $N \to \infty$

Part 1

Empirically-computed mean $\hat{M}$ = 3.9120e-04
Empirically-computed variance $\hat{V}$ = 9.9757e-04

Part 2

$$\begin{aligned} \text{True mean} &= \frac{\sum_{N_i=1}^{N} z_i}{N} \\ &= \frac{\sum_{N_i=1}^{N} \Delta z(2x_i - n)}{N} \\ &= \Delta z(2\frac{\sum_{N_i=1}^{N} x_i}{N} - n) \end{aligned}$$

Now since x is a binomial distribution with success probability p and number of trials n its mean is simply np.

Thus :=
True Mean = $\Delta z \cdot n(2p-1)$
= 0

$$\begin{aligned} \text{True mean} &= Var(z) \\ &= Var(\Delta z(2x_i - n)) \\ &= 4(\Delta z)^2 Var(x) \end{aligned}$$

Now since x is a binomial distribution with success probability p and number of trials n its variance is simply np(1-p).

Thus :=
True Variance = $4(\Delta z)^2 \cdot np(1-p)$
= 1.0000e-03

Part 3

error_mean = |True mean - Empirical mean| = 3.9120e-04
error_variance = |True variance - Empirical variance| = 2.4314e-06