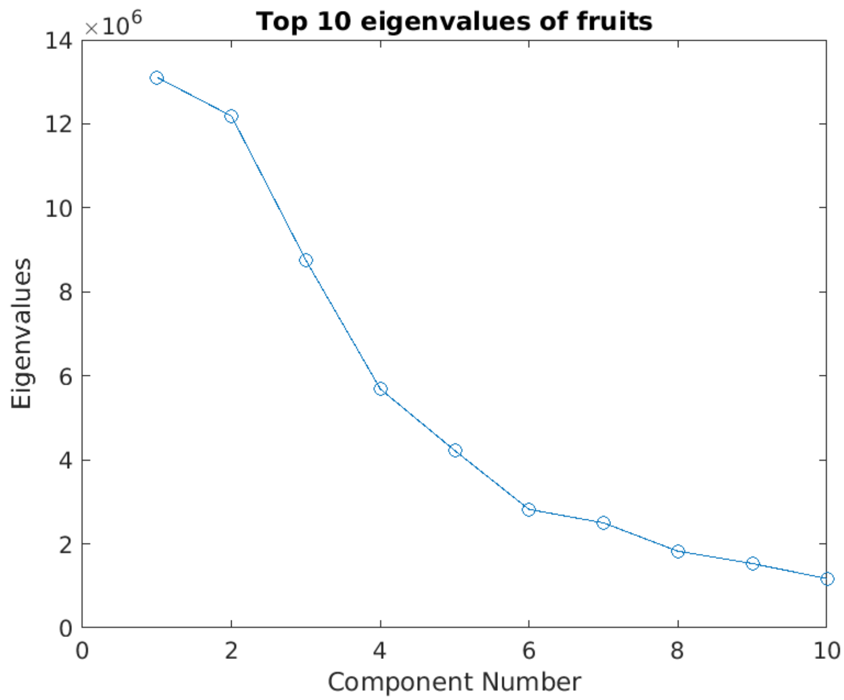
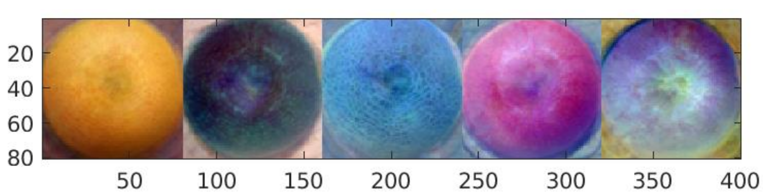
PROBLEM 6

A)



We see again that the eigenvalues decrease steeply even in the first 10 values.



B)

First, orthonormalise the 4 given eigenvectors to $\{\vec{u_1}, \vec{u_2}, \vec{u_3}, \vec{u_4}\}$. Let the given fruit image dataset be $\vec{X}$ and the mean be $\vec{\mu}$.

The problem reduces to finding reals $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ such that

$$\| \vec{X} - \vec{\mu} - \alpha_1 \vec{u_1} - \alpha_2 \vec{u_2} - \alpha_3 \vec{u_3} - \alpha_4 \vec{u_4} \|_{frob}$$

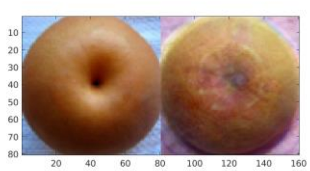
is minimised.

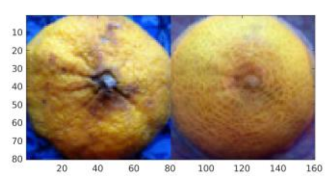
We claim that

$$\alpha_1 = (\vec{X} - \vec{\mu}) . \vec{u_1}$$
$$\alpha_2 = (\vec{X} - \vec{\mu}) . \vec{u_2}$$
$$\alpha_3 = (\vec{X} - \vec{\mu}) . \vec{u_3}$$
$$\alpha_4 = (\vec{X} - \vec{\mu}) . \vec{u_4}$$

This is because frobenius norm is square root of sum of squares of all elements. This makes it acts like Euclidean norm. So to minimise, we take the projection of the n-dimensional vector over the m-dimensional space.

Image 1 image 2 image 3





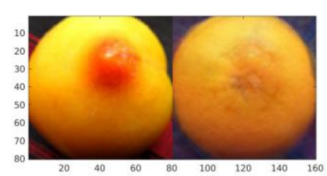


Image 4 Image 5 Image 6

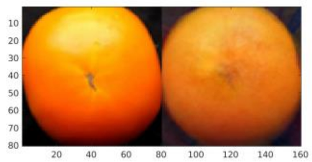


Image 7

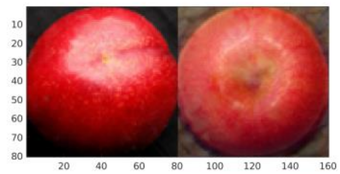


Image 8

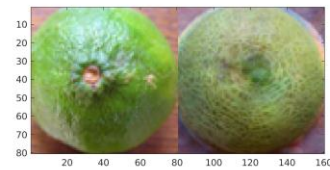


Image 9

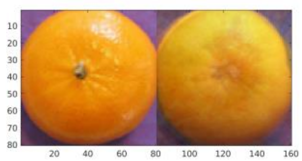


Image 10

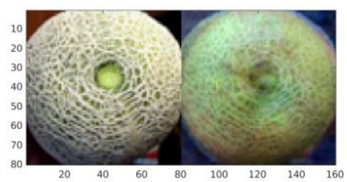


Image 11

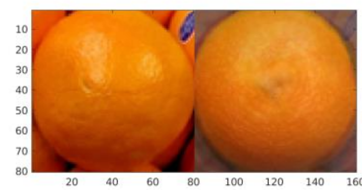


Image 12

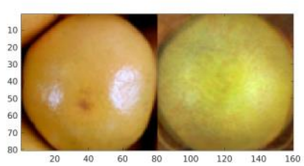


Image 13

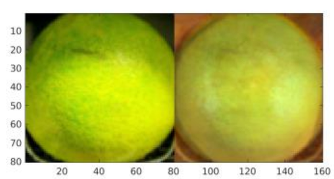


Image 14

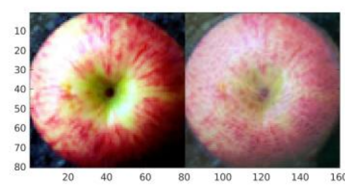


Image 15

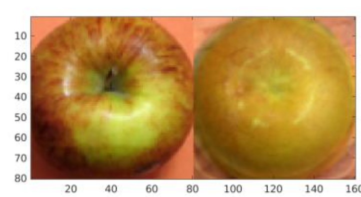
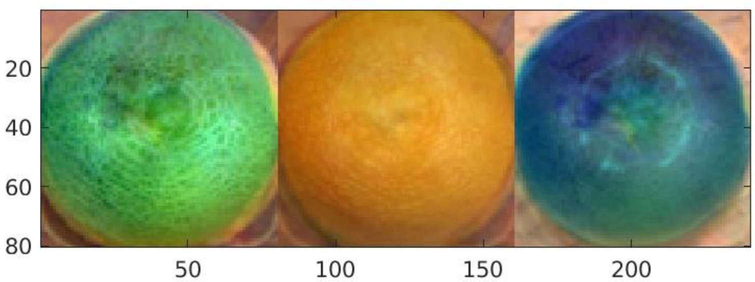


Image 16

C)

New Fruits Generated-



Algorithm-

We first find the ML-estimates for the mean μ and the covariance matrix C. Let the top 4 eigenvectors of C be $\vec{v_1}, \vec{v_2}, \vec{v_2}, \vec{v_4}$ with corresponding eigenvalues $\lambda_1, \lambda_2, \lambda_3, \lambda_4$.
We reduce the m-dimensional image to 4 principal dimensions using PCA.
So for a random image X, we get

$$\underset{m\times 1}{X} = \underset{m\times 4}{A}\,\underset{4\times 1}{W} + \mu_{m\times 1}$$

where $C = AA^T$ and W is the standardised multivaritate normal distribution.
Here, we find that

$$A_{m\times 4} = [\vec{v_1}\ \vec{v_2}\ \vec{v_3}\ \vec{v_4}]_{m\times 4} \cdot \operatorname{diag}(\sqrt{\lambda_1}, \sqrt{\lambda_2}, \sqrt{\lambda_3}, \sqrt{\lambda_4})_{4\times 4}$$

(See proof)
Using this matrix A, we calculate three values of X by using randn() in W.

Proof-

C is a real symmetric matrix. Using spectral theorem, we get $C = QDQ^T$ where D is a diagonal matrix consisting of eigenvalues of C and Q contains orthonormal eigenvalues of C.
Let $D = \operatorname{diag}(\lambda_1, \lambda_2 --, \lambda_n)$. Then define
$P = \operatorname{diag}(\sqrt{\lambda_1}, \sqrt{\lambda_2}, ---, \sqrt{\lambda_n})$
We see that $D = PP^T$.
Substituting,

$$C = QDQ^T = QPP^TQ^T = QP(QP)^T = AA^T$$

So, $A = QP$ is a solution.