Problem 2

How to run code

- My submissions are live-scripts of matlab
- They are in zip format, don't unzip them
- Open matlab go to the folder of that zip and open the *.mlx files from matlab
- They would appear as jupyter notebook with markdown, codeblock and outputs.
- Press ctrl + enter to run each code block

Part 1

(i) The Empirical function cannot be shown in the text, it is obtained in the code as a function.

k	$\hat{P}(Z = k)$
0	0.0009
1	0.0063
2	0.0224
3	0.0518
4	0.0915
5	0.1279
6	0.1492
7	0.1489
8	0.1299
9	0.1018
10	0.0708
11	0.0452
12	0.0265
13	0.0141
14	0.0070
15	0.0034
16	0.0015
17	0.0006
18	0.0002
19	0.0001
20	0.0000
21	0.0000
22	0
23	0
24	0
25	0

(ii)

Q-2) (a) (ii)

$$P_x(X=k) = \frac{\lambda_x^k e^{-\lambda_x}}{k!} \qquad P_Y(Y=b) = \frac{\lambda_y^b e^{-\lambda_y}}{b!}$$

$$P_z(Z=t) = \sum_{k=0}^{t} P(X=k, Y=t-k)$$

$$= \sum_{k=0}^{t} P(X=k)\,P(Y=t-k) \quad \left(\begin{array}{l} X \text{ and } Y \\ \text{are independent} \end{array}\right)$$

$$= \sum_{k=0}^{t} \frac{e^{-\lambda_x} \cdot \lambda_x^k}{k!} \cdot \frac{e^{-\lambda_y} \lambda_y^{t-k}}{(t-k)!}$$

$$= \frac{e^{-(\lambda_x+\lambda_y)}}{t!} \sum_{k=0}^{t} \binom{t}{k} \lambda_x^k \lambda_y^{t-k}$$

$$= \frac{e^{-(\lambda_x+\lambda_y)} (\lambda_x+\lambda_y)^t}{t!}$$

We expect $P(z)$ to be a Poisson distribution with parameter $\lambda_x + \lambda_y$.

(iii)

k	$\hat{P}(Z=k)$	$P(Z=k)$
0	0.0009	0.0009
1	0.0063	0.0064
2	0.0224	0.0223
3	0.0518	0.0521
4	0.0915	0.0912
5	0.1279	0.1277
6	0.1492	0.1490
7	0.1489	0.1490
8	0.1299	0.1304
9	0.1018	0.1014
10	0.0708	0.0710
11	0.0452	0.0452
12	0.0265	0.0263
13	0.0141	0.0142
14	0.0070	0.0071
15	0.0034	0.0033
16	0.0015	0.0014
17	0.0006	0.0006
18	0.0002	0.0002
19	0.0001	0.0001
20	0.0000	0.0000
21	0.0000	0.0000
22	0	0.0000
23	0	0.0000
24	0	0.0000
25	0	0.0000

The values obtained for various k is almost similar for the estimate $\hat{P}(Z)$ and for the PMF $P(Z)$

Part 2

(i) The Empirical function cannot be shown in the text, it is obtained in the code as a function.

k	$\hat{P}(Z = k)$
0	0.0413
1	0.1309
2	0.2082
3	0.2217
4	0.1769
5	0.1146
6	0.0608
7	0.0276
8	0.0111
9	0.0040
10	0.0013
11	0.0004
12	0.0001
13	0.0000
14	0.0000
15	0.0000
16	0.0000
17	0.0000
18	0.0000
19	0.0000
20	0.0000
21	0.0000
22	0
23	0
24	0
25	0

(ii)

(b) (ii)

$$P(Z=k) = \sum_{j=k}^{\infty} P(Y=j, Z=k) = \sum_{j=k}^{\infty} P(Z=k \mid Y=j) P(Y=j)$$

$$= \sum_{j=k}^{\infty} \frac{e^{-\lambda} \lambda^j}{j!} \binom{j}{k} p^k (1-p)^{j-k}$$

$$= e^{-\lambda} \sum_{j=k} \frac{\lambda^j}{j!} \frac{j!}{k!(j-k)!} p^k (1-p)^{j-k}$$

$$= \frac{e^{-\lambda} (\lambda p)^k}{k!} \sum_{j=k} \frac{(\lambda(1-p))^{j-k}}{(j-k)!}$$

$$= \frac{e^{-\lambda p} (\lambda p)^k}{k!}$$

We expect $P(Z)$ to be a poisson distribution with parameter λp.

(iii)

k	$\hat{P}(Z=k)$	$P(Z=k)$
0	0.0413	0.0408
1	0.1309	0.1304
2	0.2082	0.2087
3	0.2217	0.2226
4	0.1769	0.1781
5	0.1146	0.1140
6	0.0608	0.0608
7	0.0276	0.0278
8	0.0111	0.0111
9	0.0040	0.0040
10	0.0013	0.0013
11	0.0004	0.0004
12	0.0001	0.0001
13	0.0000	0.0000
14	0.0000	0.0000
15	0.0000	0.0000
16	0.0000	0.0000
17	0.0000	0.0000
18	0.0000	0.0000
19	0.0000	0.0000
20	0.0000	0.0000
21	0.0000	0.0000
22	0	0.0000
23	0	0.0000
24	0	0.0000
25	0	0.0000

The values obtained for various k is almost similar for the estimate $\hat{P}(Z)$ and for the PMF $P(Z)$