PROBLEM 1

A)

Our algorithm generates random points inside a unit-circle using rand() on $r-\theta$ coordinates, then scales the circle to the dimensions of the ellipse to get a uniform distribution.

Algorithm -

Step 1. Define r and θ to be randomly distributed numbers between $[0,1)$ and $[0,2\pi)$.

Step 2. Convert to cartesian coordinates x, y using

$$x = \sqrt{r}\cos\theta$$
$$y = \sqrt{r}\sin\theta$$

(The sqrt. of r ~~is to~~ create uniform distribution)
This creates uniform points in unit circle.

Step 3. Scale x, y to give ellipse

$$x = x \cdot \frac{\text{major-axis}}{2}$$
$$y = y \cdot \frac{\text{minor-axis}}{2}$$

Step 4. Repeat Steps 1, 2, 3 for N iterations and plot.

Proof of sqrt. factor in Step 2

$$f_{X,Y}(x,y) = f_{r,\theta}(r,\theta) \cdot |J|$$

where J is the Jacobian matrix and

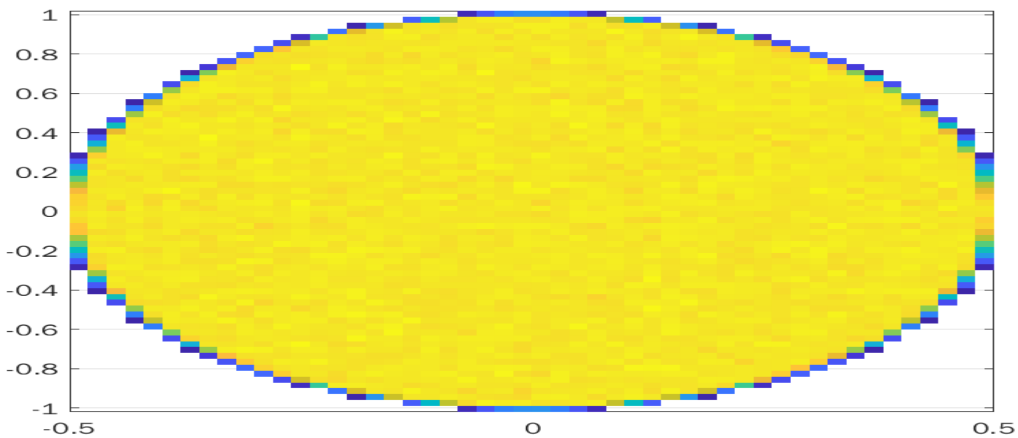
$$x = \sqrt{r}\cos\theta, \quad y = \sqrt{r}\sin\theta$$

$$|J| = \begin{vmatrix} \frac{\partial}{\partial\theta}(\sqrt{r}\cos\theta) & \frac{\partial}{\partial\theta}(\sqrt{r}\sin\theta) \\ \frac{\partial}{\partial r}(\sqrt{r}\cos\theta) & \frac{\partial}{\partial r}(\sqrt{r}\cos\theta) \end{vmatrix} = \frac{1}{2}$$

$$f_{x,y}(x,y) = f_{r,\theta}(r,\theta) \cdot \frac{1}{2}$$

Hence, creating a uniform distribution in r, θ is sufficient to imply uniform distribution in x, y.

B) UNIFORM DISTRIBUTION OF 10e7 POINTS IN ELLIPSE



C)

Our algorithm generates random points inside a parallelogram, then reflects and rotates points from one side of the diagonal to the other side to create a uniform distribution in the "filled" triangle

ALGORITHM

Step 1- Define the (2×1) vectors $\vec{a}$ and $\vec{b}$ as the direction of the other two points from a third point.

$\vec{a} = P_3 - P_1$, $\vec{b} = P_2 - P_1$

Step 2- Define two random variables u_1 and u_2 called using the rand() function s.t. $u_1, u_2 \in [0,1)$.
The point $\vec{p} = \vec{a}\,u_1 + \vec{b}\,u_2$ represents uniform distribution in the parallelogram created by $\vec{a}, \vec{b}$.

Step 3- Rotate points on one side of the diagonal by 180° about the center of parallelogram to give uniform distribution in a triangle using the tranformation $u_1' = 1 - u_1$ and $u_2' = 1 - u_2$ if $u_1 + u_2 > 1$.

Step 4- Repeat N times and plot.

D) UNIFORM RANDOM DISTRIBUTION OF 10e7 POINTS IN TRAINGLE