PROBLEM 2

A) ALGORITHM-

We know that a 2d gaussian can be represented as $X = AW + \mu$ where $C = AA^T$. Given the covariance matrix, C, we calculate its eigenvalues stored in diagonal matrix $D = \text{diag}(\lambda_1, \lambda_2, ---, \lambda_n)$ and eigenvectors in another matrix V. We then orthogonalise the eigen vectors using Grahm-Schmidt orthogonalisation.

We claim that the matrix A is given by

$$A = V_{\text{orthon}} \text{diag}(\sqrt{\lambda_1}, \sqrt{\lambda_2}, ---, \sqrt{\lambda_n}) = V_{\text{orthonormalised}} \cdot D^{1/2} \text{ (see proof)}$$

We create the multivariate spherical gaussian W using randn() giving standardised normal numbers.

Since, we know A, W and μ, we calculate $X = AW + \mu$. and then plot.

PROOF OF ALGORITHM-

C is a real symmetric matrix. Using spectral theorem, we get $C = QDQ^T$ where D is a diagonal matrix consisting of eigenvalues of C and Q contains orthonormal eigenvalues of C.

Let $D = \text{diag}(\lambda_1, \lambda_2 --, \lambda_n)$. Then define

$$P = \text{diag}(\sqrt{\lambda_1}, \sqrt{\lambda_2}, ---, \sqrt{\lambda_n})$$

We see that $D = PP^T$.

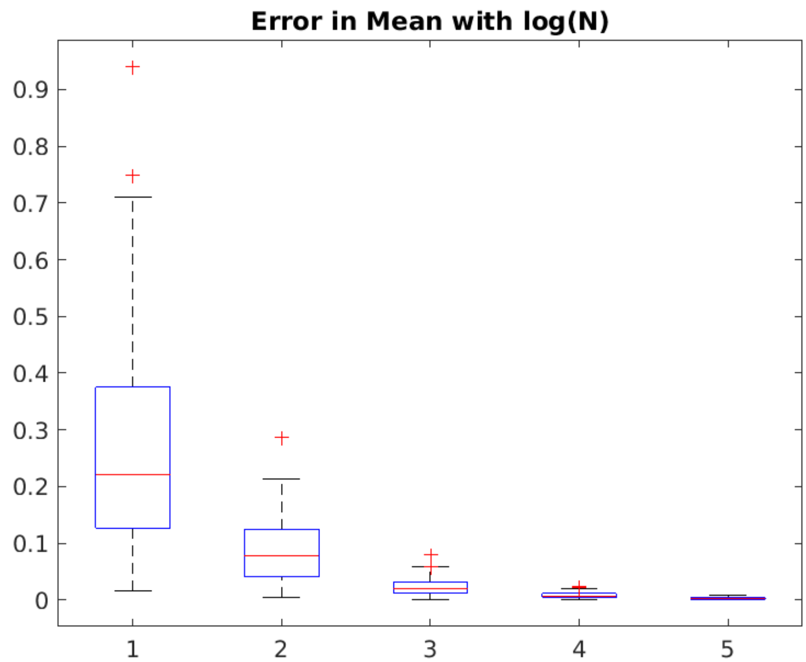
Substituting,

$$C = QDQ^T = QPP^TQ^T = QP(QP)^T = AA^T$$

So, $A = QP$ is a solution.

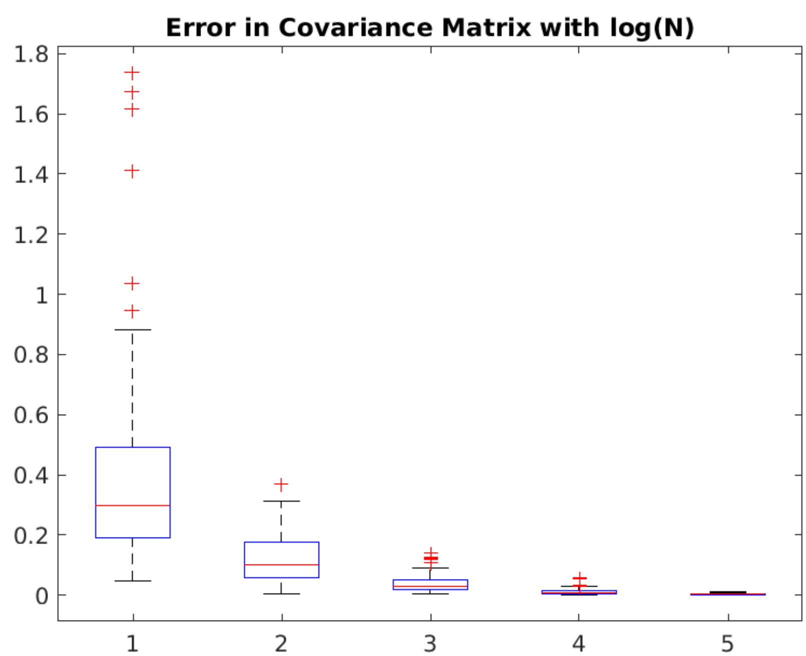
B)

Given N random vectors $X_1, X_2, ---, X_N$, the ML estimate for mean is $\hat{\mu}_N = \dfrac{X_1 + X_2 + --- + X_N}{N}$.

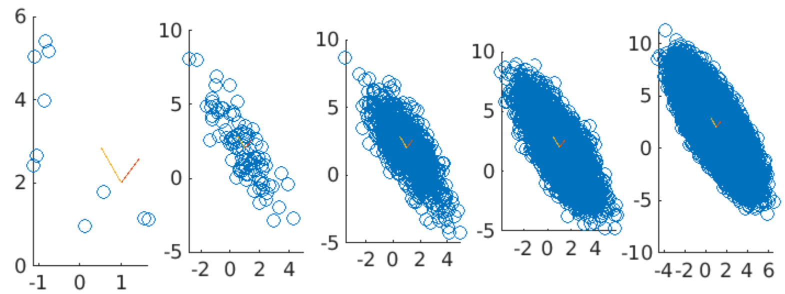


C)

Given N random vectors $x_1, x_2, \ldots, x_N$, the ML estimate for covariance matrix is $\hat{C}_N = \frac{1}{N}\sum_{i=1}^{N}(x_i - \hat{\mu}_N)(x_i - \hat{\mu}_N)^T$



D)



N=10 100 1000 10000 100000

Plot of generated data for different values of N with direction along principle modes of variation