Problem 4

How to run code

- My submissions are live-scripts of matlab
- They are in zip format, don't unzip them
- Open matlab go to the folder of that zip and open the *.mlx files from matlab
- They would appear as jupyter notebook with markdown, codeblock and outputs.
- Press ctrl + enter to run each code block

Part 1

Given PDF:

$$P_x(x) := 0 \text{ for } |x| > 1, \text{and}$$
$$P_x(x) := |x| \; for \; x \in [-1, 1]$$

We would use Invese transform sampling to generate/draw random numbers with the given custom pdf $P_x(\cdot)$.

So CDF for this distribution is:

$$F_x(c) = \int_{-\infty}^{c} P_X(t)dt$$

$$F_x(x) := \begin{cases} \int_{-\infty}^{x} 0 \cdot dt & \forall x \in (-\infty, -1) \\ \int_{-\infty}^{-1} 0 \cdot dt + \int_{-1}^{x} -t \cdot dt & \forall x \in [-1, 0] \\ \int_{-\infty}^{-1} 0 \cdot dt + \int_{-1}^{0} -t \cdot dt + \int_{0}^{x} t \cdot dt & \forall x \in [0, 1] \\ \int_{-\infty}^{-1} 0 \cdot dt + \int_{-1}^{0} -t \cdot dt + \int_{0}^{1} t \cdot dt + \int_{1}^{x} 0 \cdot dt & \forall x \in (1, \infty) \end{cases} = \begin{cases} 0 & \forall x \in (-\infty, -1) \\ \frac{1-x^2}{2} & \forall x \in [-1, 0] \\ \frac{x^2+1}{2} & \forall x \in [0, 1] \\ 1 & \forall x \in (1, \infty) \end{cases}$$

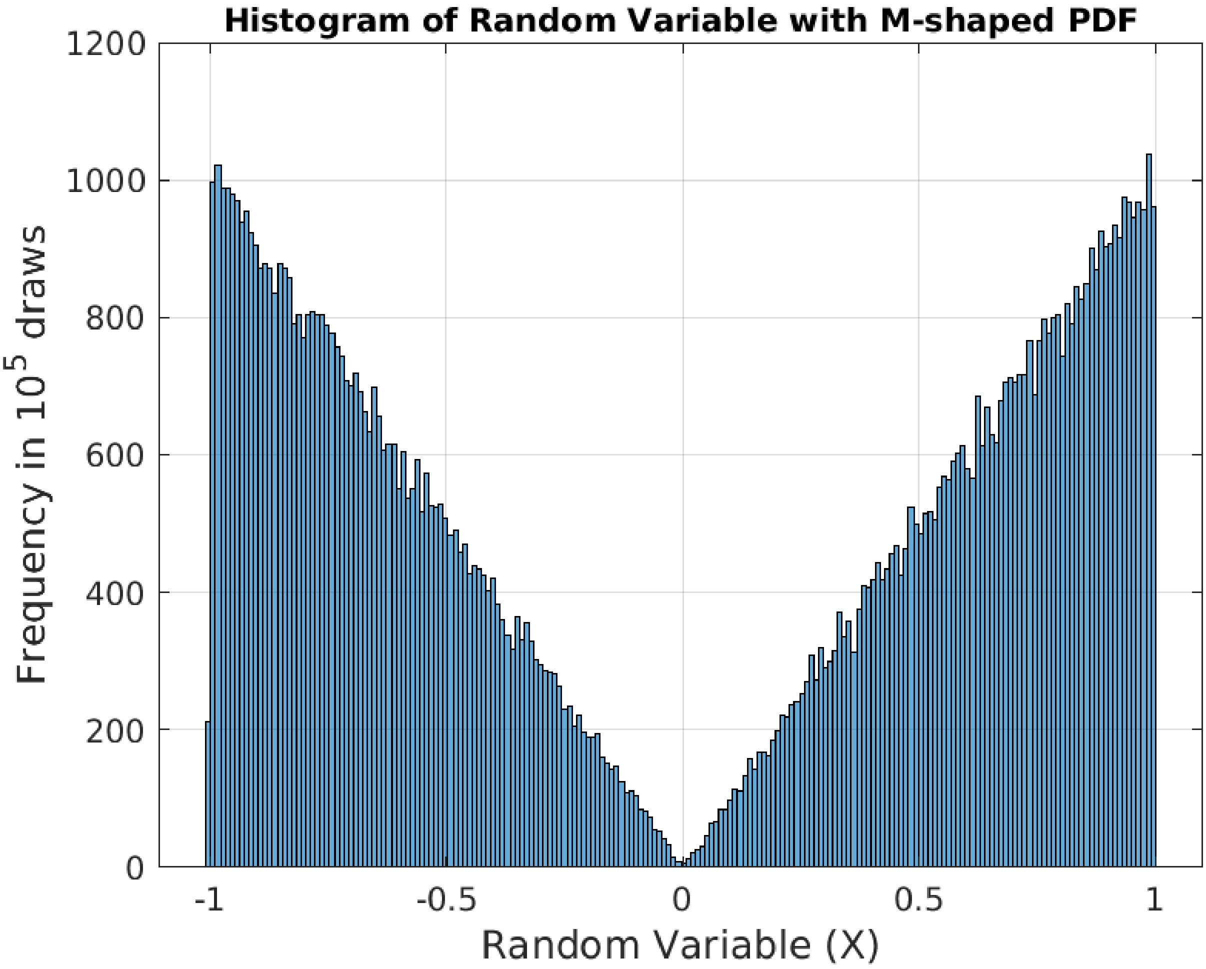
Now we would write the inverse of this cdf function $F^{-1}(x)$ in range [-1,1]

$$F_x^{-1}(x) := \begin{cases} -\sqrt{1-2x} & \forall x \in [0, \frac{1}{2}] \\ \sqrt{2x-1} & \forall x \in [\frac{1}{2}, 1] \end{cases}$$

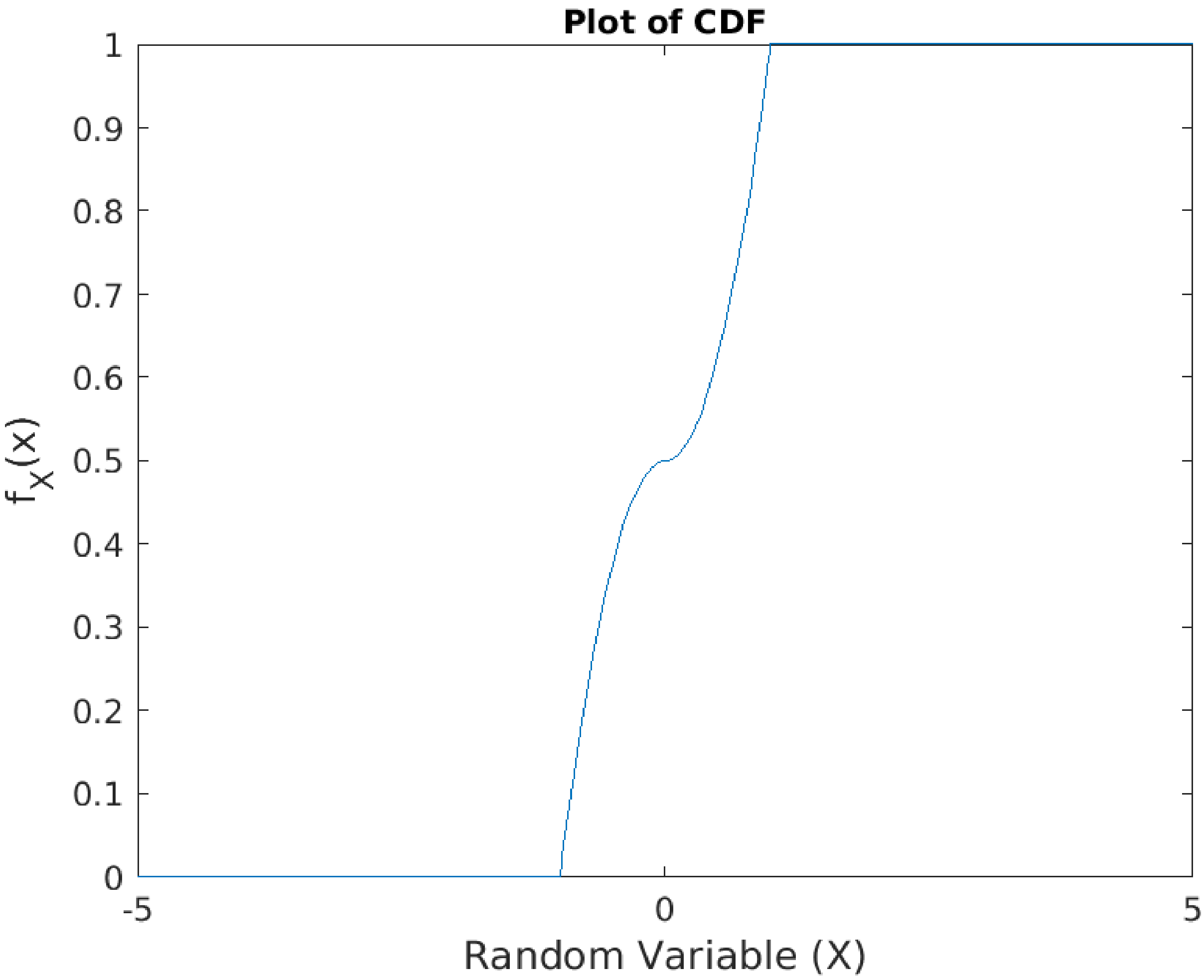
- So now we would generate random variable between 0 and 1 and then would find inverse-CDF of that random number.
- This would given us a random number from the PDF whose inverse-CDF we have used.

Generated random value :- x = -0.4074

Part 2



Histogram of Random Variable with M-shaped PDF
1200
1000
800
600
400
200
0
Frequency in 10^5 draws
-1
-0.5
0
0.5
1
Random Variable (X)



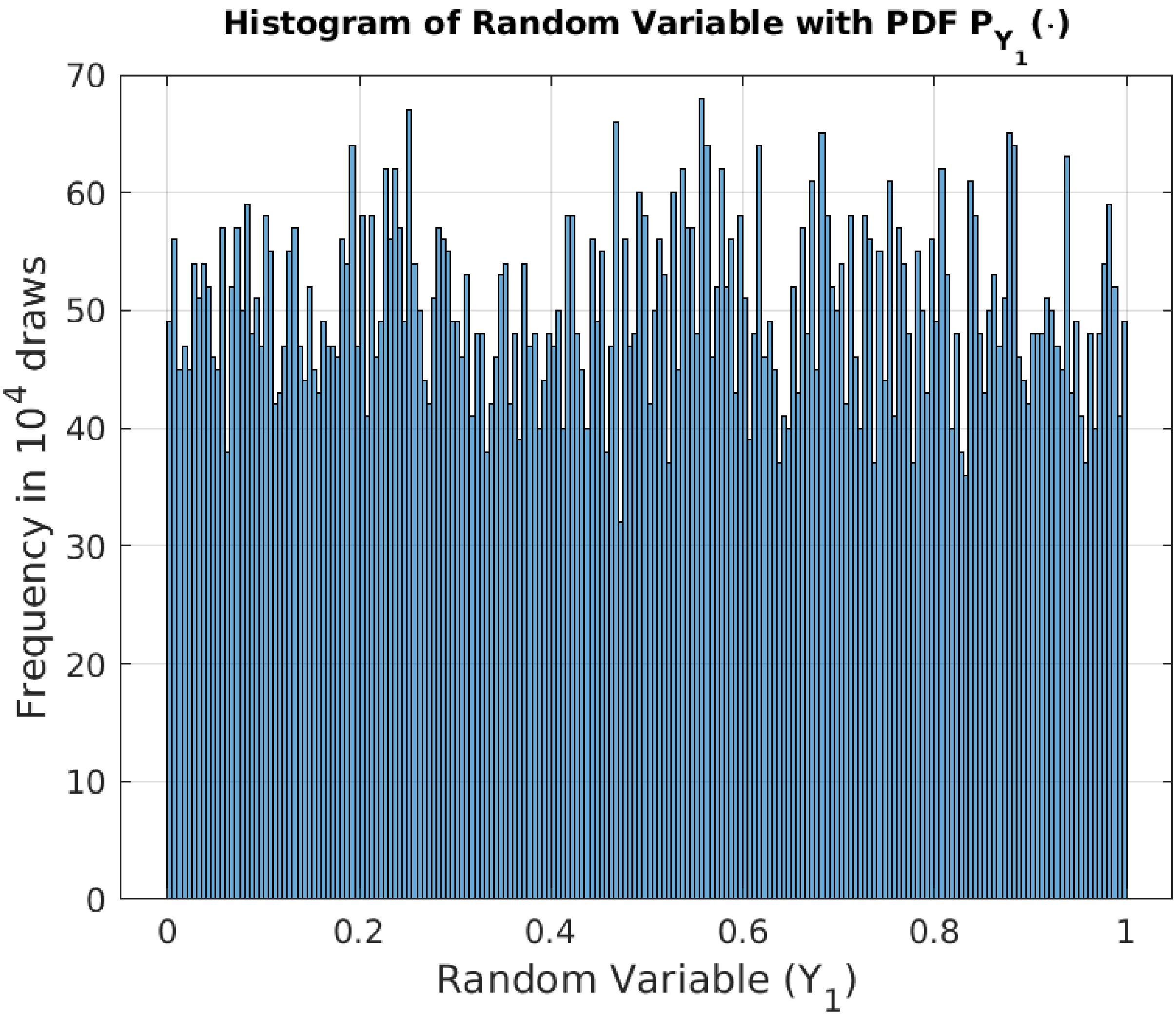
Plot of CDF
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$f_X(x)$
-5
0
5
Random Variable (X)

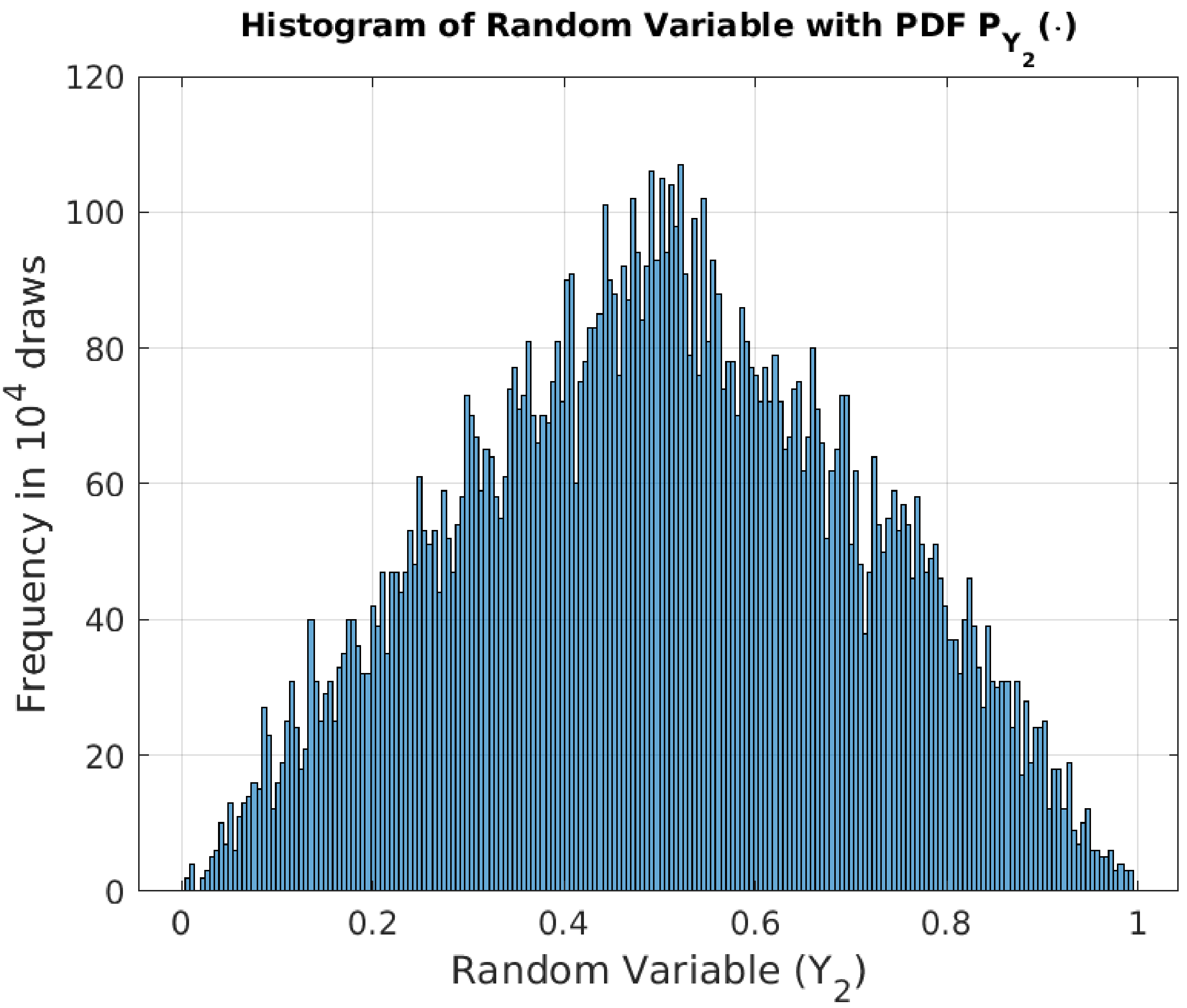
Part 3

- Draws from $P_{Y_N}(\cdot)$ can be easily found by taking N independent draws from $P_X(\cdot)$ and then taking their average

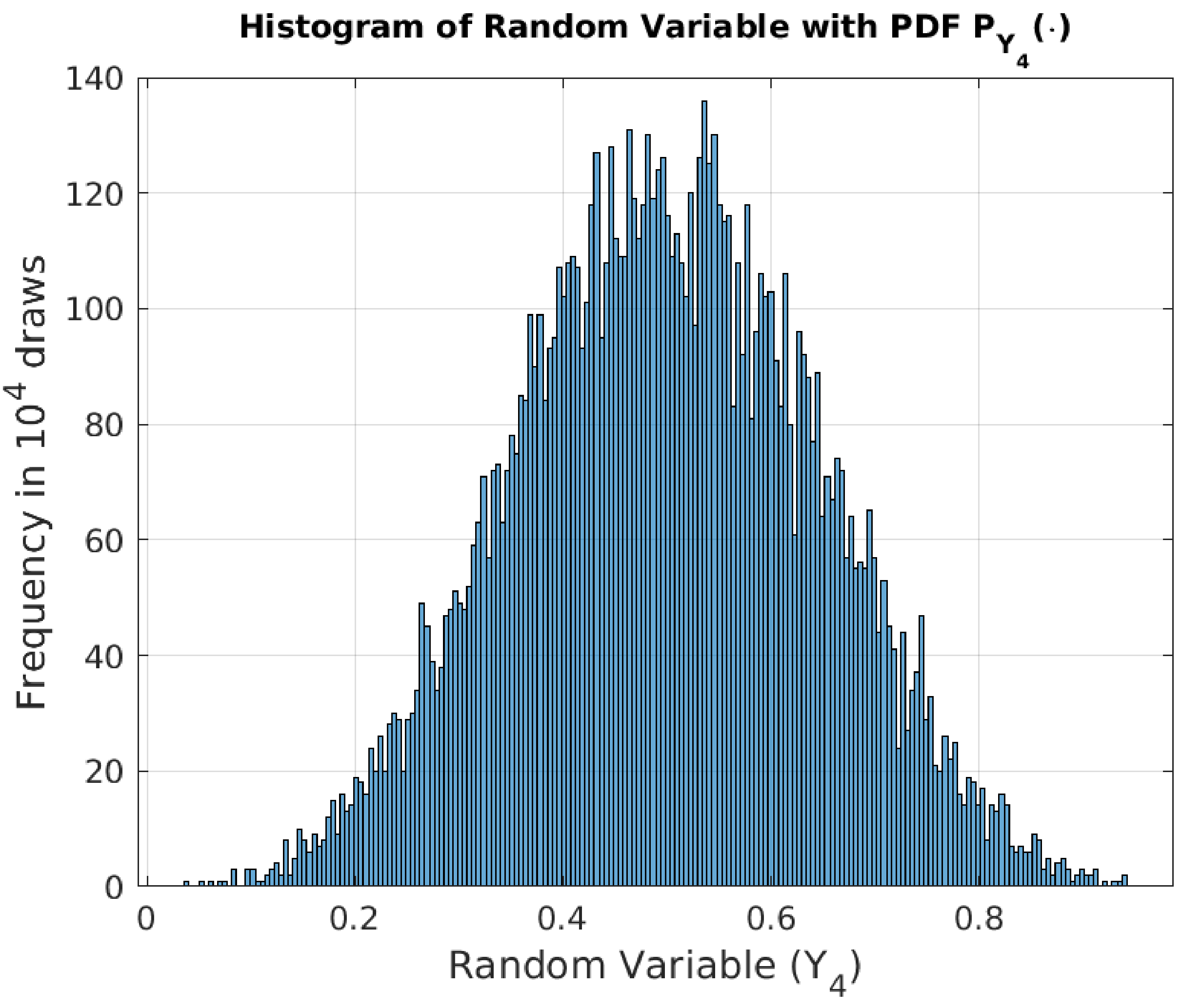
Generated random value :- Y_{100} = -0.0265

Part 4

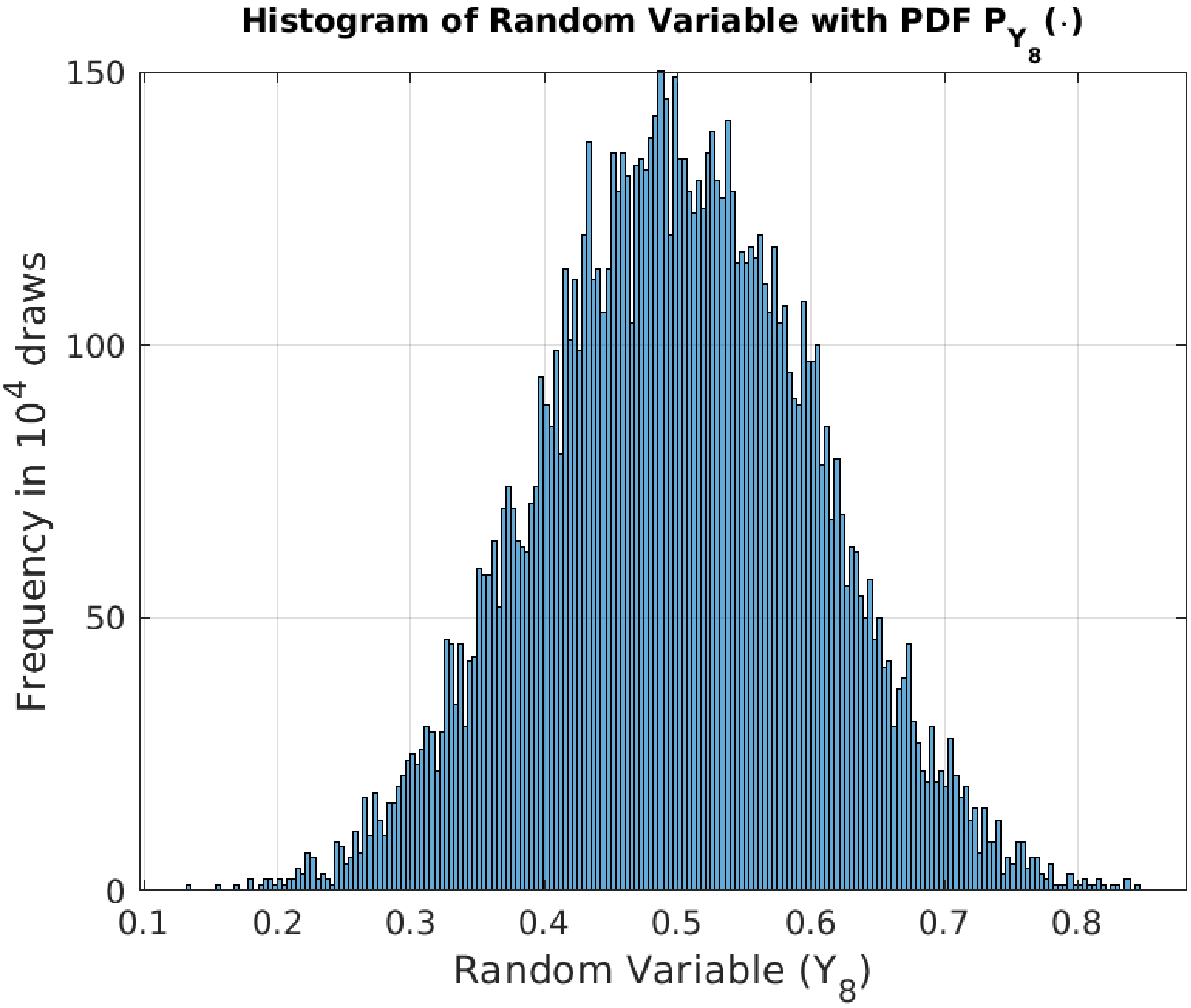


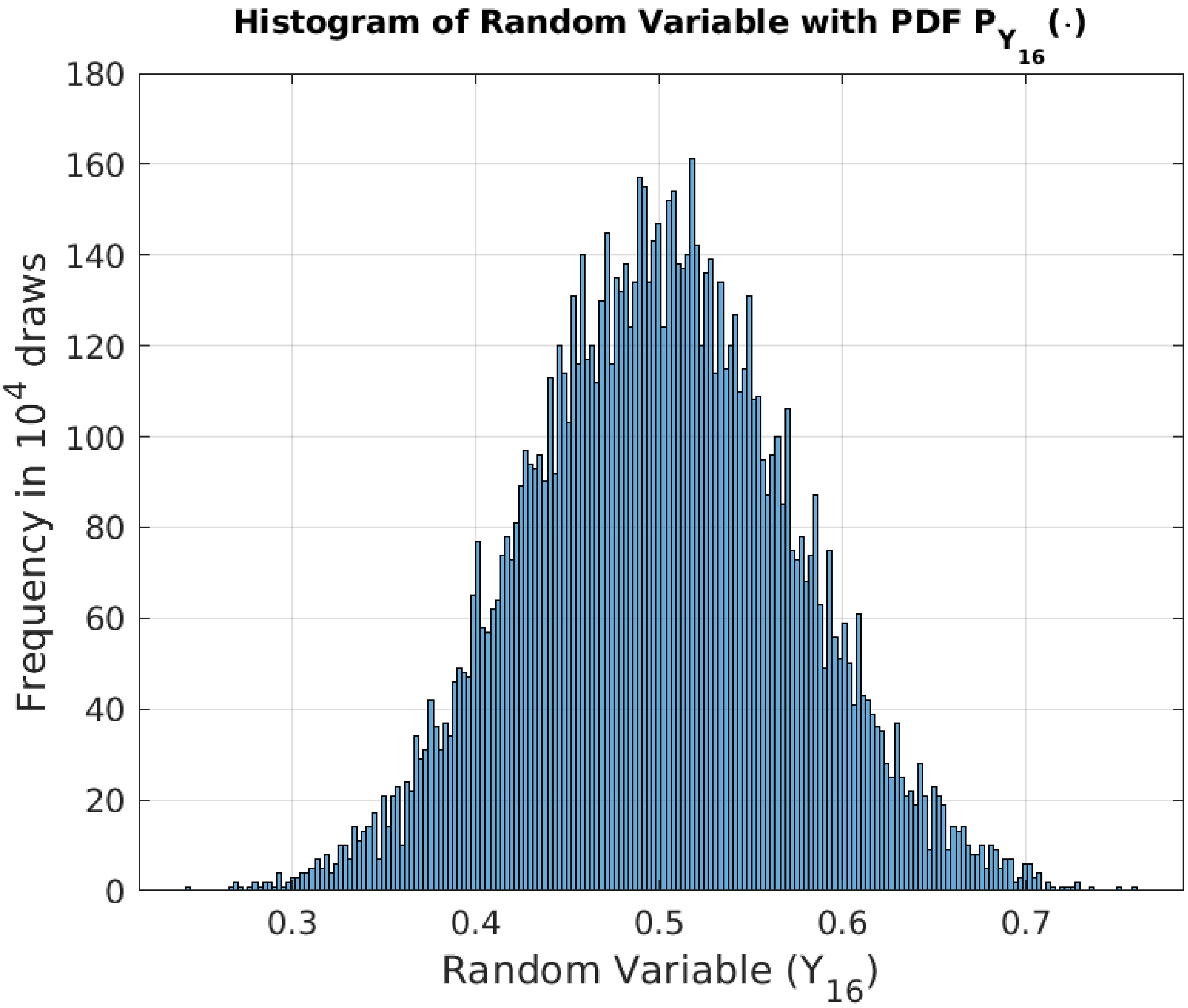


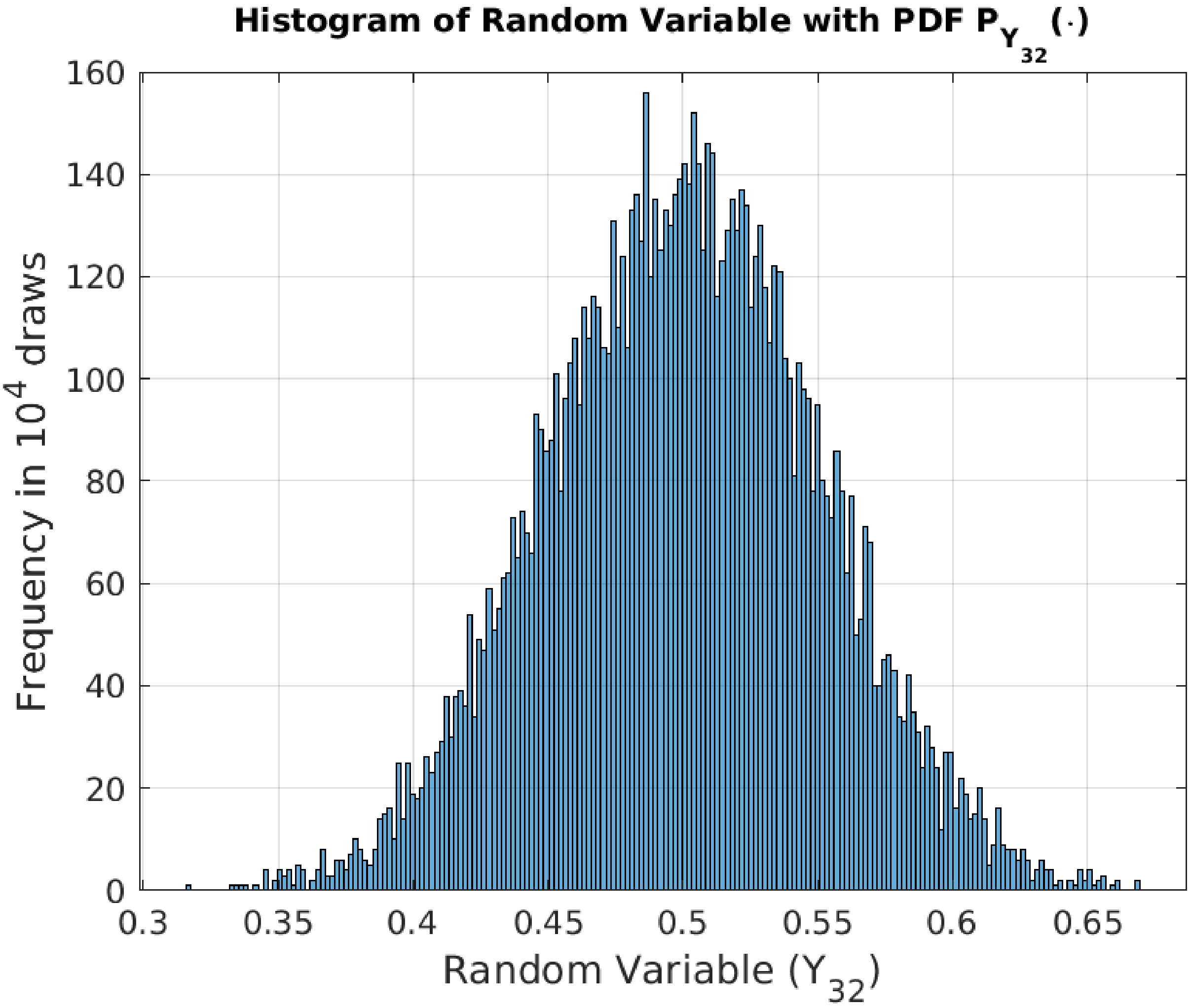
Histogram of Random Variable with PDF $P_{Y_2}(\cdot)$
Frequency in 10^4 draws
0
20
40
60
80
100
120
0
0.2
0.4
0.6
0.8
1
Random Variable (Y_2)



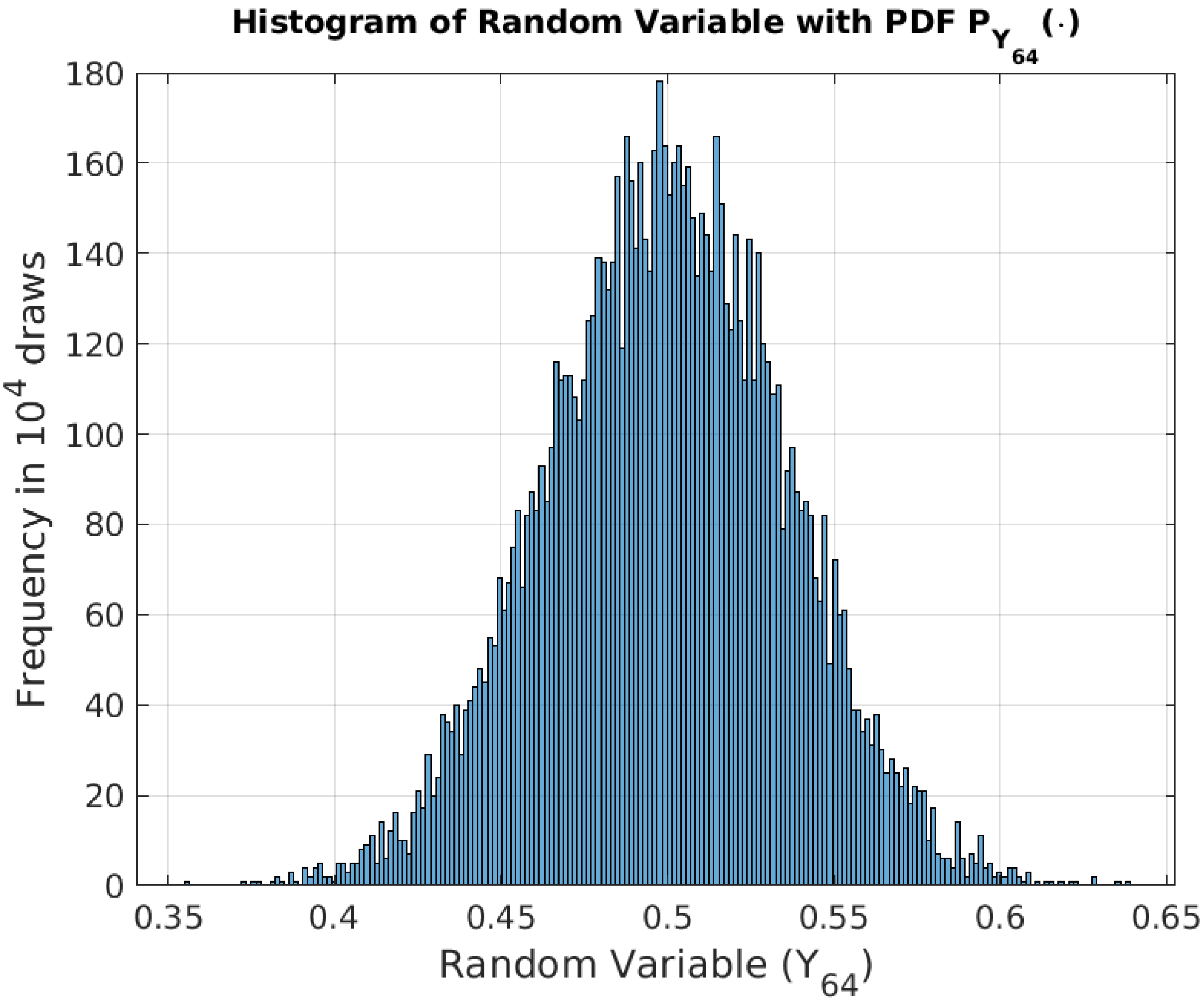
Histogram of Random Variable with PDF $P_{Y_4}(\cdot)$
Frequency in 10^4 draws
0
20
40
60
80
100
120
140
0
0.2
0.4
0.6
0.8
Random Variable (Y_4)


Histogram of Random Variable with PDF $P_{Y_8}(\cdot)$
Frequency in 10^4 draws
Random Variable (Y_8)
150
100
50
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

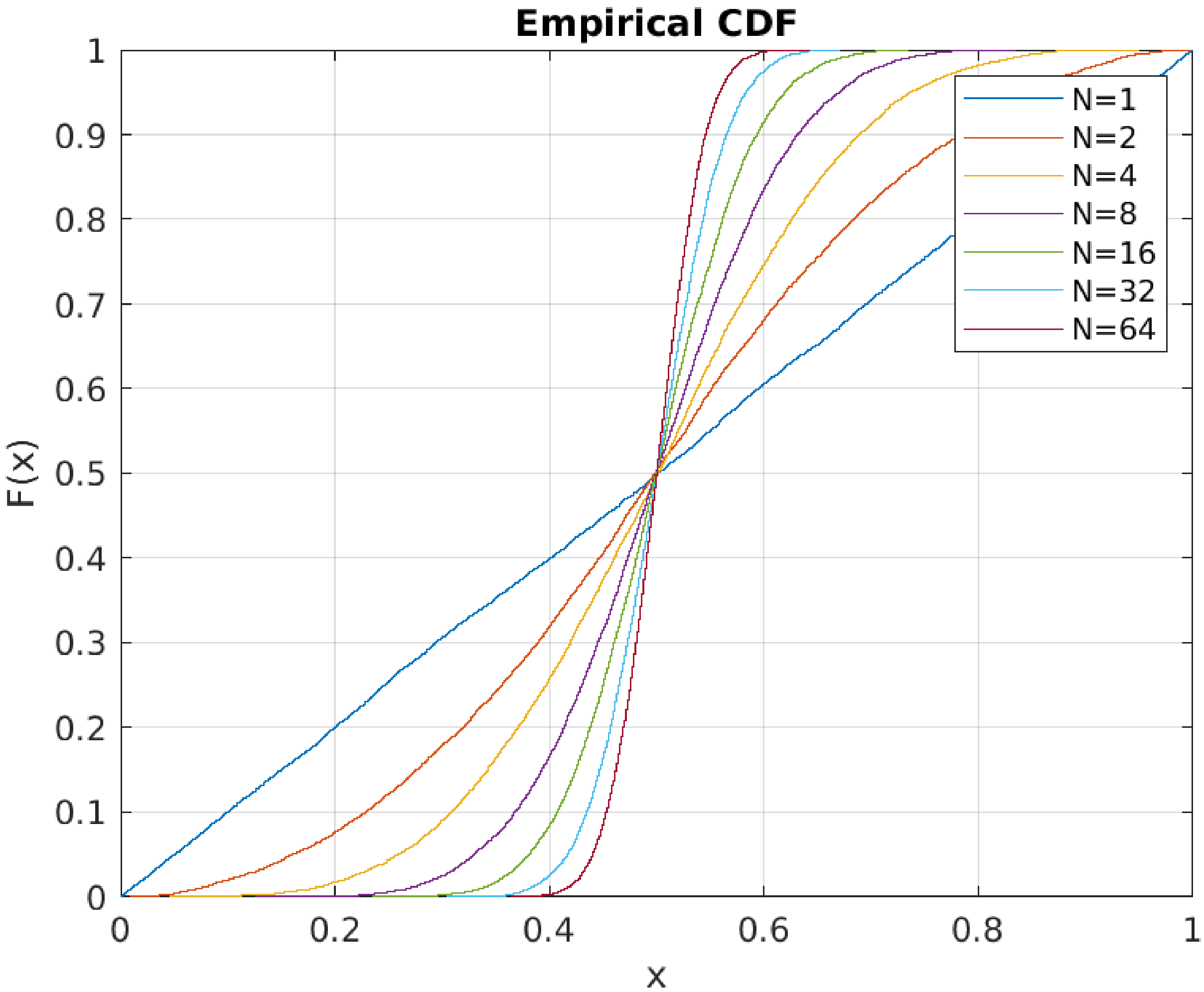

Histogram of Random Variable with PDF $P_{Y_{16}}(\cdot)$
Frequency in 10^4 draws
Random Variable (Y_{16})
180
160
140
120
100
80
60
40
20
0
0.3
0.4
0.5
0.6
0.7



Histogram of Random Variable with PDF $P_{Y_{32}}(\cdot)$
Frequency in 10^4 draws
Random Variable (Y_{32})
0
20
40
60
80
100
120
140
160
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65



Histogram of Random Variable with PDF $P_{Y_{64}}(\cdot)$
Frequency in 10^4 draws
Random Variable (Y_{64})
0
20
40
60
80
100
120
140
160
180
0.35
0.4
0.45
0.5
0.55
0.6
0.65



Observation

- Our results are in accordance with the fact specified in the slides **The Average of "M" random variables tends to Gaussian(approximately)**
- The Observation are in accordance with the central limit theorem
- As the value of N is increasing the histograms obtained are getting more and more Gaussian.