Assignment 1

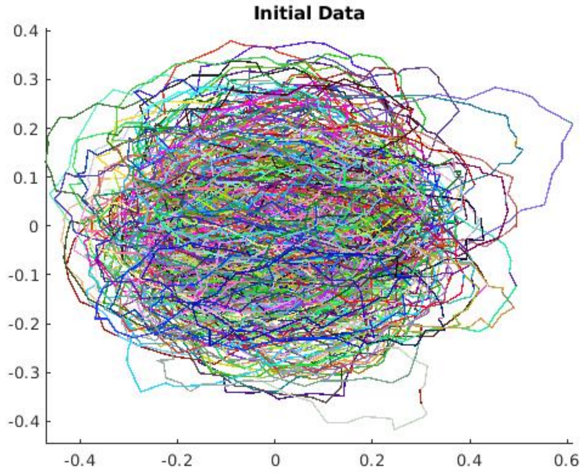
Sankalp Parashar and Utkarsh Ranjan

Spring 2022

Contents

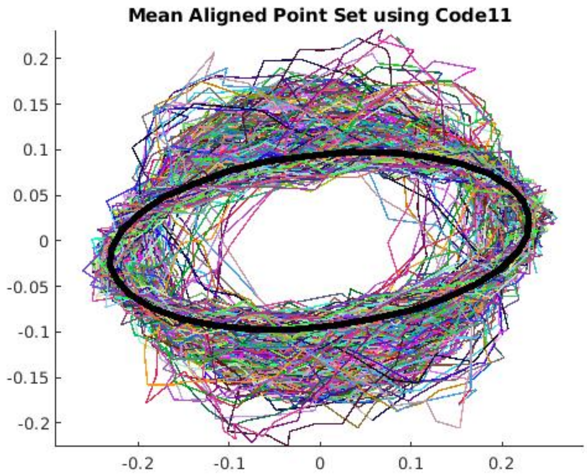
Question 1

Initial Data

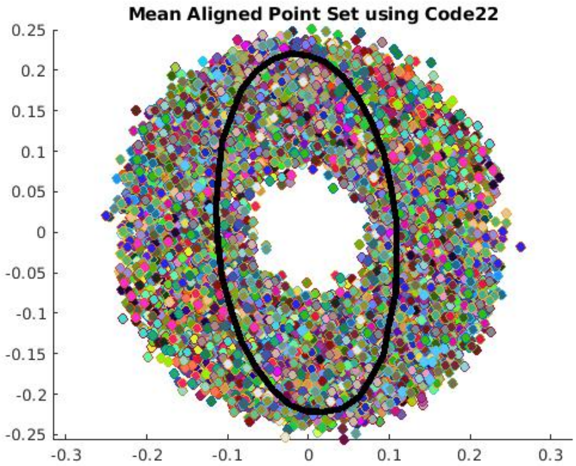


Mean Aligned Data

Using Code 11

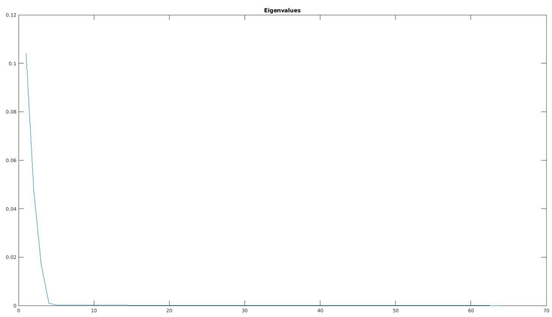


Using Code 22



Eigenvalue Plots

Using Code 11



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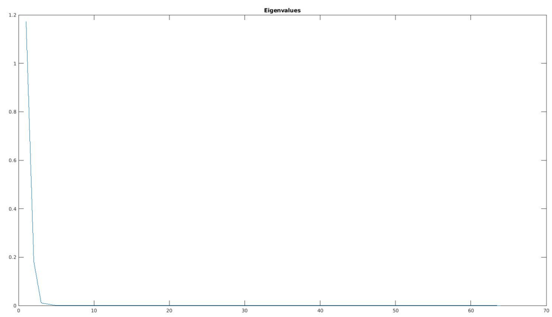
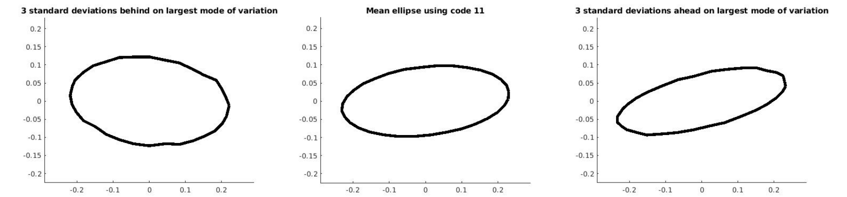
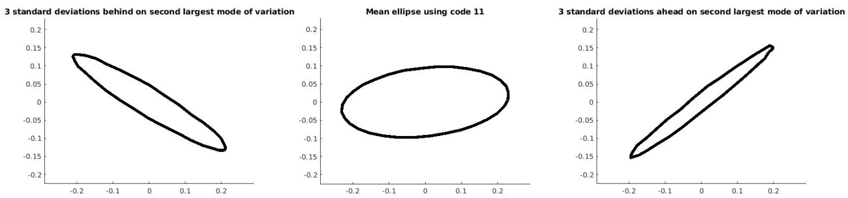
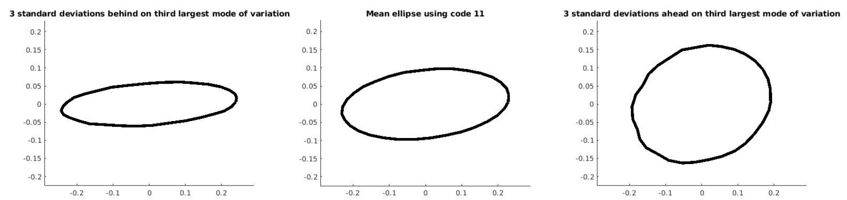


Variance along top 3 directions

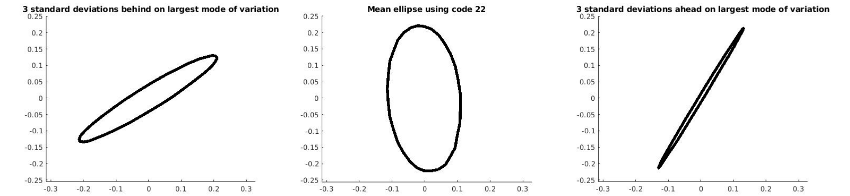
Using Code 11

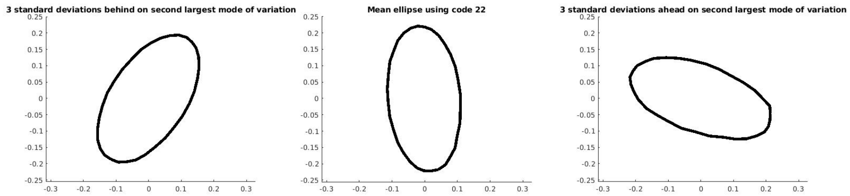


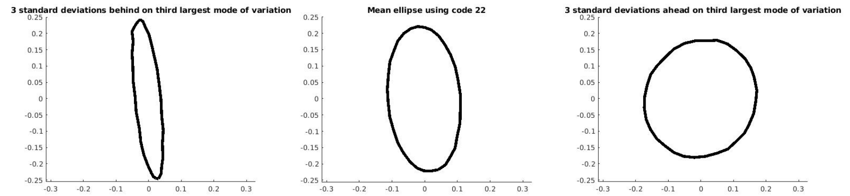




Using Code 22

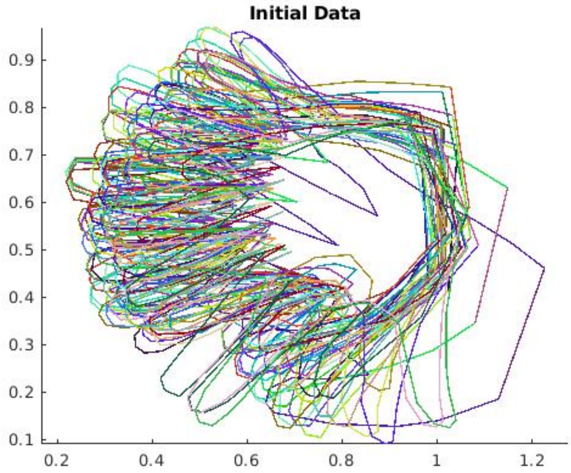

3 standard deviations behind on largest mode of variation
Mean ellipse using code 22
3 standard deviations ahead on largest mode of variation


3 standard deviations behind on second largest mode of variation
Mean ellipse using code 22
3 standard deviations ahead on second largest mode of variation


3 standard deviations behind on third largest mode of variation
Mean ellipse using code 22
3 standard deviations ahead on third largest mode of variation

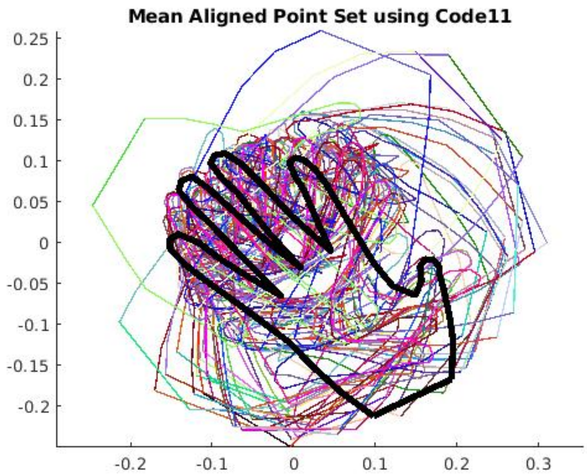
Question 2

Initial Data

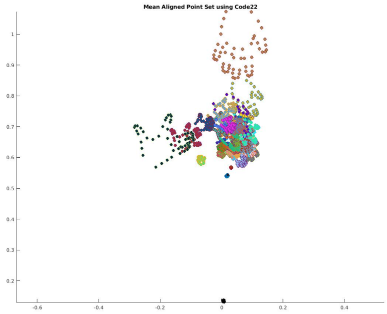


Mean Aligned Data

Using Code 11

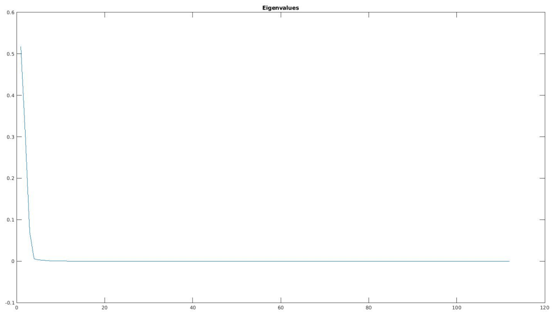


Using Code 22

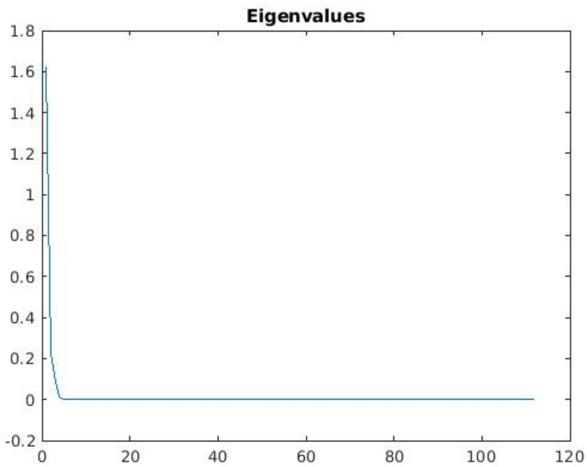


Eigenvalue Plots

Using Code 11

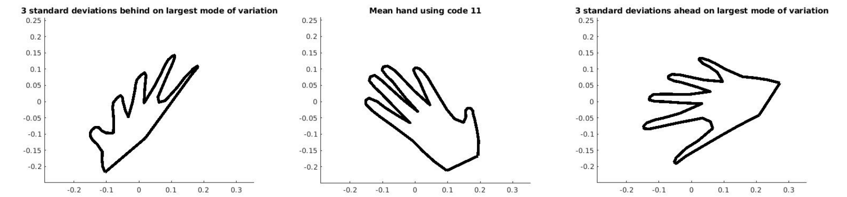


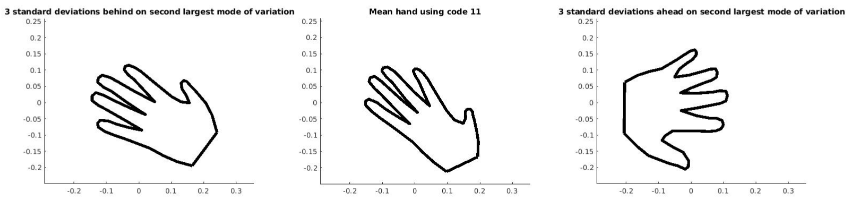
Using Code 22

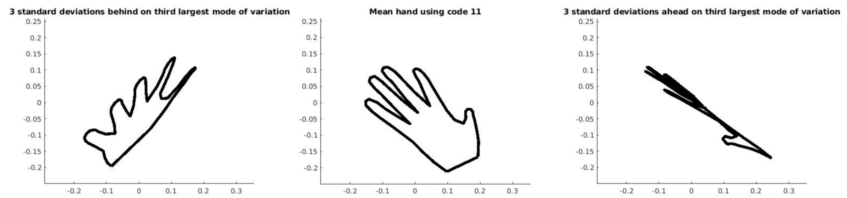


Variance along top 3 directions

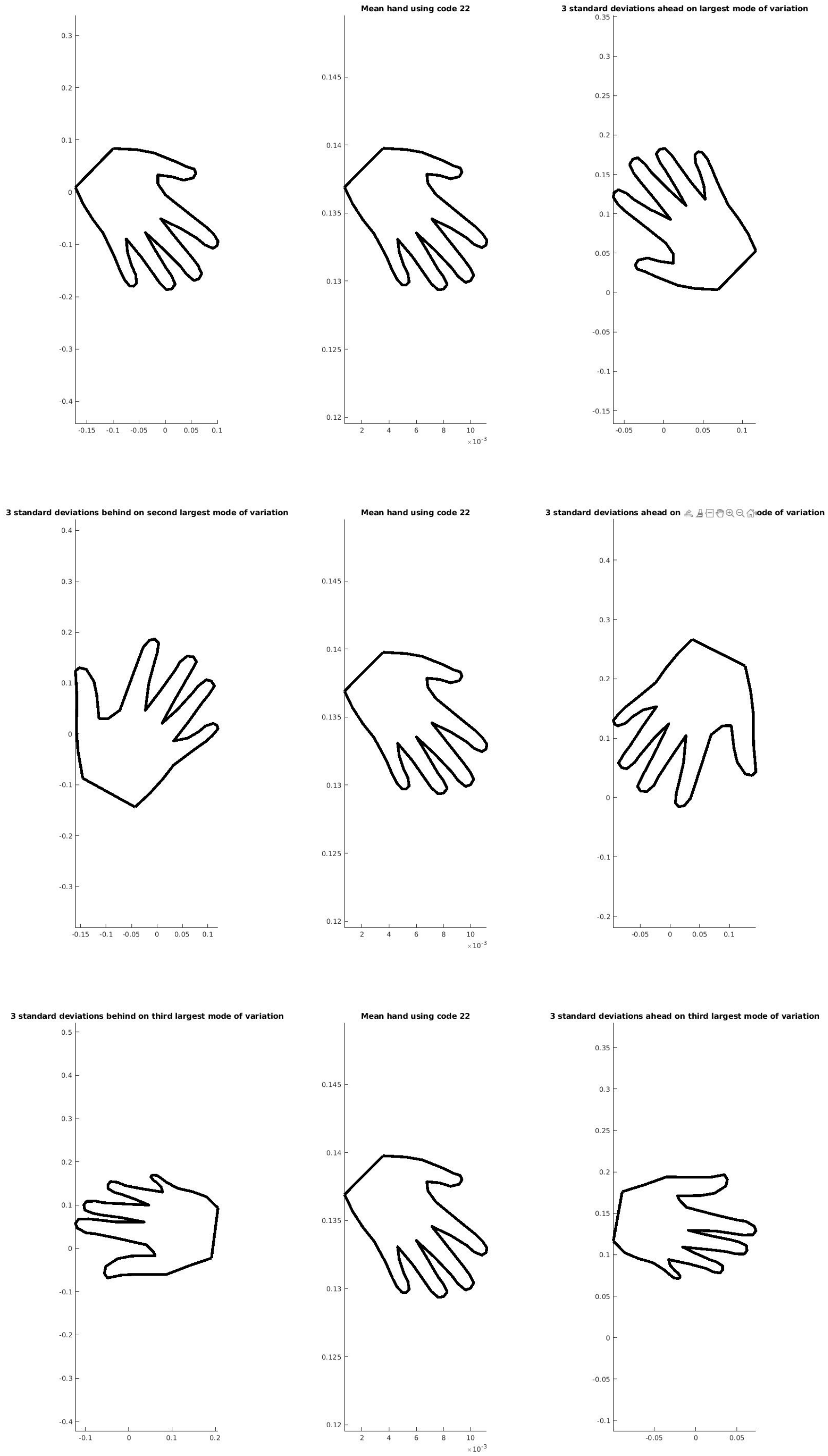
Using Code 11







Using Code 22


Mean hand using code 22
3 standard deviations ahead on largest mode of variation
3 standard deviations behind on second largest mode of variation
Mean hand using code 22
3 standard deviations ahead on
ode of variation
3 standard deviations behind on third largest mode of variation
Mean hand using code 22
3 standard deviations ahead on third largest mode of variation

Question 3

For two shapes z_1, z_2 which are not in preshape space, we can first put ~~them~~ the poinset in pre-shape space.

This can be done in two steps:-

1. Standardizing location:-
   - → Computing centroid of each pointset
   - → Subtracting centroid co-ordinates from each point co-ordinate

$$Z_1 = z_1 - \frac{\sum_{n=1}^{N} z_{1n}}{N}$$

Each z_i is a poinset : $\{ z_{in} \text{ in } R^3 : n = 1, \ldots, N \}$

2. Standardizing scale:-
   - → Re-scaling each poinset to have same scale
   - → Divide by 2-norm of the vector.

$$z_1 = \frac{z_1}{\sum_n (\|z_{1n}\|_2)}$$

3. After this we can ~~align~~ align shapes w.r.t rotation by multiplying the pointset with a rotation matrix R.

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Thus the Procrustes distance / dissimilarity between z_1, z_2 with the introduction of transformational, rotational and scale variables as :-

$$d(z_1, z_2) = \min_{\theta, T, s} d^2 (z_1, \text{similarity Transform} (z; \theta, T, s))$$

$$= \min_{\theta, T, s} \sum_{n=1,...N} \| z_{1n} - sM_\theta z_{2n} - T \|_2^2$$

Ans 2 → Objective function for k-means clustering for k-partition $S = (S_1, S_2, \ldots S_k)$ of underlying shapes, with non-empty classes can be given as:-

$$\omega(s) = \sum_{i=1}^{k} \sum_{x_j \in S_i} \| x_j - \mu_i \|^2$$

Here, ~~insted~~ instead of mean μ_i of class S_i, we have class of shapes with the mean as the mean shape and euclidean distance replaced by Procrustes distance.

$$\omega(s) = \sum_{i=1}^{k} \sum_{z_j \in S_i} d(z_j, M_i)$$

$$= \cancel{\sum_{i=1}^{k} \sum_{z_j \in S_i} \min_{\theta, T, s} \|}$$

$$\omega(S) = \sum_{i=1}^{k} \sum_{z_j \in S_i} \min_{\theta, T, s} \sum_{n=1,\ldots N} \| z_{jn} - s M_\theta M_{in} - T \|_2^2$$

Here, for each class S_i and given shape z_{jn} mean M_i can be found using the following algorithm (discussed in class):-

Minimize: $\sum_{m=1,\ldots M} \sum_{n=1,\ldots N} \| M_{in} - s_m R_m z_{mn} - T_m \|_2^2$

→ Given mean, find optimal transformation.
→ Given all transformations, find optimal mean pointset. → Averge all (aligned) pointsets.

Ans 3- Algorithm for clustering (k-Means ++):-

→ An initial estimate for the k-class mean shape is given.

(i) → Given representatives M_i, we minimize objective function $\omega(s)$ with respect to S, assigning each shape ~~point~~ z_i to the class whose mean-shape has the minimimum Procrustes distance to it.

(ii) → Given class label S, we minimize with respect to the mean-shape of class M_i for $i = 1, \ldots, k$ by finding the mean of shapes in S_i, using algo mentioned in part (b).

→ Repeat (i) and (ii) until convergence.

Termination condition:- If $\omega(s)$ doesn't decrease from its value from previous estimate to the updated estimate, then terminate.

Initialization condition:-

pick a shape z uniformly at random and set $T \leftarrow \{z\}$

while $|T| < k$:

pick $z \in Z$ at random, with probability proportional to $\text{cost}(z, T) = \min_{M \in T} d(z, M)$

$T \leftarrow T \cup \{z\}$