SUMMER-2023

UNIT-1

Q.1 a) Find Eigen values & Eigen vectors of

$$A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$$ **(6)**

b) Verify Cayley-Hamilton theorem and hence find A^{-1}.

Where $$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$ **(7)**

Q.2 a) Find inverse of matrix by partitioning Method

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$$ **(6)**

b) Investigate for what values of λ and μ the equations

$x + 2y + z = 8$

$2x + 2y + 2z = 13$

$3x + 4y + \lambda z = \mu$ have

i) No solution ii) Unique solution iii) Many solution **(7)**

UNIT-2

Q.3 a) Obtain the constant term and the coefficient of first sine & cosine terms in the Fourier expansion of y as given in the following table. **(7)**

x :	0	1	2	3	4	5
y :	9	18	24	28	26	20

b) Find the Fourier Expansion for $f(x) = \Pi^2 - x^2$, $-\Pi \le x \le \Pi$. **(6)**

Q.4 a) Obtain Fourier series for $f(x) = x^2$ in range $0 \le x \le 2\pi$.

Q.4 a) Obtain Fourier series for $f(x) = x^2$ in range $0 \le x \le 2\pi$. **(7)**

b) Find the Fourier series of $f(x) = 1 - x^2$ in the interval $(-1,1)$. **(6)**

UNIT-3

Q.5 a) Evaluate $\int_0^{\Pi/2} \frac{\sin^{2m-1}\theta.\cos^{2n-1}\theta}{\left(b\sin^2\theta + a\cos^2\theta\right)^{m+n}} d\theta$ **(5)**

b) Prove that $\int_0^{\infty} \frac{dx}{1+x^2} = \frac{\Pi}{2}$ using Beta function. **(4)**

c) Prove that $\int_a^h (x-a)^m (b-x)^n dx = (b-\alpha)^{m+n+1} \beta(m+1, n+1)$ **(5)**

Q.6 a) If $I_n = \int_{\Pi/4}^{\Pi/2} \cot^n x dx$, $(n > 2)$.

Prove that $I_n = \frac{1}{n-1} - I_{n-2}$ and hence evaluate I_6. **(5)**

b) Show that $\int_0^1 x^3 \left(1-\sqrt{x}\right)^5 dx = \frac{1}{9048}$ **(4)**

c) Prove that $\beta(m,n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$ **(5)**

UNIT-4

Q.7 a) Prove that

$\int_0^{\infty} \frac{e^{-x} - e^{-dx}}{x\sec x} dx = \frac{1}{2}\log\left(\frac{a^2+1}{2}\right)$, for $a > 0$. **(5)**

b) Trace the curve $y^2 = x^5(2a - x)$. **(4)**

c) Find the whole length of the Asteroid $x^{2/3} + y^{2/3} = a^{2/3}$. **(5)**

Q.8 a) Show that length of arc of curve
$x = \log(\sec\theta + \tan\theta) - \sin\theta$
$y = \cos\theta$ from $\theta = 0$ to $\theta = \alpha$ is $\log(\sec\alpha)$ **(5)**

b) Trace the curve $r^2 = a^2 \cos 2\theta$ **(4)**

c) If $f(a) = \int_{\pi/6a}^{\pi/2a} \frac{\sin ax}{x} dx$.
Prove that f(a) is independent of a. **(5)**

UNIT-5

Q.9 a) Change order of Integration & hence evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dx.dy$. **(6)**

b) Prove that by Double Integration the area between parabolas $y^2 = 4ax$ & $x^2 = 4ay$ is $\frac{16a^2}{3}$. **(7)**

Q.10 a) Evaluate the integration
$\iint \frac{dx.dy}{x^4 + y^2}$, where $x \geq 1$, $y \geq x^2$ **(6)**

b) Express the following integration in polar form, showing the region of integration & evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+y^2}\,dxdy$
(7)

UNIT-6

Q.11 a) Evaluate $\int_0^0 \int_0^{1-x} \int_0^{x+y} e^z dxdydz$ **(6)**

b) Find Mean value of $e^{-(x^2+y^2)}$ over the area of circle $x^2 + y^2 = 1$. **(7)**

Q.12 a) Evaluate $\iiint_V \frac{dx.dy.dz}{\sqrt{a^2 - x^2 - y^2 - z^2}}$ Over positive octant of the sphere $x^2 + y^2 + z^2 = a^2$. **(6)**

b) Find volume of the tetrahedron bounded by the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ & the co-ordinate planes $x = 0$, $y = 0$, $z = 0$. **(7)**

WINTER-2022

UNIT 1

Q.1 a) Find inverse of matrix by partitioning method **(6)**

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$$

b) Investigate for what values of λ and μ the equations

$x + 2y + z = 8,\quad 2x + 2y + 2z = 13,\quad 3x + 4y + \lambda z = \mu$ have

i) no solution ii) unique solution iii) many solution. **(7)**

Q.2 a) Find Eigen values & Eigen vectors of **(6)**

$$A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$$ **(7)**

b) Verify the Cayley–Hamilton theorem and hence find A^{-1},

where $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$. **(6)**

UNIT 2

Q.3 a) Obtain the Fourier series for $f(x) = x^2$ in range $0 \le x \le 2\pi$. **(6)**

Q.3 b) Find the Fourier series of $f(x) = 1 - x^2$ in the interval $(-1, 1)$. **(7)**

Q.4 a) Obtain the constant term and the coefficient of first sine & cosine terms in the Fourier expansion of y as given in the following table. **(7)**

X:	0	1	2	3	4	5
Y:	9	18	24	28	26	20

b) Find the Fourier Expansion for **(6)**

$$f(x) = \pi^2 - x^2, \qquad -\pi \le x \le \pi$$

UNIT 3

UNIT 3

Q.5 a) If $I_n = \int_{\pi/4}^{\pi/2} \cot^n x\,dx$, $(n > 2)$ Prove that $I_n = \frac{1}{n-1} - I_{n-2}$ and hence evaluate I_6. **(5)**

b) Show that $\int_0^1 x^3\left(1-\sqrt{x}\right)^5 dx = \frac{1}{5148}$. **(4)**

c) Prove that $\beta(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$ **(5)**

Q.6 a) Evaluate $\int_0^{\pi/2} \frac{\sin^{2m-1}\theta\cos^{2n-1}\theta}{(b\sin^2\theta + a\cos^2\theta)^{m+n}} d\theta$. **(5)**

b) Prove that $\int_0^{\infty} \frac{dx}{1+x^2} = \frac{\pi}{2}$ Using Beta function. **(4)**

c) Prove that **(5)**

$$\int_a^b (x-a)^m (b-x)^n dx = (b-a)^{m+n+1}\beta(m+1, n+1)$$

UNIT 4

Q.7 a) Prove that $\int_0^{\infty} \frac{1-e^{-ax}}{x} e^{-x} dx = \log(1+a)$ by using rule of differentiation under integral sign. **(5)**

b) Trace the curve $y^2(2a - x) = x^3$ **(4)**

c) Show that the length of the arc of the parabola $y^2 = 4ax$ cut off by the line $3y = 8x$ is $a.\left(\log 2 + \frac{15}{16}\right)$. **(5)**

Q.8 a) Show that length of arc of curve $x = \log(\sec\theta + \tan\theta) - \sin\theta$, $y = \cos\theta$ from $\theta = 0$ to $0 = \alpha$ is $\log(\sec\alpha)$. **(5)**

b) Trace the curve: $r = a(1 - \cos\theta)$. **(4)**

c) Prove that $\int_0^{\infty} \frac{e^{-ax}\sin x}{x} dx = \frac{\pi}{2} - \tan^{-1} a$ and hence show that $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$. **(5)**

UNIT 5

Q.9 a) Find area common to cariode $r = a(1 + \cos\theta)$ and $r = a(1 - \cos\theta)$. **(6)**

b) Change order of integration & Evaluate **(7)**

$$\int_0^a \int_0^{2\sqrt{ax}} x^2 dxdy.$$

Q.10 a) Evaluate $\iint xy(x + y)dx\,dy$ over the area between $y = x^2$ and $y = x$. **(6)**

b) Prove that by the double integration the area between parabolas $y^2 = 4ax$ & $x^2 = 4ay$ is $\frac{16a^2}{3}$. **(7)**

UNIT 6

Q. 11 a) Evaluate $\iiint_V \frac{dx\,dy\,dz}{\left(x^2 + y^2 + z^2\right)^{\frac{3}{2}}}$ over the area between $y = x^2$ and $y = x$. **(7)**

b) Find the mean value of $4\cos t - \sin 3t$ between range $t = 0$ & $t = \pi/6$ **(6)**

Q.12 a) Find the volume bounded by the cylinder $y^2 = x$, $x^2 = y$ and the planes $z = 0$, $x + y + z = 2$. **(7)**

b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dx\,dy\,dz.$ **(6)**

SUMMER-2022

UNIT 1

Q.1 a) Find the inverse of matrix by partitioning method where $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$. **(7)**

b) Find rank and nullity of the matrix

$$\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$$ **(6)**

Q.2 a) Determine Eigen values and Eigen vectors of matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. **(7)**

b) Verify the Cayley–Hamilton theorem and hence find A^{-1}, where $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$. **(6)**

UNIT 2

Q.3 a) Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $(0, 2\pi)$. **(6)**

Q.3 b) Find Fourier Series corresponding to the function f(x) defined in the interval (-2, 2) as follows

$$f(x) = \begin{cases} 2; & -2 \le x \le 0 \\ x; & 0 \le x \le 2 \end{cases}$$ **(7)**

Obtain half range cosine series for the function f(x) = x in the interval $(0, \pi)$.

Q.4 a) Obtain half range cosine series for the function f(x) = x in the interval $(0, \pi)$. **(6)**

b) Fine I in terms of θ upto second harmonic term using

b) Fine I in terms of θ upto second harmonic term using following table. **(7)**

θ	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
I	0	33.5	18.2	0	-33.5	-18.2	0

UNIT 3

Q.5 a) If $I_n = \int_0^\infty e^{-x} \sin^n x dx$, then prove that

$(1 + n^2)I_n = n(n - 1)I_{n-2}$. **(5)**

b) Prove that **(4)**

$$\int_a^b (x-a)^m (b-x)^n dx = (b-a)^{m+n+1} \beta(m+1, n+1)$$

c) Find the evolute of parabola $y^2 = 4ax$. **(5)**

Q.6 a) If, $I_n = \int_0^{\pi/4} \tan^n\theta \, d\theta$ Where n is positive integer, prove that: $n(I_{n-1} + I_{n+1}) = 1$ Hence prove that:

$$\int_0^a x^5 \left(2a^2 - x^2\right)^{-3} dx = \frac{1}{2}\left(\log 2 - \frac{1}{2}\right).$$ **(5)**

b) Evaluate $\int_0^1 (x \log x)^3 dx$. **(4)**

c) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **(5)**

UNIT 4

Q.7 a) Prove that $\int_0^\infty \frac{e^{-x} - e^{-ax}}{x\sec x} dx = \frac{1}{2}\log\left(\frac{a^2+1}{2}\right)$, $a > 0$. **(5)**

b) Trace the curve $x^2(x^2 + y^2) = a^2(x^2 - y^2)$. **(4)**

c) Find the length of the curve $\theta = \frac{1}{2}\left(r + \frac{1}{r}\right)$ for $r = 1$ to $r = 3$.

(5)

Q.8 a) Verify the rule of DUIS for the integral **(6)**

$$\int_a^{a^2} \log(ax)\,dx$$

b) Trace the curve: r = a(1 + cos θ). **(4)**

c) Find the whole Length of the astroid **(4)**

$$x^{2/3} + y^{2/3} = a^{2/3}$$

UNIT 5

Q.9 a) Evaluate ∬ y dx dy over the area bounded by x = 0, y = x^2, x + y = 2. **(6)**

b) Change the order of integration and hence evaluate

$$\int_0^a \int_{y^2/a}^{y} \frac{y\,dx\,dy}{(a-x)\sqrt{ax-y^2}}.$$ **(7)**

Q.10 a) Evaluate the integral $\iint \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}\,dx\,dy$ over the area of the first quadrant of the circle $x^2 + y^2 = 1$ by changing to polar co-ordinates. **(7)**

b) Use double integration, to find the area common to the Cardiodes r = a(1 – cos θ), r = (1 + cos θ). **(6)**

UNIT 6

Q. 11 a) Evaluate $\int_0^{\log 2} \int_0^{x} \int_0^{x+y} e^{x+y+z}\,dx\,dy\,dz$. **(7)**

b) Evaluate $\iiint \frac{dx\,dy\,dz}{\sqrt{a^2 - x^2 - y^2 - z^2}}$, over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$. **(6)**

Q.12 a) Find the volume cut-off from the sphere $x^2 + y^2 + z^2 = a^2$ by cylinder $x^2 + y^2 = ax$. **(6)**

b) Find the mean value and RMS value of the ordinate y of the cycloid x = a (t + sin t), y = a (1 – cos t) over the range t = - π to t = π. **(7)**

WINTER-2019

UNIT-1

Q.1 a) Find the inverse of matrix by partitioning method where **(7)**

$$A = \begin{bmatrix} 0 & -\sin\alpha & \cos\alpha \\ 0 & \cos\alpha & \sin\alpha \\ 1 & 0 & 0 \end{bmatrix}$$

b) Find the rank and nullity of matrix **(6)**

$$A = \begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$$

Q.2 a) Verify the cayley – Hamilton theorem and hence find A^{-1}, where $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ **(6)**

b) Determine eigen values and eigen vectors of matrix **(6)**

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

UNIT-2

Q.3 a) Obtain a Fourier serious for $f(x) = \frac{1}{4}(\pi - x)^2$ in the interval $(0, 2\pi)$ **(6)**

b) Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $(-\ell, \ell)$. **(7)**

Q.4) Obtain a Fourier cosine series for the function $f(x) = \sin x$ in the interval $(0, \pi)$. **(6)**

b) Find I in terms of θ upto second harmonic term using following table. **(7)**

θ	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Q.4) Obtain a Fourier cosine series for the function $f(x) = \sin x$ in the interval $(0, \pi)$. **(6)**

b) Find I in terms of θ upto second harmonic term using following table. **(7)**

θ	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
I	0	33.5	18.2	0	- 33.5	-18.2	0

UNIT-3

Q.5 a) If $I_n = \int_0^{\pi/4} \tan^n x\, dx$, prove that $I_n + I_{n-2} = \frac{1}{n-1}$, Hence find I_5. **(5)**

b) Evaluate $\int_0^1 (x \log x)^3\, dx$ **(4)**

c) Find the evaluate of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ **(5)**

Q.6 a) If $I_n = \int_0^{\infty} e^{-x} \sin^n x dx$, then prove that

$(1 + n^2)I_n = n(n - 1)I_{n-2}$ **(5)**

b) Show that $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx = \beta(m, n)$ **(4)**

c) Find the evaluate of the parabola $y^2 = 4ax$. **(5)**

UNIT-4

Q.7 a) Prove that $\int_0^{\infty} \frac{e^{-ax} - e^{-dx}}{x} dx = \log(b/a)$ **(5)**

b) Trace the curve $r = a(1 + \cos\theta)$ **(4)**

c) Find the whole Length of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

Q.8 a) Verify the rule of DUIS for the integral $\int_a^{a^2} \log(ax)\, dx$

Q.7 a) Prove that $\int_{0} \frac{\text{[illegible]}}{x}\, dx = \log(b/a)$ **(5)**

b) Trace the curve $r = a(1 + \cos\theta)$ **(4)**

c) Find the whole Length of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

Q.8 a) Verify the rule of DUIS for the integral $\int_{a}^{a^2} \log(ax)\, dx$ **(5)**

b) Trace the curve $y^2 = x^2 \frac{(x^2 + a^2)}{(a - x)(a + x)}$ **(5)**

c) Find the perimeter of the curve $r = a(1 - \cos\theta)$ **(4)**

UNIT-5

Q.9 a) Evaluate $\iint \frac{dx\, dy}{x^4 + y^2}$, where $x \geq 1$, $y \geq x^2$ **(6)**

b) Change the order of integration and hence evaluate **(6)**

$$\int_0^a \int_{y^2/a}^{y} \frac{y\, dx\, dy}{(a - x)\sqrt{ax - y^2}}$$

Q.10 a) Evaluate the integral $\int_0^2 \int_0^{\sqrt{2x - x^2}} \frac{x\, dy\, dx}{\sqrt{x^2 + y^2}}$ by changing to polar co-ordinates. **(7)**

b) Find the area common to the Cardioids $r = a(1 - \cos\theta)$, $r = (1 + \cos\theta)$ using double integration. **(6)**

UNIT-6

Q.11 a) Evaluate $\int_{-1}^{1} dz \int_{0}^{z} dx \int_{x-z}^{x+z} (x + y + z)\, dy$ **(6)**

b) Evaluate $\iiint \frac{dx\, dy\, dz}{\sqrt{a^2 - x^2 - y^2 - z^2}}$, over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$ **(7)**

Q.12 a) Find the volume bounded by the cylinders $y^2 = x$, $x^2 = y$ and the planes $z = 0$, $x + y + z = 2$ **(6)**

b) Find the mean value and RMS value of the ordinate y of the cycloid

$x = a(t + \sin t)$

$y = a(1 - \cos t)$ over the range

$t = -\pi$ to $t = \pi$ **(7)**

SUMMER-2019

UNIT 1

Q.1 a) Find inverse of the matrix by partition method.

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$ **(7)**

b) Find rank of the matrix.

$$\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$$ **(6)**

Q.2 a) Find Eigen values and eigen vectors of.

$$\begin{bmatrix} 9 & -1 & 9 \\ 3 & -1 & 3 \\ -7 & 1 & 6 \end{bmatrix}$$ **(7)**

b) Verify Cayley-Hamilton theorem and hence find A^{-1} for

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$ **(6)**

UNIT 2

Q.3 a) Find Fourier series expansion for $f(x) = \frac{\pi^2}{12} - \frac{x^2}{4}$ defined on $(0, 2\pi)$. **(7)**

b) Obtain Fourier series expansion for f(x) = sin a x where $-\pi < x < \pi$. **(6)**

Q.4 a) Find half range cosine series to represent $f(x) = x - x^2$ in (0, 1). **(7)**

b) Express y in a Fourier series up to third harmonic using following data. **(6)**

x	0	ƒ/3	2 ƒ/3	ƒ	4 ƒ/3	5 ƒ/3	2 ƒ
y	1	1.4	1.9	1.7	1.5	1.2	1.0

UNIT 3

b) Express y in a Fourier series up to third harmonic using following data. **(6)**

x	0	$f/3$	$2f/3$	f	$4f/3$	$5f/3$	$2f$
y	1	1.4	1.9	1.7	1.5	1.2	1.0

UNIT 3

Q.5 a) Evaluate $\int_0^1 \frac{x^a - x^b}{\log x} dx$, $a > 0$, $b > 0$ using differentiation under integral sign. **(5)**

b) Trace the curve $r = \frac{a}{2}(1 + \cos\theta)$. **(5)**

c) Find volume of tetrahedron formed by points (1, 1, 3), (4, 3, 2), (5, 2, 7) and (6, 4, 8). **(4)**

Q.6 a) Evaluate $\int_{\frac{\pi}{6a}}^{\frac{\pi}{2a}} \frac{\sin ax}{x} dx$ using differentiation under integral sign. **(5)**

b) Trace the curve $ay^2 = x^2(a - x)$. **(5)**

c) Prove that. $(\bar{a} \times \bar{b}) \circ \{(\bar{b} \times \bar{c}) \times (\bar{c} \times \bar{a})\} = [\bar{a} \ \ \bar{b} \ \ \bar{c}]^2$. **(4)**

UNIT 4

Q.7 a) Find length of the cycloid

$x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ between two consecutive cusp. **(5)**

b) If $I_n = \int_0^{\pi/2} x^n . \sin x \, dx$, $n > 1$

then prove that $I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$. **(5)**

c) If $\beta(n,3) = \frac{1}{3}$ then find n where n is positive integer. **(4)**

Q.8 a) Find the arc length of the curve $ay^2 = x^3$ from $x = 0$ to a point having $x = 5a$. **(5)**

then prove that $I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$. **(5)**

c) If $\beta(n,3) = \frac{1}{3}$ then find n where n is positive integer. **(4)**

Q.8 a) Find the arc length of the curve $ay^2 = x^3$ from $x = 0$ to a point having $x = 5a$. **(5)**

b) Show that $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} = \beta(m,n)$. **(5)**

c) If $U_n = \int x^n e^x \, dx$ prove that $U_n = x^n e^x - nU_{n-1}$. Hence evaluate U_4. **(4)**

UNIT 5

Q.9 a) Change the order of integration and evaluate $\int_0^2 \int_0^{x^2/4} xy \, dx \, dy$. **(7)**

b) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx\,dy}{1+x^2+y^2}$. **(6)**

Q.10 a) Express the integral in polar coordinates showing region of integration and hence evaluate

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} e^{-x^2-y^2} \, dx \, dy$$ **(7)**

b) Find by double integration the area between the curves $y^2 = 4ax$ and $x^2 = 4ay$. **(6)**

UNIT 6

Q.11 a) Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+y} e^{x+y+z} \, dx dy dz$. **(7)**

b) Change to Spherical polar coordinates and evaluate.

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \left(x^2+y^2+z^2\right) dx dy dz.$$ **(6)**

Q.12 a) Find mean value of r^2 over the area of the cardioid $r = a(1 + \cos 0)$. **(7)**

b) Obtain the volume of solid bounded by the surfaces $x = 0$, $y = 0$, $x + y + z = 1$ and $z = 0$. **(6)**

WINTER-2018

UNIT 1

Q.1 a) Find inverse of the matrix by partition method $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$. **(7)**

b) Determine λ and μ such that the equations. **(6)**

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu \qquad \text{will have}$$

i) No solution **ii)** Unique solution **iii)** Infinite Solutions

Q.2 a) Find eigen values and eigen vectors for $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. **(7)**

b) Verify Cayley-Hamilton theorem and hence find A^{-1} for $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$. **(6)**

UNIT 2

Q.3 a) Find Fourier series to represent $f(x) = x^2 - 2$ when $-2 \le x \le 2$. **(7)**

b) Analyse the current i with its constituent harmonics upto the third harmonic. The values of i and θ are as follows. **(6)**

.	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
i	0	33.5	18.2	0	-33.5	-18.2	0

Q.4 a) Find the Fourier series with period 3 to represent $f(x) = 2x - x^2$ in the range (0, 3). **(7)**

b) Find half range cosine series for $f(x) = x - x^2$ in the range (0, 1). **(6)**

UNIT 3

Q.5 a) Find constant P such that the coterminous vectors $2i - j + k$, $i + 2j - 3k$ and $3i + pj + 5k$ are coplanar. **(4)**

Q.4 a) Find the Fourier series with period 3 to represent $f(x) = 2x - x^2$ in the range (0, 3). **(7)**

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UNIT 3

Q.5 a) Find constant P such that the coterminous vectors $2i - j + k$, $i + 2j - 3k$ and $3i + pj + 5k$ are coplanar. **(4)**

b) Using differentiation under integral sign prove that, **(5)**

$$\int_0^1 \frac{x^a - 1}{x} dx = \log(1 + a), a > 1$$

c) Trace the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ **(5)**

Q.6 a) Prove that $\bar{a} \times (\bar{b} \times \bar{c}) + \bar{b} \times (\bar{c} \times \bar{a}) + \bar{c} \times (\bar{a} \times \bar{b}) = 0$. **(4)**

b) Using differentiation under integral sign prove that **(5)**

$$\int_0^{\pi/2} \frac{\log(1 + a\sin^2 x)}{\sin^2 x} dx = x[\sqrt{1+a} - 1]$$

c) Trace the curve $9ay^2 = x(x - 3a)^2$. **(5)**

UNIT 4

Q.7 a) Compute the arc length of the curve $ay^2 = x^3$ from $x = 0$ to a point having $x = 5a$. **(5)**

b) Show that $\int_0^{\pi/2} \sin^p x.dx \int_0^{\pi/2} \sin^{p+1} x.dx = \frac{\pi}{2(p+1)}$. **(5)**

c) If $In = \int_0^{\pi/4} \sec^n x\,dx$ prove that $In = \frac{(\sqrt{2})^{n-2}}{n-1} + \frac{n-2}{n-1} I_{n-2}$. **(4)**

Q.8 a) Obtain perimeter of the curve $r = a(1 + \cos\theta)$. **(5)**

b) Evaluate $\int_3^7 \sqrt{(x-3)(7-x)}dx$. **(5)**

c) If $f(m,n) = \int x^m (1-x)^n dx$, prove that

$$f(m,n) = \frac{x^{m+1}(1-x)^n}{m+n+1} + \frac{n}{m+n+1} f(m, n-1)$$ **(4)**

UNIT 5

Q.9 a) Evaluate $\iint x^3 y\,dx\,dy$ over the area bounded by the

b) Evaluate $\int_{3} \sqrt{(x-3)(7-x)}\,dx$. **(5)**

c) If $f(m,n) = \int x^m (1-x)^n dx$, prove that

$$f(m,n) = \frac{x^{m+1}(1-x)^n}{m+n+1} + \frac{n}{m+n+1} f(m,n-1)$$ **(4)**

UNIT 5

Q.9 a) Evaluate $\iint x^3 y\,dx\,dy$ over the area bounded by the positive quadrant of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **(7)**

b) Change the order of integration and evaluate. $\int_0^a \int_0^{2\sqrt{xa}} x^2 dx\,dy$. **(6)**

Q.10 a) Change the integral $\int_0^1 \int_0^x (x+y)\,dx\,dy$ to Polar coordinates and hence evaluate. **(7)**

b) Find by double integration the area common to the cardioide. **(6)**

$r = a(1 - \cos\theta)$ and $r = a(1 + \cos\theta)$

UNIT 6

Q.11 a) Evaluate $\int_{-1}^{1} \int_{0}^{z} \int_{x-z}^{x+z} (x+y+z)\,dx\,dy\,dz$. **(7)**

b) Obtain volume of solid bounded by the surface $x = 0$, $y = 0$, $x + y + z = 1$, $z = 0$. **(7)**

Q.12 a) Evaluate $\iiint (x^2 + y^2 + z^2)\,dx\,dy\,dz$ taken over volume enclosed by the sphere $x^2 + y^2 + z^2 = 1$ by changing to spherical polar coordinates. **(7)**

b) Find mean value of the cycloid $x = a(t + \sin t)$, $y = a(1 - \cos t)$ over the range $-\pi$ to π. **(6)**

SUMMER-2018

UNIT-1

Q.1 a) Find inverse of matrix by portioning **(7)**

$$A = \begin{bmatrix} 0 & -\sin\alpha & \cos\alpha \\ 0 & \cos\alpha & \sin\alpha \\ 1 & 0 & 0 \end{bmatrix}$$

b) Test for consistency and hence solve. **(6)**

$x + 5y + 7z = 15$

$2x + 3y + 4z = 11$

$x - 2y - 3z = -4$

$3x + 11y + 13z = 25$

Q.2 a) Determine the eigen values and eigen vectors of matrix A. **(7)**

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

b) Apply Cayley Hamilton theorem to $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and deduce that $A^8 = 625\,I$. **(6)**

UNIT-2

Q.3 a) Find the Fourier series of $f(x) = x.\sin x$ in the interval $(0, 2\pi)$ and hence deduce that: $\sum_{n=2}^{\infty} \frac{1}{n^2 - 1} = \frac{3}{4}$. **(7)**

b) Find the half range cosine series of $f(x) = x(\pi - x)$ in the interval $(0, \pi)$ and hence deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots\ldots = \frac{\pi^2}{12}.$$ **(6)**

Q.4 a) Find the Fourier series of $f(x) = x - x^2$ in the interval $(-1, 1)$. **(7)**

b) Obtain a Fourier series upto second harmonic for the following data: **(6)**

x :	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π

Q.4 a) Find the Fourier series of $f(x) = x - x^2$ in the interval (-1, 1). **(7)**

b) Obtain a Fourier series upto second harmonic for the following data: **(6)**

x :	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y :	16	3	−0.5	−17	−8	1

UNIT-3

Q.5 a) Prove that $\hat{i}\times(\bar{a}\times\hat{i})+\hat{j}\times(\bar{a}\times\hat{j})+\hat{k}\times(\bar{a}\times\hat{k})=2\bar{a}$. **(4)**

b) Evaluate $\int_0^\infty \frac{\tan^{-1}(ax)}{x(1+x^2)}dx$ by applying the rule of differentiation under integral sign. **(5)**

c) Trace the curve $y^2(2a - x) = x^3$. **(5)**

Q.6 a) If A, B, C, D, have co-ordinates (2, 1, 1), (3, 3, 4), (0, 1, 5) and (1, 2, 2) respectively. Obtain the volume of parallelpiped with concurrent edges AB, AC, AD. **(4)**

b) Prove that $\int_0^{\pi/2} \frac{\log(1+a\sin^2 x)}{\sin^2 x}dx = \pi(\sqrt{1+a}-1)$. **(5)**

c) Trace the curve: $r^2 = a^2.\cos 2\theta$. **(5)**

UNIT-4

Q.7 a) Evaluate: $\int_0^{2a} x\sqrt{2ax-x^2}.dx$. **(4)**

b) Prove that $\int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}}dx = \frac{1}{a^n b^m}.\beta(m,n)$. **(5)**

c) Show that the length of the arc of the curve $ay^2 = x^3$ from origin to the point whose abscissa is 'b' is

$$\frac{1}{27\sqrt{a}}.(9b+4a)^{3/2} - \frac{8a}{27}.$$ **(5)**

Q.8 a) If $I_n = \int_0^\infty e^{-x}.\sin^n x.dx$; obtain the relation between I_n

b) Prove that $\int_0 \frac{\quad}{(a+bx)^{m+n}}dx = \frac{\quad}{a^n b^m}.\beta(m,n)$. **(5)**

c) Show that the length of the arc of the curve $ay^2 = x^3$ from origin to the point whose abscissa is 'b' is

$\frac{1}{27\sqrt{a}}.(9b+4a)^{3/2} - \frac{8a}{27}$. **(5)**

Q.8 a) If $I_n = \int_0^{\infty} e^{-x}.\sin^n x.dx$; obtain the relation between I_n and I_{n-2} and hence find I_4. **(5)**

b) Prove that $\beta(m,n) = \int_0^{\infty} \frac{x^{n-1}}{(1+x)^{m+n}}dx$. **(5)**

c) Find the perimeter of the cardiod $r = a(1 + \cos\theta)$ and show that a line $\theta = \pi/3$, divides upper half of the cardiod. **(4)**

UNIT-5

Q.9 a) Evaluate: $\int_0^1 \int_0^{1-x} x^{1/2} y^{1/2} (1-x-y)^{1/2} \cdot dxdy$. **(6)**

b) Change the integral to polar coordinates and hence evaluate: $\int_0^1 \int_0^x (x+y)dxdy$. **(7)**

Q.10 a) Change the order of integration and evaluate

$\int_0^2 \int_{2-\sqrt{4-y^2}}^{2+\sqrt{4-y^2}} f(x,y)dxdy$. **(6)**

b) Find the area bounded by the parabola $y^2 = 4x$ and the line $2x - 3y + 4 = 0$. **(7)**

UNIT-6

Q.11 a) $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{1}{(x+y+z+1)^3} dxdydz$. **(6)**

b) Evaluate $\iiint \frac{dxdydz}{\sqrt{a^2-x^2-y^2-z^2}}$ over the region bounded by the sphere $x^2 + y^2 + z^2 = a^2$. **(7)**

Q.12 a) Find the volume enclosed between the cylinders $x^2 + y^2 = ax$ and $z^2 = ax$. **(6)**

b) Find the RMS value of the expression $a \sin pt + b \cos qt$ between $t = 0$, to $t = 2\pi$. **(7)**

WINTER-2017

UNIT-1

Q.1 a) Find inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$ by partitioning. **(7)**

b) Solve the system of equations by matrix method: **(6)**

$$x + y + z = 2$$
$$3x + y - 2z = 1$$
$$4x - 3y - z = 3$$
$$2x + 4y + 2z = 14$$

Q.2 a) Find the eigen values and eign vectors of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$ **(7)**

b) Verify Caley – Hamilton's theorem for the matrix **(6)**

$A = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ and hence find A^{-1}

UNIT-2

Q.3 a) A function f(x) is defined within the range $(0, 2\pi)$ by the relation.

$$f(x) = \begin{cases} x & ; \ 0 < x \le \pi \\ 2\pi - x & ; \pi \le x < 2\pi \end{cases}$$

Express f(x) as a Fourier series in the range $(0, 2\pi)$. **(7)**

b) Obtain a half range cosine series to represent $f(x) = x - x^2$ in the range (0, 1). **(6)**

Q.4 a) Determine the Fourier series of the function. **(7)**

$$f(x) = \begin{cases} 0 & ; -2 < x < -1 \\ 1 + x & ; \ -1 < x < 0 \\ 1 - x & ; \ \ 0 < x < 1 \\ 0 & ; \ \ 1 < x < 2 \end{cases}$$

b) The following table gives the variations of a periodic

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Q.4 a) Determine the Fourier series of the function. **(7)**

$$f(x) = \begin{cases} 0 & ; -2 < x < -1 \\ 1+x & ; \ -1 < x < 0 \\ 1-x & ; \ 0 < x < 1 \\ 0 & ; \ 1 < x < 2 \end{cases}$$

b) The following table gives the variations of a periodic current over a period T. **(6)**

t(sec)	0	T/6	T/3	T/2	2T/3	5T/6	T
A(Amp)	1.98	1.30	1.05	1.30	– 0.88	– 0.25	1.98

Show by numerical analysis that there is a direct current of 0.75 amp. In the variable current and obtain the amplitude of the first harmonic.

UNIT-3

Q.5 a) If vector $\bar{x}$ and the scalar λ satisfy the equations, $\bar{a} \times \bar{x} = \lambda\bar{a} + \bar{b}$ and $\bar{a}.\bar{x} = 2$, find the value of λ and $\bar{x}$ in terms of $\bar{a}$ and $\bar{b}$. Also determine them of $\bar{a} = 2i + j - k$ and $\bar{b} = -i - 2j + k$. **(5)**

b) Prove that $\int_0^{\infty} \frac{e^{-x} - e^{-ax}}{x \sec x} dx = \frac{1}{2} \log\left(\frac{a^2+1}{2}\right); a > 0$ **(5)**

c) Trace the curve $9ay^2 = x(x - 3a)^2$ **(4)**

Q.6 a) Verify the rule of differentiation under the integral sign for $\int_0^{a^2} \tan^{-1}\left(\frac{x}{a}\right) dx$. **(5)**

b) Trace the curve $r^2 = a^2 \cos 2\theta$ **(4)**

c) Find the volume of the tetrahedron with vertices at the point A (0,0,0); B(1,1,1);C (21,1) and D (1,2,1). **(5)**

UNIT-4

Q.7 a) If $I_n = \int^{\pi/4} \sin^{2n} \ldots$ Prove that

UNIT-4

Q.7 a) If $U_n = \int_{0a}^{\pi/4} \sin^{2n} x \, dx$, Prove that

$$U_n = \left(1 - \frac{1}{2n}\right) U_n - \frac{1}{n.2^{n+1}} \quad \textbf{(4)}$$

b) Show that $\int_0^{\infty} \frac{x^{m-1}}{(a+bx)^{m+n}} dx = \frac{1}{a^n b^m} \beta(n, m)$ **(5)**

c) Find the whole length of the loop of the curve: **(5)**

$3ay^2 = x(x - a)^2$

Q.8 a) If $f(m, n) = \int x^m (1 - x)^n dx$ show that. **(4)**

$$f(m, n) = \frac{x^{m+1}(1-x)^n}{m+n+1} + \frac{n}{n+n+1} f(m, n-1)$$

b) Evaluate $\int_0^1 (x \log x)^3 dx$ **(5)**

c) Find the length of the cardiode $r = a\ (1 - \cos\theta)$ lying outside the circle $r = a \cos\theta$. **(5)**

UNIT-5

Q.9 a) Evaluate $\iint \frac{dx\,dy}{x^4 + y^2}$ where $x \geq 1$ and $y \geq x^2$ **(6)**

b) Evaluate the following double integral by transforming to polar-co-ordinates. **(7)**

$$\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{dxdy}{\sqrt{a^2 - x^2 - y^2}}$$

Q.10 a) Express the following integrals as single integrals and evaluate. $\int^{1}\int^{y}(x^2 + y^2)dxdy + \int^{2}\int^{2-y}(x^2 + y^2)dxdy$ **(7)**

Q.10 a) Express the following integrals as single integrals and evaluate. $\int_0^1\int_0^y \left(x^2 + y^2\right)dxdy + \int_1^2\int_0^{2-y}\left(x^2 + y^2\right)dxdy$ **(7)**

b) Find by double integration, the area between the curve $y^2 = \frac{4a^2(2a - x)}{x}$ and its asymptote. **(6)**

UNIT-6

Q.11 a) Evaluate: $\int_0^{\log 2}\int_0^{x}\int_0^{x+\log y} e^{x+y+z}dxdydz$ **(7)**

b) Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + y^2 = z^2$ **(6)**

Q.12 a) Show that $\iiint \frac{dx\,dy\,dz}{(x+y+z+1)^3} = \frac{1}{2}\left(\log 2 - \frac{5}{8}\right)$ where integration being throughout the volume of the tetrahedron bounded by the co-ordinate planes and the plane $x + y + z = 1$. **(6)**

b) Find the r.m.s. values of the expression a sin pt + b cosqt, over the interval 0 to 2π. **(7)**

SUMMER-2017

UNIT-1

Q.1 a) Find inverse of matrix by partitioning

$$A = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$ **(7)**

b) Find the Eigen values and Eigen vectors of the following matrix: $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ **(6)**

Q.2 a) For what values of k the equations

$x + y + z = 1,$

$2x + y + 4z = k$

$4x + y + 10z = k^2$

have a solution and solve them completely in each case. **(7)**

b) Use Caley – Hamilton Theorem to find $2A^5 - 3A^4 + A^2 - 4I$, where $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$. **(6)**

UNIT-2

Q.3 a) Find a Fourier series to represent $f(x) = x + x^2$ from $x = -\pi$ to $x = \pi$. **(7)**

b) If $f(x) = lx - x^2$ in the range $(0, l)$, show that the half range sine series for f(x) is $\frac{8l^2}{\pi^3}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^3}\sin\left[\frac{(2n+1)\pi x}{l}\right]$. **(6)**

Q.4 a) Obtain the Fourier series as far as the second harmonic to represent the function given by **(7)**

x:	45°	90°	135°	180°	225°	270°	315°	360°
y:	4.0	3.8	2.4	2.0	1.5	0.0	2.8	3.4

b) Find the Fourier series for the function $f(x) = 1 - x^2$ in the interval $-1 < x < 1$. **(6)**

UNIT-3

Q.5 a) Show that the four points whose position vectors given below are co-planer $3i - 2j + 4k$, $6i + 3j + k$, $5i + 7j + 3k$, $2i + 2j + 6k$. **(4)**

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y:	4.0	3.8	2.4	2.0	1.5	0.0	2.8	3.4

b) Find the Fourier series for the function $f(x) = 1 - x^2$ in the interval $-1 < x < 1$. **(6)**

UNIT-3

Q.5 a) Show that the four points whose position vectors given below are co-planer $3i - 2j + 4k$, $6i + 3j + k$, $5i + 7j + 3k$, $2i + 2j + 6k$. **(4)**

b) Evaluate $\int_0^\infty \frac{e^{-ax} \sin x}{x} dx$ and hence show that $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$. **(5)**

c) Trace the curve $y^2(x + a) = x^2(3a - x)$. **(5)**

Q.6 a) Show that $[\bar{a} \times \bar{b} \quad \bar{c} \times \bar{d} \quad \bar{p} \times \bar{q}] = [\overline{abd}][\overline{cpq}] - [\overline{abc}][\overline{dpq}]$ **(4)**

b) Prove that $\int_0^{\pi/2} \frac{\log(1 + \sin^2 x)}{\sin^2 x} dx = \pi(\sqrt{1 + a} - 1)$. **(5)**

c) Trace the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$. **(5)**

UNIT-4

Q.7 a) If $I_n \int_0^{\pi/4} \sec^n\theta \, d\theta$, prove that $I_n = \frac{(\sqrt{2})^{n-2}}{n-1} + \frac{n-2}{n-1} I_{n-2}$. **(5)**

b) Prove that $\Gamma\left(-\frac{3}{2}\right) = \frac{4\sqrt{\pi}}{3}$. **(4)**

c) For the curve $x = e^\theta \cos\theta$, $y = e^\theta \sin\theta$, show that the length of the curve from $\theta = 0$ to $\theta = \frac{\pi}{2}$ is $\sqrt{2}(e^{\pi/2} - 1)$. **(5)**

Q.8 a) Evaluate $\int_0^4 x^3 \sqrt{4x - x^2} \, dx$. **(5)**

b) Prove that, :$\beta(m, n) = \beta(m + 1, n) + \beta(m, n + 1)$ **(4)**

c) Find the length of the curve $\theta = \frac{1}{2}\left(r + \frac{1}{r}\right)$ for $r = 1$ to $r = 3$.

length of the curve from $\theta = 0$ to $\theta = \frac{\pi}{2}$ is $\sqrt{2}(e^{\dots} - 1)$. **(5)**

Q.8 a) Evaluate $\int_0^4 x^3 \sqrt{4x - x^2}\, dx$. **(5)**

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c) Find the length of the curve $\theta = \frac{1}{2}\left(r + \frac{1}{r}\right)$ for $r = 1$ to $r = 3$. **(5)**

UNIT-5

Q.9 a) Evaluate $\iint y\, dx\, dy$ over the area bounded by $x = 0$, $y = x^2$, $x + y = 2$. **(6)**

b) Find by double integration, the area common to the cardiodes $r = a(1 + \cos\theta)$ and $r = a(1 - \cos\theta)$. **(7)**

Q.10 a) Change the order of integration and hence evaluate

$$\int_0^a \int_0^{2\sqrt{xa}} x^2 dx\, dy.$$ **(7)**

b) Change to polar co-ordinates and evaluate

$$\int_0^a \int_y^a \frac{x^2 dx\, dy}{\sqrt{x^2 + y^2}}.$$ **(6)**

UNIT-6

Q.11 a) Evaluate $\int_0^2 \int_x^{4-x} \int_{\frac{3x}{2}-y}^{3} dx\, dy\, dz$. **(6)**

b) Find the volume bounded by the xy-plane, the paraboloid $x^2 + y^2 = 2z$ and the cylinder $x^2 + y^2 = 4$. **(7)**

Q.12 a) Find the mean value of the expression $(4 \cos t - \sin 3t)$ over the interval 0 to $\pi/6$. **(6)**

b) Find $\iiint xyz\, dx\, dy\, dz$ over the positive octant of sphere $x^2 + y^2 + z^2 = 1$, using spherical co-ordinates. **(7)**

WINTER-2016

UNIT 1

Q.1 a) Using matrix method, show that the equation $3x + 3y + 2z = 1$, $x + 2y = 4$, $10y + 3z = -2$, $2x - 3y - z = 5$ are consistent and hence obtain the solution for x, y, z. **(6)**

b) Verify Cayley-Hamilton theorem for the matrix:

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$ Hence compute A^{-1}. **(7)**

Q.2 a) Find the inverse of matrix by partitioning:

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 3 & 2 \\ 2 & 4 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$ **(7)**

b) Determine the rank of the matrix:

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$ **(6)**

UNIT 2

Q.3 a) Obtain the Fourier Series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$. **(6)**

b) Obtain Fourier Series for the function

$$\begin{aligned} f(x) &= \pi x & 0 \le x \le 1 \\ &= \pi(2 - x) & 1 \le x \le 2 \end{aligned}$$ **(7)**

Q.4 a) Find Fourier expansion for the function $f(x) = x - x^2$, $-1 < x < 1$ **(6)**

b) The turning moment T unit of the Crank shaft of a steam engine is given for a series of value of the Crank-angle θ in degrees.

θ	0	30	60	90	120	150	180
T	0	5224	8097	7850	5499	2626	0

Find the first four terms in a series of series of sines to represent T. Also calculate T when $\theta = 75°$. **(7)**

UNIT 3

b) The turning moment T unit of the Crank shaft of a steam engine is given for a series of value of the Crank-angle θ in degrees.

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T	0	5224	8097	7850	5499	2626	0

Find the first four terms in a series of series of sines to represent T. Also calculate T when $\theta = 75°$. **(7)**

UNIT 3

Q.5 a) Prove that $\left[\bar{a}+\bar{b}, \bar{b}+\bar{c}, \bar{c}+\bar{a}\right] = 2\left[\bar{a}\,\bar{b}\,\bar{c}\right]$ **(4)**

b) Evaluate $\int_0^\infty \frac{\log\left(1+a^2x^2\right)}{1+b^2x^2}dx$ by using Differentiation under the integral sign theorem. **(6)**

c) Trace the Curve: $x^{2/3} + y^{2/3} = a^{2/3}$ **(4)**

Q.6 a) Find the volume of the tetrahedron, whose coterminous edges are $3i - j + 2k$, $2i + j - k$ and $i - 2j + 2k$. **(4)**

b) Verify the rule of differentiation under integral:

$$\int_a^{a^2} \log(ax)\,dx.$$ **(6)**

c) Trace the curve $y^2 (a^2 + x^2) = x^2 (a^2 - x^2)$. **(4)**

UNIT 4

Q.7 a) If $f(m, n) = \int x^m (1 - x)^n\, dx$ prove that:

$$f(m,n) = \frac{x^{m+1}(1-x)^n}{m+n+1} + \frac{n}{m+n+1} f(m, n-1)$$ **(4)**

b) Prove that: $\dfrac{2^n \Gamma\left(n+\frac{1}{2}\right)}{\sqrt{\pi}} = (2n-1)(2n-3)......5.3.1$ **(4)**

c) Find the length of the arc of the parabola $y^2 = 4ax$ cut off by the line $3y = 8x$. **(5)**

Q.8 a) If $I_n = \int_0^{\pi/4} \sec^n \theta\, d\theta$ Prove that:

Q.8 a) If $I_n = \int_0^{\pi/4} \sec^n \theta d\theta$ Prove that:

$$I_n = \frac{\left(\sqrt{2}\right)^{n-2}}{n-1} + \frac{(n-2)}{(n-1)} I_{n-2} \text{ find } I_4.$$ **(5)**

b) Prove that $\beta(m,m).\beta\left(m+\frac{1}{2}, m+\frac{1}{2}\right) = \frac{\pi}{m} 2^{1-4m}$ **(4)**

c) Find the length of arch of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$. **(5)**

UNIT 5

Q.9 a) Evaluate $\iint_R y\, dx\, dy$, where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. **(7)**

b) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{x^2+y^2} dy\, dx$ by changing to polar co-ordinates. **(6)**

Q.10 a) Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\, dx\, dy$ and hence evaluate the same. **(7)**

b) Find by double integration, the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **(6)**

UNIT 6

Q.11 a) Evaluate: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}} dx\, dy\, dz$ **(7)**

b) Prove that the mean distance of points, within a circular area of radius a from the fixed point on the circumference is, $\frac{32a}{9\pi}$. **(6)**

Q.12 a) Find the volume of the tetrahedron bounded by the co-ordinate planes and the plane, $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. **(7)**

b) Prove that the mean distance of points, within a circular area of radius a from the fixed point on the circumference is, $\frac{32a}{9\pi}$. **(6)**

Q.12 a) Find the volume of the tetrahedron bounded by the co-ordinate planes and the plane, $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. **(7)**

b) Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+\log y} e^{x+y+z}\,dz\,dy\,dx$. **(6)**

SUMMER-2016

UNIT 1

Q. 1 a) Find inverse of matrix by partitioning:

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 8 & 1 & 0 \\ 4 & 1 & 1 \end{bmatrix} \quad \textbf{(7)}$$

b) Verify Caley-Hamilton theorem for the following matrix. Also find A^{-1}.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix} \quad \textbf{(6)}$$

Q.2 a) Find the Eigen values and Eigen vectors of the following matrix;

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & -1 \\ 0 & 0 & -3 \end{bmatrix} \quad \textbf{(6)}$$

b) For what values of λ the following set of equations are consistent and solve then.

$X_1 + 2X_2 + X_3 = 3,\qquad X_1 + X_2 + X_3 = \lambda$ and

$3X_1 + X_2 + 3X_3 = \lambda^2$ **(7)**

UNIT 2

Q.3 a) Find the Fourier series for f(x) if,

$$f(x) = \begin{cases} -\pi & -\pi \le x \le 0 \\ x & 0 \le x \le \pi \end{cases} \quad \textbf{(7)}$$

b) Prove that: $\frac{\ell}{2} - x = \frac{\ell}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{2n\pi n}{\ell}\right)$, $0 < x < l$ **(6)**

Q.4 a) Find the Fourier series to represent, $f(x) = x^2 - 2$, when $-2 \le x \le 2$. **(6)**

b) Express y in a Fourier series upto third harmonic using following data. **(7)**

x	0	60°	120°	180°	240°	300°	360°
y	1.98	2.15	2.77	-0.22	-0.31	1.43	1.98

UNIT 3

b) Express y in a Fourier series upto third harmonic using following data. **(7)**

x	0	60°	120°	180°	240°	300°	360°
y	1.98	2.15	2.77	-0.22	-0.31	1.43	1.98

UNIT 3

Q.5 a) Prove that the four points 4i + 5j + k, –(j + k), 3i + 9j + 4k and 4(–i + j + k) are coplanar. **(4)**

b) Prove that $\int_0^{\infty}\frac{e^{-x}-e^{-ax}}{x\sec x}dx = \frac{1}{2}\log\left(\frac{a^2+1}{2}\right)$ given that a > 0. **(5)**

c) Trace the Curve: $r = \frac{a}{5}(1+\cos\theta)$ **(5)**

Q.6 a) Prove that $\bar{d}\times(\bar{a}\times\bar{b}).(\bar{a}\times\bar{c}) = [\bar{a}.(\bar{b}\times\bar{c})]\,(\bar{a}.\bar{d})$ **(5)**

b) Prove that: $\int_0^{\pi/2}\frac{\log(1+a\sin^2 x)}{\sin^2 x}dx = \pi\left(\sqrt{1+a}-1\right)$ **(5)**

c) Trace the curve $y^2(x^2+y^2)+a^2(x^2-y^2)=0$ **(5)**

UNIT 4

Q.7 a) If $u_n = \int_0^{\pi/4}\sin^{2n}x\,dx$, prove that:

$$u_n = \left(1-\frac{1}{2n}\right)u_{n-1} - \frac{1}{n.2^{n+1}}$$ **(5)**

b) Prove that: $\left(\int_0^{\pi/2}\sin^p x\,dx\right)\left(\int_0^{\pi/2}\sin^{p+1}x\,dx\right) = \frac{p}{2(p+1)}$ **(4)**

Q.8 a) Evaluate: $\int_0^4 x^3\sqrt{4x-x^2}\,dx$ **(4)**

b) Prove that $\int_0^{\infty}\frac{x^{m-1}}{(a+bx)^{m+n}}dx = \frac{1}{a^n b^m}\beta(m,n)$ and hence find the value of $\int_0^{\infty}\frac{x^5}{(2+3x)^{16}}dx$. **(5)**

c) Show that the length of the loop of the curve:

b) Prove that $\int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}} dx = \frac{1}{a^n b^m} \beta(m,n)$ and hence find the value of $\int_0^\infty \frac{x^5}{(2+3x)^{16}} dx$. **(5)**

c) Show that the length of the loop of the curve:

$$\int_0^\infty \frac{x^5}{(2+3x)^{16}} dx.$$ **(5)**

UNIT 5

Q.9 a) Evaluate $\iint\limits_A xy\, dx\, dy$, where A is the domain bounded by the x-axis, ordinate x = 2a and the curve $x^2 = 4ay$. **(7)**

b) Evaluate by transforming to polar coordinates:

$$\int_0^1 \int_{x^2}^{\sqrt{x}} \int_0^{2-x-y} dx\, dy\, dz$$ **(6)**

Q.10 a) Change the order of integration and evaluate:

$$\int_0^2 \int_0^{x^2/4} xy\, dx\, dy$$ **(7)**

b) Find by double integration, the area common to the cardioids, r = a (1 + cos θ) and r = a (1 − cos θ) **(6)**

UNIT 6

Q.11 a) Evaluate $\int_0^1 \int_{x^2}^{\sqrt{x}} \int_0^{2-x-y} dx\, dy\, dz$ **(6)**

b) Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$ **(7)**

Q.12 a) Evaluate the integral $\iiint (x^2 + y^2 + z^2) dx\, dy\, dz$ taken over the volume enclosed by the sphere $x^2 + y^2 + z^2 = 1$. **(6)**

b) Find the mean value of r^2 over the area of the cardioid r = a (1 + cos θ). **(7)**

WINTER-2015

UNIT 1

Q.1 a) Find Inverse of a matrix by partitioning method:

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 3 & 2 \\ 2 & 4 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad \textbf{(6)}$$

b) Find Eigen values and Eigen vectors of: $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ **(7)**

Q.2 a) Find the values of a and b for which the equations: are consistent,

$$x + ay + z = 3,$$
$$x + 2y + 2z = b$$
$$x + 5y + 3z = 9 \quad \text{are consistent.} \quad \textbf{(6)}$$

b) Verify Cayley-Hamilton theorem and find inverse of the matrix,

$$A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix} \quad \textbf{(7)}$$

UNIT 2

Q.3 a) Find the Fourier series for the functions:

$$f(x) = \begin{cases} -1 & \text{for } -\pi < x < -\pi/2 \\ 0 & \text{for } -\pi/2 < x < \pi/2 \\ 1 & \text{for } \pi/2 < x < \pi \end{cases} \quad \textbf{(6)}$$

b) Find the half range cosine series expansion of the function:

$$f(x) = \begin{cases} 0 & 0 \le x \le \ell/2 \\ \ell - x & \ell/2 \le x \le \ell \end{cases} \quad \textbf{(7)}$$

Q.4 a) Expand $f(x) = 2x - x^2$ in $(0, 3)$; in terms of Fourier series and hence show that:

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots\ldots = \frac{\pi^2}{12} \quad \textbf{(6)}$$

b) Compute the first two harmonics of the Fourier series of f(x) given in the following table: **(7)**

$$f(x)=\begin{cases}0 & 0\le x\le \ell/2\\ \ell-x & \ell/2\le x\le \ell\end{cases}$$ **(7)**

Q.4 a) Expand $f(x) = 2x - x^2$ in $(0, 3)$; in terms of Fourier series and hence show that:

$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}\ldots\ldots=\frac{\pi^2}{12}$$ **(6)**

b) Compute the first two harmonics of the Fourier series of f(x) given in the following table: **(7)**

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

UNIT 3

Q.5 a) Find a such that the vectors $2i - j + k$; $i + 2j - 3k$ and $3i + aj + 5k$ are coplanar. **(4)**

b) Use rule of differentiation under integral sign: $\int_0^a \frac{\log(1+ax)}{1+x^2}dx$ and hence show that $\int_0^1 \frac{\log(1+ax)}{1+x^2}dx=\frac{\pi}{8}.\log 2$ **(6)**

c) Trace the curve $r = a(1 - \cos\theta)$ **(4)**

Q.6 a) Prove that: $\bar{a}\times\left[\bar{b}\times\left(\bar{c}\times\bar{d}\right)\right]=\bar{b}.\bar{d}(\bar{a}\times\bar{c})-\bar{b}.\bar{c}(\bar{a}\times\bar{d})$. **(4)**

b) Prove that: $\int_0^1 \frac{x^a-1}{\log x}dx=\log(1+a)\ (a>1)$. **(5)**

c) Trace the curve: $9ay^2 = x(x - 3a)^2$ **(5)**

UNIT 4

Q.7 a) Find a reduction formula for, $\int e^{ax}\sin^n x\,dx$.

Hence evaluate $\int e^x \sin^3 x\,dx$. **(5)**

b) Evaluate: $\int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}}dx$. **(4)**

c) Prove that the length of the curve $ay^2 = x^3$ from the vertex to the point whose abscissa is:

$$\frac{1}{27\sqrt{a}}(9b-4a)^{3/2}-\frac{8a}{27}$$ **(5)**

Q.8 a) If $I_n=\int x^n e^x dx$, Show that: $I_n+nI_{n-1}=x^n e^x$. Hence find I_4. **(4)**

b) Evaluate: $\int_0^{\infty} \frac{x^{m-1}}{(a+bx)^{m+n}} dx.$ **(4)**

c) Prove that the length of the curve $ay^2 = x^3$ from the vertex to the point whose abscissa is:

$$\frac{1}{27\sqrt{a}}(9b-4a)^{3/2} - \frac{8a}{27}$$ **(5)**

Q.8 a) If $I_n = \int x^n e^x dx$, Show that: $I_n + nI_{n-1} = x^n e^x$. Hence find I_4. **(4)**

b) Evaluate: $\int_0^{\pi/2} \frac{\sin^{2m-1}\theta . \cos^{2n-1}\theta}{\left(a\cos^2\theta + b\sin^2\theta\right)^{m+n}}$ **(5)**

c) Find the length of the cycloid: $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$ between two cusp. **(5)**

UNIT 5

Q.9 a) Change the order of integration: $\int_0^1 \int_{x^2}^{\sqrt{2-x^2}} f(x, y)dxdy$. **(4)**

b) Find by double integration the area lying inside the circle $r = a\sin\theta$ and cardiod $r = a(1 - \cos\theta)$. **(7)**

Q.10 a) Evaluate by transforming to polar co-ordinates:

$$\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} \left(x^2 + y^2\right)dxdy$$ **(7)**

b) Evaluate: $\iint \sqrt{xy(1-x-y)}.dxdy$ In the area bounded by $x = 0,\ y = 0$ and $x + y = 1$. **(6)**

UNIT 6

Q.11 a) Prove that: $\int_0^{\pi/2} \int_0^{2\sin\theta} \int_0^{\frac{a^2-r^2}{a}} r dr\, d\theta\, d\phi = \frac{5\pi a^3}{64}$ **(6)**

b) Find the volume bounded by cylinder $x^2 + y^2 = 4$ and the plan $y + z = 4$ and $z = 0$. **(7)**

Q.12 a) Change to spherical polar co-ordinates and evaluate: $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^3}} \left(x^2 + y^2 + z^3\right)dxdydz$ **(7)**

b) Obtain the root mean square value of

$f(t) = 3\sin 2t + 4\cos 2t$ over the range $0 \le t \le \pi$. **(6)**

SUMMER-2015

UNIT 1

Q.1 a) Find inverse of a matrix by Partitioning:

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 2 \\ -1 & 2 & 1 \end{bmatrix}$$ **(7)**

b) Discuss the consistency of the following system of equations and solve them whenever possible,

$x_1 + 2x_2 + 2x_3 = 1$, $2x_1 + 2x_2 + 3x_3 = 3$, and

$x_1 - x_2 + 3x_3 = 5$. **(6)**

Q.2 a) Find Eigen values and Eigen vectors of the matrix:

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ **(7)**

b) Show that the matrix: $A = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ Satisfies its characteristic equation and hence find A^{-1} **(6)**

UNIT 2

Q.3 a) Obtain Fourier series expansion of f(x) defined as follows:

$$f(x) = \begin{cases} x + \frac{\pi}{2} & -\pi \le x \le 0 \\ \frac{\pi}{2} - x & 0 \le x \le \pi \end{cases}$$ **(7)**

b) Expand $\sin^2 x$ in the range $0 < x < \pi$ in a cosine series. **(6)**

Q.4 a) Analyze Harmonically the data given below and express y in Fourier series upto second harmonic: **(7)**

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1	1.4	1.9	1.7	1.5	1.2	1

b) If $f(x) = lx - x^2$ in the range $(0, l)$, show that the half range

Q.4 a) Analyze Harmonically the data given below and express y in Fourier series upto second harmonic: **(7)**

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1	1.4	1.9	1.7	1.5	1.2	1

b) If $f(x) = \ell x - x^2$ in the range $(0, \ell)$, show that the half range sine series for f(x) is,

$$= \frac{8\ell^2}{\pi^3} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^3} \sin \frac{(2n+1)\lambda x}{\ell}$$ **(6)**

UNIT 3

Q.5 a) Prove that: $i \times (\bar{a} \times i) + j \times (\bar{a} \times j) + k \times (\bar{a} \times k) = 2\bar{a}$. **(4)**

b) Evaluate $\int_0^{\infty} \frac{e^{-ax} \sin x}{x} dx$ and hence show that $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$. **(6)**

c) Trace the curve $r = 2a \cos \theta$. **(5)**

Q.6 a) Show that the following four points having position vectors $3i - 2j + 4k$, $6i + 3j + k$, $5i + 7j + 3k$ and $2i + 2j + 6k$ are coplanar. **(4)**

b) Show that $\int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x} dx = \log\left(\frac{b}{a}\right)$ **(5)**

c) Trace the curve $a^2x^2 = y^3 (2a - y)$. **(5)**

UNIT 4

Q.7 a) Show that:

$$\int \cos^{2n}\phi \, d\phi = \frac{1}{2n} \tan\phi \, \cos^{2n}\phi + \left(1 - \frac{1}{2n}\right) \int \cos^{2n-2}\phi \, d\phi$$ **(5)**

b) Prove that: $\int_1^{\infty} \frac{dx}{x^{p+1} (x-1)^q} = \beta(p+q, 1-q)$, if $-p < q < 1$. **(5)**

c) Show that the perimeter of the cardioide $r = a(1 + \cos\theta)$ is 8a. **(4)**

Q.8 a) Prove that: $\int_a^b (x-a)^m (b-x)^n dx = (b-a)^{m+n+1} \beta(m+1, n+1)$ **(5)**

b) Show that the length of the arc of the parabola $y^2 = 4ax$

$\int \cos \phi \, d\phi = \frac{\;}{2n} \tan\phi \cos \phi + \left(1 - \frac{\;}{2n}\right) \int \cos \phi \, d\phi$ **(5)**

b) Prove that: $\int_1^\infty \frac{dx}{x^{p+1}(x-1)^q} = \beta(p+q, 1-q)$, if $-p < q < 1$. **(5)**

c) Show that the perimeter of the cardioide $r = a\,(1 + \cos\theta)$ is $8a$. **(4)**

Q.8 a) Prove that: $\int_a^b (x-a)^m (b-x)^n dx = (b-a)^{m+n+1} \beta(m+1, n+1)$ **(5)**

b) Show that the length of the arc of the parabola $y^2 = 4ax$ cut off by the line $3y = 8x$ is $a\left(\log 2 + \frac{15}{16}\right)$. **(5)**

c) If $\beta(n,3) = \frac{1}{3}$ and n is +ve integer, find n. **(4)**

UNIT 5

Q.9 a) Change to polar co-ordinates and evaluate $\iint \frac{x^2 - y^2}{(x^2 + y^2)^{3/2}} dxdy$ over the region of circle $x^2 + y^2 = 2ax$ in the first quadrant. **(7)**

b) Change the order and evaluate: $\int_0^\infty \int_0^x x\, e^{\frac{-x^2}{y}} dy\, dx$ **(6)**

Q.10 a) Evaluate, $\int_0^1 \int_0^{\sqrt{x-x^2}} \frac{4xy}{x^2 + y^2} e^{-(x^2+y^2)} dxdy$. **(7)**

b) Find the area common to circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = 2ax$. **(6)**

UNIT 6

Q.11 a) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{x+y} e^z dz\, dy\, dx$ **(7)**

b) Obtain the volume of solid bounded by the surface $x = 0$, $y = 0$, $x + y + z = 1$, and $z = 0$. **(6)**

Q.12 a) $\int_0^{2a} dx \int_{-\sqrt{2ax - x^2}}^{\sqrt{2ax - x^2}} dy \int_0^{\sqrt{4a^2 - x^2 - y^2}} dz$ **(7)**

b) Find mean value of $e^{-(x^2+y^2)}$ over the area within the circle $x^2 + y^2 = 1$. **(6)**

WINTER-2014

UNIT 1

Q. 1 a) Find inverse by Partitioning $C = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -4 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix}$ **(7)**

b) Find rank of matrix, $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 3 & 1 & -3 & -1 \\ 1 & 1 & -3 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ **(6)**

Q.2 a) Determine λ and μ such that the equations,

$$x + y + z = 6,\ x + 2y + 3z = 10 \text{ and } x + 2y + \lambda z = \mu$$

Have **i)** No solution, **ii)** unique solution

iii) Many solutions. **(6)**

b) Find Eigen values and Eigen vectors of matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 6 \end{bmatrix}$$ **(7)**

UNIT 2

Q.3 a) If $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the range $(0, 2\pi)$, show that in this range. $f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$, And hence obtain the following relations:

i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots.. = \frac{\pi^2}{6}$ **ii)** $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots.. = \frac{\pi^2}{12}$

iii) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots.. = \frac{\pi^2}{8}$ **(7)**

b) Expand $f(x) = x^2$ as a Fourier series in $(-l, l)$ and deduce that,

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots.. = \frac{\pi^2}{12}$$ **(6)**

Q.4 a) Obtain Fourier expansion of $\sqrt{1 - \cos x}$ in the interval

$$\frac{1}{1^2} \quad \frac{1}{3^2} \quad \frac{1}{5^2} \cdots\cdots = \frac{\pi^2}{8}$$

b) Expand f(x) = x^2 as a Fourier series in $(-l, l)$ and deduce that,

$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}\cdots\cdots=\frac{\pi^2}{12}$$ **(6)**

Q.4 a) Obtain Fourier expansion of $\sqrt{1-\cos x}$ in the interval $0 \le x \le 2\pi$ and hence deduce that, $\sum_{n=1}^{\infty}\frac{1}{4n^2-1}=\frac{1}{2}$ **(7)**

b) Expand $f(x)=a\left(1-\frac{x}{\ell}\right)$ in the range $(0, \ell)$ in a half range sine series. **(6)**

UNIT 3

Q.5 a) Find the volume of tetrahedron whose vertices are the points A(2, -1, -3), B(4, 1, 3), C(3, 2, 1) and D(1, 4, 2). **(4)**

b) Prove that, $\int_0^1 x^p(\log x)^n dx=\frac{(-1)^n n!}{(p+1)^{n+1}}$, where n is positive integer and p > -1. **(5)**

c) Trace the curve $ay^2 = x(a^2 + x^2)$. **(5)**

Q.6 a) The concurrent edges of length a, b, c of a parallelepiped are along the lines.

$\frac{x}{1}=\frac{y}{2}=\frac{z}{3};\frac{x}{2}=\frac{y}{1}=\frac{z}{3};\frac{x}{3}=\frac{y}{1}=\frac{z}{2}$ respectively.

Using scalar triple product prove that the volume of parallelepiped is $\frac{3\sqrt{14}}{98}abc$. **(4)**

b) Trace the curve $r(1 + \cos\theta) = 2a$. **(5)**

c) Prove by using differentiation under integral sign,

$$\int_0^1 \frac{x^a - x^b}{\log x}dx=\log\left(\frac{a+1}{b+1}\right), a>0, b>0.$$ **(5)**

UNIT 4

UNIT 4

Q.7 a) If $f(m,n) = \int x^m (1-x)^n dx$,

show that, $f(m,n) = \dfrac{x^{m+1}(1-x)^n}{m+n+1} + \dfrac{n}{m+n+1} f(m, n-1)$ **(5)**

b) Prove: $\beta(x+1, y) = \dfrac{x}{x+y}\beta(x,y)$ **(4)**

c) Find the length of the curve $\theta = \dfrac{1}{2}\left(r + \dfrac{1}{r}\right)$ for r = 1 to r = 3. **(5)**

Q.8 a) Prove that $\int_0^\infty \dfrac{x^{m-1}}{(a+bx)^{m+n}} dx = \dfrac{1}{a^n b^m}\beta(n,m)$ **(5)**

b) Find the length of the loop of the curve, $x = t^2, y = t\left(1 - \dfrac{t^3}{3}\right)$ **(5)**

c) Prove that: $\Gamma\left(\dfrac{-3}{2}\right) = \dfrac{4\sqrt{\pi}}{3}$ **(4)**

UNIT 5

Q.9 a) Evaluate $\int_0^a dx \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \dfrac{dy}{\sqrt{a^2 - x^2 - y^2}}$ by changing to polar co-ordinates. **(7)**

b) Change the order and evaluate,

$$\int_0^a \int_{x^2/a}^{2a-x} xy\, dx\, dy$$ **(6)**

Q.10) Show by double integration that the area between the parabolas $y^2 = 4\,ax$ and $x^2 = 4ay$ is $\dfrac{16}{3}a^2$. **(7)**

b) Evaluate the integral, $\iint \dfrac{dx\, dy}{x^4 + y^2}$, where $x \geq 1$ and $y \geq x^2$.

Q.10) Show by double integration that the area between the parabolas $y^2 = 4\,ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. **(7)**

b) Evaluate the integral, $\iint \frac{dx\,dy}{x^4 + y^2}$, where $x \geq 1$ and $y \geq x^2$.

(6)

UNIT 6

Q.11 a) Show that $\iiint \frac{dx\,dy\,dz}{(x+y+z+1)^3} = \frac{1}{2}\left(\log 2 - \frac{5}{8}\right)$ integration being taken throughout the volume of the tetrahedron bounded by the co-ordinate planes and the plane $x + y + z = 1$. **(7)**

b) Change to spherical polar co-ordinates and evaluate,

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}$$ **(6)**

Q.12 a) Find mean value of xy over the area of positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ **(6)**

b) Find by double integration the volume of the sphere $x^2 + y^2 + z^2 = a^2$, cut off by the plane $z = 0$ and the cylinder $x^2 + y^2 = ax$. **(7)**

SUMMER-2014

UNIT 1

Q.1 a) Determine Eigen values and Eigen vectors of the matrix:

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \qquad \textbf{(7)}$$

b) If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ use Caley-Harmilton theorem to express:

$A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2$ as a linear polynomial in A. **(6)**

Q.2 a) Find the inverse of the matrix by partitioning method:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \qquad \textbf{(7)}$$

b) Find the rank of the matrix: $\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$ **(6)**

UNIT 2

Q.3 a) Find the half-range sine series for the function $f(x) = x(\pi - x)$ in the interval $0 \le x \le \pi$. **(7)**

b) Find a Fourier series to represent $f(x) = x^2$ from $x = -l$ to $x = l$ and hence deduce that: $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots\ldots = \frac{\pi^2}{12}$. **(7)**

Q.4 a) Obtain the Fourier series for the function f(x) given by:

$$f(x) = \begin{cases} 1 + \dfrac{2x}{\pi} & -\pi \le x \le 0 \\ 1 - \dfrac{2x}{\pi} & 0 \le x \le \pi \end{cases}$$

Q.4 a) Obtain the Fourier series for the function f(x) given by:

$$f(x)=\begin{cases}1+\dfrac{2x}{\pi} & -\pi\le x\le 0\\[2mm] 1-\dfrac{2x}{\pi} & 0\le x\le \pi\end{cases}$$

and deduce that: $\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+\ldots..=\dfrac{\pi^2}{8}$ **(7)**

b) Obtain Fourier series if:

$$f(x)=\begin{cases}x^2 & \text{if} -\pi<x<0\\ -x^2 & \text{if } 0<x<\pi\end{cases}$$ **(7)**

UNIT 3

Q.5 a) Show that: $\bar{a}\times(\bar{b}+\bar{c})+\bar{b}\times(\bar{c}+\bar{a})+\bar{c}\times(\bar{a}+\bar{b})$ **(5)**

b) Trace the curve: $27ay^2 = 4(x - 2a)^3$ **(3)**

c) By applying rule of differentiation under integral sign, Evaluate, $\int_a^{a^2} \log ax\,dx.$ **(5)**

Q.6 a) By applying Rule of differentiation under integral sign prove that: $\int_0^{\infty}\dfrac{\tan^{-1}(ax)}{x(1+x^2)}dx=\dfrac{\pi}{2}\log(1+a)$ Given that a > 0. **(5)**

b) Find the volume of tetrahedron formed by the points: (1, 1, 3), (4, 3, 2), (5, 2, 7) and (6, 4, 8). **(4)**

c) Trace the curve: $r = a(1 + \cos\theta)$ **(4)**

UNIT 4

Q.7 a) If $I_n = \int^{\pi/4} \tan^n\theta\, d\theta$

UNIT 4

Q.7 a) If, $I_n = \int_0^{\pi/4} \tan^n \theta \, d\theta$

Where n is positive integer, prove that: $n(I_{n-1} + I_{n+1}) = 1$

Hence prove that: $\int_0^a x^5 (2a^2 - x^2)^{-3} dx = \frac{1}{2}\left(\log 2 - \frac{1}{2}\right)$ **(7)**

b) Prove that: $\beta(m, n) = \beta(m + 1, n) + \beta(m, n + 1)$ **(6)**

Q.8 a) Prove that: $\beta(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1 + x)^{m+n}} dx$ **(7)**

b) Compute the arc-length of the curve: $ay^2 = x^3$ from $x = 0$ to a point having $x = 5a$. **(6)**

UNIT 5

Q.9 a) Change the order of integration and hence evaluate:

$$\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dx.dy$$ **(7)**

b) Change the following integral to polar co-ordinate and evaluate: $\int_{-a}^{a} \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2 + y^2} \, dx \, dy$ **(7)**

Q.10 a) $\iint x^3 y \, dx \, dy$ over the area bounded in positive quadrant of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **(7)**

b) Change the order of integration and hence evaluate:

$$\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2 + y^2}} dx \, dy$$ **(7)**

UNIT 6

Q.11 a) Change to spherical polar co-ordinates and

$$\int_0 \int_y \frac{\quad}{\sqrt{x^2+y^2}}\,dx\,dy \qquad (7)$$

UNIT 6

Q.11 a) Change to spherical polar co-ordinates and evaluate:

$$\int_0^{2a} dx \int_{-\sqrt{2ax-x^2}}^{\sqrt{2ax-x^2}} dy \int_0^{\sqrt{4a^2-x^2-y^2}} dz \qquad \textbf{(7)}$$

b) Find the mean value of $e^{-\left(x^2+y^2\right)}$ within the area $x^2 + y^2 = 1$. **(6)**

Q.12 a) Find the mean value and RMS value of the ordinates y of the cycloid: $x = a(t + \sin t)$ and $y = a(1 - \cos t)$ **(7)**

b) Evaluate: $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z}\,dx\,dy\,dz$ **(6)**

WINTER-2013

UNIT 1

Q.1 a) Using Caley-Hamilton Theorem, find A^8 if $A=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$ **(7)**

b) Find the Rank of the following Matrix $A=\begin{bmatrix}0 & 1 & -3 & -1\\ 1 & 0 & 1 & 1\\ 3 & 1 & 0 & 2\\ 1 & 1 & -2 & 0\end{bmatrix}$ **(6)**

Q.2 a) Find the inverse of the matrix by partitioning method;

$$A=\begin{bmatrix}1 & 1 & 1\\ 4 & 3 & -1\\ 3 & 5 & 3\end{bmatrix}$$ **(7)**

b) Find characteristic roots and characteristic vectors for the matrix.

$$A=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$$ **(5)**

UNIT 2

Q.3 a) If $f(x)=\begin{cases}\pi x & 0\le x\le 1\\ \pi(2-x) & 1\le x\le 2\end{cases}$ Show that in the interval $0 \le x \le 2,\ f(x)=\frac{\pi}{2}-\frac{\pi}{4}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\cos(2n+1)\pi x$ **(7)**

b) Analyses the current I with its constituents harmonics as for as the Third harmonics. The values of I and θ being as follows: **(7)**

θ	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
I	0	33.5	18.2	0	-33.5	-18.2	0

Q.4 a) Find Fourier series for the function:

$$f(x)=\begin{cases}0, & -2<x<-1\\ 1+x, & -1<x<0\\ 1-x, & 0<x<1\\ 0, & 1<x<2\end{cases}$$ **(7)**

1	0	33.5	18.2	0	-33.5	-18.2	0

Q.4 a) Find Fourier series for the function:

$$f(x) = \begin{cases} 0, & -2 < x < -1 \\ 1+x, & -1 < x < 0 \\ 1-x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases} \qquad \textbf{(7)}$$

b) Find the Fourier Half Range cosine series and Fourier Half range sine series for the function $f(x) = x^2 \quad 0 \le x \le \pi$. **(7)**

UNIT 3

Q.5 a) Using differentiation under integral sign,

Evaluate, $\int_0^\infty \frac{e^{-\alpha x} \sin x}{x} dx$ Hence show that $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$ **(5)**

b) Prove that: $(\bar{a} \times \bar{b}) \cdot \{(\bar{b} \times \bar{c}) \times (\bar{c} \times \bar{a})\} = [\bar{a}\,\bar{\mathbf{b}}\,\bar{c}]^2$ **(5)**

c) Trace the curve: $x^{2/3} + y^{2/3} = a^{2/3}$ **(3)**

Q.6 a) Trace the curve: $r = a\,(1 - \cos\theta)$ **(3)**

b) $\bar{i} \times (\bar{a} \times \bar{i}) + \bar{j} \times (\bar{a} \times \bar{j}) + \bar{k}(\bar{a} \times \bar{k}) = 2\bar{a}$, Prove this. **(4)**

c) Prove that: $\int_0^1 \frac{x^a - x^b}{\log x} dx = \log\left(\frac{a+1}{b+1}\right)$ **(5)**

UNIT 4

Q.7 a) Show that:

$$\int_a^b (x-a)^m (b-x)^n dx = (b-a)^{m+n+1} \beta(m+1, n+1) \qquad \textbf{(6)}$$

b) If $I_n = \int_0^\infty e^{-x} \sin^n x \, dx (n \ge 2)$ Then Prove that:

$(1 + n^2)\, I_n = n(n-1) I_{n-2}$ **(7)**

Q.8 a) Prove that: $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx = \beta(m, n)$ **(7)**

b) For the curve: $x = e^0 \cos$ and $y = e^0 \sin\theta$, Show that the length of the arc from $\theta = 0$ to $\theta = \frac{\pi}{2}$ is $\sqrt{2}\left(e^{\pi/2} - 1\right)$ **(6)**

b) If $In = \int_{0} e^{-x} \sin^n x \, dx (n \geq 2)$ Then Prove that:

$(1 + n^2) I^n = n(n-1) I_{n-2}$ **(7)**

Q.8 a) Prove that: $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx = \beta(m, n)$ **(7)**

b) For the curve: $x = e^0 \cos$ and $y = e^0 \sin 0$, Show that the length of the arc from $\theta = 0$ to $\theta = \frac{\pi}{2}$ is $\sqrt{2}\left(e^{\pi/2} - 1\right)$ **(6)**

UNIT 5

Q.9 a) Change the integral to polar co-ordinates and hence evaluate, it $\int_0^1 \int_0^x (x + y) dx.dy$ **(6)**

b) Show by double integration, the area between the parabolas $y^2 = 4$ ax, and $x^2 = 4$ ay is $\frac{16a^2}{3}$. **(7)**

Q.10 a) Evaluate: $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy \, dx}{1 + x^2 + y^2}$ **(6)**

b) Show the region of integration and change the order of integration for, $\int_{-2}^{1} \int_{x^2}^{2-x} f(x, y) dx \, dy$ **(7)**

UNIT 6

Q.11 a) Evaluate: $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \left(x^2 + y^2 + z^2\right) dx \, dy \, dz$ **(7)**

b) Find the Root Mean square values of the expression a sin (pt) + b cos (qt) over the interval from 0 to 2π. **(7)**

Q.12 a) Evaluate: $\int_0^{\log 2} \int_0^x \int_0^{x+y} e^{x+y+z} dx \, dy \, dz$ **(7)**

b) Find the volume cut-off from the parabolic $x^2 + \frac{1}{4} y^2 + z = 1$ Plane $z = 0$. **(5)**

SUMMER-2013

UNIT 1

Q.1 a) Using portioning method, find the inverse of the matrix:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{bmatrix} \qquad \textbf{(7)}$$

b) Determine for what values of λ and μ, the system of equations, $2x + 3y + 5z = 9$ $\quad 7x + 3y - 2z = 8$

$$2x + 3y + \lambda z = \mu$$

have **i)** a unique solution, **ii)** no solution and,

iii) an infinite number of solution. **(6)**

Q.2 a) Find Eigenvalue Eigen vector of the matrix:

$$A = \begin{bmatrix} 9 & -1 & 9 \\ 3 & -1 & 3 \\ -7 & 1 & -7 \end{bmatrix} \qquad \textbf{(6)}$$

b) Verify Cayley-Hamilton Theorem and hence find A^{-1}, where,

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix} \qquad \textbf{(7)}$$

UNIT 2

Q.3 a) Find the Fourier series for f(x), where

$$f(x) = \begin{cases} x^2 & -\pi \le x \le 0 \\ -x^2 & 0 \le x \le \pi \end{cases} \qquad \textbf{(7)}$$

b) Obtain the constant terms and the coefficients of the first and cosine terms in the Fourier series of y as given in the following table: **(6)**

x	0	1	2	3	4	5
y	9	18	24	38	26	20

Q.4 a) Find a Fourier series with period 3 to represent

$$f(x) = 2x - x^2 \text{ in } 0 < x < 3 \qquad \textbf{(7)}$$

b) Find a Fourier sine series for the function:

the following table: **(6)**

x	0	1	2	3	4	5
y	9	18	24	38	26	20

Q.4 a) Find a Fourier series with period 3 to represent

$f(x) = 2x - x^2$ in $0 < x < 3$ **(7)**

b) Find a Fourier sine series for the function:

$$\text{For } f(x) = \begin{cases} mx & \text{for } 0 < x \le \frac{\pi}{2} \\ m(\pi - x) & \text{for } \frac{\pi}{2} < x < \pi \end{cases}$$ **(6)**

UNIT 3

Q.5 a) Prove that: $(\bar{a} + \bar{b} \quad \bar{b} + \bar{c} \quad \bar{c} + \bar{a}) = 2(\bar{a}\,\bar{b}\,\bar{c})$ **(4)**

b) Using differentiation under integral sign theorem, evaluate,

$$\int_0^{\pi/2} \frac{\log(1 + a\sin^2 x)}{\sin^2 x}\,dx$$ **(6)**

c) Trace the curve $r^2 = a^2 \cos 2\theta$ **(4)**

Q.6 a) Trace the curve: $x = a(\theta + \sin\theta)$ and

$y = a(1 - \cos\theta)$ **(4)**

b) Prove that: $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a}.\bar{c})\bar{b} - (\bar{a}.\bar{b})\bar{c}$ **(4)**

c) Using differentiation under integral Sign theorem, Prove that;

$$\int_0^{\infty} \frac{\tan^{-1}(ax)}{x(1 + x^2)}\,dx = \frac{\pi}{2}\log(1 + a)$$ **(6)**

UNIT 4

Q.7 a) If $I_a = \int_0^{\infty} e^{-x}\sin^n x$, $n \ge 2$, prove that,

$(1 + n^2)\, I_n = n(n - 1) I_{n-2}$ **(5)**

b) Find the length of the cardioid $r = a(1 + \cos\theta)$. **(4)**

c) Evaluate $\int_0^{\infty} \frac{x^a}{a^x}\,dx$ **(5)**

b) Find the length of the cardioid $r = a(1 + \cos\theta)$. **(4)**

c) Evaluate $\int_0^{\infty} \frac{x^a}{a^x} dx$ **(5)**

Q.8 a) Evaluate $\int_0^{\infty} \frac{x^{m-1}}{(a+bx)^{m+n}}$ **(5)**

b) Find the total length of asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ **(5)**

c) Prove that: $\int_0^{\pi/2} \cos^n x . \cos nx \; dx = \frac{\pi}{2^{n+1}}$ **(4)**

UNIT 5

Q.9 a) Change the order of integration and evaluate:

$$\int_0^a \int_0^x \frac{\sin y \, dx \, dy}{\sqrt{(a-x)(x-y)(4-5\cos y)^2}}$$ **(7)**

b) Find the total area of the loop of the curve, $y^2 = x^2 \frac{a^2 - x^2}{a^2 + x^2}$. **(6)**

Q.10 a) Express as a single integral and evaluate

$$\int_0^{\pi/2} \int_0^x \cos k(x^2 + y^2) dx \, dy + \int_{a/\sqrt{2}}^{a} \int_0^{\sqrt{a^2 - x^2}} \cos k(x^2 + y^2) dx dy$$

By changing the polar coordinates. **(7)**

b) Find the area between parabolas $y^2 = 4x$ and $x^2 = 4y$. **(6)**

UNIT 6

Q.11 a) Find the root mean square value of the expression $a \cos pt + b \sin qt$ over the interval 0 to 2π. **(6)**

b) Evaluate $\iiint_V \frac{dx \, dy \, dz}{(1+x+y+z)^3}$, where V is the volume bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$.

(7)

Q.12 a) Find the volume bounded by the cylinder, $x^2 + y^2 = 4$ and the plane $y + z = 4$, $z = 0$. **(6)**

b) Evaluate $\iiint_V \frac{dx\,dy\,dz}{(1+x+y+z)^3}$, where V is the volume bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$.

(7)

Q.12 a) Find the volume bounded by the cylinder, $x^2 + y^2 = 4$ and the plane $y + z = 4$, $z = 0$. **(6)**

b) Evaluate $\iiint_V \frac{dx\,dy\,dz}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}$, where V is volume bounded by the sphere $x^2 + y^2 + z^2$ and $x^2 + y^2 + z^2 = b^2$, $(a > b)$. **(7)**