

# DREAM STUDY

## Smart Way of Easy Learning

### Elements of Statistics

(EL)

[www.dreamstudy.tk](http://www.dreamstudy.tk)



[dreamstudy123@gmail.com](mailto:dreamstudy123@gmail.com)

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BCA - 2<sup>nd</sup> Year

3<sup>rd</sup> semester

Statistics

## UNIT - 1

### Population, Sample, Data Condensation

#### A. Definition of Statistics :-

Statistics in the plural sense, Acc/ to W.L. Bowley, "Statistics are numerical statement of facts in any department of Inquiry placed in relation to each other.

✓ According to this definition the characteristics of statistics are :-

- 1)- It facts must be in the form of numerical terms.
- 2)- The facts must be capable of being placed in relation to each other.
- 3)- It may be related to any field like physical, social or nature.

#### B. Characteristics / scope of Statistics :-

- 1- Statistics are numerically expressed.
- 2- Statistics are aggregate of fact.
- 3- Statistics are estimated according to reasonable standard of accuracy.
- 4- Statistics is affected to a marked extent by multiplicity of causes.

- 5- Statistics are collected in a systematic manner.
- 6- Statistics are placed in relation to each other.

### \* functions of Statistics :-

- 1- Simplifies the data.
- 2- Comparison of fact.
- 3- Formation of policies and decision making.
- 4- Correction.
- 5- Forecasting for the future.
- 6- Testing a hypothesis.

### \* Population :-

According to G. Kalton In statistical usage the term population thus not necessarily refer to people but it is a technical term used to describe the complete group of person or objects for which the result are to apply.

### \* Sample :- It is a part of population selected by some Sampling procedure

the no. of units in the sample is called the size of the sample.

\* collection of data :-

There are two types of data :-

1- Primary data - Primary data are based on primary source of information these collected by ~~Investigator~~, either on his own through some agencies for the purpose. It is also known as raw data. The investigator gets the primary data which not being earlier processed grouped, average and summarized.

✓ Difference b/w Primary data and Secondary data -

1- Originality :-

- Primary data are collected originally.
- Secondary data are already available and thus they are not original.

### (ii) Collection :-

- Collected by the Investigator himself.
- Collected by some other people.

### (iii) Objective :-

- Collected by the Investigator according to the object of the Survey.
- Used and adjusted by the person according to the object.

### (iv) Time :-

- It is a time consuming to collect a primary data.
- less time consuming.

### (v) Point of Shape :-

- A primary data is in the shape of raw material.
- A secondary data is in the shape of either working process or finished material.

Ques:- C.I

F

1-10

5

11-20

7

21-30

8

31-40

10

C.I

F

CF

Tally Mark

0.5 - 10.5

5

5

||||

10.5 - 20.5

7

12

||||||

20.5 - 30.5

8

20

|||||||

30.5 + 40.5

10

30

|||||||

Q:- C.I

F

less than 10

2

"

20

"

30

4

40

"

50

C.I.

F

C.F.

0 - 10

2

2

10 - 20

10

12

20 - 30

12

24

30 - 40

5

29

40 - 50

1

30

## UNIT-2.

### \* Measurement of Central Tendency!

#### 1- Concept of Central Tendency:-

There are two types of average -

- i) Mathematical average.
- ii) Positional average.

#### 1- Mathematical average:-

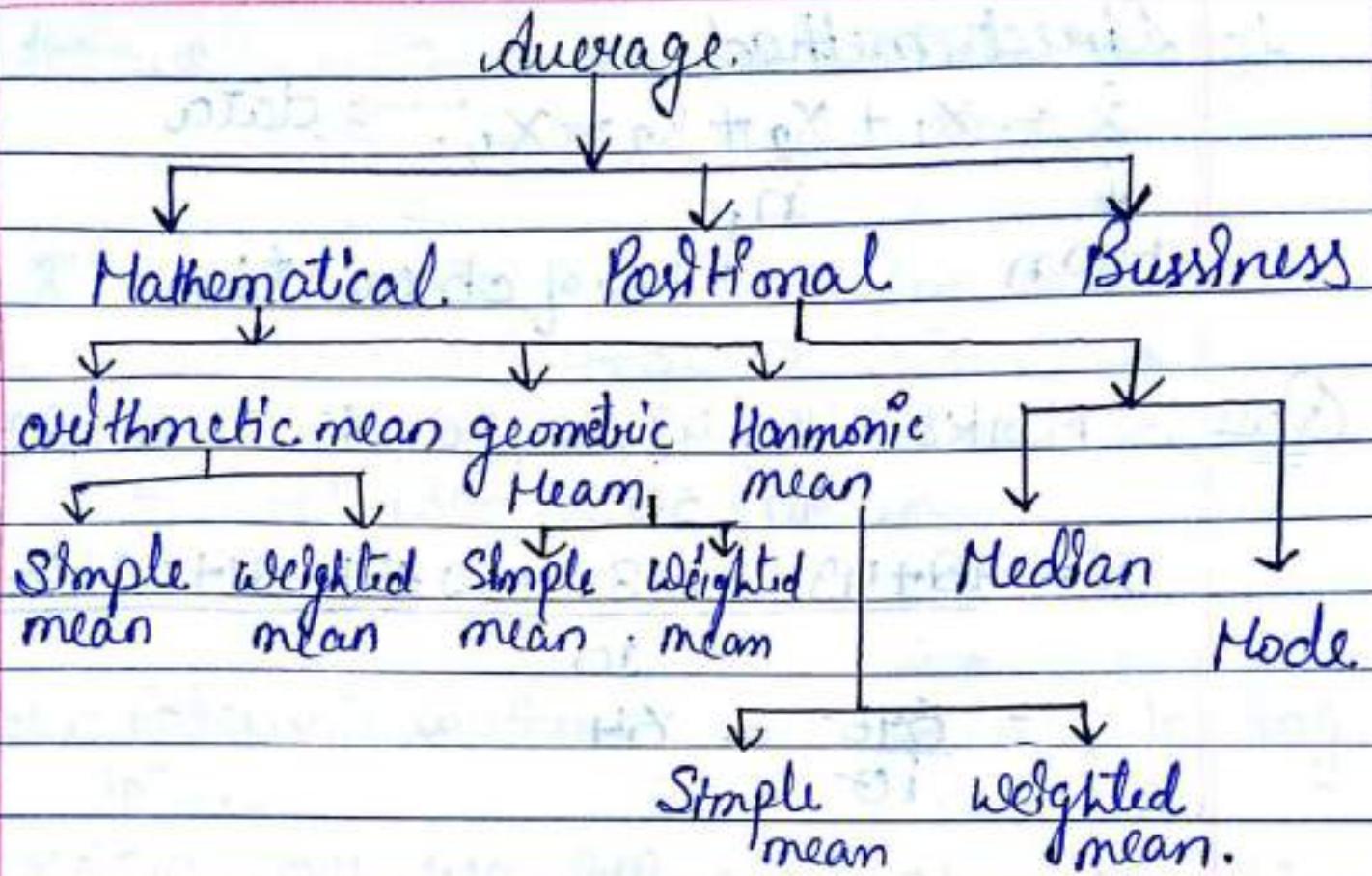
These are calculated values of the series that is why they are classified under this they are -

- a) Arithmetic mean /Average.
- b) Geometric mean.
- c) Harmonic mean.

#### 2- Positional average:-

They are locate the position of the average in the series that is why is called positional average.

- a) Median
- b) Mode



\* Essential of a good central Tendency :-

- 1- Well define.
- 2- Easy to understand.
- 3- Based on all item of series.
- 4- Good representation of data.
- 5- Free from the effect of extreme value.

• Calculation of arithmetic mean in Individual Series :-

1- Direct method.

$$\bar{X} = \frac{x_1 + x_2 + x_3 + x_4}{n} \rightarrow \text{data}$$

↓                  ↓  
Mean              No. of observation

Ques :- Marks:- 78, 49, 82, 38, 69, 71, 81, 82, 40, 50.

$$\bar{X} = \frac{78+49+82+38+69+71+81+82+40+50}{10}$$

$$= \frac{640}{10} = 64$$

Ques:- Calculate the arithmetic mean from the following data by testing the lifetime of bulb.

Lifetime:- 967, 889, 940, 922, 952.

$$\bar{X} = \frac{967+889+940+922+952}{5}$$

$$= \frac{4670}{5} = 934$$

Ques :- Calculate the arithmetic mean from the following data.

Income:- 4780, 5760, 4690, 4750, 4840,  
4920, 4100, 4850, 4850, 5050,  
5950.

$$\bar{x} = \frac{4780 + 5760 + 4690 + 4750 + 4840 + 4920 + 4100 + 4850 + 5050 + 5950}{10}$$

$$\bar{x} = \frac{49,730}{10} = 4,973 \text{ Ans.}$$

Ques :- Calculate arithmetic mean of the following data.

$$x = 0.01, 0.11, 1.10, 2.02, 2.23, 2.91, 3.12, 3.19, 3.90, 4.02.$$

$$\bar{x} = \frac{0.01 + 0.11 + 1.10 + 2.02 + 2.23 + 2.91 + 3.12 + 3.19 + 3.90 + 4.02}{10}$$

$$= \frac{22.61}{10} = 2.261 \text{ Ans.}$$

2- Shortcut method:-

$$\bar{x} = A + \sum d u$$

N 50.1

21.1

20.6

82.6

20.8

20.5

11.6

20.5

20.5

0

0

0

Ques:- Calculate arithmetic mean by Shortcut method.

S.no.	X.
1	1.82
2	1.96
3	1.01
4	1.81
5	1.90
6	2.09
7	2.11
8	2.98
9	3.05
10	3.99

S.no.	X.	$d_u(x-A)$
1	1.82	-1.00
2	1.96	-1.06
3	1.01	-0.89
4	1.81	-0.09
A = 5	1.90	0
6	2.09	0.19
7	2.11	0.21
8	2.98	1.08
9	3.05	1.15
10	3.99	2.09
$\sum d_u = 1.72$		

$$\bar{x} = 1.90 + \frac{16.72}{1000}$$

$$= 1.90 + 0.0172$$

$$= 2.072 \text{ Ans}$$

Ques:- Calculate the arithmetic mean by shortcut method.

RSI no.	Marks	$du(x-A)$
1	30	11
2	20	1
3	20	9
4	33	14
A = 5	19	0
6	22	3
7	26	7
8	18	-1
9	20	1
10	14	-5
		$\sum du = 40$

$$\bar{x} = A + \frac{\sum du}{N} = 19 + \frac{40}{10} = 23 \text{ Ans}$$

Ques Calculation of arithmetic mean in discrete Series.

i) Direct method :-

$$\bar{x} = \frac{x_1 f_1 + x_2 f_2 + x_3 f_3 - \dots}{\sum f}$$

$$\bar{x} = \frac{\sum x f}{\sum f}$$

Ques- Find the mean from the following data

X	F	X	F	XF
8	3	8	3	24
9	5	9	5	45
10	8	10	8	80
11	12	11	12	132
12	7	12	7	84
13	5	13	5	65
			$\sum f = 40$	$\sum xf = 430$

$$\bar{x} = \frac{430}{40} = \frac{43}{4} = 10.75$$

● Shortcut method in discrete Series:-

$$\bar{x} = A + \frac{\sum f d u}{\sum f}$$

Q:- Height No. of Student du(x-A). fdu

60	2	-7	-14
61	3	-6	-18
63	4	-4	-16
64	9	-3	-27
66	18	-1	-18
67	20 = A	0	0
68	17	1	17
69	14	2	28
70	5	3	15
71	3	4	12
72	3	5	15
73	2	6	12
$\sum f = 100$		$\sum f d u = 6$	

$$\bar{x} = A + \frac{\sum f d u}{N} = 67 + \frac{6}{100}$$

$$= 67 + 0.06 = 67.06.$$

Ques :- Find the A.M. from the following data by Shortcut method.

X	f	dn	Fdn	
9	2	4	8	
8	3	3	09	
7	4	2	8	$\bar{x} = A + \frac{\sum Fdn}{\sum f}$
6	6	1	6	
5	11	0	0	$= 5 - \frac{19}{50}$
4	10	-1	-10	
3	6	-2	-12	$= 5 - 0.38$
2	5	-3	-15	$= 0.12$ Ans
1	2	-4	-8	
0	1	-5	-5	
	50		-19	

Ques :- From the following information calculate mean by Shortcut method.

Marks	No. of Student	dn	Fdn
20	8	-20	-160
30	12	-10	-120
40	20	0	0
50	10	10	100
60	6	20	120
70	4	30	120
	60		60

$$\bar{x} = A + \frac{\sum fdm}{\sum f}$$

$$= 40 + \frac{60}{60} = 40 + 1 = 41 \text{ Ans.}$$

Ques:- Calculation of A.M. In Continuous Series -

i) Direct Method.

$$\bar{x} = \frac{\sum fx}{\sum f}$$

CF	F	$x = \frac{L+U}{2}$	$fx$	
0-10	9	5	10	
10-20	7	15	105	$\bar{x} = \frac{\sum fx}{\sum f}$
20-30	11	25	275	
30-40	15	35	525	= 45.70
40-50	25	45	1125	1050
50-60	22	55	1210	= 45.70
60-70	0	65	520	
70-80	5	75	375	
80-90	5	85	425	
	100		4570	

Ques :- From the following data calculate A.M.

C.I	F	X	FX	
0-10	5	5	25	
10-20	10	15	150	$\bar{X} = \frac{\sum f d u}{\sum f}$
20-30	25	25	625	
30-40	30	35	1050	= 33.00
40-50	20	45	900	100
50-60	10	55	550	
	100		3300	= 33.00

Ques:- Calculation of A.M. by Shortcut method  
In Continuous series.

$$\bar{X} = A + \frac{\sum f d u}{\sum f}$$

C.I	F	X	$du = (X - A)$	$f du$	
0-10	10	5	-40	-400	01-08
10-20	10	15	-30	-300	08-15
20-30	14	25	-20	-280	15-22
30-40	36	35	-10	-360	22-29
40-50	44	45	0	0	29-36
50-60	30	55	10	300	36-43
60-70	24	65	20	480	43-50
70-80	12	75	30	360	50-57
80-90	16	85	40	640	57-64
90-100	4	95	50	200	64-71
	200			640	

$$\bar{X} = \frac{45 + 645}{200} = \frac{45 + 3.2}{200} = 40.2 \text{ Ans}$$

Ques:- Calculate arithmetic mean by shortcut method.

C-I	F	X	dn	f.dn	
0-10	4	5	-30	-120	
10-20	8	15	-20	-160	
20-30	11	25	-10	-110	$\bar{X} = A + \frac{\sum f.dn}{\sum f}$
30-40	15	35	0	0	
40-50	11	45	10	110	$= 35 - \frac{20}{60}$
50-60	7	55	20	140	
60-70	4	65	30	120	$= 35 - 0.33$
	60			-20	$= 34.67 \text{ Ans}$

Ques:- Shortcut method

C-I	F	X	dn	f.dn	
0-10	5	5	-20	-100	
10-20	10	15	-10	-100	$\bar{X} = 25 + \frac{800}{100}$
20-30	25	25	0	0	
30-40	30	35	10	300	$= 25 + 0$
40-50	20	45	20	400	$= 33 \text{ Ans}$
50-60	10	55	30	300	
	100			800	

Ques: Calculate d.M. from the following data by Shortcut and direct method.

C-I	F	X	d <sub>n</sub>	f <sub>d<sub>n</sub></sub>	FX
0-5	15	2.5	-20	-300	37.5
5-10	24	7.5	-15	-360	180
10-15	28	12.5	-10	-280	350
15-20	40	17.5	-5	-200	700
20-25	50	22.5	0	0	1125
25-30	30	27.5	5	150	825
30-35	25	32.5	10	250	8125
35-40	20	37.5	15	300	750
40-45	10	42.5	20	200	425
45-50	8	47.5	25	200	380
	250			-40	5505

(i) By shortcut method,

$$\bar{x} = 22.5 + \frac{40}{250} = 22.5 - 0.16.$$

$$\bar{x} = 22.34.$$

(ii) By direct method,

$$\bar{x} = \frac{\sum f d_n}{\sum f} = \frac{5505}{250} = 22.34.$$

Ques:- Calculate A.M. by direct method and Shortcut method.

C-I	F	X	d <sub>n</sub>	f <sub>dn</sub>	f <sub>x</sub>
45-50	18	47.5	-25	-450	855
50-55	21	52.5	-20	-420	1102.5
55-60	31	57.5	-15	-465	1702.5
60-65	45	62.5	-10	-450	2812.5
65-70	85	67.5	-5	-175	2362.5
70-75	26	72.5	0	0	1805
75-80	22	77.5	5	110	1705
80-85	19	82.5	10	180	1485
85-90	40	87.5	15	600	3500
90-95	5	92.5	20	100	462.5
95-100	5	97.5	25	125	487.5
	266			-845	18440

$$(i) \bar{x} = \frac{18444}{266} = 69.323 \text{ Ans}$$

$$(ii) \bar{x} = A + \frac{\sum f d u}{\sum f} = 72.5 - \frac{845}{266}$$

$$= 72.5 - 3.17 = 69.33 \text{ Ans}$$

- Calculation of A.M. In continuous series by Step-division method.

$$\bar{x} = A + \frac{\sum fd'u}{\sum f}$$

Salary	No. of employees	$\times$	$du$	$d'u$	$fd'u$
1500 - 2500	13	2000	-4000	-4	-52
2500 - 3500	12	3000	-3000	-3	-36
3500 - 4500	20	4000	-2000	-2	-40
4500 - 5500	32	5000	-1000	-1	-32
5500 - 6500	16	6000	0	0	0
6500 - 7500	15	7000	1000	1	15
7500 - 8500	12	8000	2000	2	24
8500 - 9500	8	9000	3000	3	24
9500 - 10,500	4	10,500	4000	4	16
	150				23

$$\bar{x} = 6000 + \frac{23 \times 1000}{150}$$

$$= 6000 + 23 \times 100 = 6000 + 153.333$$

$$= 6,153.333 \text{ Ann.}$$

Ques :- Calculate A.M. by step deviation method.

C-I	F	X	diu	cl'u	Fcl'u
0-10	5	5	-20	-2	-10
10-20	10	15	-10	-1	-10
20-30	25	25	0	0	0
30-40	30	35	10	1	30
40-50	20	45	20	2	40
50-60	10	55	30	3	30
	100				80

$$\bar{x} = 25 + \frac{80 \times 10}{100} = 33 \text{ Am}$$

Ques :- Calculation of A.M. in case of unequal interval.

C-I	F	X	FX	
5-10	15	7.5	112.5	
10-18	22	14	308	$\bar{x} = \frac{\sum fX}{\sum f}$
20-40	30	30	900	= 33.220.5
50-100	24	75	1800	153
100-250	22	175	3850	
250-500	20	375	7500	= 217.12
500-1000	15	750	11250	
1000-2000	5	1500	7500	
	153		33,220.5	

Ques. Calculation of A.M. In case of Inclusive Series.

Marks	F	C-I	X	FX
10-19	5	9.5-19.5	14.5	72.5
20-29	8	19.5-29.5	24.5	196
30-39	12	29.5-39.5	34.5	414
40-49	8	39.5-49.5	44.5	356
50-59	7	49.5-59.5	54.5	381.5
	40			1420.0

$$\bar{x} = \frac{1420}{40} = \frac{71}{2}$$

$$= 35.5 \text{ Ans.}$$

Ques 5:- Calculation of A.M. by Step deviation method

C-I	F	X	d <sub>u</sub>	d <sub>u</sub>	Fd <sub>u</sub>
0-10	4	5	-20	-2	-8
10-20	6	15	-10	-1	-6
20-30	10	25	0	0	0
30-40	15	35	10	1	15
40-50	8	45	20	2	16
50-60	7	55	30	3	21
	50		0.21		38

$$\bar{x} = \frac{25 + 38}{50} = 32.6 \text{ Ans.}$$

Ques:- C-S

	F	C-S	$fX$	$fX$	
below - 10	4	0-10	5	20	
10 - 30	6	10-30	20	120	
30 - 60	10	30-60	40	400	
60 - 100	7	60-100	80	560	
above - 100	3	100-150	125	375	
	30			1475	

$$\bar{X} = \frac{\sum f u}{\sum f} = \frac{1475}{30} = 49.16. \text{ Ans.}$$

- Arithmetic mean in case of cumulative frequency distribution -

Height	No. of tree (cf)	f	C-S	X	$fX$
less than 7	25	25	0-7	8.5	212.5
" 14	57	32	7-14	10.5	336
" 21	92	35	14-21	17.5	612.5
" 28	134	42	21-28	24.5	1029
" 35	216	82	28-35	31.5	2583
" 42	287	71	35-42	38.5	2733.5
" 49	341	54	42-49	45.5	2457
" 56	360	19	49-56	52.5	997.5
		360			10,836

di	f di
-21	-525
-14	-440
-7	-245
0	0
7	574
14	994
21	1134
28	532
	<del>6055</del>
	2016

$$\text{iii) } \bar{x} = \frac{\sum f x}{\sum f} = \frac{10836}{360} = 30.1 \text{ Am.}$$

$$\text{iv) } \bar{x} = A + \frac{\sum f d_i}{\sum f}$$

$$\bar{x} = 24.5 + \frac{2016}{360} = 24.5 + 5.6 = 30.1 \text{ Am}$$

Ques :- calculate A.H. from the following data.

C.I.	F	X	FX	
10-20	4	15	60	
20-40	10	30	300	$\bar{x} = \frac{\sum f x}{\sum f}$
40-70	26	55	1430	
70-120	8	95	760	
120-200	2	160	320	
	50		2,870	

Ques :-

No. of Students	f	C-I	X	fx	du	Fdu	
above - 0	80	3	0-10	5	15	-40	-120
" - 10	77	5	10-20	15	75	-30	-150
" - 20	72	7	20-30	25	175	-20	-140
" - 30	65	10	30-40	35	350	-10	-100
" - 40	55	12	40-50	45	540	0	0
" - 50	43	15	50-60	55	825	10	150
" - 60	28	12	60-70	65	780	20	240
" - 70	16	6	70-80	75	450	30	180
" - 80	10	2	80-90	85	170	40	80
" - 90	8	8	90-100	95	760	50	400
above - 100	0	0	100-110	105	0	60	0
	80				4,140		540

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{4140}{80} = 51.75$$

$$\bar{x} = A + \frac{\sum Fdu}{\sum f} = 45 + \frac{540}{80} = 45 + 6.75$$

$$= 51.75 \text{ Ans}$$

Ques:- Calculation of A.M. from the following method, direct method, shortcut method and step deviation.

C-I	F	X	fx	cfu	fdu	$d'u = \frac{cfu}{f}$	fd'u.
45-50	18	47.5	855	-25	-450	-5	-90
50-55	21	52.5	1102.5	-20	-420	-4	-84
55-60	31	57.5	1790.5	-15	-465	-3	-93
60-65	45	62.5	2,812.5	-10	-450	-2	-90
65-70	35	67.5	2,362.5	-5	-175	-1	-35
70-75	26	72.5	1,885	0	0	0	0
75-80	22	77.5	1,705	5	110	1	22
80-85	10	82.5	1,405	10	100	2	36
85-90	14	87.5	1,225	15	90	3	42
90-95	5	92.5	462.5	20	100	4	20
95-100	5	97.5	487.5	25	125	5	25
	240		16,165		1235		-247

i) Direct method; -  $\bar{x} = \frac{\sum fx}{\sum f} = \frac{16,165}{240} = 67.354$ .

ii) Shortcut method; -  $\bar{x} = A + \frac{\sum fdu}{\sum f} = 72.5 - \frac{1235}{240} = 67.355$

$$\bar{x} = 67.355$$

iii) Step-deviation method; -

$$\begin{aligned}\bar{x} &= A + \frac{\sum d'u}{\sum f} \times c = 72.5 + \left( \frac{-247}{240} \right) \times 5 \\ &= 72.5 - 5.14 = 67.36\end{aligned}$$

Ques :- Calculate A.M. from the following data by step deviation method.

Mid values(x)	F	d <sub>n</sub>	d' <sub>n</sub> = $\frac{d_n}{20}$	f d' <sub>n</sub>
115	2	-60	-3	-6
135	4	-40	-2	-8
155	6	-20	-1	-6
175	9	0	0	0
195	10	20	1	10
215	9	40	2	18
235	7	60	3	21
255	4	80	4	16
	50			45

$$\bar{x} = A + \frac{\sum f d' n x^2}{\sum f d' n} = 175 + \frac{45 \times 20}{45} = 175 + 10 = 193 \text{ Ans}$$

Ques :- Calculate mean from the shortcut method.

Mid values(x)	F	c <sub>n</sub>	f c <sub>n</sub>
5	4	-30	-120
15	8	-20	-160
25	11	-10	-110
35	15	0	0
45	11	20	110

55	7	20	140
65	4	30	120
	60		20

$$\bar{x} = A + \frac{\sum f d_n}{\sum f} = 35 - \frac{20}{60} = 35 - 0.33 \\ = 34.67 \text{ dm.}$$

Ques:- Calculate A.M. by shortcut and step deviation method.

C-I	F	X	d <sub>n</sub>	f d <sub>n</sub>
0-10	5	5	-37.5	-187.5
10-20	10	17.5	-25	-250
20-30	16	30	-12.5	-120
30-40	30	42.5	0	0
40-50	19	59.5	17	323
50-60	10	74	31.5	315
60-70	5	81.5	39	195
70-80	5	92.5	50	250
	100			270.5

$$(b) \bar{x} = A + \frac{\sum f d_n}{\sum f} = 42.5 + \frac{270.5}{1000} = 42.5 + 0.2705 \\ = 42.705$$

Ques:- Wages in ₹	F	
less than 10	30	
" " 20	70	
20 - 30	50	
20 - 40	98	
40 and above	332	
50 "	308	
60 - 70	132	
70 and above.	14	

Wages in ₹	F	C-I	F	
less than 10	30	0-10	30	
" " 20	70	10-20	40	
20-30	50	20-30	50	
20-40	98	30-40	48	
40 and above	332	40-50	24	
50 and above	308	50-60	162	
60-70	132	60-70	132	
70 and above	14	70-80	14	

Ques:- Calculate mean from the following distribution -

C-I	F	X	fx
0-10	5	5	25
10-25	10	17.5	175
25-35	16	30	480
35-50	30	42.5	1275
50-65	19	57.5	1130.5
65-70	10	67.5	675
70-85	5	77.5	387.5
85-100	5	92.5	462.5
	100		4695.5

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{4695.5}{100}$$

$$\bar{x} = 46.95 \text{ dm}$$

Ques:- Find the A.M. by Shortcut method.

C-I	F	C-I	X	du	fdu
below-4	3	0-4	2	-6	-18
4-12	7	4-12	8	0	0
12-24	15	12-24	18	10	150
24 and above	20	24-40	32	24	480
	45				612

$$\bar{x} = A + \frac{\sum f d u}{\sum f} = 8 + \frac{612}{45} = 8 + 13.6 \\ = 21.6 \text{ days}$$

Ques:- Find mean from the three-method.

Below-10	5	C-I
10-20	10	0-10
20-30	20	10/20
30-40	40	20-30
40-50	30	30/40
50-60	20	40-50
60-70	10	50-60
70 and above	4	60-70

C-I	f	X	fx	dn	fdn	$d'u = \frac{dn}{10}$	$fd'u$
0-10	5	5	25	-30	-150	-3	-15
10-20	10	15	150	-20	-200	-2	-20
20-30	20	25	500	-10	-200	-1	-20
30-40	40	35	1400	0	0	0	0
40-50	30	45	1350	10	300	1	30
50-60	20	55	1100	20	400	2	40
60-70	10	65	650	30	300	3	30
70-80	4	75	300	40	160	4	16
	139	55	5475		610		61

$$\text{Q1) } \bar{x} = \frac{\sum f x}{\sum f} = \frac{5475}{139} = 39.388.$$

$$\text{Q2) } \bar{x} = A + \frac{\sum f d_u}{\sum f} \times l = 35 + \frac{610}{139} = 39.388$$

$$\text{Q3) } \bar{x} = A + \frac{\sum f d'_u \times l}{\sum f} = 35 + \frac{61 \times 10}{130} = 39.388.$$

Ques:- Find the A.M. from step-deviation method.

below -20	3
20-30	5
30-40	9
40-50	5
50 and above	4

C-I	F	X	d <sub>u</sub>	d' <sub>u</sub>	f d' <sub>u</sub>
10-20	3	15	-20	-2	-6
20-30	5	25	-10	-1	-5
30-40	8	35	0	0	0
40-50	5	45	10	1	5
50-60	4	55	20	2	8
	25				9

$$\begin{aligned} \bar{x} &= A + \frac{\sum f d'_u \times l}{\sum f} = 35 + \frac{9 \times 10}{25} \\ &= 35 + \frac{90}{25} = 35.8 \text{ Ans.} \end{aligned}$$

Q To find unknown frequency or value of the variable.

C-I	F	X	CF	FX
60-62	15	61	15	915
63-65	54	64	69	3456
66-68	A	67	63+A	67A
69-71	81	70	150+A	5670
72-74	27	73	177+A	1971
				12012+67A

$$\text{Mean} = 67.45$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$67.45 = \frac{12012+67A}{177+A}$$

$$67.45(177+A) = 12012+67A$$

$$67.45A + 11938.65 = 12012+67A$$

$$0.45A = 73.35$$

$$A = \frac{73.35}{0.45} = 163 \text{ adm.}$$

Add tools = 8 adms.

AE = 911

<u>Ques:-</u>	X	F	fx
20	8	160	
30	12	360	
A	20	20A	
50	10	500	
60	6	360	
70	4	280	
	60	1660+20A	

$$\text{Mean} = 41$$

$$41 = \frac{1660+20A}{60}$$

$$2460 = 1660 + 20A$$

$$800 = 20A$$

$$A = 40 \text{ A/m}$$

<u>Ques:-</u>	C-I	No. of Shop	X	fx
0-10	12	5	60	
10-20	10	15	270	
20-30	27	25	675	
30-40	A	35	35A	
40-50	17	45	765	
50-60	6	55	330	
	80+A		2100+35A	

$$\text{Mean} = 28$$

$$28 = \frac{2100+35A}{80+A}$$

$$2240+98A = 2100+35A$$

$$140 = 7A$$

$$A = 20 \text{ A/m}$$

Ques :- By using mean = 16.41 calculate missing frequency.

C-I	F	X	F	
0-5	3	2.5	7.5	Mean = 16.41
5-10	4	7.5	30	$16.41 = \frac{507.5 + 12.5A}{29+A}$
10-15	A	12.5	12.5A	
15-20	10	17.5	175	$475.89 + 16.41A = 507.5 + 12.5A$
20-25	7	22.5	157.5	$3.91A = 31.61$
25-30	5	27.5	137.5	$A = 8.00$ , <del>days</del>
			$507.5 + 12.5A$	
				$29+A$

Median =

Median no. =  $\frac{N+1}{2}$  in Individual series.

Median no. =  $\frac{N}{2}$  in continuous series.

$$\text{Median} = l_1 + \frac{f}{2} \left( \frac{n}{2} - c \right)$$

Mode

$$M_O = m_1 = \text{modality}$$

$$(f_C - f_O) / (f_C + f_O) = \text{modality}$$

Ques

Marks	F	CF
0-7	19	19
7-14	25	44
14-21	36	80
21-28	72	152
28-35	51	203
35-42	43	246
42-49	28	274
	274	

$$\text{Median no.} = \frac{N}{2}$$

$$\therefore \text{Median} = \frac{274}{2}$$

$$\text{Median no.} = 137 \text{ Item}$$

$$\text{Median} = l_1 + \frac{\frac{N}{2} - C}{f} \times h$$

$$= 21 + \frac{7}{72} (137 - 80)$$

$$= 21 + \frac{7}{72} (57)$$

$$= 21 + 39.9$$

$$= 21 + 5.54$$

C-I	F	CF
0-10	12	12
10-20	18	30
20-30	27	57
30-40	20	77
40-50	17	94
50-60	6	100

$$\text{Median} = 26.54$$

$$\text{Median no.} = \frac{100}{2} = 50$$

$$\text{Median} = 20 + \frac{10}{27} (50 - 30)$$

$$= 20 + \frac{10 \times 20}{27} = 20 + \frac{200}{27}$$

$$\begin{aligned}
 &= 20 + 7.40 \\
 &= 27.40 \text{ Ans.}
 \end{aligned}$$

Ques:- Find out missing frequency if median is 52.

C-I	F	CF	Median = 52.
0-10	5	5	$52 = 50 + \frac{1}{2} \left( \frac{72+A - 32+A}{2} \right)$
10-20	8	13	$20$
20-30	7	20	$= 50 + \frac{1}{2} \left( \frac{72+A - 64-2A}{2} \right)$
30-40	12	32	
40-50	A	32+A	$= 50 + \frac{1}{2} \left( \frac{8-A}{2} \right)$
50-60	20	52+A	
60-70	10	62+A	$52 = 50 + \frac{8-A}{4}$
70-80	10	72+A	
	$72+A$		$2 = \frac{8-A}{4}$

$$8 = 8 - A$$

$$A = 0 \text{ } \boxed{A=0} \text{ Ans.}$$

Ques:- From the following data, Compute median:-

C.I.	f	c.f.	$M \cdot no. = \frac{N}{2} \Rightarrow \frac{200}{2} = 100$
0-25	30	30	
25-50	50	80	$M = 50 + \frac{25}{80} (100 - 80)$
50-75	80	160	
75-100	40	200	$= 50 + \frac{25}{40} (80)$
	200		$= 50 + 6.25 = 56.25 \text{ km}$

Ques:- Find the median from the following data-

S.NO.	f	disc.	
1.	745	465	$M \cdot no. = \frac{N}{2} = \frac{10+1}{2} = 5.5$
2.	465	595	
3.	956	650	Median = $\frac{5^{\text{th}} \text{ observation} + 6^{\text{th}} \text{ observation}}{2}$
4.	750	745	
5.	1050	750	$= \frac{750 + 870}{2}$
6.	650	870	
7.	900	900	$= \frac{1620}{2} = 810 \text{ km}$
8.	870	956	
9.	575	1050	
10.	1250	1250	

Ques. Find out median from the following series-

mid values	f	C.I.	C.F.	M.no = $\frac{N}{2} = \frac{250}{2}$
5	15	0-10	15	
15	20	10-20	35	= 125
25	25	20-30	60	Median = $L + \frac{f}{2} \left( \frac{N}{2} - C \right)$
35	24	30-40	84	
45	12	40-50	96	= $50 + \frac{10}{31} (125 - 96)$
55	31	50-60	127	31
65	71	60-70	198	= $50 + \frac{10}{31} (29)$
75	52	70-80	250	
	- 250			= 50 + 9.35
				= 59.35 <u>dmr.</u>

Ques. Find the median from the following distribution-

C-I	f	C.F.	M.no. = $\frac{33}{2} = 36.5$ item.
0-4	3	3	
4-8	9	12	Median = $12 + \frac{3}{20} (36.5 - 30)$
8-12	18	30	
12-16	20	50	= $12 + \frac{1}{5} \times 6.5$
16-20	16	66	
20-24	7	73	= $12 + 1.3 = 13.3$ <u>dmr.</u>

## \* Mode :-

- Mode In Individual Series -

Ex :- 5, 8, 10, 11, 15, 18, 20, 22, 25, 30.

Sol :- No mode.

Ques :- 4, 8, 10, 12, 13, 10, 15, 12, 13, 12  
Mode = 12.

- Mode In Continuous Series -

$$\text{Mode} = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

<u>Ques.</u>	C-I	$f$	Mode = $l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$
10-20	10		
20-25	35		
25-30	45		
30-35	70	$f_0$	
35-40	105	$f_1$	
40-45	74	$f_2$	
45-50	51		
50-55	30		

Mode  
class

Ques :- Calculation of mode in Inclusive series -

C.I.	f	C.I.	Mode = $\frac{43.5 + 80 - 65 \times 4}{2 \times 20 - 65 - 51}$
32-35	10	31.5-35.5	
36-39	31	35.5-39.5	$= \frac{43.5 + 15}{160 - 114} \times 4$
40-43	65	39.5-43.5	
44-47	10	43.5-47.5	$= \frac{43.5 + 60}{46} \times 4$
48-51	51	47.5-51.5	
52-55	35	51.5-55.5	$= 43.5 + 1.36$
56-59	10	55.5-59.5	$= 44.86$ <del>Ans</del>
60-63	4	59.5-63.5	

Ques :- calculate mode -

X	58	60	61	62	63	64	65	66	68	70
f	4	6	5	10	20	22	24	6	2	1

$$\text{Mode} = 65$$

Ques & find mode -

no. of days no. of Students(f).

0-5 29

5-10 195 50

10-15 241 51

15-20 117 52

20-25 52

25-30 10

$$\begin{array}{ll}
 30-35 & 6 \\
 35-40 & 3 \\
 40-45 & 2
 \end{array}
 \quad \text{Mode} = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c$$

$$= 10 + \frac{241 - 195}{2 \times 241 - 195 - 117} \times 5$$

$$= 10 + \frac{46}{482 - 312} \times 5$$

$$= 10 + \frac{46}{170} \times 5$$

$$\text{Mode} = 10 + 1.35 = 11.35$$

Ques. Find mode -

C-5	f	Mode = $l_0 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c$
0-10	5	$= 40 + \frac{32 - 20}{2 \times 32 - 20 - 14} \times 10$
10-20	15	$= 40 + \frac{12}{64 - 34} \times 10$
20-30	20	$= 40 + \frac{12}{20} \times 10$
30-40	20	$= 40 + 12 = 44$
40-50	32	
50-60	14	
60-70	14	
70-80	5	

\* Mode in unequal distribution -

C.I	6	C.I	6	$\text{Mode} = 20 + \frac{12 - 6}{2 \times 12 - 10 - 7} \times 10.$
0-5	5	0-10	9	$= 20 + \frac{2}{7} \times 10.$
5-10	4	10-20	10	$24.29$
10-20	10	20-30	12	$= 20 + \frac{2}{7} \times 10.$
20-22	3	30-40	7	$24.29$
22-26	4	40-50	7	$24.29$
26-30	5	50-60	5	$= 20 + 2.85$
30-40	7			$= 22.85 \text{ Ans}$
40-50	7			
50-55	3			
55-60	2			

Ques: Mode in Inclusive series -

C.I	6	C.I	$\text{Mode} = 43.5 + \frac{80 - 65}{2 \times 80 - 65 - 51} \times 4$
32-35	10	31.5-35.5	$= 43.5 + \frac{15}{160 - 116} \times 4$
36-39	37	35.5-39.5	$= 43.5 + 15 \times 4$
40-43	65	39.5-43.5	$= 43.5 + 15 \times 4$
44-47	80	43.5-47.5	$= 43.5 + 15 \times 4$
48-51	51	47.5-51.5	$= 43.5 + 1.4$
52-55	35	51.5-55.5	$= 44.9 \text{ Ans}$
56-59	10	55.5-59.5	
60-63	4	59.5-63.5	

\* Determination of mode from mean and median -

$$\text{Mode} = 3\text{median} - 2\text{mean}$$

Ques:- If mean is 26, mode is 28.5, find median.

$$\text{Mode} = 3\text{median} - 2\text{mode}$$

$$28.5 = 3M - 2 \times 26$$

$$M = \frac{28.5 + 52}{3}$$

$$M = \frac{80.5}{3} = 26.83 \text{ dms}$$

Ques 8- Mode = 32.1, median = 35.4, find mean.

$$\text{Mode} = 3\text{Median} - 2\text{Mode}$$

$$32.1 = 3 \times 35.4 - 2M$$

$$32.1 = 106.2 - 2M$$

$$M = \frac{74.1}{2} = 37.05 \text{ dms}$$

Ques 9- Find out mode from the following data -

$$\text{Mode} = 40 + \frac{32-20}{64-34} \times 10$$

$$= 40 + \frac{12}{30} \times 10$$

$$= 40 + \frac{12 \times 10}{30} = 40 + \frac{120}{30} = 44 \text{ dms}$$

$$= 40 + 4 = 44 \text{ dms}$$

C.S	b
0-10	5
10-20	15
20-30	20
30-40	20 $f_0$
40-50	32 $f_1$
50-60	14 $f_2$
60-70	14
70-80	5

Ques 1:- Find mean, median, mode from the following data -

C.S	f	n	$\sum f_n$	C.F.
10-20	5	15	15	5
20-30	8	25	200	13
30-40	20	35	700	33
40-50	12	45	540	45
50-60	15	55	825	60
60-70	20	65	1300	80
70-80	14	75	1050	94
80-90	10	85	850	104
		104	5540	

$$\text{Mean} = \frac{\sum f_n}{\sum f} = \frac{5540}{104} = 53.26$$

$$\text{M. no.} = \frac{N}{2} = \frac{104}{2} = 52$$

$$\text{Median} = l_1 + \frac{1}{f} \left( \frac{N}{2} - C \right)$$

$$= 50 + \frac{10}{15} (52 - 45)$$

$$= 50 + \frac{10}{15} \times 7 = 50 + 4.60 = 54.60$$

$$\text{Mode} = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

Ques:- Find mode from the following data -

C-I	8	C-I	16
0-2	4	0-6	12
2-6	0	6-12	24
6-9	10	12-18	36
9-12	14	18-24	38
12-16	16	24-30	37
16-18	20	30-36	6
18-20	24		
20-24	14		
24-26	16		
26-28	16		
28-30	10	C-I	F
30-36	6	0-9	2

Ques:- Find out median and mode from the following data.

10-19	10
20-29	18
30-39	20
40-49	38
50-59	25
60-69	16
70-79	10
80-89	8
90-99	3
	150

Ques: In a Series mean and median are 10 and 11. Find out mode.

$$\text{Mode} = 3 \times 11 - 2 \times 10 = 33 - 20 = 13 \text{ Ans.}$$

Ques: (i) Find out median if mean = 20 and mode = 10,

(ii) Median and mean are 24 and 26. Find mode.

(iii) Mean is 26 and mode is 28.5. Find median.

(iv) Mode =  $3 \times \text{Median} - 2 \times \text{Mean}$ .

$$10 = 3M - 2 \times 20$$

$$10 = 3M - 40$$

$$50 = 3M$$

$$\text{Median} = \frac{50}{3} = 16.67 \text{ Ans.}$$

(ii) Mode =  $3 \times 24 - 2 \times 26 = 72 - 52 = 20 \text{ Ans.}$

(iii) Mode =  $3 \times \text{Median} - 2 \times 26$ .

$$28.5 = 3M - 52$$

$$3M = 80.5 \Rightarrow \text{median} = 26.83 \text{ Ans.}$$

\* Missing frequency in mode :-

Median and mode of data is 27 and 26.

C.F	f	Cf.
0-10	3	3
10-20	A	11
20-30	20	31
30-40	12	43
40-50	B	43+B
		43+B

Median = 27 and mode = 26  
is lies in class 20-30.  
Hence it is median class  
as well as mode class.

$$\text{Mode} = 20 + \frac{20+A}{2} \times 10$$

$$2 \times 20 + A - 12$$

$$26 = 20 + \frac{20+A}{40-A-12} \times 10$$

$$6 = \frac{200-10A}{18-A}$$

$$16A - 6A = 200 - 10A$$

$$4A = 32$$

$$\boxed{A = 8}$$

$$\text{Median no.} = \frac{N}{2} = \frac{43+B}{2}$$

$$\text{Median} = 20 + \frac{1}{2} \left( \frac{43+B-11}{2} \right)$$

$$27 = 20 + \frac{1}{2} \left( \frac{43+B-22}{2} \right)$$

$$7 \times 4 = 21 \times B$$

$$B = 28 - 21$$

$$\boxed{B = 7}$$

Ques:- Find out mixing frequency if mode = 24.

C.S.	b
0-10	14
10-20	A
20-30	27
30-40	B
40-50	15
	100

$$\text{Mode} = M + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times l$$

$$24 = 20 + \frac{27 - A}{2 \times 7 - A - B} \times 10$$

$$4 = \frac{270 - 10A}{54 - A - B}$$

$$216 - 4A - 4B = 270 - 10A$$

$$6A - 4B = 54$$

~~3A - 2B = 27~~

$$3A - 2B = 27 \quad (1)$$

$$14 + 27 + 15 + A + B = 100$$

$$A + B = 44 - (2)$$

$$A + B = 44$$

from eqn (1) and (2)

$$28 + B = 44$$

$$3A - 2B = 27$$

$$\boxed{B = 21}$$

$$2A + 2B = 88$$

$$5A = 115$$

$$\boxed{A = 23}$$

Ques:- obtain mean, median and mode from the following data class:-

C.I	b	n	f <sub>u</sub>	C.F.	Mean = $\frac{\sum f u}{\sum f} = \frac{2520}{100}$
0-10	13	5	65	13	
10-20	22	15	330	35	= 25.2
20-30	30	25	750	65	Median no. = $\frac{N}{2} = \frac{100}{2}$
30-40	30	35	700	85	
40-50	15	45	75	100	= 50
	100	2520			

$$\text{Median} = 20 + \frac{10}{30} (50 - 35) = 20 + \frac{1}{3} (15) = 20 + 5 = 25$$

$$\text{Mode} = 20 + \frac{3(22 - 20)}{60 - 22 - 20} = 20 + \frac{6}{18} \times 10 = 20 + 4.44 = 24.44$$

Ques:- find out missing frequency when median = 25 and mode = 25.55.

C.I	b	C.F.	Mode = $20 + \frac{2(22 - 20)}{40 - A - B} \times 10$
0-10	5	5	
10-20	A	5+A	
20-30	20	25+A	$25.55 = 20 + \frac{2(200 - 10A)}{40 - A - B}$
30-40	B	25+A+B	
40-50	3	28+A+B	

$$5.55(40-A-B) = 200 - 10A$$

$$222 - 5.5A - 5.5B = 200 - 10A$$

$$5.55A - 5.55B = -22 - 10A$$

$$4.5A - 5.55B = -22 \quad (1)$$

$$\text{M. no.} = N = \frac{28+A+B}{2}$$

$$\text{Median} = 20 + \frac{10}{20} \left( \frac{28+A+B-S-A}{2} \right)$$

$$25 = 20 + \frac{1}{2} \left( \frac{28+A+B-10-2A}{2} \right).$$

$$5 \times 4 = -A+18+B$$

$$20 = -A+B+18$$

$$A-B = -2 \quad (2)$$

From eqn (1) and (2)

$$A = 9.9 = 10$$

$$B = 11.9 = 12 \cancel{16.2}$$

Ques & Mode and median are 33.5 and 34.

C-I	$\frac{1}{f}$
0-10	4
10-20	16
20-30	A
30-40	B
40-50	C
50-60	6
60-70	4

\* Geometric mean -

$$G.M = \sqrt[N]{(u_1) \times (u_2) \times (u_3) \dots}$$

Ques :- Find geometric mean of - 3, 27

$$G.M. = \sqrt[3]{3 \times 27}$$

$$= \sqrt[3]{3 \times 3 \times 3 \times 3}$$

$$= 9 \text{ cms.}$$

Ques :- Find geometric mean of - 4, 8, 16

$$G.M. = \sqrt[3]{4 \times 8 \times 16}$$

$$= \sqrt[3]{2 \times 2 \times 2}$$

$$= 8$$

16, 25, 36

$$G.M. = \sqrt[3]{16 \times 25 \times 36}$$

$$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 2 \times 2 \times 3 \times 3}$$

$$= 2 \times 2 \sqrt[3]{5 \times 5 \times 3 \times 3}$$

$$= 4 \sqrt[3]{225} \text{ cms.}$$

\* Uses of geometric mean -

- 1- It is the most useful in the study of social and economical phenomena, where small numbers are to be given importance.

2- If some items of series are very large in magnitude and others are small then geometric mean is the most representative average.

3- The most important use of geometric mean is case of Index number. Fisher Ideal Index number is based on geometric mean.

### \* Harmonic mean -

$$H.M. = \frac{N}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots}$$

Ques :- Calculate harmonic mean from the following data - 2, 4, 5, 6.

$$\begin{aligned} H.M. &= \frac{4}{\frac{1}{2} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}} \\ &= \frac{4}{\frac{30+15+12+10}{60}} = \frac{240}{67} = 3.58 \text{ days} \end{aligned}$$

### \* Merits of Harmonic Mean -

- 1- Its value is defined and proper.
- 2- It is based on the observation.

- 3- It gives highest weight to smallest unit.  
By this larger unit do not influence its value.
- 4- It is specially used in finding average of rate or times
- 5- Its algebraic treatment is possible.

\* Demerits of Harmonic mean :-

- 1- It is used in specific case only.
- 2- It is not easy to understand and involve complicated calculations.
- 3- It cannot be calculated if the value of any item is unknown.  
It may not present in the series.

\* Relationship between A.M., G.M. and H.M.

(i) In case of series are ~~individually~~ identical -

$$A.M. = G.M. = H.M.$$

(ii) If the items are not equal -

$$A.M. \geq G.M. \geq H.M.$$

(iii) If there are two items -

$$GM = \sqrt{AM \times HM}$$

$$HM = \sqrt{AM \times GM}$$

$$AM = \sqrt{GM \times HM}$$

UNIT-3Measurement of dispersion.

Definition of dispersion :- According to A. L. Bowley dispersion is the measurement of the variation of the items. "According to L.R. Conner dispersion is the measure of the extent to which the individual items vary."

\* Objective of measure of dispersion -

- 1- To judge the variability of average.
- 2- To make a comparative study of the variability of two or more series.
- 3- To control variability.
- 4- To facilitate further mathematical analysis.
- 5- To help in quantity control.

\* Type of measure of dispersion.

- 1) Absolute dispersion.
- 2) Relative dispersion.

1) Absolute dispersion :- Absolute measure of variation is measured in the same

\* Statistical unit in which original data exist.

For example:- If the data is given in rupees or kilogram the result may be measure in terms of money and kilogram.

2- Relative dispersion - Absolute dispersion is not suitable for comparative study of variability between two series specially when the statistical unit is not same.

### \* Methods of measuring dispersion :-

Method

Mathematical

Graphical

Methods of  
limit

Methods of  
moments

Range Inter-Quartile  
range

Mean Quantile  
dispersion deviation

Standard deviation

1- Range :-

Range = L-S → Smallest Item.  
 ↓  
 Largest Item

Coefficient of Range  $\Rightarrow \frac{L-S}{L+S}$

Ques B- Find the Range and Coefficient of range in individual series.

Series - 11, 7, 9.5, 7.2, 10.3, 8.4, 6.7, 5.6, 7.5, 8.7

dsr. - 5.6, 6.7, 7, 7.4, 7.5, 8.4, 8.7, 9.5, 10.3, 11.

$$\begin{aligned}\text{Range} &= L-S \\ &= 11-5.6 \\ &= 5.4 \text{ dm}\end{aligned}$$

$$\begin{aligned}\text{Coefficient of range} &= \frac{L-S}{L+S} = \frac{5.4}{11+5.6} = \frac{54}{166} \\ &= \frac{27}{83} = 0.32\end{aligned}$$

\* Range in discrete series.

Variables	F
82	6
87	7
92	9
97	13
102	12

107	5	Range = L-S = 112 - 82 = 30.
112	3	

Coefficient of range =  $\frac{30}{194} = 0.1 \text{ gtr}$

\* Range in continuous series :-

C-I	F	Range = L-S = 60 - 0 = 60
0-10	2	
10-20	4	
20-30	6	Coefficient of range = $\frac{60}{60} = 1$
30-40	8	
40-50	11	
50-60	4	

Ques:- Find range and its coefficient from the following data.

C-I	F	C-I	Range = 40.5 - 10.5 = 30
10-15	3	10.5-15.5	
15-20	5	15.5-20.5	
20-25	6	20.5-25.5	
25-30	9	25.5-30.5	
30-35	10	30.5-35.5	
35-40	7	35.5-40.5	

### \* Merits of range -

- 1- Range is the simplest measure of dispersion.
- 2- Range is the rigidly define.

### \* Demerits of range -

- 1- It is based on the 2 extreme values.
- 2- It take low account of the form of distribution with in the two extreme values.

Ques:- Find the minimum value if the maximum value is 80 and the coefficient of range is 60%.

Solution :- Max = L = 80

$$C.O.R = 60\% = \frac{60}{100} = 0.6$$

$$C.O.R = \frac{L-S}{L+S}$$

$$\frac{0.6}{10} = \frac{80-S}{80+S}$$

$$480 + 6S = 800 - 10S$$

$$S = 320$$

$$S = \frac{320}{16} = 20$$

$S = 20$

$$\begin{aligned}\text{Range} &= L - S \\ &= 80 - 20 \\ &= 60 \text{ days}\end{aligned}$$

\* Quartile deviation -

$$\text{Quartile dev.} = \frac{Q_3 - Q_1}{2}$$

$$\text{Inter Quartile range} = Q_3 - Q_1$$

$$\text{Coefficient of Quartile deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

Its range from 0 to 1.

where  $Q_3$  is third Quartile or upper Quartile and  $Q_1$  is first Quartile or lower Quartile.

Ques:- Quartile deviation in individual series

Series	Ans.
87	36
36	49
95	50
99	52
52	63
63	65
49	70

70	87	$Q_1 \Rightarrow \text{size of } \left(\frac{N+1}{4}\right) \text{ item.}$
50	95	
65	98	$= 11+1 = \frac{12}{4} = 3 \text{ item}$
98	99	

$$Q_1 = 50$$

$$Q_3 = 3 \left( \frac{N+1}{4} \right)^{\text{th}} \text{ item.}$$

$$= 3 \times 3 = 9 \text{ item}, \boxed{Q_3 = 95}$$

$$\text{Quartile deviation} = \frac{95 - 50}{2} = \frac{45}{2} = 22.5$$

$$\text{range} = 95 - 50 = 45$$

$$\text{coefficient} = \frac{45}{145} = \frac{9}{29} = 0.31$$

Ques:-	Series	ass.	$Q_1 = \frac{N+1}{4} = \frac{11+1}{4} = \frac{12}{4}$
	48	37	
	45	38	$Q_1 = 3$
	54	41	$\boxed{Q_1 = 41}$
	43	42	$Q_3 = 3 \times 3 = 9$
	51	43	$\boxed{Q_3 = 49}$
	49	45	
	38	46	

41	48	$Q_D = \frac{Q_3 - Q_1}{2} = \frac{49 - 41}{2}$
37	49	
42	51	$= \frac{8}{2} = 4.$
46	54	

Range = 8.

$$\text{Coefficient of } Q_D = \frac{8}{49+41} = \frac{8}{90} = 0.08$$

$$= 0.08 \text{ dms.}$$

Ques:- Quartile deviation in discrete series.

X	F	CF	$Q_1 = \frac{68+1}{4} = \frac{69}{4} = 17.25$
2	2	2	
3	3	5	$Q_1 = 6$
4	5	10	$Q_3 = \frac{3 \times 69}{4} = 3 \times 17.25$
5	6	16	
6	8	24	$= 51.75$
7	12	36	$Q_3 = 9$
8	16	52	
9	7	59	Quartile range = $Q_3 - Q_1$
10	5	64	$= 9 - 6 = 3$
11	4	68	$Q.D. = \frac{Q_3 - Q_1}{2} = \frac{9 - 6}{2} = 1.5$
	68		

$$\text{Coefficient of Q.D.} = \frac{3}{9+6} = \frac{1}{5} = 0.2$$

## Ques 8 - Quartile deviation in Continuous Series -

C.I.	F	C.F.
0-5	29	29
5-10	95	124
10-15	225	349
15-20	93	442
20-25	29	471
25-30	7	478
30-35	9	487
35-40	6	493
40-45	4	497
45-50	3	500

$$Q_1 = \frac{N+1}{4} = \frac{500+1}{4} = \frac{501}{4}$$

= 125.25 Item.

$$Q_1 = l_i + \frac{i}{f} \left( \frac{n}{2} - cf \right).$$

$$= 10 + \frac{5}{225} \left( \frac{500}{2} - 124 \right).$$

$$= 10 + \frac{1}{45} (250 - 124)$$

$$= 10 + \frac{1}{45} \times 126$$

$$= 10 + \frac{126}{45} = 10 + 2.7$$

$$\boxed{Q_1 = 12.7}$$

$$Q_3 = 3 \left( \frac{n+1}{4} \right) = 3 \times 125.25 = 375.75$$

$$Q_3 = 15 + \frac{5}{93} (250 - 349)$$

$$= 15 + \frac{5}{93} \times (-\frac{33}{31}) = 15 + \frac{5}{31} \times -33$$

$$\boxed{Q_3 = 9.63}$$

$$Q_2 = \frac{Q_3 - Q_1}{2} = \frac{9.6 - 12.7}{2} = 3.1$$

$$Q_2 = 1.55.$$

$$\text{Range} = Q_3 - Q_1 = 3.1$$

$$\text{Coefficient of Range} = \frac{3.1}{22.3} = \frac{31}{223} = 0.13.$$

<u>Ans</u>	C.F	C.F	F	CF	$Q_1 = \frac{n+1}{4} = \frac{50+1}{4}$
11-20	10.5-20.5	4	4		
21-30	20.5-30.5	8	12		$Q_1 = 12.75$
31-40	30.5-40.5	20	32		$Q_3 = 3 \times 12.75$
41-50	40.5-50.5	12	44		$Q_3 = 38.25$
51-60	50.5-60.5	6	50		
		50			

$$Q_1 = l_1 + \frac{f}{6} \left( \frac{n}{2} - CF \right)$$

$$= 30.5 + \frac{10}{20} \left( \frac{50}{4} - 12 \right)$$

$$= 30.5 + \frac{1}{2} \left( \frac{50}{4} - 40 \right)$$

$$= 30.5 + \frac{1}{4}$$

$$= 30.5 + 0.25$$

$$Q_1 = 30.75$$

$$Q_3 = 40.5 + \frac{10}{12} \left( \frac{50}{4} - 32 \right)$$

$$= 40.5 + \frac{10}{12} \cdot (50 - 120)$$

$$= 40.5 + \frac{5}{12} \times (-78) \quad \text{Ans}$$

$$= 40.5 + 5 \times (-13)$$

$$Q_3 = 105.5$$

$$Q_2 = \frac{105.5 - 38.25}{2}$$

$$Q_1 = \frac{67.25}{2} = 33.625$$

$$\text{Range} = 67.25$$

$$\text{Coefficient} = \frac{67.25}{143.75} = 0.4.$$

PAGE NO. / /

DATE / /

<u>Quartile</u>	X	F	C.F	C.S	$Q_1 = \frac{N+1}{4} = \frac{176}{4} = 44$
5	12	12	0-10		
15	15	27	10-20		$Q_1 = 20 + \frac{10}{18} (175 - 27)$
25	18	45	20-30		
35	21	66	30-40		$Q_1 = 20 + \frac{5}{9} (175 - 100)$
45	36	102	40-50		
55	22	124	50-60		$= 20 + \frac{5}{9} (57)$
65	17	141	60-70		
75	13	154	70-80		$= 20 + \frac{335}{36}$
85	10	164	80-90		
95	11	175	90-100		$Q_1 = 28.3$
	175				

$$Q_3 = 3 \times 44 = 132$$

$$Q_2 = \frac{66.32 - 29.3}{2}$$

$$Q_3 = 60 + \frac{10}{17} (175 - 124)$$

$$Q_2 = \frac{32.02}{2} = 16.51$$

$$= 60 + \frac{10}{17} \left( 175 - 59.6 \right)$$

$$Q_2 = 32.02$$

$$= 60 + \frac{2105}{34}$$

$$C.O.R = \frac{32.02}{95.62} = 0.33$$

$$= 60 + 6.32$$

$$= 66.32$$

A. Mean deviation & or average deviation :-

$$\text{Mean deviation} = \frac{1}{N} \sum |x - M|$$

$$\text{Coefficient of M.D.} = \frac{M.D.}{\text{Mean}}, \frac{M.D.}{\text{Median}}, \frac{M.D.}{\text{Mode}}$$

Ques Find mean deviation from mean from the following data.

Series	3	5	6	7	8	10	11	14	
$x - M$	-5	-3	-2	-1	0	2	3	6	
$ x - M $	5	3	2	1	0	2	3	6	$\sum  x - M  = 22$

$$\bar{x} = \frac{\sum x}{N} = \frac{64}{8} = 8.$$

$$\text{Mean} = 8.$$

$$M.D. = \frac{1}{N} \sum |x - M| = \frac{1}{8} \times 22 = \frac{11}{4} = 2.75$$

$$C.O.M.D. = \frac{M.D.}{\text{Mean}} = \frac{2.75}{8} = 0.34.$$

Ques Find out mean deviation from median from the following data.

Series.	5	7	9	10	12	13	15	
$x - M$	-5	-3	-1	0	2	3	5	
$ x - M $	5	3	1	0	2	3	5	19

$$\text{Median} = 10, \text{ Median no.} = \frac{N+1}{2} = \frac{8}{2} = 4$$

$$M.D. = \frac{1}{N} \sum |x - M|$$

$$= \frac{1}{7} \times 19 = \frac{19}{7} = 2.71$$

$$C.Q.M.D. = \frac{2.71}{1000} = 0.27.$$

Ques 8 - Find out mean deviation from mode from the following data.

Series	$x - M$	$ x - M $	
8	0	0	Mode. = 8.
3	-5	5	$M.D. = \frac{1}{10} \times 21 = \frac{21}{10} = 2.1$
5	-3	3	
9	0	0	$C.Q.M.D. = \frac{21}{8} = 0.26$
7	-1	1	
9	1	1	
5	-3	3	
8	0	0	
6	-2	2	
2	-6	6	
		21	

## \* Calculation of M.D in discrete series-

$$M.D. = \frac{1}{N} \sum f |x - M|$$

Ques:- Find out mean deviation from median and its coefficient from the following data.

X	F	CF	X-M	X-M	FX-M	
10	3	3	-2	2	6	
11	12	15	-1	1	12	M. no. = $\frac{48}{2} = 24$ .
12	18	33	0	0	0	
13	12	45	1	1	12	Median = 12.
14	3	48	2	2	6	
		48			36	

$$M.D = \frac{1}{48} (36) = \frac{3}{4} = 0.75$$

$$\text{Coefficient of M.D} = \frac{0.75}{12} = 0.62.$$

Ques:- Determine Quantile deviation from the following.

C-I	F	CF	
0-5	5	5	$Q_1 = \frac{N+1}{4} = \frac{51}{4} = 12.75$
5-10	8	13	
10-15	10	23	$Q_2 = 3 \times 12.75$
15-20	12	35	$Q_3 = 38.25$
20-25	8	43	
25-30	7	50	

$$Q_1 = l_1 + \frac{f}{Q} (n - c)$$

$$= 5 + \frac{10}{8} \left( \frac{50 - 5}{4} \right)$$

$$= 5 + \frac{10}{8} \left( \frac{50 - 20}{4} \right)$$

$$= 5 + \frac{10}{8} \left( \frac{30}{4} \right)$$

$$= 5 + \frac{75}{8}$$

$$Q_1 = 14.37$$

$$Q.D. = \frac{Q_3 - Q_1}{2} = \frac{56.25 - 14.37}{2}$$

$$Q.D. = \frac{41.88}{2} = 20.94$$

$$\text{Range} = 41.88$$

$$\text{Coefficient of Range} = \frac{41.88}{70.62} = 0.5$$

Ques:- Calculate mean deviation and its coefficient from mean median and mode.

X	F	FX	CF	X-N	X-N1	X-N2	<del>X-N3</del>
12	1	12	1	-5	5	-4	
14	2	28	3	-3	3	-2	
16	3	48	6	-1	1	0	
18	2	36	8	1	1	2	
22	1	22	9	5	5	6	
24	1	24	10	7	7	8	
	10	170			22	22	

$$\text{Mean} = \frac{\sum f_x}{N} = \frac{170}{10} = 17$$

$$\text{N.o.} = \frac{10}{2} = 5, \text{ Median} = 16$$

$$\text{Mode} = 16$$

$$(i) \text{ N.O.} = \frac{1}{N} \sum |x-N|$$

for mean

$$= \frac{1}{10} \times 22 = 2.2$$

$$\text{C.Q. N.O.} = \frac{2.2}{170} = 0.12$$

$$(ii) \text{ N.O.} = \frac{1}{N} \times 22$$

for median 10

$$= 2.2$$

$$\text{C.Q. N.O.} = \frac{2.2}{160}$$

$$= 0.13.$$

$$(iii) \text{ N.O. for Mode} = \frac{1}{N} \times 22 = 2.2$$

$$\text{C.Q. N.O.} = \frac{2.2}{160} = 0.13$$

Ques 1:- Calculation of Mean deviation in Continuous series, Mean deviation from mean.

Marks.	F	X	FX	X-N	X-N	F X-N
140-150	4	145	580	26.2	26.2	104.8
150-160	6	155	930	16.2	16.2	97.2
160-170	10	165	1650	6.2	6.2	62
170-180	18	175	3150	-3.8	3.8	68.4
180-190	9	185	1665	-13.8	13.8	124.2
190-200	3	195	585	-23.8	23.8	71.4
	50		8560			528

$$\text{Mean} = \frac{\sum f_x}{N} = \frac{8560}{50} = 171.2$$

$$M.D = \frac{1}{N} \sum f |x - N| = \frac{1}{50} \times 528 = 10.56$$

$$\text{Coefficient of M.D.} = \frac{M.D.}{\text{Mean}} = \frac{10.56}{171.2} = 0.06$$

Ques 2:- find out mean deviation from median.

C-I	CF	F	X-N	X-N	X	F X-N
9-10	10	10	-25	25	5	250
10-20	12	2	-15	15	15	30
20-30	25	13	-5	5	25	65
30-40	35	10	5	5	35	50
40-50	40	5	15	15	45	75
50-60	50	10	25	25	55	250

$$\text{M.no.} = \frac{50}{2} = 25$$

$$\text{Median} = 20 + \frac{10}{13} (25 - 12)$$

$$= 20 + \frac{10}{13} (15)$$

$$\text{Median} = 20 + 10 = 30$$

$$\text{M.D} = \frac{1}{N} \sum f |x - M|$$

$$= \frac{1}{50} \times 720 = 14.4.$$

Ques:- Find out mean deviation from mode.

C-I	F	C.F.	X-M	X	$f x - M $
0-10	5	5	-39	5	195
10-20	15	20	-29	15	435
20-30	20	40	-19	25	380
30-40	20	60	9	35	180
40-50	32	92	1	45	32
50-60	14	106	11	55	154
60-70	14	120	21	65	294
70-80	5	125	31	75	155
	125				1025

$$\text{Mode} = l_1 + f_1 - f_0 \times k$$

$$2f_1 - f_0 - f_2$$

$$= 40 + \frac{32-20}{64-20-14} \times 10$$

$$= 40 + \frac{12}{34} \times 10$$

$$= 40 + \frac{12}{34}$$

$$= 40+4 = 44.$$

$$M.D. = \frac{1}{N} \sum f |x - M|$$

$$= \frac{1}{125} \times 12.25 = \frac{73}{5} = 14.6$$

A. Standard deviation :-

Standard deviation in individual series:-

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{N}} \text{ or } \sqrt{\frac{\sum d^2}{N}}$$

$$\text{or } \sqrt{\frac{\sum x^2}{N} - \frac{\sum x^2}{N}}$$

$$\text{Coefficient of S.D.} = \frac{\text{S.D.}}{\text{Mean}}$$

$$\text{Coefficient of variation} = \frac{\text{S.D.}}{\text{Mean}} \times 100$$

Calculate SD. and its coefficient from the following data -

<u>Ques:-</u>	$x$	$x - \bar{x}$	$(x - \bar{x})^2$	
10	-20	400		$\text{Mean} = \frac{300}{10} = 30$
12	-18	324		
18	-12	144		$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{N}}$
22	-8	64		
26	-4	16		$= \sqrt{\frac{2202}{10}} = 46.92$
30	0	0		
32	2	4		$= 14.84$
45	15	225		$\text{Co. of S.D.} = \frac{14.84}{30} = 0.494.$
50	20	400		
55	25	625		
<u>300</u>		<u>2205</u>		

Ques:- Calculate S.D. from the following information.

$$N=7, \sum x=84, \sum x^2=1130$$

$$\sigma = \sqrt{\frac{\sum x^2 - (\sum x)^2}{N}} = \sqrt{\frac{1130 - (84)^2}{7}}$$

$$= \sqrt{\frac{1130 - 7056}{49}} = \sqrt{\frac{7910 - 1056}{49}}$$

$$= \sqrt{\frac{854}{49}} = \sqrt{17.42} = 4.17 \text{ Ans}$$

\* Shortcut method in individual series -

$$S.D = \sqrt{\frac{\sum d^2}{N} - \left(\frac{\sum d}{N}\right)^2}$$

Ques:  $x - d = x - A$   $d^2$

10 3 9

11 4 16

17 10 100

25 18 324

$A = \boxed{7} \quad 0 \quad 0$

13 6 86

21 14 196

10 3 9

12 5 25

14 7 49

70 764

$$\sigma = \sqrt{\frac{1}{10} \sum d^2 - \left(\frac{\sum d}{10}\right)^2} = \sqrt{\frac{7640 - 4900}{100}}$$

$$= \sqrt{\frac{2740}{10}} = \frac{52.34}{10} = 5.23$$

Ques: Calculate S.D. and its coefficient from the following.

$$x \quad d(x-A) \quad d^2$$

$$30 \quad -70 \quad 4900$$

$$50 \quad -50 \quad 2500$$

$$60 \quad -40 \quad 1600$$

$$70 \quad -30 \quad 900$$

$$90 \quad -10 \quad 100$$

$$A = 100 \quad 0 \quad 0$$

$$105 \quad 5 \quad 25$$

$$120 \quad 20 \quad 400$$

$$125 \quad 25 \quad 625$$

$$130 \quad 30 \quad 900$$

$$150 \quad 50 \quad 2500$$

$$170, \quad 70 \quad 4900$$

$$\underline{14350}$$

$$\sigma = \sqrt{\frac{\sum d^2}{N} - \left(\frac{\sum d}{N}\right)^2}$$

$$= \sqrt{\frac{14350}{12}} = 0$$

$$= 40.15 \text{ $d.m$}$$

$$\text{Co. q. S.D.} = \frac{\text{S.D.}}{\text{Mean}}$$

$$= \frac{40.15}{100} = 4.015 \text{ $d.m$}$$

Ques:- Calculate S.D. & its Coefficient -

$$x \quad (x-\bar{x}) \quad (x-\bar{x})^2$$

$$25 \quad -19 \quad 361 \quad \bar{x} = \frac{\sum x}{N} = \frac{440}{10} = 44.$$

$$34 \quad -10 \quad 100$$

$$48 \quad 4 \quad 16$$

$$86 \quad -8 \quad 64$$

$$42 \quad -2 \quad 4$$

$$70 \quad 26 \quad 676$$

$$30 \quad -14 \quad 196$$

$$60 \quad 16 \quad 256$$

$$45 \quad 1 \quad 1$$

$$50 \quad 6 \quad 36$$

$$\sigma = \sqrt{\frac{\sum (x-\bar{x})^2}{N}}$$

$$= \sqrt{\frac{1774}{10}} = \sqrt{177.4}$$

$$= \sqrt{177.4} = \sqrt{177.4}$$

$$= \sqrt{177.4} = \sqrt{177.4}$$

$$= 13.12$$

$$\text{C. q. S.D.} = \frac{13.12}{44} = 0.3$$

$$= 0.3$$

Ques: -  $x \quad (x-\bar{x}) \quad (x-\bar{x})^2$

$$155 - 9.6 \quad 92.16$$

$$158 - 6.6 \quad 43.56$$

$$163 - 1.6 \quad 2.56$$

$$169 \quad 4.4 \quad 19.36$$

$$178 \quad 13.6 \quad 184.48$$

$$0.2 \quad 336.12$$

$$\bar{x} = \frac{823}{5} = 164.6$$

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{N}} = \sqrt{\frac{336.12}{5}}$$

$$= \sqrt{67.22} = 8.19$$

$$C.O.F = \frac{8.19}{164.6} = 0.04$$

A. Standard deviation in discrete series -

$$S.D. = \sqrt{\frac{1}{N} \sum f(x-\bar{x})^2} \text{ or } \sqrt{\frac{\sum f d^2}{N}}$$

Ques:-	X	F	$d = x - A$	$d^2$	$fd^2$
	10	5	-30	900	4500
	20	15	-20	400	6000
	30	30	-10	100	3000
	40	35	0	0	0
	50	25	10	100	2500
	60	17	20	400	6800
	70	8	30	900	7200
					30000
					135

$$S.D. = \sqrt{\frac{\sum f d^2}{N}} = \sqrt{\frac{30000}{135}} = \sqrt{\frac{6000}{27}} = 6000$$

Ques:- Standard deviation in shortcut method of discrete series -

$$(S.D.) = \sqrt{\frac{\sum f d^2}{N} - \left(\frac{\sum f d}{N}\right)^2}$$

<u>Class</u>	X	F	d	$d^2$	$fd$	$fd^2$
21	5	-2	4	20	-10	
22	8	-1	1	8	-8	
23	12	0	0	0	0	
24	8	1	1	8	8	
25	7	2	4	28	14	
	<u>40</u>			<u>64</u>	<u>4</u>	

$$S.D. = \sqrt{\frac{64}{40} - \left(\frac{4}{40}\right)^2}$$

$$= \sqrt{\left(\frac{64}{40} - \frac{16}{1600}\right)} = \sqrt{\frac{16}{10} - \frac{1}{100}}$$

$$= \sqrt{\frac{160-1}{100}} = \sqrt{\frac{159}{100}} = \sqrt{1.59}$$

$$S.D. = 1.26$$

Ques: Find out S.D. by Shortcut method from the following information.

Series	F	d	fd	$d^2$	$fd^2$
10	1	-10	-10	100	100
20	2	0	0	0	0
30	4	10	40	100	400
40	3	20	60	400	1200
	10		<u>90</u>		<u>1700</u>

$$S.D = \sqrt{\frac{700 - (\bar{x})^2}{10}} = \sqrt{70 - 81} \\ = \sqrt{89} = 9.44 \text{ cm}$$

Ques:- Find out S.D. from the step-deviations method

$x$  8

Q 2

9 4

10 5

11 8

12 6

13 3

14 2

30

\* Direct method in continuous series -

$$\sigma = \sqrt{\frac{\sum f(x-\bar{x})^2}{N}} \text{ or } \sqrt{\frac{\sum fd^2}{N}}$$

	C.F.	X	f <sub>n</sub>	(x- $\bar{x}$ ) <sup>2</sup>	(x- $\bar{x}$ ) f <sub>n</sub>	f(x- $\bar{x}$ ) <sup>2</sup>
0-10	1	5	5	361	-19	361
10-20	2	15	30	81	-9	162
20-30	4	25	100	1	1	4
30-40	3	35	105	121	11	363
	10		240			890

$$\bar{x} = \frac{\sum f_n}{\sum f} = \frac{240}{10} = 24.$$

$$\sigma = \sqrt{\frac{\sum f(x-\bar{x})^2}{N}} = \sqrt{\frac{890}{10}} = \sqrt{89}$$

$$\sigma = 9.43 \text{ S.D.}$$

Ques: Find out S.D. by Shortcut method in continuous series -

C.F.	b	X	c1(*-A)	d <sup>2</sup>	fd	fd <sup>2</sup>
0-10	1	5	-20	400	-20	400
10-20	2	15	-10	100	-20	200
20-30	4	25	A	0	0	0
30-40	3	35	10	100	80	300
	10				-10	900

$$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left( \frac{\sum fd}{N} \right)^2}$$

$$= \sqrt{\frac{900}{10} - \left( \frac{-10}{10} \right)^2} = \sqrt{89} = 9.43 \text{ S.D.}$$

Ques:- Find out S.D. by shortcut method -

C.F.	f	x	d(x-A)	d <sup>2</sup>	fd	fd <sup>2</sup>
0-10	3	5	-20	400	-60	1200
10-20	5	15	-10	100	-50	500
20-30	8	25	0	0	0	0
30-40	3	35	10	100	30	300
40-50	1	45	20	400	20	400
		20			60	2400

$$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$$

$$= \sqrt{\frac{2400}{26} - \left(\frac{60}{20}\right)^2} = \sqrt{120 - 9}$$

$$= \sqrt{111} = 10.53 \text{ for.}$$

Ques:- Calculate S.D. from direct and shortcut method -

C.F.	f	x	d	fd	d <sup>2</sup>	fd <sup>2</sup>
0-20	40	10	-60	-2400	3600	14400
20-40	64	30	-40	-2560	1600	102400
40-60	56	50	-20	-1120	400	22400
60-80	96	70	0	0	0	0
80-100	56	90	20	1120	400	22400
100-120	24	110	40	960	1600	38400
120-140	40	130	60	2400	3600	144000

140-160 24 150 80 1920 6400 153600  
400 800 497600.

(ii) Obiect method -

$$\sigma = \sqrt{\frac{\sum fd^2}{N}} = \sqrt{\frac{497600}{400}}$$

$$= \sqrt{1244} = 35.27$$

(iii) Shortcut method -

$$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left( \frac{\sum fd}{N} \right)^2}$$

$$= \sqrt{\frac{497600}{400} - \left( \frac{320}{400} \right)^2}$$

$$= \sqrt{1244 - 64} = \sqrt{1244 - 0.64}$$

$$= \sqrt{1243.36} = 35.26$$

## UNIT-4

### • Combination :-

Selection of the arrangements of the object is called combination.

For example :-

ABC and BCA are same combination. The total nos. of combination of 'r' object select from n is denoted by:-

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

### • Relationship b/w Permutation and Combination :-

$${}^n C_r = \frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!} = {}^n P_r$$

$${}^n C_1 = \frac{1}{1!} = 1 = {}^n P_1$$

$${}^n C_r = \frac{{}^n P_r}{r!}$$

Ques:- In how many ways can a cricket 11 be chosen out of the batch of 15 players.

- There is no restriction on the selection.
- A particular player is always chosen.
- " player never be chosen.

$$\begin{aligned}
 \text{i) } {}^{15}C_{11} &= \frac{115}{11 \ 15-11} = \frac{115}{11 \ 4} \\
 &= \frac{15 \times 14 \times 13 \times 12 \times 11}{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \\
 &= 105 \times 13 \\
 &= 1365 \text{ Ans.}
 \end{aligned}$$

$$\begin{aligned}
 \text{ii) } {}^{14}C_{10} &= \frac{114}{110 \ 14} = \frac{14 \times 13 \times 12 \times 11 \times 10}{110 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \\
 &= 77 \times 13 \\
 &= 1001 \text{ Ans.}
 \end{aligned}$$

$$\begin{aligned}
 \text{iii) } {}^{14}C_{11} &= \frac{114}{11 \ 14-11} = \frac{14 \times 13 \times 12 \times 11}{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \\
 &= 28 \times 13 \\
 &= 364 \text{ Ans.}
 \end{aligned}$$

Ques:- A Committee of 5 is to be form out of 6 Gentlemen and 4 ladies In how many ways this each can be done,

- i) Atleast two ladies are included.
- ii) Atleast two ladies are included.

i) Total Gentleman = 6.

" ladies = 4

$${}^4C_2 = \frac{4!}{2!(4-2)!} = \frac{4 \times 3 \times 12}{2 \times 1} = 2 \times 3 = 6.$$

$${}^6C_3 = \frac{6!}{3!3!} = \frac{8 \times 7 \times 6 \times 5 \times 4}{3 \times 2 \times 1} = 4 \times 5 = 20$$

$${}^4C_2 \times {}^6C_3 = 6 \times 20 = 120$$

b)  ${}^4C_3 \times {}^6C_2$

$$\frac{4!}{3!1!} \times \frac{16}{12!4!}$$

$$\frac{4 \times 12}{18} \times \frac{3 \times 6 \times 5 \times 4}{12 \times 10 \times 8 \times 6}$$

$$4 \times 15 = 60.$$

(iii)  ${}^4C_4 \times {}^6C_1$

$$\frac{4!}{4!0!} \times \frac{16}{15!1!}$$

$$\frac{1}{15} \times \frac{6 \times 18}{18}$$

$$1 \times 6 = 6.$$

$${}^7C_4 = 120 + 60 + 6 = 186 \text{ Ans.}$$

ii) at most 2 Ladies:-

a)  ${}^4C_1 \times {}^6C_3$

$$\frac{4!}{3!1!} \times \frac{16}{12!4!} = \frac{4 \times 3 \times 12}{12 \times 10 \times 8 \times 6} \times \frac{6 \times 5 \times 4}{18 \times 16 \times 14 \times 12}$$

$$= 6 \times 20 = 120.$$

b)  ${}^4C_1 \times {}^6C_4$

$$\frac{4!}{3!1!} \times \frac{16}{12!4!} = \frac{4 \times 12}{12} \times \frac{6 \times 5 \times 4}{14 \times 2}$$

$$= 4 \times 15 = 60.$$

$$\text{c) } {}^4C_6 \times {}^6C_5$$

$$\frac{14}{10} \times \frac{16}{11 \times 15}$$

$$\frac{14}{14} \times \frac{6 \times 18}{18} = 1 \times 6 = 6$$

$${}^7C_{31} = 120 + 60 + 6 = 186 \text{ Ans.}$$

Ques:- A bag contains 5 black, 3 white and 2 Red balls. In how many ways?

- i) 3 balls be drawn.
- ii) 3 black balls are drawn.
- iii) 2 black and 2 White balls are drawn.
- iv) 3 black and 2 white and 1 red balls are drawn.

$$\text{i) } {}^{10}C_3 = \frac{10}{17 \times 13} = \frac{10 \times 9 \times 8 \times 12}{17 \times 8 \times 2 \times 1}$$

$$\text{ii) } {}^5C_3 = \frac{15}{13 \times 12} = \frac{5 \times 4 \times 18}{13 \times 2 \times 1} = 10 \text{ Ans.}$$

$$\text{iii) } {}^5C_2 * {}^3C_2$$

$$\frac{15}{13 \times 12} \times \frac{13}{12 \times 11} = \frac{5 \times 4 \times 12}{13 \times 2 \times 1} \times \frac{3 \times 12}{12}$$

$$10 \times 3 = 30 \text{ Ans.}$$

$$\text{iv) } {}^5C_3 \times {}^3C_2 \times {}^2C_1$$

$$\frac{15}{13} \times \frac{13}{12} \times \frac{12}{11}$$

$$\frac{5 \times 4 \times 18}{13 \times 2 \times 1} \times \frac{3 \times 2}{12} \times \frac{2 \times 1}{1}$$

$$10 \times 3 \times 2 = 60 \text{ Ans.}$$

### \* Permutation :-

The word permutation means only of the arrangement in which a set of object can be arranged.

$${}^n P_r = \frac{n!}{(n-r)!}$$

Q1:- Evaluate :-  ${}^{12}P_4$

$${}^{12}P_4 = \frac{12}{12-4} = \frac{12}{8} = \frac{12 \times 11 \times 10 \times 9 \times 8}{28}$$

$$= 11880 \text{ Ans.}$$

$${}^{75}P_2 = \frac{75}{75-2} = \frac{75 \times 74 \times 73}{73}$$

$$= 5550 \text{ Ans.}$$

$${}^8P_8 = \frac{8}{8-8} = \frac{8}{0} = \frac{8}{1}$$

$$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 40320 \text{ Ans.}$$

Ques :- The word 'DELHI' contains . How many words with or without meaning can be formed by using the letters 'DELHI'

$${}^5P_5 = \frac{15}{15-5} = \frac{15}{10} = \frac{15}{1}$$

$$= 5 \times 4 \times 3 \times 2 \times 1$$

$$= 20 \times 6 = 120 \text{ Ans.}$$

Ques:- In how many ways can 8 students be seated?

i) In circle

ii) In a line.

$${}^8P_7 = \frac{18}{18-7} = \frac{18}{11} = \frac{18}{1}$$

$$= 40320 \text{ Ans.}$$

$${}^8P_8 = \frac{18}{18-8} = \frac{18}{10} = \frac{18}{1}$$

$$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 40320 \text{ Ans.}$$

Ques :- How many permutations can be made out of the letter 'TRIANGLE'

$${}^8P_8 = \frac{18}{18-8} = \frac{18}{10} = \frac{18}{1}$$

$$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \\ = 40320 \text{ Ans}$$

ii) How many arrangement can be done that start with T and end with E.

$${}^6P_6 = \frac{16}{16-6} = \frac{16}{0} = \frac{16}{1}$$

$$= 6 \times 5 \times 4 \times 3 \times 2 \times 1 \\ = 30 \times 24 = 720 \text{ Ans}$$

Ques:- How many words can be formed of the word 'DAUGHTER'. So, that

- i) The vowels always come together.
- ii) The vowels are never together.
- iii) AUEOGHTER

$${}^6P_6 = \frac{16}{16-6} = \frac{16}{10} = \frac{16}{1}$$

$$= 5 \times 4 \times 3 \times 2 \times 1$$

$$= 30 \times 24 = 720 \text{ Ans}$$

$$\text{iv) } {}^8P_8 = \frac{18}{10} = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \\ = 40320 \text{ Ans}$$

Ques:- In a lottery each ticket has 5 digit no. 0-9 on it.

- You win if your ticket has the digit in any order. What are your chances of winning?
- You could win only if your ticket has the digit in the required order. What are the chances winning.

a) 0-9, total = 10

$$n = 10$$

$$r = 5$$

$${}^{10}C_5 = \frac{10!}{5!5!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 1}{15 \times 8 \times 7 \times 6 \times 5 \times 4} = 6 \times 42 = 252 \text{ Am}$$

$$b) {}^{10}P_5 = \frac{10!}{5!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5}{15} = 720 \times 42 = 30240 \text{ Am.}$$

Ques:- A museum has 7 paintings by M.H. R and want to arrange 3 of them on the same wall. How many ways this can be done.

$${}^7P_3 = \frac{7!}{7-3!} = \frac{7 \times 6 \times 5 \times 4}{12}$$

$$= 42 \times 5 = 210 \text{ Am}$$

Ques P- A bag contains 10 black ball, 6 yellow ball and 4 blue ball. In how many ways:-

- 7 balls be drawn.
- 5 black balls be drawn.
- 3 black balls 3 Yellow ball be drawn.
- 4 black ball 4 yellow ball and 3 blue ball are drawn.

$$a) {}^{20}C_7 = \frac{120}{7 \times 13} = \frac{20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 14 \times 13}{13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= 60 \times 19 \times 17 \times 16 = 2,32,560 \text{ Ans}$$

$$b) {}^{10}C_5 = \frac{110}{5 \times 4 \times 3 \times 2 \times 1} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5}{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= 10 \times 9 \times 8 \times 7 \times 6 = 252 \text{ Ans}$$

$$c) {}^{10}C_3 \times {}^6C_3$$

$$\frac{10}{7 \times 6 \times 5} \times \frac{16}{4 \times 3 \times 2 \times 1} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} \times \frac{16 \times 15 \times 14 \times 13}{4 \times 3 \times 2 \times 1}$$

$$120 \times 20 = 2400 \text{ Ans.}$$

$$d) {}^{10}C_4 \times {}^6C_4 \times {}^4C_3$$

$$\frac{10}{4 \times 3 \times 2 \times 1} \times \frac{16}{4 \times 3 \times 2 \times 1} \times \frac{14}{3 \times 2 \times 1}$$

$$\begin{array}{r}
 \cancel{9} \cancel{8} \cancel{3} \\
 \cancel{10} \times \cancel{9} \times \cancel{8} \times \cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \\
 \cancel{16} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1} \quad \cancel{2} \times \cancel{1} \times \cancel{1} \times \cancel{1} \\
 30 \times 7 \times 15 \times 4 \\
 210 \times 60 = 12600 \text{ soln.}
 \end{array}$$

Ques:- How many 5 letters words can be form the letter 'EQUATIONS'

$$9P_5 = \frac{19}{19-5} = \frac{19}{14}$$

$$= 9 \times 8 \times 7 \times 6 \times 5 \times \cancel{4}$$

$$= \cancel{24}$$

$$= 72 \times 42 \times 5$$

$$= 360 \times 42$$

$$= 15120 \text{ soln.}$$

Ques:- How many words with or without can be form using all the letters of the word 'EQUATIONS' using each letters exactly one.

$$8P_8 = \frac{19}{19-8} = \frac{19}{10} = 19$$

$$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 40320 \text{ soln.}$$

UNIT-5

## Probability , Sample Space , events

## ★ Sample space:-

The set of all possible outcomes of a random experiment is called the sample space or outcome space. It is denoted by 'S'. The elements of sample space are called sample point.

For example :-

- 1- If a coin is toss, the two outcome namely head or tail may occur, thus a sample space contain two points head or tail.

$$S = \{H, T\}$$

- 2- If a dice is tossed the following sample space will be form -

$$S = \{1, 2, 3, 4, 5, 6\}$$

- 3- If two coins are tossed the sample space contain -

$$S = \{(H, H), (H, T), (T, H), (T, T)\}$$

- 4- If two dice are throw -

$$(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6).$$

$$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6).$$

$$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)$$

$$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$$

(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)  
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

★ Event :- The meaning of event is very clear and to the point when a random experiment is conducted we extract some results - this result is called event.

For ex:- If we toss a coin either head will occur or tail will occur, then tossing a coin is an experiment and occurrence of head and tail is an event.

Types of event :-

- 1- Simple and Compound event
- 2- Independent and dependent event
- 3- Complementary event.
- 4- Sure and Impossible event.

★ Probability :-

Probability =  $\frac{\text{No. of favourable cases}}{\text{Total no. of cases}}$

Ques:- A bag contains 4 red, 5 green balls. A ball is drawn at random, what is the probability that it is red in colour.

$$\text{Total no. of balls} = 4+5=9$$

$$\text{No. of red balls} = 4 \text{ balls}$$

$$P = \frac{\text{no. of favourable cases}}{\text{total no. of cases}}$$

$$P = \frac{4}{9}$$

Ques:- There are 38 balls in a bag in which 15 are white, 13 are pink, 10 are black. If a ball is drawn determine the probability of

a)- white ball.

b)- pink ball.

c)- black ball.

d)- black or white ball.

$$\text{a). Total no. of balls} = 38.$$

$$\text{No. of white balls} = 15$$

$$\text{P. of white ball } P(A) = \frac{15}{38}$$

$$\text{b). P. of pink ball } P(B) = \frac{13}{38}$$

$$\text{c). P. of black ball } P(C) = \frac{10}{38} = \frac{5}{19}$$

d) P. q. block on white ball.

$$P(D) = \frac{10+15}{38} = \frac{25}{38}$$

Ques- A bag contain 8 balls of which 5 are red and 3 are black. 2 balls are drawn at random. What is probability that both are black?

Total no. of ways so that 2 balls are drawn =  ${}^8C_2$

$$\begin{aligned} {}^8C_2 &= \frac{8!}{6! 2!} = \frac{8 \times 7 \times 6!}{6! 2} = 4 \times 7 \\ &= 28 \end{aligned}$$

$$\begin{aligned} \text{Ways of getting 2 black balls} &= {}^3C_2 \\ &= 3. \end{aligned}$$

$$P. \text{ that 2 balls are black} = \frac{3}{28}$$

Ques- There are 100 cards. These cards are no. from 1 to 100, one card is drawn at random. What is the probability that the no. on the card is a square.

Total no. of ways =  ${}^{100}C_1 = 100$

Square = 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

$$P(A) = \frac{10}{100} = \frac{1}{10} \text{ or}$$

Ques:- Two dice are thrown and the sum of the no. of the faces up are added. Find the probability that the sum is 10.

Total no. of outcomes = 36

$$P(\text{Sum is } 10) = \frac{3}{36} = \frac{1}{12} \text{ or}$$

Ques- Find the probability of 53 Sunday in a year selected at random.

Total no. of days = 365

Total no. of weeks in a year = 52 weeks 1 day

$$P. \text{ of } 53 \text{ Sunday in a year} = \frac{1}{7}$$

Ques:- A bag contains 8 white balls and 4 red balls. 5 balls are drawn at random. What is the probability that two of them are red and 3 are white.

Then no. Total no. of balls =  $8 + 4 = 12$ .

Ways of getting white ball =  ${}^8C_3 = \frac{8!}{3!5!}$

$$= \frac{8 \times 7 \times 6 \times 5!}{3 \times 2 \times 1 \times 5!} = 56.$$

Ways of getting red balls =  ${}^4C_2 = \frac{4!}{2!2!}$

$$\frac{2 \times 1 \times 2!}{2 \times 1 \times 2!} = 6.$$

$\therefore P(\text{getting 3 white and 2 red balls})$

$$= \frac{{}^8C_3 \times {}^4C_2}{{}^{12}C_5} = \frac{56 \times 6}{\frac{12!}{7!5!}}$$

$$= \frac{56 \times 6}{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= \frac{56 \times 6!}{12 \times 6 \times 11 \times 3!} = \frac{14}{33}$$

Ques:- 3 horses A, B and C are in a race. A is twice as likely to win as B is twice as likely to win as C. What are their respective probability of winning?

Let the probability of winning C =  $n$ .

$$\text{", " } \quad B = 2n$$

$$\text{" ", " } \quad A = 4n$$

ATQ :- Total Probability = 1

$$n + 2n + 4n = 1$$

$$7n = 1$$

$$n = \frac{1}{7}$$

$$A = \frac{1}{7}, B = \frac{2}{7}, C = \frac{4}{7} \text{ Ans.}$$

Ques :- A and B throw with 2 dice. If A throws 9. Find the B chances of throwing a highest no. than A.

Total ways when 2 dice are thrown = 36

A's sum = 9.

B's probability more than 9 = 10, 11, 12

$P[B] = (5,5), (6,4), (4,6), (6,5), (5,6), (6,6)$

$P[B]$  getting more than 9 =  $\frac{6}{36} = \frac{1}{6}$  Ans

Ques :- (i) If a pair of dice is thrown. Find the probability. -

- That the sum is neither 8 nor 10.
- The sum is greater than 12.

Q) Sum 8 and 10 = (2, 6), (6, 2), (4, 4), (3, 5),  
 (5, 3), (5, 5), (6, 4), (4, 6)

P. of sum is 8 and 10 =  $\frac{8}{36}$

$$P(\text{sum is neither 8 nor 10}) = 1 - \frac{8}{36}$$

$$= \frac{28}{36} = \frac{7}{9} \text{ Ans}$$

(b)  $P = \frac{0}{36} = 0$

(iii) If a pair of dice is thrown. Find the chances of throwing exactly 10 in one throw with 3 dice.

Total ways when 3 dice are thrown = 216

Ques:- 3 coins are tossed simultaneously, what is the probability that they will fall 2 Head and 1 tail.

Total no. of ways of 3 coins =  $2 \times 2 \times 2 = 8$   
 ways of getting 2 Head and 1 tail = 3  
 $(H, H, T), (T, H, H), (H, T, H)$

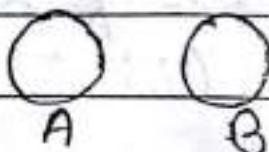
P of getting 2 Head and 1 tail =  $\frac{3}{8}$  Ans.

Ques:- Is the statement "The probability of 3 mutually exclusive events A, B and C are  $\frac{2}{3}, \frac{1}{4}$  and  $\frac{1}{6}$  respectively" is true?

$$\frac{2}{3} + \frac{1}{4} + \frac{1}{6} = \frac{8+3+2}{12} = \frac{13}{12} = 1.09 > 1$$

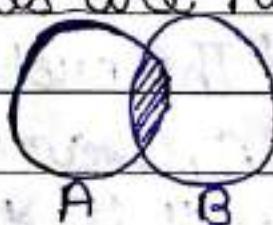
No, because the total probability is more than 1.

\* Addition of theorem of probability :-  
 If two event A, B are mutually exclusive  
 - the probability of occurrence of either  
 A or B is the sum of the individual  
 probability of A and B.



$$P(A \text{ or } B) = P(A) + P(B).$$

- When events are not mutually exclusive



$$P(A \text{ or } B) = P(A) + P(B) - P(AB).$$

Ques :- A bag contains 30 balls no. from 1 to 30  
 1 ball is drawn at random, find the  
 probability that the no. of the ball will be  
 multiple of 5 or 9.

Soln :- Multiple of 5 = 5, 10, 15, 20, 25, 30.  
 $P(\text{multiple of 5}) = \frac{6}{30} = \frac{1}{5}$

Multiple of 9 = 9, 18, 27

$$= \frac{3}{30} = \frac{1}{10}$$

P of multiple of 5 or 9 =  $P(\text{multiple of } 5) + P(\text{multiple of } 9)$

$$= \frac{1}{5} + \frac{1}{10}$$

$$= \frac{2+1}{10} = \frac{3}{10}$$

Ques 8- What is the probability of drawing a card of heart or an ace in a single draw from a standard pack of 52 cards?

Sol- Total cards = 52

$$P\text{ of heart} = \frac{13}{52}$$

$$P\text{ of ace} = \frac{4}{52}$$

$$P(\text{Hearts or ace}) = P(H) + P(Ace) - P(\text{Heartace})$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \frac{16}{52} = \frac{4}{13}$$

Ques 8- From a pack of 52 cards, A card is drawn at random. What is the probability of getting the card of heart or club or seven.

Total cards = 52

$$P \text{ of hearts} = \frac{13}{52}$$

$$P \text{ of club} = \frac{13}{52}$$

$$P \text{ of seven} = \frac{4}{52}$$

$$P(\text{Heart or club or seven}) = \frac{13}{52} + \frac{13}{52} + \frac{4}{52} - \frac{2}{52}$$

$$= \frac{28}{52} = \frac{7}{13}$$

Ques:- A bag contain 100 pages no. from 1 to 100. A page is open at random and is selected. Find the probability of the page is multiple of 6 or 10.

Multiple of 6 = 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96.

$$P(\text{multiple of 6}) = \frac{16}{100}$$

Multiple of 10 = 10, 20, 30, 40, 50, 60, 70, 80, 90, 100

$$P(\text{multiple of 10}) = \frac{10}{100}$$

$$P(\text{multiple of } 6 \text{ or } 10) = \frac{16}{100} + \frac{10}{100} - \frac{3}{100}$$

$$= \frac{23}{100}$$

Ques:- From a pack of 52 card . 1 card is drawn at random . What is the probability that the card is of spade or an owner.

Total cards = 52.

$$P \text{ of spade} = \frac{13}{52}$$

$$P \text{ of owner} = \frac{16}{52}$$

$$P(\text{spade or owner}) = P(S) + P(O) - P(\text{spade and owner})$$

$$= \frac{13}{52} + \frac{16}{52} - \frac{4}{52}$$

$$= \frac{25}{52}$$

Ques:- A dice is thrown . What is the probability of getting a no. less than 5 or an odd no.

Total no. of ways in a dice = 6.

Number less than 5 = 1, 2, 3, 4.

$$P(\text{less than } 5) = \frac{4}{6}$$

odd no. = 1, 3, 5

$$P(\text{odd no.}) = \frac{3}{6}$$

$P(\text{getting no. less than } 5 \text{ or odd no.}) =$

$$P(\text{less than } 5) + P(\text{odd no.}) - P(\text{odd no. less than } 5).$$

$$= \frac{4}{6} + \frac{3}{6} - \frac{2}{6} = \frac{5}{6}$$

Ques 3- Out of no. 1 to 150 cards, 1 no. is selected at random. What is probability that it is divisible by 3 or 5.

No. divisible by 3 = 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, 63, 66, 69, 72, 75, 78, 81, 84, 87, 90, 93, 96, 99, 102, 105, 108, 111, 114, 117, 120, 123, 126, 129, 132, 135, 138, 141, 144, 147, 150.

$$P(\text{divisible by } 3) = \frac{49}{150}$$

No. divisible by 5 = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 105, 110, 115, 120, 125, 130, 135, 140, 145, 150.

135, 140, 145, 150.

$$P(\text{divisible by 5}) = \frac{30}{150}$$

$$\begin{aligned} P(\text{divisible by 3 or 5}) &= \frac{49}{50} + \frac{30}{150} - \frac{10}{150} \\ &= \frac{69}{150} = \frac{23}{50} \end{aligned}$$

Ques:- From 30 Tickets mark with the first 30 no. one is drawn at random. It is then replaced and second draw is made. Find the chances that:-

- In the first draw it is a multiple of  $\frac{5}{7}$
- In the second it is multiple of 3 or 11.

a) Multiple of 5 = 5, 10, 15, 20, 25, 30

$$P(\text{M. of 5}) = \frac{6}{30}$$

Multiple of 7 = 7, 14, 21, 28.

$$P(\text{M. of 7}) = \frac{4}{30}$$

$$P(\text{M. of 5 or 7}) = \frac{6}{30} + \frac{4}{30} = \frac{10}{30} = \frac{1}{3}$$

(b) Multiple of 3 = 3, 9, 6, 12, 15, 18, 21, 24, 27, 30

Multiple of 11 = 11, 22.

$$P(\text{Lcm of } 5 \text{ or } 11) = \frac{10}{30} + \frac{2}{30} = \frac{12}{30} = \frac{2}{5}$$

**★ Multiplication theorem :-** If two events A and B are independent the probability that they will both occur is equal to the product of the individual probability. Symbolically, probability of A and B is equal to probability of multiplication of A and B.

$$P(A \text{ and } B) = P(A) \times P(B).$$

Ques:- If a coin is thrown what is the probability of getting Head on third and fourth tosses.

$$P(A) = \frac{1}{2}$$

$$P(B) = \frac{1}{2}$$

$$P(\text{Head on 3rd and 4th toss}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Ques:- 13 cards, each are distributed to 4 players from a pack of 52 cards. What is the probability that particular player got all the 4 ace.

$$\text{Total cards} = 52$$

$$\text{Ways of getting 13 cards} = \mathbf{52C_{13}}$$

$$4 \text{ card distributed to 4 players} = 4C_4$$

$$\text{Remaining cards} = 9$$

$$= 48C_9$$

$$P. \text{ of getting all ace} = \frac{4C_4 \times 48C_9}{52C_{13}}$$

$$= \frac{1 \times 48!}{39! \cdot 9!}$$

$$\frac{52!}{39! \cdot 13!}$$

$$= \frac{48! \times 39! \times 13!}{39! \times 9! \times 52!}$$

$$= \frac{48! \times 13 \times 12 \times 11 \times 10 \times 9!}{9! \times 52 \times 51 \times 50 \times 49 \times 48!}$$

$$= \frac{11}{4165} \quad \underline{\text{Ans.}}$$

Ques:- A bag contain 10 red and 6 green balls and 4 successive draw are made without replacement. Find the

probability that they are alternative  
of same colour.

$$P. \text{ of drawing first ball} = \frac{10}{16}$$

$$P. " " \text{ Second } " = \frac{6}{15}$$

$$" " " \text{ third } " = \frac{9}{14}$$

$$" " " \text{ fourth } " = \frac{5}{13}$$

$$\text{Required probability} = \frac{10}{16} \times \frac{6}{15} \times \frac{9}{14} \times \frac{5}{13}$$

$$= \frac{45}{728}$$

Ques: If 'A' speaks truth in 75% and 'B' in 90% of cases. In what percentage cases are they likely to contradict in each other in stating the same fact.

'A' speaks truth in 75% cases.

$$P(A) = \frac{75}{100} = \frac{3}{4}$$

$$\text{A not speak truth in cases} = 1 - \frac{1}{3} \\ = \frac{2}{3}$$

'B' speaks truth in 20% cases  $P(B) = \frac{9}{10} = \frac{9}{10}$

B not speaks truth in cases  $P(\bar{B}) = 1 - \frac{9}{10} = \frac{1}{10}$

P. of contradiction  $P(A) \times P(\bar{B}) = \frac{3}{4} \times \frac{1}{10} = \frac{3}{40}$

P. of contradiction  $P(\bar{A}) \times P(B) = \frac{1}{4} \times \frac{9}{10} = \frac{9}{40}$

Total Probability  $= \frac{3}{40} + \frac{9}{40}$

$$= \frac{12}{40} = \frac{3}{10}$$

$$= 30\% \text{ Ans}$$

## UNIT-6

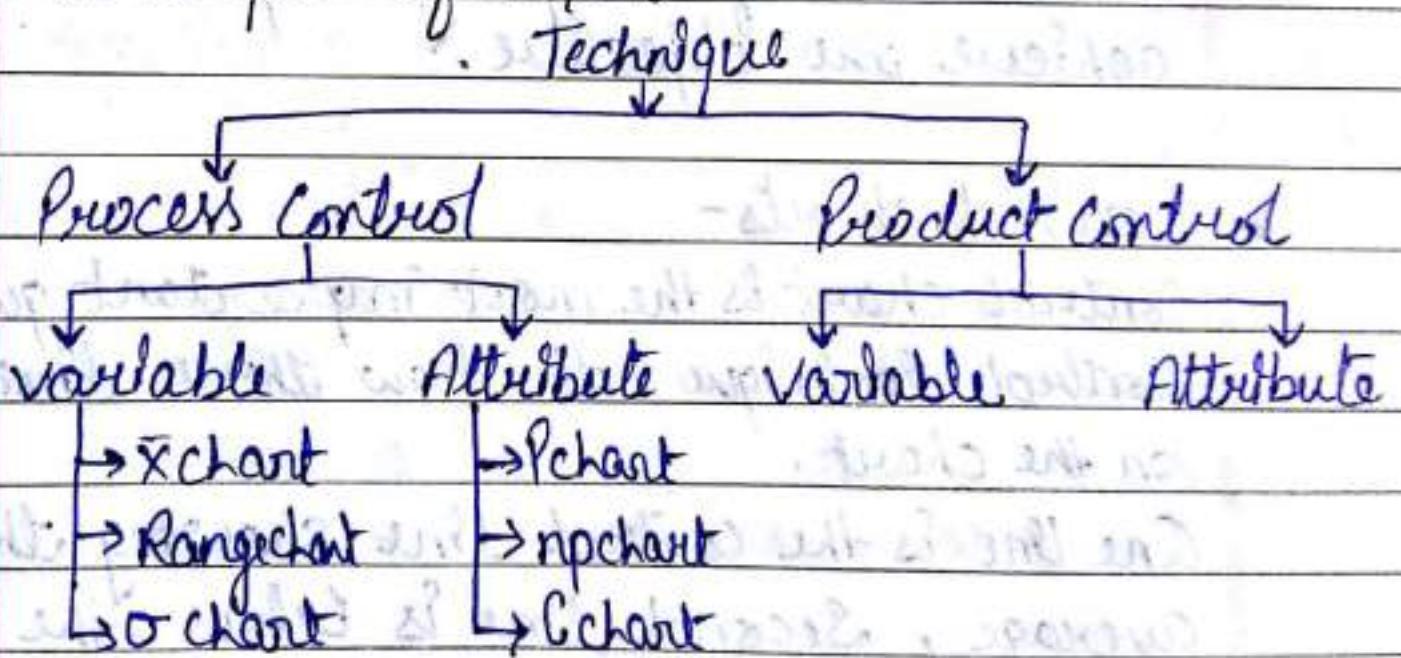
# Statistical Quality Control

- Statistical Quality Control :- It is the application of statistical techniques to determine how far the product conform to the standard of quality and to what extent quality is below the standard quality. The purpose of Statistical Quality Control is to discover and correct only those forces which are responsible for variation outside the stable pattern.
  
- Objective of Quality Control :-
- 1- To locate and identify the process fault, in order to control the defective waste.
- 2- To make necessary corrective measure to maintain the quality of the product.
- 3- To ensure that sub-standard products do not reach to the customer.
- 4- To achieve better utilization of man material and equipment.

\* Advantages of statistical quality control -

- 1- To ensure control, maintenance and improvement in the quality standard.
- 2- To provide better quality assurance at lower inspection cost.
- 3- It reduces the wastage of time and material to the minimum as it reduces the inspection and manufacturing cost and enhance profit.
- 4- It provided a basis for resolving the difference among the various interest in the organisation.

\* Techniques of SQC :-



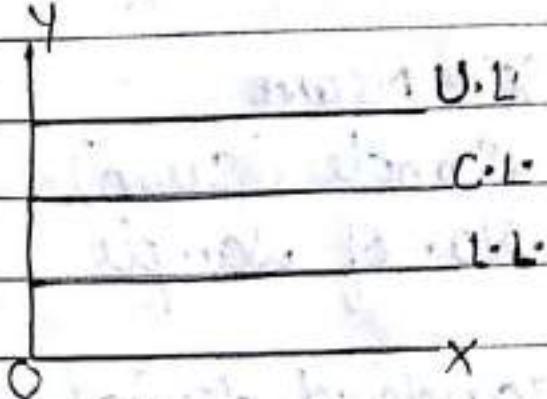
- 1- Process control :- It is concern with controlling the quality of the product during the production process. It ensure that a product of only standard is produce and make use of control charts.
- 2- Product Control :- It is concern with controlling the quality of the product by critical examination at strategic point. It is concern with inspection of goods already produce to a certain know whether they are fit to be dispatched or not. It make use of sampling inspection to achieve the objective.

- Control charts -

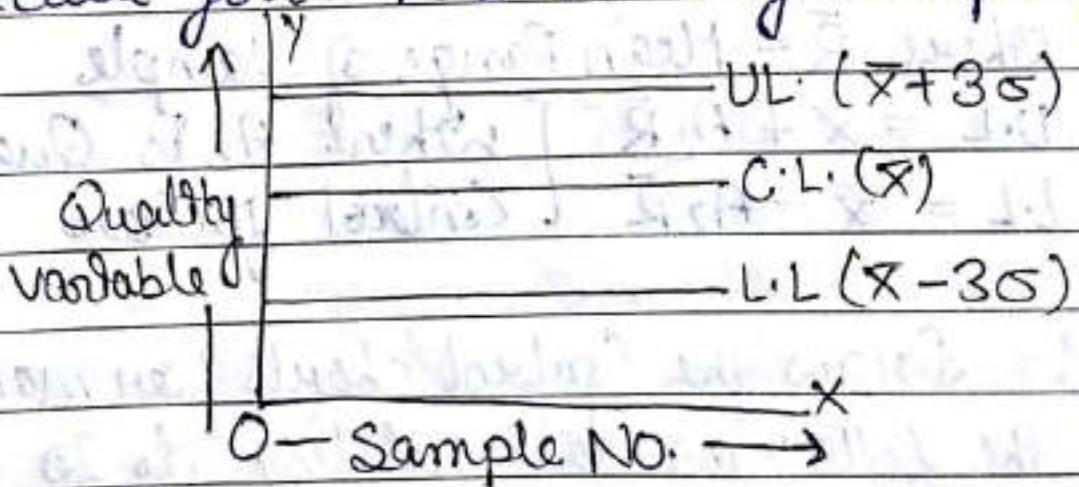
Control chart is the most important quality control technique. It show three lines on the chart.

One line is the control line showing the average, Second line is below the first line indicates the limit of tolerances, Third line is above the

first line which show the limit of higher tolerance.



- \* Acceptance Sampling :- Under this technique a sample is selected at random to determine whether it conform to the standard laid down. It can be assumed that a certain percentage of goods will not conform to the standard. So, a certain percentage of defective goods in a lot may be specifying.



- \* Control charts for mean :- ( $\bar{x}$  chart) :-

$$\text{Central Line } (\bar{x}) = \frac{\sum x}{K}$$

where  $\bar{x}$  = Mean

$\sum x$  = Sample (sum).

$K$  = No. of sample

- When standard deviation are known :-

$$U.L = \bar{x} + \frac{3\sigma_p}{\sqrt{N}} \quad [\sigma_p = S.D. \text{ of Population}]$$

$$L.L = \bar{x} - \frac{3\sigma_p}{\sqrt{N}} \quad [N = \text{No. of items in sample}]$$

$$U.L = \bar{x} + \frac{3\sigma_p}{\sqrt{N}}$$

- When S.D. are not known :-

$$\sigma_p = \frac{\bar{R}}{d_2}$$

Where  $\bar{R}$  = Mean Range of Sample

$$U.L = \bar{x} + A_2 \bar{R} \quad [\text{where } A_2 \text{ is Quality}]$$

$$L.L = \bar{x} - A_2 \bar{R} \quad [\text{control factor}]$$

Ques:- Draw the Control charts for man from the following data relating to 20 Sample. Each of size 5.

Sample No.	Mean	Range	Sample No.	Mean	Range
1	38.2	15	11	32.6	31
2	33.8	1	12	22.8	12
3	24.4	22	13	21.6	29
4	36.6	24	14	28.8	22
5	27.4	18	15	24.4	16
6	30.6	33	16	30.4	19
7	31.2	21	17	25.4	20
8	27	29	18	25.4	34
9	24	29	19	27.8	19
10	29.4	18	20	31.4	17

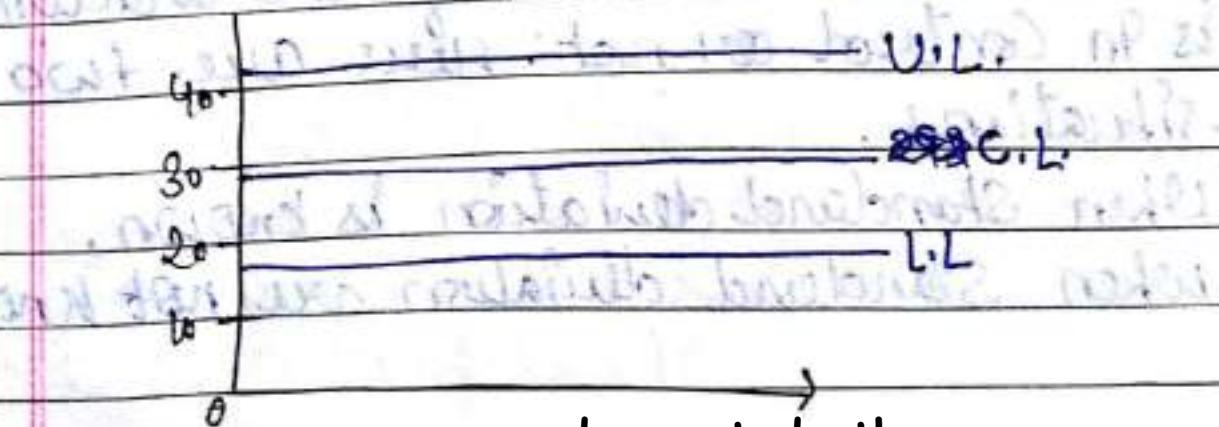
$$D_2 = 2.326$$

$$D_3 = 0.864$$

Solution :-  $C\bar{L}m(\bar{x}) = \frac{\sum x_i}{K} = \frac{583.1}{20} = 29.155$

$$U.L. = \bar{x} + \frac{3\bar{\sigma}_p}{\sqrt{N}} = 29.16 + \frac{2.326}{\sqrt{5}} = 41.44$$

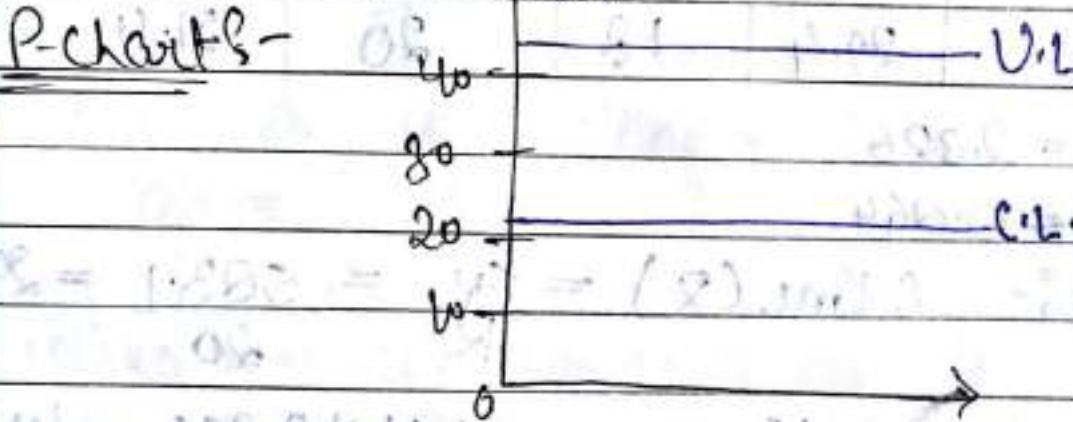
$$L.L. = \bar{x} - \frac{3\bar{\sigma}_p}{\sqrt{N}} = 29.16 - \frac{2.326}{\sqrt{5}} = 17.26$$



$$\bar{R} = \frac{\sum R}{k} = \frac{420}{20} = 21.$$

$$\begin{aligned} UCL &= \bar{R} + 3SR \\ &= 21 + 3 \times 7.8 \quad (SR = 7.8) \\ &= 21 + 22.4 \\ &= 43.4. \end{aligned}$$

$$\begin{aligned} LCL &= \bar{R} - 3SR \\ &= 21 - 3 \times 7.8 \\ &= 21 - 22.4 \\ &= 21 - 14 = 0. \end{aligned}$$



★ Control chart for Range (R chart) :-

The Range chart is used to see whether the Standard deviation of the characteristics is in control or not. Here are two situations.

- 1- When Standard deviation is known.
- 2- when Standard deviation are not known.

- When S.D. are known :-

Range = Highest - Lowest

Mean of Range =  $\frac{\sum R}{K}$  [K = total no. of sample]

Central Line  $\Rightarrow (\bar{R}) = d_2 \bar{O_p}$

$$U.L. = d_2 \bar{O_p} + 3\bar{O_p} \times d_3$$

$$L.L. = d_2 \bar{O_p} - 3\bar{O_p} \times d_3$$

- When S.D. are not known :-

Control Chart =  $\frac{\sum R}{K}$

$$U.L. = D_4 \bar{R}$$

$$L.L. = D_3 \bar{R}$$

\* Control charts attribute :-

P Chart -

Central Line =  $\frac{\sum p}{n \times K}$

$$U.L. = \bar{p} + 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$L.L. = \bar{p} - 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

(n is sample size, K is total no. sample)  
(p = central line).

nPchart :-

$$\text{Central Unit} = \frac{\sum p}{K}$$

$$U.\text{ Control Unit} = n\bar{p} + 3\sqrt{np(1-p)}$$

$$L.\text{ " } = n\bar{p} - 3\sqrt{np(1-p)}$$

C-chart :-

$$\text{Central Line} = \frac{\text{Total defect}}{\text{Total no. of items inspected}}$$

$$UCL = \bar{C} + 3\sqrt{C}$$

$$LCL = \bar{C} - 3\sqrt{C}$$

If lower limit is negative it is taken to be equal to 0.

R-chart :-

$$\bar{R} = \frac{\sum R}{K}$$

$$UCL = \bar{R} + 3\sigma_R$$

$$LCL = \bar{R} - 3\sigma_R$$

Ques :- The following table give the result of inspection of 20 sample of 100 items each taken in 20 working day. Draw P-chart.

Sample No.	No. of defective	Sample no.	No. of defective
1	6	11	10
2	2	12	4
3	4	13	6
4	1	14	11
5	20	15	22
6	6	16	8
7	10	17	0
8	19	18	3
9	4	19	23
10	21	20	10

Total no. of item inspected = No. of Sample × unit inspected each day

$$= 100 \times 20$$

$$= 2000$$

Average fraction =  $\frac{\text{Total no. of defective}}{\text{Total no. of items inspected}}$

$$= \frac{200}{2000} = \frac{1}{10}$$

$$CL = 0.10$$

$$UL = \bar{P} + 3\sqrt{\bar{P}(1-\bar{P})}$$

$$= 0.10 + 3\sqrt{\frac{0.10(1-0.10)}{100}}$$

$$= 0.19$$

PAGE NO.	8
DATE	20/10/19

$$L.L. = 10 - 3 \int_{0}^{10} \frac{0.10(1-0.10)}{100} = 01.$$

• 19 ..... U.L

• 18 ..... C.L.

• 17 ..... C.L.

• 16 ..... C.L.

• 15 ..... C.L.

• 14 ..... C.L.

• 13 ..... C.L.

• 12 ..... C.L.

• 11 ..... C.L.

• 10 ..... C.L.

• 9 ..... C.L.

• 8 ..... C.L.

• 7 ..... C.L.

• 6 ..... C.L.

• 5 ..... C.L.

• 4 ..... C.L.

• 3 ..... C.L.

• 2 ..... C.L.

• 1 ..... C.L.

0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30



# END

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