Reinforcement Learning Assignment-1

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1. Histogram obtained after sampling $N = 100$ samples:

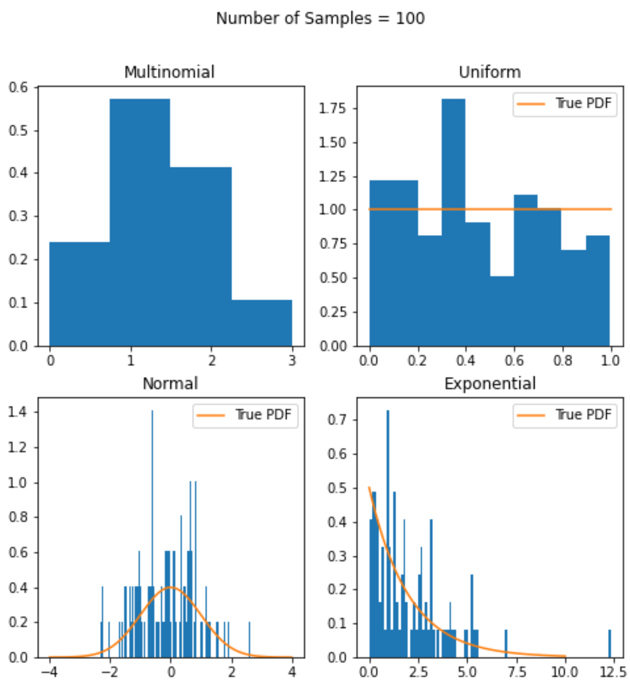

Figure 1: Histogram obtained after sampling $N = 100$ samples.

Histogram obtained after sampling $N = 1000$ samples:

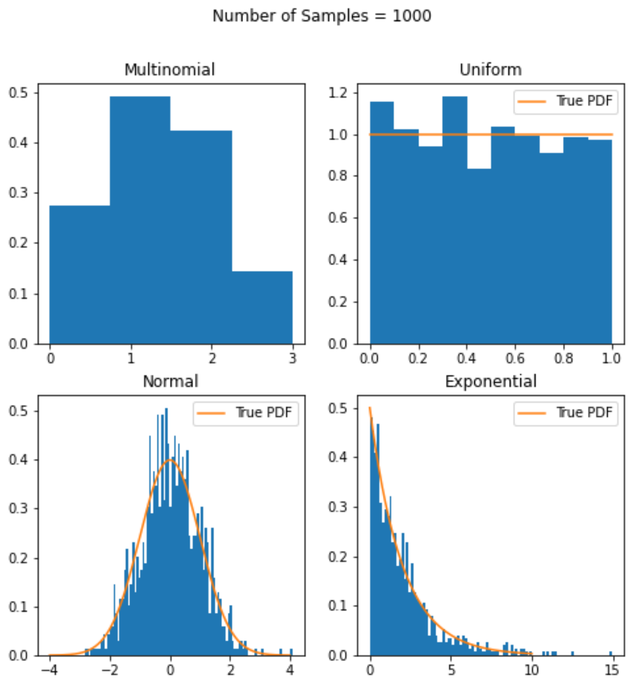

Figure 2: Histogram obtained after sampling $N = 1000$ samples.

Histogram obtained after sampling $N = 10000$ samples:

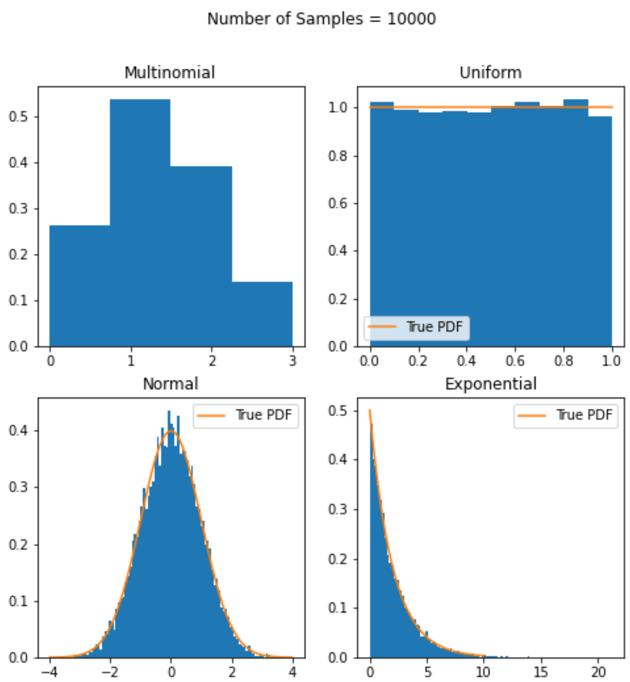

Figure 3: Histogram obtained after sampling $N = 10000$ samples.

2. Let $U \sim Unif(0,1)$ and $\Phi(x)$ denote the standard normal CDF. Then, we can show that if $X = \Phi^{-1}(U)$, then $X \sim \mathcal{N}(0,1)$. This is because $P(X \leq t) = P(\Phi^{-1}(U) \leq t) = P(U \leq$

$\Phi(t)) = \Phi(t)$. Now, since the CDF X is $\Phi(x)$, hence, $X \sim \mathcal{N}(0,1)$.

Let u be a sample from $Unif(0,1)$. Then using the above fact, we generate samples standard normal random samples as $x = \Phi^{-1}(u)$. Then, to generate normal samples with mean μ and variance σ^2 we can simply transform as $y = \sigma x + \mu$, where y is the sample from normal random variable with mean μ and variance σ^2.

3. **Idea:** In order to compute the integral numerically, we can represent the integration as an expectation of a random variable. Then using Law of Large Numbers, we can compute the expectation, by sampling from the distribution of that random variable.

   Let's $X \sim Unif(0, \pi)$ and $f(x)$ be it's PDF i.e.,

$$f(x) = \begin{cases} \frac{1}{\pi} & if 0 \leq x \leq \pi \\ 0 & otherwise \end{cases}$$

   (a) Let $g(X) = \sqrt{(}sin(X))$, where X is as defined above. Then

$$\mathbb{E}[g(X)] = \int_{-\infty}^{\infty} g(x)f(x)\,dx = \frac{1}{\pi}\int_{0}^{\pi} g(x)\,dx \tag{1}$$

   Let $X_1, X_2, ..., X_N \overset{i.i.d.}{\sim}$ Unif$(0, \pi)$ and $Y_i = g(X_i)$ for $i \in 1, 2, ...N$. Then according to Law of Large Numbers, we have

$$\lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} Y_i = \mathbb{E}[g(X)]$$

   Now,

$$\int_{0}^{\pi} \sqrt{(}sin(x))\,dx = \pi\left(\lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} Y_i\right) \tag{2}$$

   (b) We can follow the same thing as above but by defining $g(X) = \sqrt{(}sin(X))exp(-x^2)$.