CHAPTER 1 Motion

Motion is one of the more common events in your surroundings. You can see motion in natural events such as clouds moving, rain and snow falling, and streams of water moving, etc. Motion can also be seen in the activities of people who walk, jog, or drive various vehicles from place to place. Motion is so common that you may think that everyone understand the concepts of motion. But history indicates that it was only during the past three hundred years or so that people began to understand motion correctly. The foundations for the study of motion were laid down more than 300 years ago by Galileo in Italy and later by Isaac Newton in England.

The study of motion comes in the branch of physics called '**mechanics**'. It is broken down into two parts, kinematics and dynamics. **Kinematics** is the "how" of motion, that is, the study of how objects move, without concerning that why they move. **Dynamics** is the "why" of motion. In dynamics, we are concerned with the causes of motion, which is the study of forces.

1.1 Position and reference point

Suppose you leave in near Allen career institute (Samanvaya building), Kota. Fig.1 shows the aerial view (top view) of your locality. To reach Allen's Samanvaya building, you follow a path shown in the fig.1. Your house is the starting place for you to find the location, or position, of Allen's Samanvaya building.

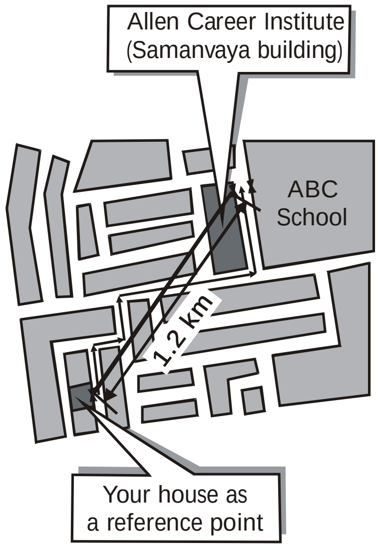


Fig.1 A reference point is needed in order to describe the location of a an object.

- A **reference point** is a starting point used to describe the position of an object. A reference point is also called the **origin**.

To describe an object's position, three things must be included in the description : (i) a reference point, (ii) a direction from the reference point, (iii) distance from the reference point (the length of the line segment joining the reference point and the object).

For example, in the fig.1, choose your house as the reference point. Next, choose a direction from the reference point, let it be 'toward the ABC school' (see fig.1). Finally, give the distance from the reference point: let it be 1.2 km.

How to describe the reference direction ?

One way of indicating the direction is to use a plus (+) or a minus (–) sign. The plus sign can be the direction from the reference point is in the reference direction (see fig.2). A minus sign means the direction is opposite to the reference direction. For example, plus (+) sign can be used to indicate 'toward the school' in the fig.1 on previous page and minus (–) sign to indicate 'away from the school'.

- The position of an object can be described as a distance from the origin together with a plus or minus sign that indicates the direction.
- Cartesian system of representing position of a particle is shown in fig.3

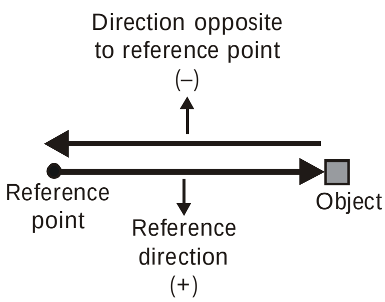


Fig.2 Describing the reference direction : sign conventions.

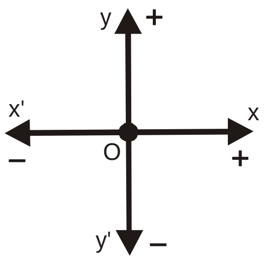


Fig.3 Cartesian system : Sign convention for position

NUMERICAL CHALLENGE 1.1

In the fig.4, a car is parked at 100 m from the centre of a crossing. Also a tree is located 75 m from the car as shown in fig.4. The reference direction and its sign is mentioned in the fig.4. How will you express the car's position if

(i) when the centre of the crossing is taken as reference point (or origin),

(ii) when the tree is taken as reference point (or origin).

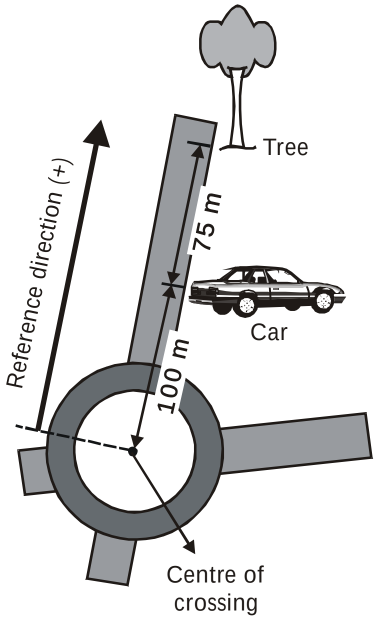


Fig.4

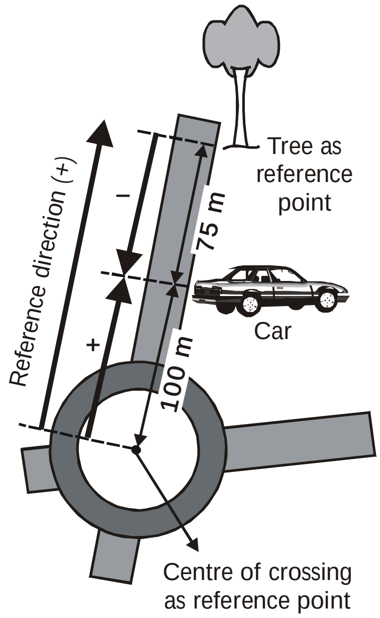


Fig.5

Solution

(i) Here, the reference point is centre of the crossing. The car is 100 m away from this reference point. If draw a direction from this reference point to the car, this direction is same as the reference direction (see fig.5). Thus, the sign of this direction must be taken positive. Hence, the position of car is expressed or represented as **+ 100 m**.

(ii) Here, the reference point is the tree. The car is 75 m away from this reference point. If draw a direction from this reference point to the car, this direction is opposite to the reference direction (see fig.5). Thus, the sign of this direction must be taken negative. Hence, the position of car is expressed or represented as **– 75 m**.

Thus, an object's position is its location compared to other things. Position of an object is not absolute, it is a variable that gives location of an object **relative** to a reference point or origin.

1.2 Understanding motion

Consider a ball that you notice one morning in the middle of a lawn. Later in the afternoon, you notice that the ball is at the edge of the lawn, against a fence, and you wonder if the wind or some person moved the ball. You do not know if the wind blew it at a steady rate, or even if some children kicked it all over the yard. All you know for sure is that the ball has been moved because it is in a different position after some time passed. These are the two important aspects of motion :

(1) A change of position (2) The passage of time

Moving involves a change of position during some time period. Motion is the act or process of something changing position. The motion of an object is usually described with respect to a stationary object. Such a stationary object is said to be 'at rest'.

■ **Motion** is a change in an object's position compared to a fixed object. If you ride in a car, your position changes compared to a tree or an electric pole.

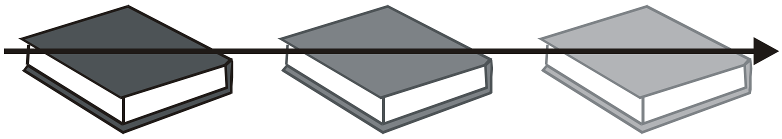
- An object is said to be at **rest** if it does not change its position with time.

Rest and motion are relative terms

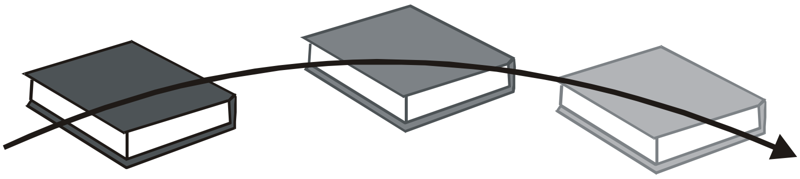
Imagine that you are traveling in an automobile with another person. You know that you are moving across the land outside the car since your location on the highway changes from one moment to another. Observing your fellow passenger shows that there is no change of position. You are in motion relative to the ground but you are not in motion relative to your fellow passenger. The motion of any object or body is the process of a change in position 'relative' to some reference object or location.

Translational motion (or translatory motion)

Motion of a body in which all the points in the body follow parallel paths is called 'translational motion'. It is a motion in which the orientation of an object remains the same throughout the journey . The path of a translatory motion can be straight or curved (see fig.6).



(a) A book moved along a straight path without changing its orientation



(b) A book moved along a curved path without changing its orientation

Fig.6 Translational motion

On the basis of the path travelled by an object, the translational motion can classified as :

(1) **Rectilinear motion :** If an object moves in a straight line, its motion is called rectilinear motion or one dimensional motion. Motion of car along a straight path, motion of a piston in the cylinder are examples of rectilinear motion.

(2) **Curvilinear motion :** If an object moves along a curved path without change in its orientation, its motion is called curvilinear motion. Motion of a car along a curved or circular path, motion of an athlete on a circular track are examples of curvilinear motion.

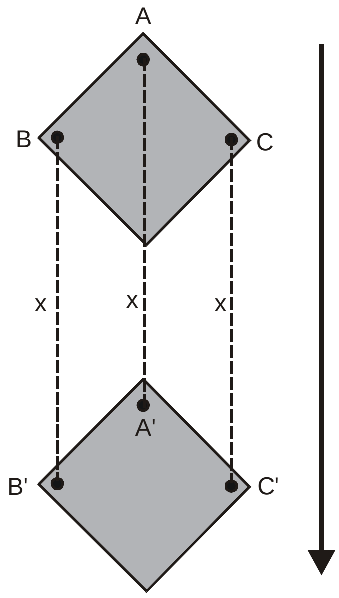


Fig.7 Translational motion : The particles of the object shown cover same distance in a given time.

Rotational motion (Rotatory motion)

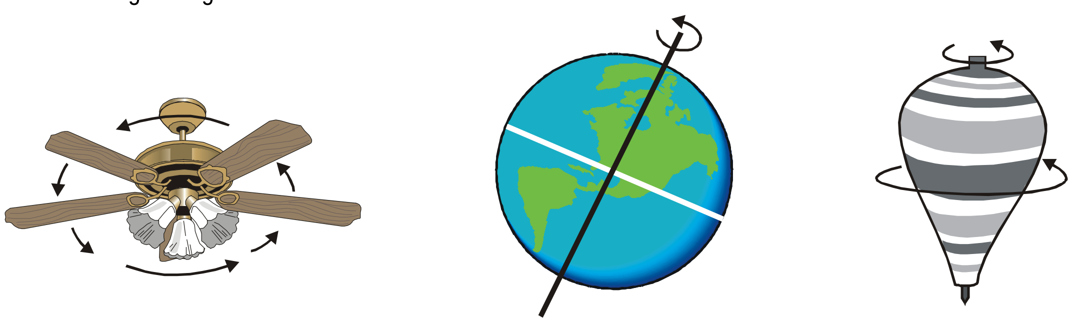
Motion of a body turning about an axis is called rotational motion. In other words, 'a motion in which an object spins about a fixed axis is called rotational motion'. It is a motion in which the orientation of an object continuously changes throughout the motion. The path of an object in a rotational motion is always circular.

Some examples of rotational motion are :

(1) The Earth's spin on its axis. (2) Motion of a fan or motor.

(3) Motion of blades of windmill. (4) Motion of a spinning top.

(5) Motion of a grinding stone.



(a) Motion of a ceiling fan (b) Motion of Earth about its axis (c) Motion of a spinning top

Fig.8 Some examples of rotational motion

■ In rotational motion, the particles of the object move through the unequal distances in a given time depending on their location in the object (see fig.9). The particle which is located near the axis of rotation, covers less distance as compared to the particle that is located far away from the axis.

- In translational motion at any instant of time every particle of the body has the same velocity while in rotational motion at any instant of time particles of the body have different velocities depending on their position from the axis of rotation.
- In rotation of a body about a fixed axis, every particle of the body moves in a circle, which lies in a plane perpendicular to the axis and has its centre on the axis.

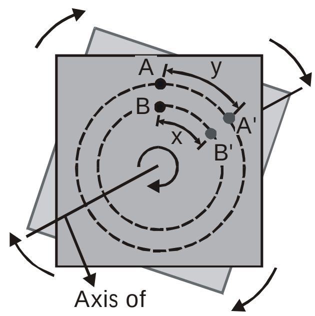


Fig.9 Rotational motion : Particles cover unequal distances in a given time.

CONCEPTUAL CHALLENGE 1.1

In fig.10, motion a frying pan used in kitchen is shown. Is the motion of the frying pan a translational motion ? Can it be considered as rotational motion ? Explain.

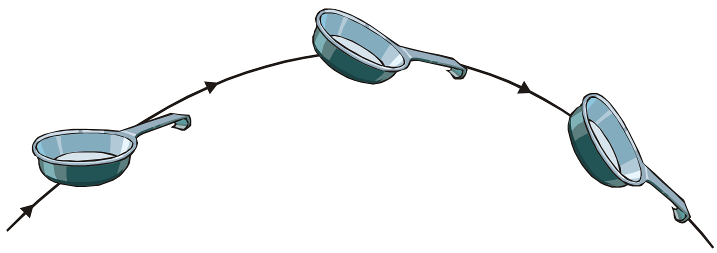


Fig.10 Conceptual challenge 2.1

Explanation

The motion of frying pan shown in fig.10 cannot be considered as translational motion though it is moving along a curved path. This is because its orientation is changing during its journey. Also, the motion of frying pan cannot be considered as rotational motion though it is spinning. This is because, rotation means spinning of an object about a fixed axis. Here, the flask is not spinning about a fixed axis. This type of motion is ‘a combination of translational motion and rotational motion’.

■ Motion of a car or cycle wheels is a combination of translational and rotational motion (see fig.11).

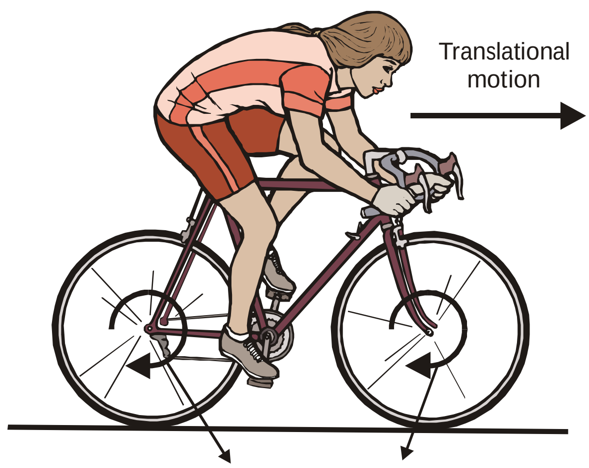


Fig.10 Motion of a cycle wheel is a combination of translational and rotational motion. Motion of a wheel is also called ‘rolling motion’.

1.3 Uniform and non-uniform motion

Uniform motion

If a body covers equal distances in equal intervals of time in a particular direction, its motion is called 'uniform motion'. In other words, 'if the velocity of a body is constant, its motion is called uniform motion'.

■ A uniform motion always takes place in straight line. Any motion along a curved path is not a uniform motion.

Examples : (i) A car moving with a constant speed in straight line (ii) Motion of an athlete along straight path with constant speed.

- In real world, we rarely see uniform motion. A body can be in uniform motion for a short time period then we have to take turn, reduce speed or increase speed according to our need.

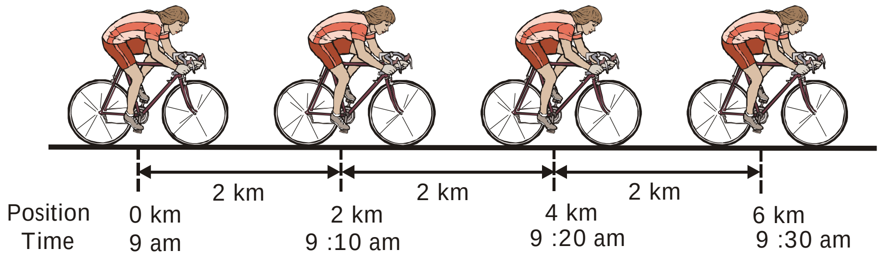


Fig.11 Motion of this cyclist is uniform motion ; she is covering equal distances in equal intervals of time; she is moving in straight line in a particular direction.

Non-uniform motion

If velocity of a body is variable, its motion is called non-uniform motion.

■ For non-uniform motion,

(a) Magnitude of velocity is variable, or

(b) Direction of velocity is variable, or

(c) Both the magnitude as well as direction the velocity is variable.

- If a body covers unequal distances in equal intervals of time, its motion is called 'non-uniform motion'.
- Even if a body covers equal distances in equal intervals of time but it changes its direction, still its motion is said to be 'non-uniform'.
- Motion of a particle along a curved path is always a non-uniform motion. If particle changes its direction during the journey, its motion is always non-uniform.

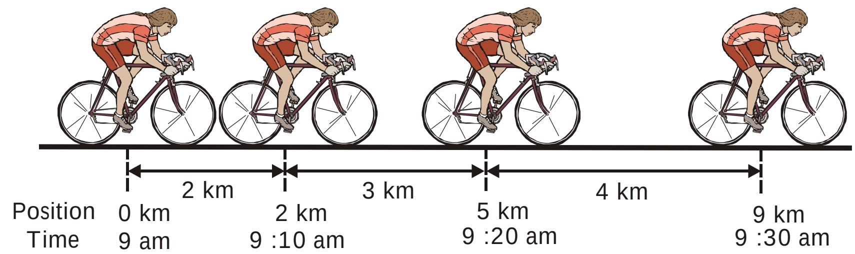


Fig.12 Motion of this cyclist is 'non-uniform' ; she is covering unequal distances in equal intervals of time; she is moving in straight line in a particular direction.

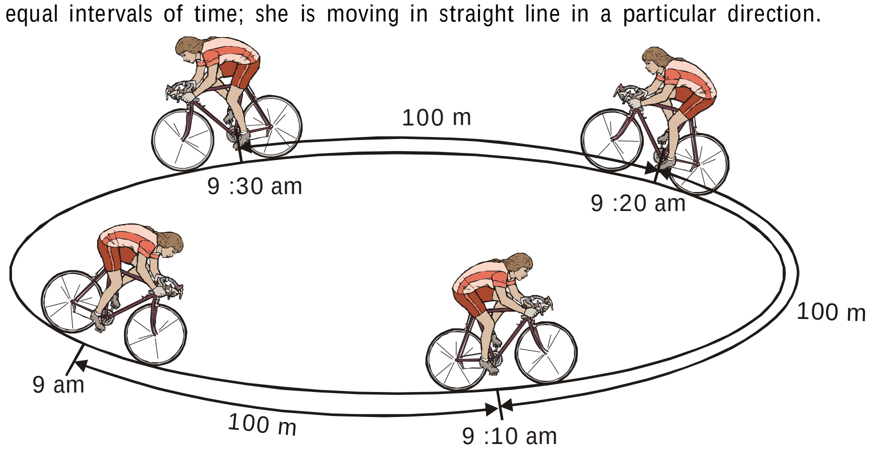


Fig.13 This cyclist is moving along a circular path ; she is covering equal distances in equal intervals of time ; still this is a non-uniform motion as this is not in a straight line.

1.4 Distance and displacement

Distance

The length of the actual path between initial and final positions of a moving object is called 'distance'.

- Distance is a scalar quantity.
- Distance depends on the path.
- Distance is always taken positive.

Unit of distance : In S.I. system unit of distance is metre (m). Some other popular units are millimetre (mm), centimetre (cm), kilometre (km).

Displacement

The shortest distance between the initial position and the final position of the particle is called displacement.

It is also defined as the change in the position of the particle.

Displacement = $x_f - x_i$

Where, x_f = final position ; x_i = initial position.

- Displacement is a vector quantity, its direction is always taken from initial position to final position.
- Displacement depends only on initial position and final position, does not depend on path.
- Displacement of a particle in motion can be positive, negative or even zero.

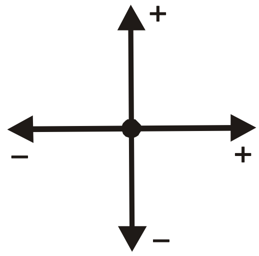


Fig.14 Sign convention for displacement.

Unit of displacement : Units of distance and displacement are same as both represent some length. Thus, in S.I. system unit of displacement is metre (m). Some other popular unts are millimetre (mm), centimetre (cm), kilometre (km).

Let us understand the distance and displacement using some real life situation. Suppose a boy walks in a park, as shown in fig.15. His initial position is A. He first walks a distance of 30 m due east. Then, he walks 40 m due north. Here, the distance travelled by him is AB + BC = 30 m + 40 m = **70 m**.

His displacement is given by,

BC = $\sqrt{AB^2 + BC^2}$ = $\sqrt{(30)^2 + (40)^2}$ = $\sqrt{2500}$ = **50 m**

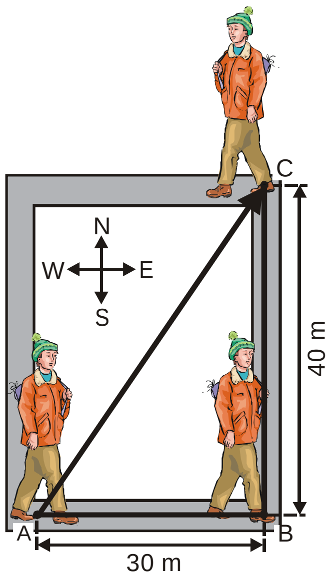


Fig.15 Understanding distance and displacement

- ■ Distance is always greater than or equal to the magnitude of displacement.
  - Whenever a particle changes its direction or follows a curved path, distance is always greater than the magnitude of displacement.
  - Distance is exactly equal to displacement (i) when it follows a straight path without changing its direction (ii) when its is in uniform motion.

NUMERICAL CHALLENGE 1.2

In the fig.16, a car moves on the road from the 20 km mark (its initial position) to the 100 km mark. After that, it reverses and moves back to the 50 km mark (its final position). Find the displacement and distance travelled by the car.

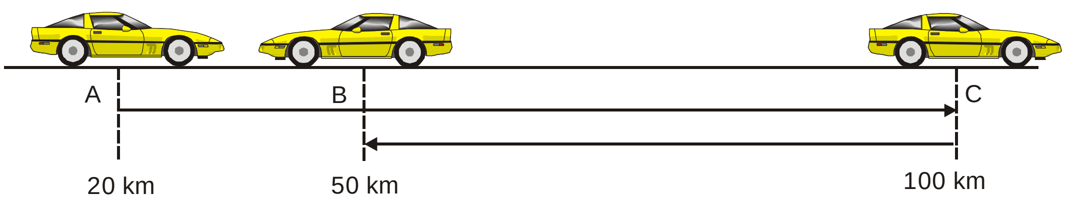


Fig.16 Numerical challenge 2.2

Solution

Given, initial position, x_i = + 20 km ; final position, x_f = +50 km

Displacement = $x_f - x_i$ = (+50) – (+20) = **+ 30 km**

Now, distance travelled by car from A to C, AC = 100 – 20 = 80 km

Distance travelled by car from C to B, BC = 100 – 50 = 50 km

Total distance travelled by car = AB +BC = 80 + 50 = **130 km**

1.5 Speed and velocity

Speed

The distance travelled by a particle per unit time is called speed.

$$\boxed{\text{Speed} = \frac{\text{Distance}}{\text{time}}}$$

- Speed is a scalar quantity.
- Speed depends on the path.
- $\boxed{1 \text{ km / h} = \frac{5}{18} \text{m / s}}$
- Speed gives no idea about the direction of motion of the object.
- Speed can never be negative ; in motion, it is taken positive ; at rest, it is zero.

Unit of speed : C.G.S.system - centimetre/second (cm/s) ; S.I. system - metre/second (m/s).

Uniform speed : An object is said to be moving with a uniform speed, if it covers equal distances in equal intervals of time. That is, magnitude of speed is constant.

Non uniform speed : An object is said to be moving with a variable speed if it covers unequal distances in equal intervals of time. That is, magnitude of speed is variable.

Average Speed : When an object is moving with a variable speed, then the average speed of the object is thought to be that constant speed with which the object covers the same distance in a given time interval as it does while moving with variable speed during the same time interval.

Average speed is the ratio of the total distance travelled by the object to the total time taken.

$$\boxed{\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}}$$

Instantaneous speed : The speed of the body at any instant of time is called instantaneous speed.

- **Speedometer** of the vehicle measures its instantaneous speed.
- In uniform motion of a particle, the instantaneous speed is equal to its average speed.

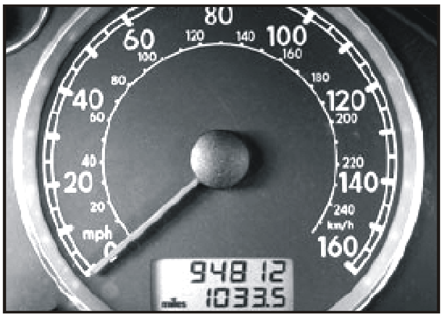


Fig.17 A speedometer measures speed but not velocity.

Velocity

The rate of change of displacement is called velocity.

- Velocity is a vector quantity.
- Velocity can be negative, positive or zero.
- The direction of average velocity is same as that of the total displacement.
- If average velocity for a journey is positive, it may have a negative instantaneous velocity at some point of time during the journey and vice-versa.

Unit of velocity : C.G.S.system - centimetre/second (cm/s) ; S.I. system - metre/second (m/s).

Instantaneous velocity : It is the velocity at some particular instant of time.

Average velocity : It is the ratio of total displacement to the total time taken.

$$\boxed{\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}}}$$

Uniform Velocity : A particle is said to have uniform velocity, if the magnitude as well as the direction of its velocity remains constant. It is possible only when the particles moves in straight line without changing its direction.

Non-uniform Velocity : A particle is said to have non-uniform velocity, if either of magnitude or direction of its velocity changes (or both changes).

- In uniform motion of a particle, the instantaneous velocity is equal to its average velocity.

- Average speed is always greater than or equal to the magnitude of average velocity.
  - Whenever a particle changes its direction or follows a curved path, average speed is always greater than the magnitude of average velocity.
  - Average speed is exactly equal to average velocity when it follows a straight path without changing its direction.

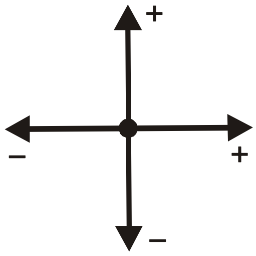


Fig.18 Sign convention for velocity

- If body covers distances x_1, x_2, x_3,----- with speeds v_1, v_2, v_3,----- respectively in same direction then average speed/average velocity of body is given by,

$$\boxed{v_{average} = \frac{x_1 + x_2 + x_3 + ---}{\frac{x_1}{v_1} + \frac{x_2}{v_2} + \frac{x_3}{v_3} + ---}}$$

  - **Case of half journey :** If body covers equal distances with different speeds i.e, $x_1 = x_2 = x$ (let),

$$v_{average} = \frac{x + x}{\left(\frac{x}{v_1} + \frac{x}{v_2}\right)} = \frac{2x}{x\left(\frac{1}{v_1} + \frac{1}{v_2}\right)} = \frac{2}{\left(\frac{v_2 + v_1}{v_1 v_2}\right)} = \boxed{\frac{2v_1v_2}{v_1 + v_2}}$$

  - If a body covers three equal distances with speeds v_1, v_2 and v_3 respectively, then average speed is given by,

$$v_{average} = \frac{x + x + x}{\left(\frac{x}{v_1} + \frac{x}{v_2} + \frac{x}{v_3}\right)} = \frac{3x}{x\left(\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3}\right)} = \frac{3}{\left(\frac{v_1v_2 + v_2v_3 + v_3v_1}{v_1v_2v_3}\right)} = \boxed{\frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}}$$

- If a body travels with speeds v_1, v_2, v_3 ,---- during time intervals t_1, t_2, t_3, ---- respectively then the average speed of the body is given by,

$$\boxed{v_{average} = \frac{v_1t_1 + v_2t_2 + v_3t_3 + ---}{t_1 + t_2 + t_3 + ---}}$$

  - If the two given time intervals are same i.e., $t_1 = t_2 = t$ (let), then,

$$v_{average} = \frac{v_1t + v_2t}{t + t} = \frac{(v_1 + v_2)t}{2t} = \boxed{\frac{v_1 + v_2}{2}}$$

  - If the three given time intervals are same i.e., $t_1 = t_2 = t_3 = t$ (let), then,

$$v_{average} = \frac{v_1t + v_2t + v_3t}{t + t + t} = \frac{(v_1 + v_2 + v_3)t}{3t} = \boxed{\frac{v_1 + v_2 + v_3}{3}}$$

NUMERICAL CHALLENGE 1.3

An auto travels at a rate of 25 km/hr for 4 min. then 50 km/hr for 8 min., finally at 20 km/hr for 2 min., find the distance travelled in km and the average speed for complete trip in m/s.

Solution

Given, v_1 = 25 km/hr ; v_2 = 50 km/hr ; v_3 = 20 km/hr ;

t_1 =4 min = (4/60) hr ; t_2 = 8 min = (8/60) hr ; t_1 = 2 min = (2/60) hr

Distance travelled, $s = v_1t_1 + v_2t_2 + v_3t_3$

$$= 25 \times \frac{4}{60} + 50 \times \frac{8}{60} + 20 \times \frac{2}{60} = \frac{100 + 400 + 40}{60} = \frac{540}{60} = \mathbf{9\ km}$$

Total time, $t = t_1 + t_2 + t_3 = \frac{4}{60} + \frac{8}{60} + \frac{2}{60} = \frac{14}{60} = \frac{7}{30}$ hr

Average speed, $v = \frac{s}{t} = \frac{9\text{ km}}{(7/30)\text{hr}} = \frac{9 \times 30}{7}$ km / hr $= \frac{9 \times 30}{7} \times \frac{5}{18}$ m / s $= \frac{75}{7}$ m / s = **10.7 m/s**

NUMERICAL CHALLENGE 1.4

On a 60 km track, a train travels the first 30 km at a uniform speed of 30 km/hr. How fast must the train travel the next 30 km so as to average 40 km/hr for entire trip ?

Solution

Given, speed for first 30 km, v_1 = 30 km/hr ; speed for next 30 km, v_2 = ? ;
average speed, $v_{average}$ = 40 km/hr.
This is a case of half journey, therefore, we can apply the formula for half journey directly.

$$v_{average} = \frac{2v_1v_2}{v_1+v_2} \quad \text{or} \quad 40 = \frac{2(30)v_2}{30+v_2} \quad \text{or} \quad 40 = \frac{60v_2}{30+v_2} \quad \text{or} \quad 2 = \frac{3v_2}{30+v_2}$$

or $2(30 + v_2) = 3v_2$ or $60 + 2v_2 = 3v_2$
or v_2 = **60 km/hr**

1.6 Acceleration

- The rate of change of velocity is called acceleration.
  - It is a vector quantity. Its direction is same as that of change in velocity and NOT of the velocity.
  - It is NOT the rate of change of speed. For example, when a body moving with constant speed along a circular path, there is no change in its speed but there is a change in velocity as its direction is changing continuously at every point. Thus, there must be some acceleration of the body.
  - A change in velocity occurs when (i) only its direction changes, e.g. uniform circular motion. (ii) only its magnitude changes. e.g. a ball dropped from a certain height under gravity (iii) both magnitude as well as direction changes, e.g. a projectile motion. In all these cases, there MUST be some acceleration present in the motion.
  - Whenever velocity and acceleration are in same direction, the velocity of a particle increases. Such motion is called accelerated motion. Such an acceleration for numericals is usually taken 'positive acceleration'.

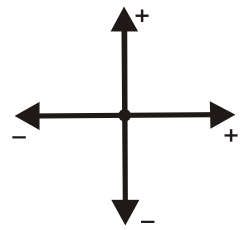


Fig.19 Sign convention for acceleration

  - Whenever velocity and acceleration are in opposite direction, the velocity of a particle decreases. Such motion is called retarded motion. Such an acceleration for numericals is usually taken 'negative acceleration' and also called 'retardation' or 'deceleration'.

Acceleration, $\boxed{a = \frac{v-u}{t}}$

Unit of acceleration : C.G.S.system - centimetre/(second)^2 (cm/s^2) ; S.I. system - metre/(second)^2 (m/s^2).

Non-uniform motion with constant acceleration (uniformly accelerated motion)

It is a motion in which acceleration is constant in both magnitude as well as direction.

- It is a non-uniform motion.

Equations of motion for a uniformly accelerated motion are :

(i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$ (iii) $v^2 = u^2 + 2as$ (iv) $s = \left(\frac{v+u}{2}\right)t$ (v) $v_{average} = \frac{v+u}{2}$

Where, u = initial velocity ; v = final velocity ; s = distance travelled ; t = time taken, a = acceleration.

- Distance travelled in nth second (i.e., in a particular second) is given by,

$$\boxed{s_{nth} = u + \frac{1}{2}a(2n-1)}$$

NUMERICAL CHALLENGE 1.5

A body travels 200 cm in first two seconds and 220 cm in next four seconds. What will be the velocity at the end of the seventh second.

Solution

Let u be the initial velocity, a be the acceleration of the body.

For first two seconds, distance travelled is 200 cm i.e., for t = 2 ; s = 200 cm.

Using second equation of motion, $s = ut + \frac{1}{2}at^2$, we get,

$$200 = u(2) + \frac{1}{2}a(2)^2 \quad \text{or} \quad 200 = 2u + 2a$$

or u + a = 100 ----- (1)

For next four seconds, distance travelled is 220 cm. This means for first (2 + 4) second i.e., first 6 seconds, the distance travelled is 200 + 220 = 420 cm. Here, at t = 6 s ; s = 420 cm. Again using second equation of motion, we get,

$$420 = u(6) + \frac{1}{2}a(6)^2 \quad \text{or} \quad 420 = 6u + 18a$$

or u + 3a = 70 ----- (2)

Subtracting eq.(1) from eq.(2), we get,

$$\begin{array}{r} u + 3a = 70 \\ -\quad u + a = 100 \\ \hline 2a = -30 \end{array}$$

or $a = -15\ \text{cm/s}^2$

Putting the value of a in eq.(1), we get, u – 15 = 100 or u = 115 cm/s

Now, we have to find velocity at the end of seventh second. Using first equation of motion, v = u + at we get,

v = 115 + (–15)(7) = 115 – 105 = **10 cm/s**

NUMERICAL CHALLENGE 1.6

A particle moving with constant acceleration form A to B in straight line AB has velocities 'u' and 'v' at A and B respectively. Find the velocity at C, the mid point of AB.

Solution

Since C is the mid point of AB,

AC = CB = s (let)

Velocity at A, $V_A = u$; Velocity at B, VB = v ;

Velocity at C, V_C = ?

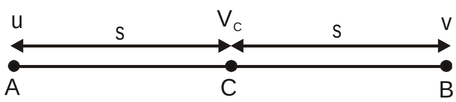


Fig.20 Numerical challenge 2.6

Applying third equation of motion between A and C, we get,

$V_C^2 = V_A^2 + 2as$ or $V_C^2 = u^2 + 2as$ ----- (1)

Applying third equation of motion between C and B, we get,

$V_B^2 = V_C^2 + 2as$ or $v^2 = V_C^2 + 2as$ or $V_C^2 = v^2 - 2as$ ----- (2)

Adding eq.(1) + eq.(2), we get,

$V_C^2 + V_C^2 = (u^2 + 2as) + (v^2 - 2as)$

$2V_C^2 = v^2 + u^2$ or $V_C^2 = \frac{v^2 + u^2}{2}$

or $V_C = \sqrt{\frac{v^2 + u^2}{2}}$

NUMERICAL CHALLENGE 1.7

A particle moving with uniform acceleration in a straight line covers 3 m in the 8th second and 5 m in the 16th second of its motion. Find the distance travelled by it from the beginning of the 6th second to the end of the 15th second.

Solution

Let u be the initial velocity, a be the acceleration of the particle.

Distance covered by the particle in 8th second is 3 m. Using the equation for s_{nth},

$3 = u + \frac{1}{2}a(2\times 8 - 1)$ or $3 = u + \frac{1}{2}a(15)$ or $2u + 15a = 6$ ----- (1)

Distance covered by the particle in 16th second is 5 m. Again, using the equation for s_{nth},

$5 = u + \frac{1}{2}a(2\times 16 - 1)$ or $5 = u + \frac{1}{2}a(31)$ or $2u + 31a = 10$ ----- (2)

Eq.(2) – eq.(1) $\Rightarrow$ (2u + 31a) – (2u + 15a) = 10 – 6

or 16a = 4 or a = (1/4) m/s^2

Using eq.(1), we get, $2u + 15 \times \frac{1}{4} = 6$ or $2u = 6 - \frac{15}{4} = \frac{9}{4}$ or u = (9/8) m/s

Now, we have to find the distance covered by the particle from the beginning of the 6th second to the end of the 15th second. At the beginning of the 6th second, total time elapsed is 5 second. First, we will find the velocity at the end of 5th second using first equation of motion,

v = u + at or $v = \frac{9}{8} + \left(\frac{1}{4}\right)(5) = \frac{9}{8} + \frac{5}{4} = \frac{19}{8}$ m/s

Now time taken between the beginning of the 6th second to the end of the 15th second is actually 10 seconds (6th, 7th, 8th, 9th, 10th, 11th, 12th, 13th, 14th, 15th). [**Caution :** If you subtract 15 – 6, you will get 9 seconds while actual time elapsed is 10 seconds]

Now, using second equation of motion, $s = ut + \frac{1}{2}at^2$, we get,

$$s = \left(\frac{19}{8}\right)(10) + \frac{1}{2}\left(\frac{1}{4}\right)(10)^2 = \frac{190}{8} + \frac{100}{8} = \frac{290}{8} = 36.25 \text{ m}$$

1.7 Free fall (motion under gravity)

Till 1600 AD, the teachings of the Greek philosopher Aristotle (384–322 BC) had held that *heavier objects fall faster than lighter ones*. The Italian physicist Galileo Galilei gave the present day ideas of falling objects. Now, it is an established fact that, in the absence of air resistance, all objects dropped near the Earth's surface fall with the same constant acceleration under the influence of the Earth's gravity.

- Free fall is the motion of an object subject only to the influence of gravity. An object is in free fall as soon as it is dropped from rest, thrown downward or thrown upward.

  Acceleration due to gravity : The constant acceleration of a freely falling body is called the acceleration due to gravity.

  - The acceleration due to gravity is the acceleration of an object in free fall that results from the influence of Earth's gravity. Its magnitude is denoted with the letter g. The value of g on the surface of Earth is nearly 9.8 m/s^2. In C.G.S. system, g = 980 cm/s^2 ; in F.P.S. system, g = 32 ft/s^2.

- Earth's gravity always pulls downward, so the acceleration (g) of an object in free fall is always downward and constant in magnitude, regardless of whether the object is moving up, down, or is at rest, and independent of its speed.

  - If the object is moving downward, the downward acceleration makes it speed up; if it is moving upward, the downward acceleration makes it slow down.

Equations of motion of freely falling body

There are two main assumptions in free fall :

(1) Acceleration due to gravity (g) is constant throughout the motion and it acts vertically downwards.

(2) Air resistance is negligible.

Case 1 : An object thrown vertically upward and it returns after some time (see fig.21).

Let us consider an object thrown vertically upward with an initial velocity u, the acceleration due to gravity g is acting vertically downward on it. Let after a time interval t, it achieves an height h and final velocity v.

Initial velocity = + u ; acceleration, a = – g ; distance travelled, s = + h

From first equation of motion, we have, $v = u + at$

or $v = (+u) + (-g)t$ or $v = u - gt$ ----- (1)

From second equation of motion, we have, $s = ut + \frac{1}{2}at^2$

or $+h = (+u)t + \frac{1}{2}(-g)t^2$ or $h = ut - \frac{1}{2}gt^2$ ----- (2)

From third equation of motion, we have, $v^2 = u^2 + 2as$

or $v^2 = (+u)^2 + 2(-g)(+h)$ or $v^2 = u^2 - 2gh$ ----- (3)

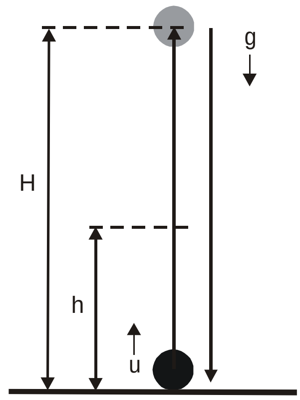


Fig.21

Time taken to reach maximum height :

At maximum height, v = 0

From eq.(1), we get, $0 = u - gt$ or $u = gt$ or $\boxed{t = \frac{u}{g}}$

Total time of journey :

Since g is constant throughout the motion, time taken to reach maximum height from the ground is equal to time taken to reach ground from the maximum height. That is, total time (T) of journey,

$$T = 2t = \frac{2u}{g} \quad \text{or} \quad \boxed{T = \frac{2u}{g}}$$

Maximum height achieved by the object :

Let the maximum height achieved be H. At maximum height, v = 0

From eq.(3), we get, $(0)^2 = u^2 - 2g(H)$ or $u^2 = 2gH$ or $\boxed{H = \frac{u^2}{2g}}$

- Here, the total distance covered, $s = 2H = 2\left(\frac{u^2}{2g}\right) = \frac{u^2}{g}$ while, the total displacement is zero.

Case 2 : An object is thrown vertically downward from a certain height H (see fig.22)

Let us consider an object thrown vertically downward with an initial velocity u, the acceleration due to gravity g is acting vertically downward on it. Let after a time interval t, it falls through a distance y and achieves a final velocity v.

Initial velocity = – u ; acceleration, a = – g ; distance travelled, s = – y ; final velocity = – v

From first equation of motion, we have, $v = u + at$

or $(-v) = (-u) + (-g)t$ or $-v = -u - gt$

or $-v = -(u + gt)$ or $v = u + gt$ ----- (1)

From second equation of motion, we have, $s = ut + \frac{1}{2}at^2$

or $-y = (-u)t + \frac{1}{2}(-g)t^2$ or $-y = -ut - \frac{1}{2}gt^2$

or $-y = -\left(ut + \frac{1}{2}gt^2\right)$ or $y = ut + \frac{1}{2}gt^2$ ----- (2)

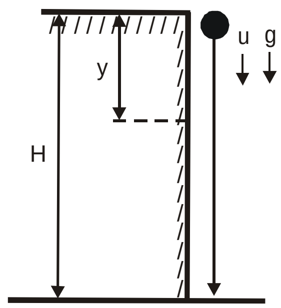


Fig.22

From third equation of motion, we have, $v^2 = u^2 + 2as$

or $(-v)^2 = (-u)^2 + 2(-g)(-y)$ or $v^2 = u^2 + 2gy$ ----- (3)

Velocity at ground : When the object reaches the ground, y = –H, then, from third equation of motion,

$v^2 = u^2 + 2as$ or $(-v)^2 = (-u)^2 + 2(-g)(-H)$

or $v^2 = u^2 + 2gH$ or $v = \sqrt{u^2 + 2gH}$

Time taken to reach the ground : When the object reaches the ground, y = – H, then, from second equation of motion,

$$s = ut + \frac{1}{2}at^2 \quad \text{or} \quad -H = (-u)t - \frac{1}{2}gt^2$$

or $H = ut + \frac{1}{2}gt^2$. This is a quadratic equation that can be solved by factorisation or using quadratic formula.

- For numericals, we can assume acceleration due to gravity as + g for downward while –g for upward motion.

■ If an object is dropped from certain height, its initial velocity is taken zero i.e., u = 0. In such case the eqs.(1),(2),(3) will reduce to,

$$\boxed{v = gt \ ; \ y = \frac{1}{2}gt^2 \ ; \ v^2 = 2gy}$$

- **Velocity at ground :** When particle reaches the ground, y = H, then,

  $v^2 = 2gH$ or $v = \sqrt{2gH}$

- **Time taken to reach the ground :** When particle reaches the ground, y = H, then,

  $H = \frac{1}{2}gt^2$ or $t = \sqrt{\frac{2H}{g}}$.

Case 3 : An object thrown up from a certain height H or dropped from a rising balloon/helicopter :

Let us consider an object thrown vertically upward (see fig.23) from a certain height H with an initial velocity u, the acceleration due to gravity g is acting vertically downward on it. Also, if an object is dropped from a hot air balloon or a helicopter which is rising up into the atmosphere, the case will remain the same. This is because the initial velocity of a body dropped from a moving object is equal to the velocity of the moving object. In both cases, the object rises first, reaches a maximum height, then it moves downwards and finally reaches the ground.

Let after a time interval t, it moves a distance y and achieves a final velocity v.

Initial velocity = + u ; acceleration, a = – g ; distance travelled, s = y ; final velocity = v.

From first equation of motion, we have,

$v = u + at$

or $v = (+u) + (-g)t$

or $v = u - gt$ ----- (1)

- In the eq.(1), if v comes positive, it means that object is moving upwards. If v comes negative, it means that object is moving downwards.

From second equation of motion, we have,

$$s = ut + \frac{1}{2}at^2 \quad \text{or} \quad y = (+u)t + \frac{1}{2}(-g)t^2$$

or $y = ut - \frac{1}{2}gt^2$ ----- (2)

- In the eq.(2), if y comes positive, it means that object is above the initial point. If y comes negative, it means that object is below the initial point.

  From third equation of motion, we have, $v^2 = u^2 + 2as$

  or $(v)^2 = (+u)^2 + 2(-g)(y)$ or $v^2 = u^2 - 2gy$ ----- (3)

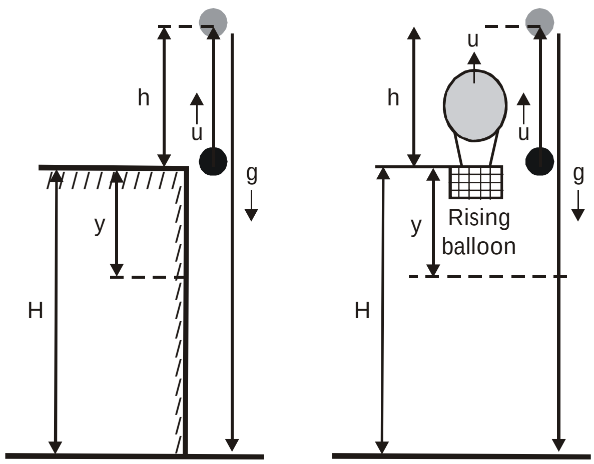


Fig.23

Velocity at ground : When particle reaches the ground, y = –H, then, from third equation of motion,

$v^2 = u^2 + 2as$ or $(-v)^2 = (-u)^2 + 2(-g)(-H)$

or $v^2 = u^2 + 2gH$ or $v = \sqrt{u^2 + 2gH}$

Time taken to reach the ground : When particle reaches the ground, y = – H, then, from second equation of motion,

$$s = ut + \frac{1}{2}at^2 \quad \text{or} \quad -H = (+u)t - \frac{1}{2}gt^2$$

or $H = -ut + \frac{1}{2}gt^2$. This is a quadratic equation that can be solved by factorisation or using quadratic formula.

- Let three balls 1, 2, and 3 are allowed to fall under gravity from the same height. Ball 1 is thrown vertically upward with speed u and it reaches the ground (see fig.24) in time t_1. Ball 2 is thrown vertically downward with the same speed u and it reaches the ground in time t_2. Ball 3 is dropped (i.e., u = 0) from the same height and it reaches ground in time t_3. Then, the relationship between t_1, t_2 and t_3 is given by,

$$\boxed{t_3 = \sqrt{t_1 t_2}}$$

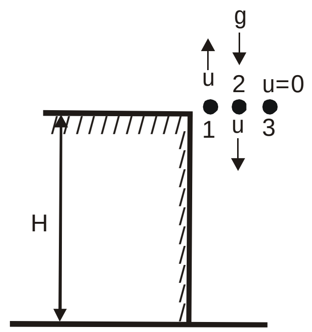


Fig.24

Case 4 : An object is dropped in a well and the sound of splash in water is heard after a certain time t

Let us consider a well in which water level is present at a depth 'd' from the ground level. An object is dropped in it. When the object strikes the water surface, a splash (sound) is produced which reaches our ear after a very short time period (see fig.25).

Downward motion of object : It is a case of free fall i.e., motion under gravity. Initial velocity, u = 0 ; distance travelled, s = depth of well = d ; time taken, $t = t_1$

Now, from second equation of motion, $s = ut + \frac{1}{2}gt^2$

$$\text{or} \quad d = \frac{1}{2}gt_1^2 \quad \text{or} \quad t_1 = \sqrt{\frac{2d}{g}}$$

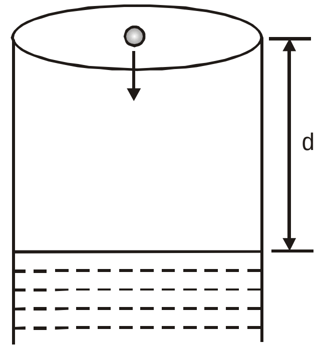


Fig.25

Upward motion of sound : There is no effect of gravity in the propagation of sound i.e, always the formula of uniform motion is used for sound or any other wave.

Speed of sound, $v = \frac{d}{t_2}$

(Distance travelled by sound = depth of well = d)

or $t_2 = \frac{d}{v}$

Total time taken to hear the splash, $T = t_1 + t_2 = \sqrt{\frac{2d}{g}} + \frac{d}{v}$

NUMERICAL CHALLENGE 1.8

A person, on the top of a building, throws one stone vertically upwards with a velocity 'u'. He throws an another stone from the same place in the downward direction with a velocity 'u'. Find the ratio of velocities of two stones on the bottom of the building.

Solution

For the stone thrown upward (see fig.24), Initial velocity = + u ; acceleration, a = – g ; distance travelled, s = – H ; final velocity = – v_1.

From second equation of motion, we have, $v^2 = u^2 + 2as$

or $(-v_1)^2 = (+u)^2 + 2(-g)(-H)$ or $v_1^2 = u^2 + 2gH$ or $v_1 = \sqrt{u^2 + 2gH}$ ----- (1)

For the stone thrown downward (see fig.24), Initial velocity = – u ; acceleration, a = – g ; distance travelled, s = – H ; final velocity = – v_2.

$v^2 = u^2 + 2as$ or $(-v_2)^2 = (-u)^2 + 2(-g)(-H)$

or $v_2^2 = u^2 + 2gH$ or $v_2 = \sqrt{u^2 + 2gH}$ ----- (2)

From eq.(1) and eq.(2), we get that $v_1 = v_2$, therefore $v_1 : v_2 =$ **1 : 1**

NUMERICAL CHALLENGE 1.9

A balloon is ascending at the rate of 5 m/s at a height of 100 m above the ground when a packet is dropped from the balloon. After how much time does it reach the ground ? (g = 10 m/s^2)

Solution

Since the balloon is ascending with velocity 5 m/s, the initial velocity of the packet dropped from the balloon is u = + 5 m/s ; a = – g = – 10 m/s^2 ; displacement, s = – 100 m.

From first equation of motion, we have, $s = ut + \frac{1}{2}at^2$ or $-100 = (+5)t + \frac{1}{2}(-10)t^2$

or $-5t^2 + 5t + 100 = 0$ or $t^2 - t - 20 = 0$ or $t^2 - 5t + 4t - 20 = 0$

or $(t - 5)(t + 4) = 0$ or t = 5 s and t = – 4 s (negative time not possible)

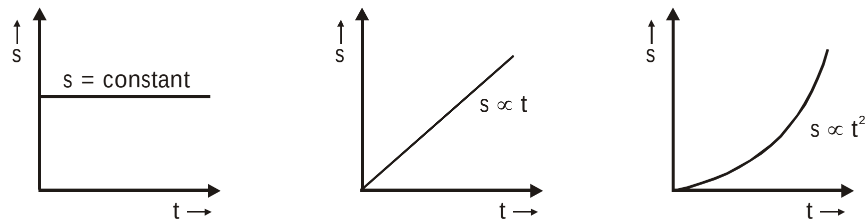
Thus, **t = 5 s**. The packet will reach the ground after 5 seconds.

1.8 Graphs in motion

Usually distance-time, position-time, displacement-time, speed-time, velocity-time, acceleration-time graphs are used in understanding motion.

Distance - time graph

Here, distance is taken on y-axis and time is taken on x-axis.



A body at rest
(s = constant)
(v = 0)

A body in uniform motion
(s = v × t)

A body in uniformly accelerated motion
($s = ut + \frac{1}{2}at^2$)

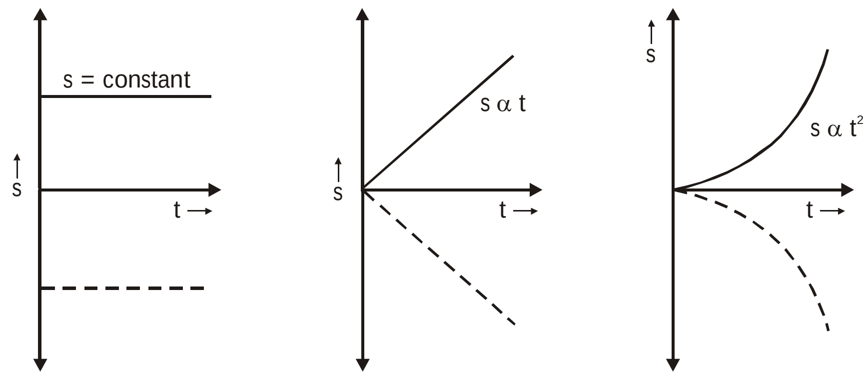
Fig.26 Distance-time graphs for different states of motion

- Distance-time graph is always positive, it is always increasing NEVER decreasing.

Displacement-time graph

Here, displacement is taken on y-axis and time is taken on x-axis.

- Displacement-time graph can be positive or negative, it can be increasing or decreasing.



A body at rest
(s = constant)
(v = 0)

A body in uniform motion
(s = v × t)

A body in uniformly accelerated motion
($s = ut + \frac{1}{2}at^2$)

Fig.27 Displacement-time graphs for different states of motion

Speed-time graph

Here, speed is taken on y-axis and time is taken on x-axis.

- Speed-time graph is always positive, it can be increasing or decreasing.

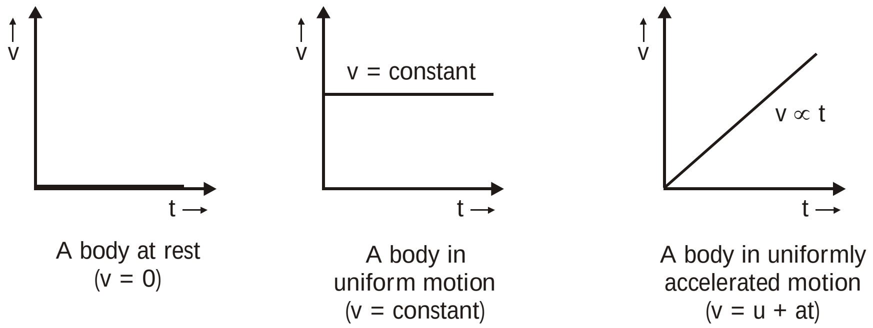


Fig.28 Speed-time graphs for different states of motion

Velocity-time graph

Here, velocity is taken on y-axis and time is taken on x-axis.

- Velocity-time graph can be positive or negative, it can be increasing or decreasing.

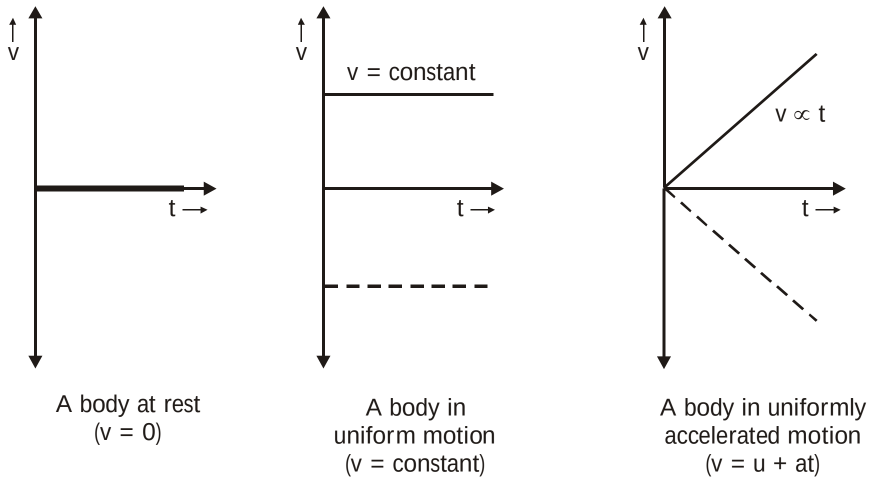


Fig.29 Velocity-time graphs for different states of motion

Acceleration-time graph

Here, acceleration is taken on y-axis and time is taken on x-axis.

- Acceleration-time graph can be positive or negative, it can be increasing or decreasing.

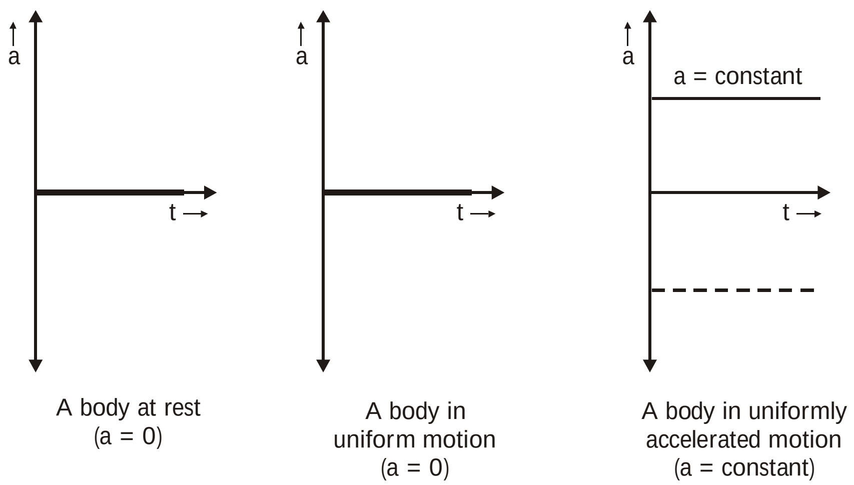


Fig.30 Acceleration-time graphs for different states of motion

1.9 Significance of graphs in motion

Slope of a graph

Slope of a graph is given by,

Slope = $\tan\theta = \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{p}{b}$

Where, θ is the angle made by the graph with positive x-axis.

- Slope of a graph can be zero, positive, negative or even infinite (∞).
  (1) For θ = 0°, slope is zero (e.g. a horizontal line).
  (2) For θ = 90°, slope is infinite (e.g. a vertical line).
  (3) For 0° < θ < 90°, slope is positive (e.g. a line making acute angle with the positive x-axis).
  (4) For 90° < θ < 180°, slope is negative (e.g. a line making obtuse angle with the positive x-axis).
  - More the value of θ, more will be the value of $\tan\theta$ i.e., more will be the slope of the graph.

Slope of a straight line graph :

A straight line has a constant slope (see fig.31)

Slope = $\tan\theta = \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{p}{b}$

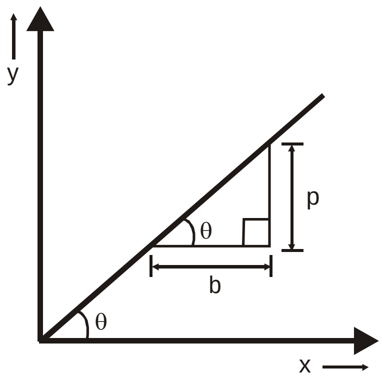


Fig.31 Slope of a straight line graph

Slope of a curved line graph :

The process of finding slopes is more challenging for a curved-line graph because the slope of the curve line changes with the change in the values of variable like x (or time t in motion).

- The slope of a curve line at any point on it is found by making a tangent at that point. If θ be the angle made by that tangent with the positive x-axis, than $\tan\theta$ will be the slope of the curve line at that point (see fig.32).

In the fig.33 (a), the slope of the graph is increasing with the increase in value of x (a concave graph) while in fig.33(b), the slope of the graph is decreasing with the increase in value of x.

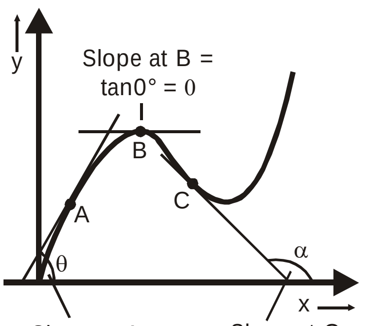


Fig.32 Slope of a curve line graph

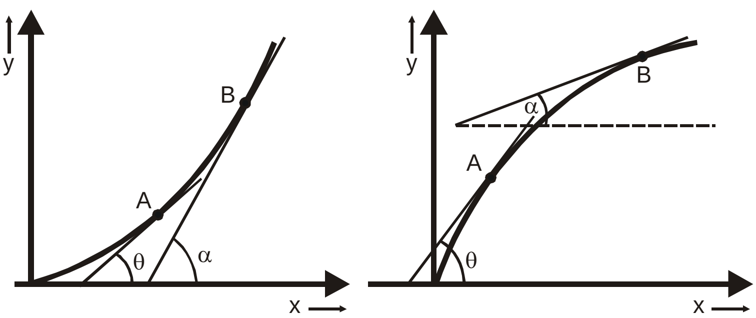


Fig.33 Slope of a curve line graph can be increasing or decreasing

- Slope of distance-time graph gives speed. Slope of displacement-time graph gives velocity.
  - Fig.34 shows a s-t graph in which slope of A is more than slope of B, thus, $v_A > v_B$.
  - From the s-t graph shown in fig.35, we can find the value of v.

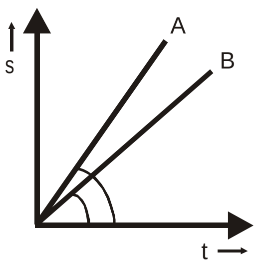


Fig.34

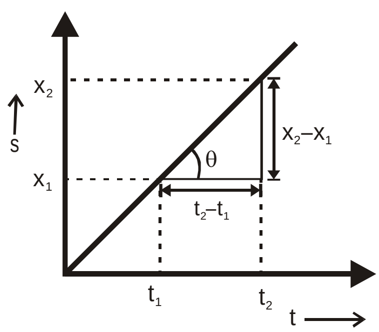
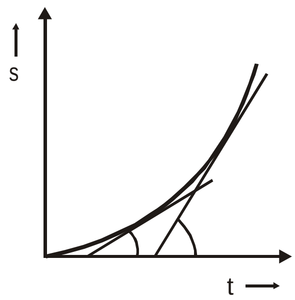


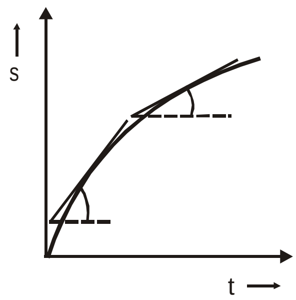
Fig.35

$$\boxed{v = \frac{p}{b} = \frac{x_2 - x_1}{t_2 - t_1}}$$

- In the graphs shown in fig.36, graph 1 represents accelerated motion i.e., v increasing with time. This is because the slope of the graph is increasing with time. Graph 2 represents retarded motion i.e., v decreasing with time. This is because the slope of the graph is decreasing with time.



(a) Graph 1
(v increasing with time)
Accelerated motion



(b) Graph 2
(v decreasing with time)
Retarded motion

Fig.36 Using the concept of slope in s-t graph

- Slope of speed-time graph or velocity-time graph gives acceleration.
  - Fig.37 shows a v-t graph in which, slope of 1 is more than slope of 2, thus, $\mathbf{a_1 > a_2}$.
  - From the v-t graph shown in fig.38, we can find the value of a.

$$\boxed{a = \frac{p}{b} = \frac{v_2 - v_1}{t_2 - t_1}}$$

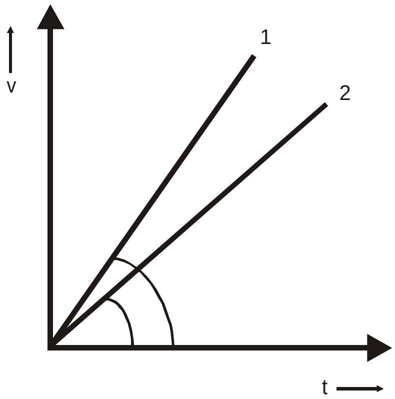


Fig.37

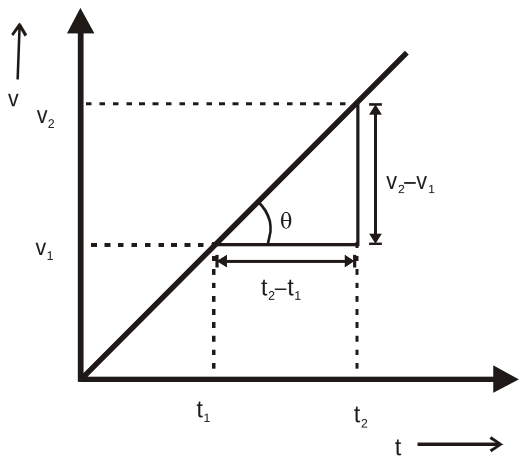
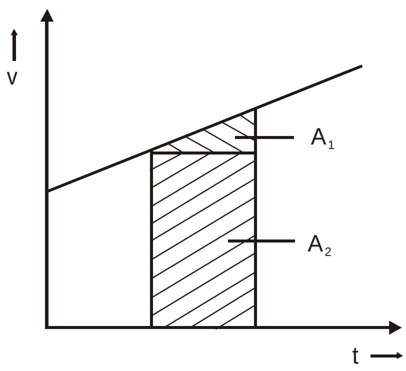


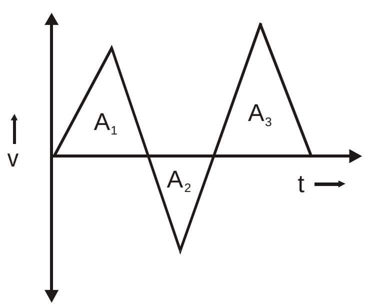
Fig.38

- Total area under the speed-time graph or velocity-time always gives total distance travelled by the body during a given time interval. We can also find displacement using a velocity-time graph [see fig.39(b)].



Distance travelled = $A_1 + A_2$

(a)



Distance travelled = $A_1 + A_2 + A_3$
Displacement = $A_1 - A_2 + A_3$

(b)

Fig.39 Area under v-t graph gives distance travelled by the body.

1.10 Graphs of motion under gravity

We know that upward motion of an object is a retarded motion while downward motion is an accelerated motion. Let us try to make graphs for an object which is thrown upward and returns back to the same height (see fig.40).

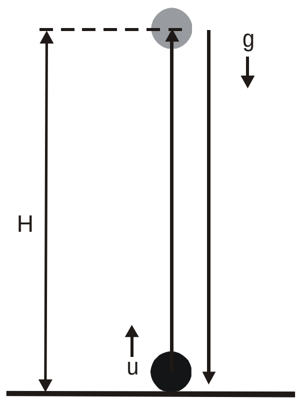
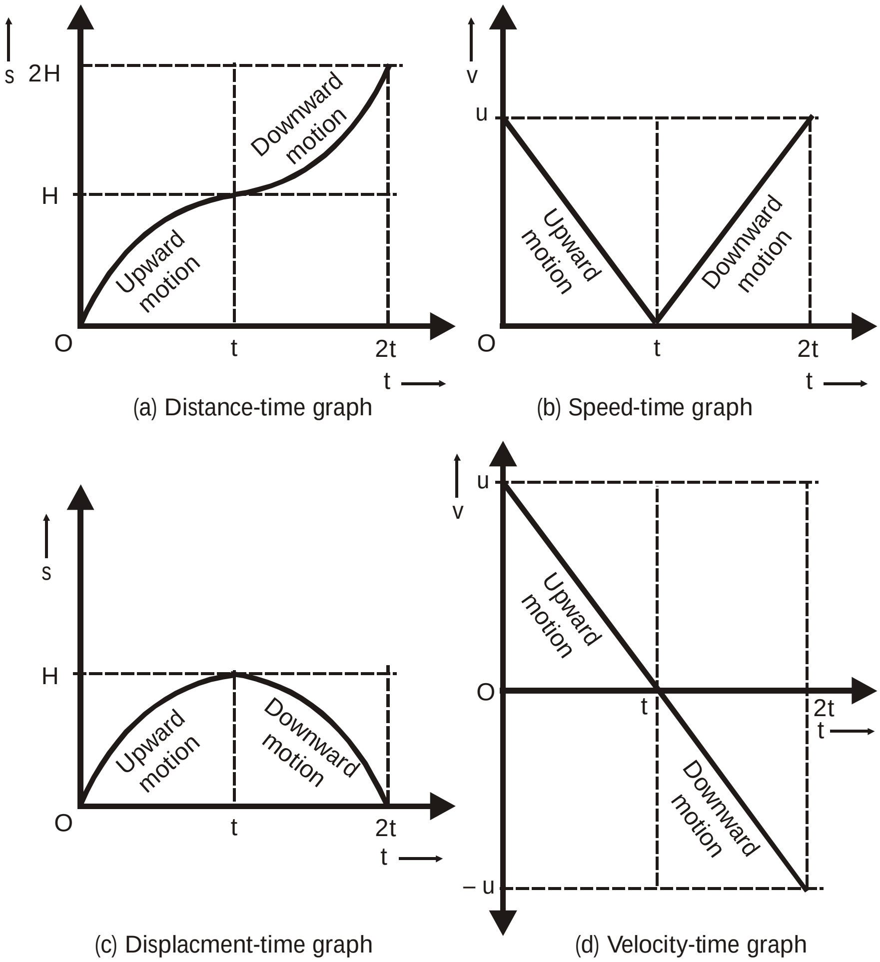
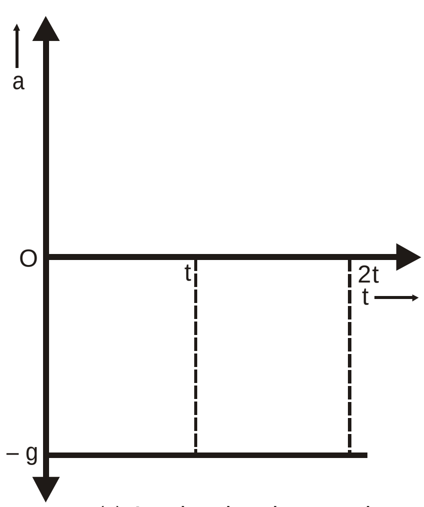


Fig.40

(a) Distance-time graph (b) Speed-time graph

(c) Displacment-time graph (d) Velocity-time graph (e) Acceleration-time graph

Fig.41 Graphs of motion under gravity

- The area under the acceleration-time graph gives change in velocity during a given time interval.

1.11 Circular motion

When a particle moves along a circular path, its motion is called circular motion.

- **A circular motion is always a non-uniform motion** i.e., accelerated motion because the direction of velocity change continuously.
  - Velocity of a particle in circular motion is always tangential to the circular path (see fig.42) i.e., velocity and radius are always $\perp$ to each other.

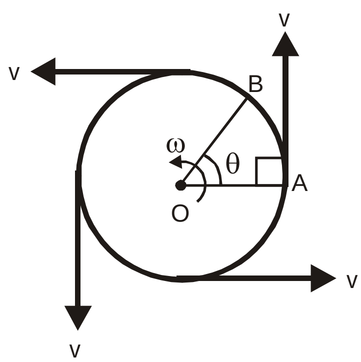


Fig.42 Velocity of a particle along a circular path is tangential to the path

Angular displacement (θ) : The angle described by particle moving along a circular path is called **angular displacement**.

- S.I. unit of angular displacement is **radian**.

π radian = 180° 1 radian = 180°/π = 57.3°

Angular velocity (ω) : The rate of change of angular displacement is called angular velocity.

Formula for ω : $\boxed{\omega = \dfrac{\theta}{t}}$

S.I. unit of ω : radian per second or rad s^{-1}.

Relation between angular velocity (ω) and linear speed (v) :

$\boxed{v = r\omega}$ (r = radius of circular path)

Angular acceleration (α)

The rate of change of angular velocity is called **angular acceleration**.

Formula for α : $\boxed{\alpha = \dfrac{\omega_2 - \omega_1}{t}}$

S.I. unit of α : radian/(second)^2 or rad s^{-2}.

Relation between angular acceleration (α) and linear (tangential) acceleration (a_t) :

$\boxed{a = r\alpha}$ (r = radius of circular path)

Uniform circular motion

Motion of a particle along the circumference of a circle with a constant speed is called uniform circular motion.

- In uniform circular motion :
  - Linear speed, v = constant
  - Angular velocity, ω = constant
  - Angular acceleration, α = 0

Here, linear speed can also be found by formula, $\boxed{v = \dfrac{2\pi r}{T}}$ (T = time period of 1 revolution)

Also, angular velocity ω can be found using formula, $\boxed{\omega = \dfrac{2\pi}{T}}$

- Uniform circular motion is always an accelerated motion. It has a radially inward acceleration called **centripetal acceleration**.

Formula for centripetal acceleration : $\boxed{a_c = \dfrac{v^2}{r} = r\omega^2}$

Centripetal acceleration (a_c) and velocity (v) are always perpendicular to each other.

Centripetal force

It is the radially inward force that is required to move an object along a circular path.

Formula for centripetal force : $\boxed{F = ma_c = \dfrac{mv^2}{r} = mr\omega^2}$

- Centripetal force is always supplied by a real force, the nature of which depends on the situation. While turning a motorcycle on a horizontal circular path, friction provides the necessary centripetal force. The electron moves in a circle around nucleus due to centripetal force provided by the electrostatic force of attraction between positive nucleus and negative electron.
  - While whirling a stone tied with a string, the tension in the string provides the centripetal force. Earth revolves round the Sun due to the centripetal force provided by the gravitational force between the Earth and the Sun.

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Non-uniform circular motion

Motion of a particle along the circumference of a circle with a variable speed is called non-uniform circular motion.

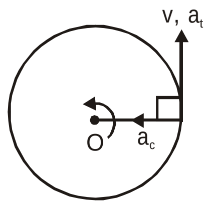


Fig.43 Non-uniform circular motion

- In non-uniform circular motion :
  - Linear speed, $v \neq$ constant
  - Angular velocity, $\omega \neq$ constant
  - Angular acceleration, $\alpha \neq 0$
  - There are two linear accelerations :

    (i) centripetal acceleration (radiailly inward)

    (ii) tangential acceleration (along the tangent or in the direction of velocity)

Non-uniform circular motion with constant angular acceleration

Equations of motion for the above motion are :

(i) $\omega_2 = \omega_1 + \alpha t$

(ii) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

(iii) $\omega_2^2 = \omega_1^2 + 2\alpha\theta$

(iv) $\theta = \left(\frac{\omega_2 + \omega_1}{2}\right)t$

(v) $\omega_{average} = \frac{\omega_2 + \omega_1}{2}$

Where, ω_1 = initial angular velocity ; ω_2 = final angular velocity ; θ = distance travelled ; t = time taken.

Angular Displacement in the nth second (i.e., in a particular second) is given by,

$$\boxed{\theta_{nth} = \omega_1 + \frac{1}{2}\alpha(2n-1)}$$

Angular speed, $\omega = 2\pi n$, where, n is number of revolutions per second or the frequency of revolution.

Also, angular speed, $\omega = \frac{2\pi}{T}$, where, T is time one revolution (called time period or period).

If a particle is making N revolution per minute (denoted as rpm), angular speed, $\omega = \frac{2\pi N}{60}$

NUMERICAL CHALLENGE 1.10

What is the angular velocity in rad/s of the hour, minute and second hand of clock ?

Solution

Time period of revolution of hour hand, T_1 = 12 hours = 12 x 60 x 60 s

Angular velocity of hour hand, $\omega_1 = \frac{2\pi}{T_1} = \frac{2\pi}{12\times60\times60} = \frac{\pi}{21600}$ rad/s

Time period of revolution of minute hand, T_2 = 1 hour = 1 x 60 x 60 s

Angular velocity of hour hand, $\omega_2 = \frac{2\pi}{T_2} = \frac{2\pi}{1\times60\times60} = \frac{\pi}{1800}$ rad/s

Time period of revolution of second hand, T_3 = 1 minute = 1 x 60 s

Angular velocity of hour hand, $\omega_3 = \frac{2\pi}{T_3} = \frac{2\pi}{1\times60} = \frac{\pi}{30}$ rad/s

NUMERICAL CHALLENGE 1.11

A child pushes a merry-go-round from rest to a final angular speed of 0.50 rev/s with constant angular acceleration. In doing so, the child pushes the merry-go-round 2.0 revolutions. What is the angular acceleration of the merry-go-round ?

Solution

Given, initial angular speed, $\omega_1 = 0$; final no. of revolution/sec or frequency, n = 0.50 rev/s

final angular speed, $\omega_2 = 2\pi n = 2\pi(0.5) = \pi$ rad/s ; angular acceleration, α = ?

Total no. of revolution = 2

$\therefore$ Total angle covered, $\theta = 2 \times 2\pi = 4\pi$ rad

Now, using third equation of motion, $\omega_2^2 = \omega_1^2 + 2\alpha\theta$, we get,

$(\pi)^2 = (0)^2 + 2\alpha(4\pi)$

or $\pi^2 = 8\alpha\pi$

or **$\alpha = \pi/8$ rad/s**

CHAPTER 2 Force & Newton's laws of motion

Human life would be dull without social interactions. Similarly, the physical universe would be dull without physical interactions. Social interactions with friends and family change our behaviour; physical interactions change the "behaviour" (e.g. motion) of matter. An interaction between two objects can be described and measured in terms of two forces, one exerted on each of the two interacting objects. A force is a push or pull that one object exerts on another.

2.1 Kinds of forces

Force can be classified in many ways. For example, force can be divided in two kinds :

(1) Continuous forces (2) Momentary forces

Continuous forces

A force constantly applied to an object is called a continuous force. There are many examples of continuous forces. The downward force of Earth's gravity on all objects is a continuous force called weight. All the automobile engines, aeroplane engines, rocket engines provide continuous forces to run them continuously.

Momentary forces

Not all forces are continuous. A moving object sometimes collides with another object, causing a change in speed or direction. This is called a momentary force.

A force applied to an object for a moment only is called a momentary force.

One example of a momentary force is when a cricket player hits a cricket ball with a bat. First, the bowler throws the ball toward the batsman. Then, the batsman swings the bat. When the bat makes contact with the ball, the momentary force applied by the bat causes the ball to move in another direction. The force applied by the bat is limited and does not keep continuously acting on the ball. However, it is important to remember that other forces, such as gravitational force, do act on the ball continuously.

Force can also be divided in two kinds :

(1) Contact forces (2) Non-contact force (or action-at-a-distance forces).

Contact forces

When you press the keys on a computer keyboard, your fingers exert a force on the keys. This force can be exerted only when your fingers are touching the keys.

A force that is exerted only when two objects are touching is a contact force.

A contact force can be small, such as the force you exert to push a pencil across a sheet of paper, or large, such as the force exerted by a traffic crane as it pulls a car along a street. Contact forces include muscular forces, tension, friction, normal forces.

Non-contact force (Action-at-a-distance force)

When you jump up in the air, you are pulled back to the ground, even though nothing seems to be touching you. Forces can be exerted by one object on another even though they aren't touching each other. The force pulling you down to Earth is the gravitational force exerted by Earth. This force is a non-contact force.

A non-contact force is a force that one object exerts on another when they are not touching. Non-contact forces include the gravitational force, the electric force, and the magnetic force.

Conservative force

A force is conservative if work done by the force on a particle that moves through any round trip (complete cycle) is zero. e.g. gravitational forces, electrostatic forces, elastic forces are conservative in nature.

Non-conservative force

A force is non-conservative if work done by the force on a particle that moves through any round trip (complete cycle) is not zero. e.g. frictional forces are non-conservative in nature.

2.2 Inertia

It is 'the natural tendency of an object to remain at rest or in motion at a constant speed along a straight line'. It is the tendency of an object to resist any attempt to change in its velocity.

- The mass of an object is a quantitative measure of inertia. More the mass, more will be the inertia of an object and vice-versa.

Inertia of an object can be of three types :

(1) **Inertia of rest**, the tendency of an object to remain at rest. This means an object at rest remains at rest until a sufficiently large external force is applied on it.

(2) **Inertia of motion**, the tendency of an object to remain in the state of uniform motion. This means an object in uniform motion remains continue to move uniformly until an external force is applied on it.

(3) **Inertia of direction**, the tendency of an object to maintain its direction. This means an object moving in a particular direction remains continue to move in that until an external force is applied to change it.

Newton's first law of motion (Galileo's law of inertia)

'Every object continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed upon it'.

2.3 Linear momentum (or momentum)

When someone asks you whether you would hit more by a piece of chalk or by a cork ball at first, the answer seems to be the cork ball. But if the chalk was moving at 250 m/s, then you're basically dealing with a bullet, and the choice becomes obvious.

- Momentum is an important concept when considering impacts, collisions, and how objects in general interact. It is not just an object's mass or an object's velocity that is important; it is the product of its mass and velocity.

The product of the mass (m) & velocity ($\vec{v}$) is called linear momentum.

$$\boxed{\vec{p} = m\vec{v}}$$

Linear momentum is a vector quantity. Its direction is 'the direction along the velocity'.

The linear momentum of a particle is directly proportional to (i) its mass (ii) its velocity.

Unit of linear momentum : SI unit : kg m/s or kg m s^{-1} or Newton-second (N-s)

C.G.S. unit : g cm/s or g cm s^{-1} or Dyne-second

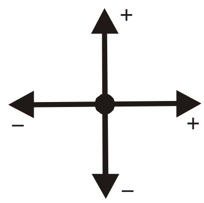


Fig.1 Sign convention for linear momentum

- Linear momentum can be positive or negative depending on its direction.

For a given velocity, the momentum is directly proportional to the mass of the object ($p \propto m$). This means more the mass, more will be the momentum and vice-versa. If a car and a truck has same velocity, then, the momentum of truck is more than the momentum of car as the mass of a truck is greater than the mass of a car.

For a given mass, the momentum is directly proportional to the velocity of the object ($p \propto v$). This means more the velocity, more will be the momentum and vice-versa. If two bodies with same masses move with different velocities then, the body having more velocity will have more momentum.

For a given momentum, the velocity is inversely proportional to the mass of the object $(v \propto 1/m)$. This means smaller the mass, more will be the velocity of an object and vice-versa. If a car and a truck has same momentum, the velocity of car will be more than the velocity of truck as the mass of a car is smaller than the mass of a truck.

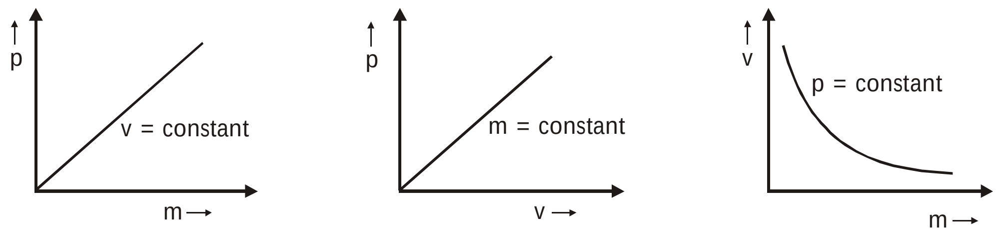


Fig.2 Different graphs related to momentum

- When a object is moving along a circular path, its velocity is tangential to the circular path hence, its momentum is also tangential to the circular path.

Momentum (p) of a photon

A photon is considered as massless, chargeless particle of an electromagnetic wave like visible light, X rays, ultraviolet rays, radio waves, etc. but it carries energy,

$$\boxed{p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}}$$

Where, E = energy carried by a photon = $h\nu$; h = Planck's constant = 6.63×10^{-34} J s ;

ν = frequency of electromagnetic wave ; λ = wavelength of electromagnetic wave.

2.4 Newton's second law of motion

'The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts'. Mathematically, it can be represented as,

$$\boxed{F = ma = \frac{p_2 - p_1}{t} = \frac{m(v-u)}{t}}.$$

- If force is constant i.e., F = ma = constant, then, the acceleration produced in the body is inversely proportional to its mass, i.e, $a \propto 1/m$. This means, if same force F is applied to masses m_1 and m_2 and the resulting accelerations in them are a_1 and a_2 respectively, then, $m_1a_1 = m_2a_2$

or $\boxed{\frac{a_2}{a_1} = \frac{m_1}{m_2}}$.

- 1 newton is the amount of force that produces an acceleration of 1 m s^{-2} in an object of 1 kg mass. Similarly, 1 dyne is the amount of force that produces an acceleration of 1 cm s^{-2} in an object of 1 g mass.

1 N = 10^5 dynes

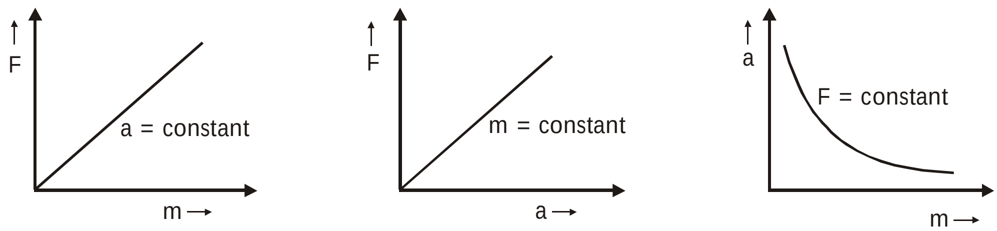


Fig.3 Graphs related to force, mass and acceleration

Impulse (J)

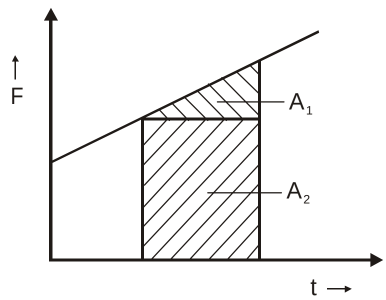
The product of force and time is called 'impulse'. It is also the change in momentum of the body. It is a vector quantity.

$$\boxed{J = F \times t = \Delta p = p_2 - p_1 = m\,(v-u)}$$

A large force acting for a short time that produces a significant change in momentum is called an **impulsive force**.

If force F acting is variable then, impulse, $\boxed{J = F_{av} \times t}$

- Area under the force-time graph gives impulse (see fig.4).



Impulse = $A_1 + A_2$

Fig.4 Area under F-t graph gives impulse

Newton's third law of motion

Whenever one body exerts a force on a second body, the second body exerts an oppositely directed force of equal magnitude on the first body'. '**To every action, there is always an equal and opposite reaction**'.

Forces always exist in pairs : When two objects interact, two forces will always be involved. One force is the action force and the other is the reaction force.

- Consider a pair of bodies A and B. According to the Newton's third law, $\mathbf{F_{AB} = -F_{BA}}$

  Where, F_{AB} = force on A due to B and F_{BA}= force on B due to A

  Though action-reaction pair are equal in magnitude and opposite in direction but the reaction force always acts on a different object than the action force. Thus, these forces do not cancel out each other. Hence, there can be an acceleration in an object.

  Newton's third law is applicable to non-contact forces also. For example, the Earth pulls an object downwards due to gravity. The object also exert the same force on the Earth but in upward direction. But, we hardly see the effect of the stone on the Earth because the Earth is very massive and the effect of a small force on its motion is negligible. That is, the acceleration of Earth is negligible due to its huge mass.
- Even though the action and reaction forces are always equal in magnitude, these forces may not produce accelerations of equal magnitudes. This is because each force acts on a different object that may have a different masses.

2.5 Conservation of linear momentum

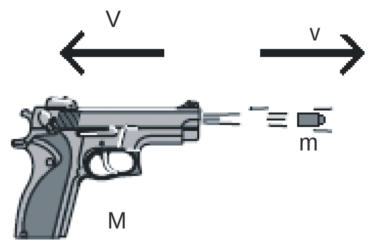
When the net external force on a system of objects is zero, the total linear momentum of the system remains constant'. In other words 'the total momentum of an isolated system of objects remains constant'.

The term '**collision**' is used to represent the event of two particles coming together for a short time and thereby producing 'impulsive forces' on each other. These forces are assumed to be much greater than any external forces present because they act for a very short time interval.

Momentum is conserved for all types of collisions that take place in real world in the absence of any external force.

- Rocket propulsion or the recoil of gun are based on law of conservation of momentum as well as Newton's third law. This is because the law of conservation of momentum is derived using Newton's third law.

Initial momentum = 0



Final momentum = mv – MV = 0

Fig.5 Recoil of a gun or pistol

Solving problems on conservation of momentum

Recoil of a gun : Initial momentum = Final momentum

or 0 = MV – mv or $\boxed{V = \frac{m}{M}v}$ (see fig.5)

- A bullet is fired on a wooden block and it gets embedded in it, after that they move together with a common velocity (see fig.6).

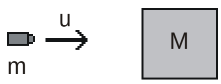
  Initial momentum = Final momentum

  or mu = (M +m) V or $\boxed{V = \frac{mu}{M+m}}$

  A bomb of mass M explodes in two parts having masses m_1 and m_2 (see fig.7).

  Final momentum = initial momentum

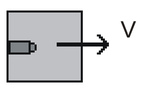
  or $m_2v_2 - m_1v_1 = 0$ or $\mathbf{m_2v_2 = m_1v_1}$



U = 0

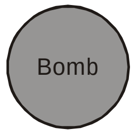
Initial momentum =
m × u + M × 0 = mu

M + m

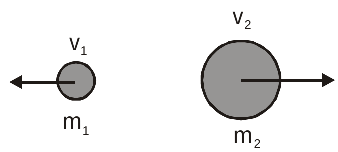


Final momentum =
(M+m)V

Fig.6



Initial velocity, u = 0
Mass of bomb = M
Initial momentum = M × 0 = 0
(Before explosion)



Final momentum =
$m_2v_2 - m_1v_1$
(After explosion)

Fig.7

NUMERICAL CHALLENGE 2.1

A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of 12 m s^{-1}. If the mass of the ball is 0.15 kg, determine the change in momentum. Also, find the impulse imparted to the ball. (Assume linear motion of the ball).

Solution

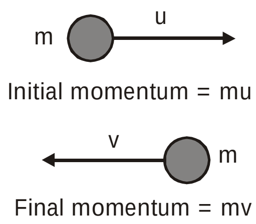


Fig.8 Numerical challenge 3.1

Initial velocity, u = + 12 m s^{-1} ; final velocity, v = – 12 m s^{-1} ; mass, m = 0.15 kg

Change in momentum = mv – mu = m(v – u)

= 0.15 × [(–12) – (+12)] = 0.15 × –24

= **–3.6 kg m s^{-1}**

Impulse = change in momentum = **3.6 kg m s^{-1}**

NUMERICAL CHALLENGE 2.2

A body of mass 1 kg moving with a speed of 50 m/s hits a wall and rebounds with the same speed. If the contact time is (1/25) s, find the force applied on the wall.

Solution

Given, mass of the body, m = 1 kg ; initial velocity of the body, u = + 50 m/s ;

final velocity of the body, v = – 50 m/s ; contact time, t = (1/25) s

Initial momentum of the body, p_1 = mu = 1 x (+50) = +50 kgm/s

Final momentum of the body, p_2 = mu = 1 x (–50) = –50 kgm/s

Change of momentum of the body, $\Delta p = p_2 - p_1$ = (–50) – (+50) = – 100 kgm/s

Force applied on the body, $F = \frac{\Delta p}{t} = \frac{-100}{(1/25)}$ = – 2500 N

Force applied on the wall = – Force applied on the body (Action-reaction pair)

= – (– 2500 N) = **+ 2500 N**

NUMERICAL CHALLENGE 2.3

A bullet of mass 0.04 kg moving with a speed of 90 m s^{-1} enters a heavy wooden block and is stopped after a distance of 60 cm. What is the average resistive force exerted by the block on the bullet ?

Solution

Mass of bullet, m = 0.04 kg ; initial speed, u = 90 ms^{-1} ; final speed, v = 0 ;

distance, s = 60 cm = (60/100) m = 0.6 m

From third equation of motion, $v^2 = u^2 + 2as$ or $(0)^2 = (90)^2 + 2a(0.6)$

or $a = -\frac{(90)^2}{2 \times 0.6}$ = – 6750 ms^{-2}

∴ Force, F = ma = (0.04) (–6750) = **– 270 N**

NUMERICAL CHALLENGE 2.4

A shell of mass 0.020 kg is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 m/s, what is the recoil speed of the gun ?

Solution

Given, mass of bullet, m_1 = 0.020 kg ; muzzle speed (speed of bullet), v_1 = 80 m/s ;

mass of gun, m_2 = 100 kg ; recoil speed, v_2 = ?

Initial momentum, p_1 = 0

Final momentum, $p_2 = m_1v_1 - m_2v_2$

By conservation of momentum,

Final momentum = Initial momentum

or $p_2 = p_1$

or $m_1v_1 - m_2v_2 = 0$

or $(0.020)(80) - (100)(v_2) = 0$ or $1.6 - 100v_2 = 0$

or v_2 = (1.6/100) m/s = **1.6 x 10^{-2} m/s**

2.6 Tension in strings

Strings are assumed to be inextensible i.e., they cannot be stretched. Due to this assumption 'acceleration of masses connected through a string is always same. They are assumed to be massless unless it is mentioned. Due to this assumption 'tension in the string is same every where'.

If the string has mass, tension at different points will be different. It is maximum at the end at which force is applied and minimum at the other end connected to a mass.

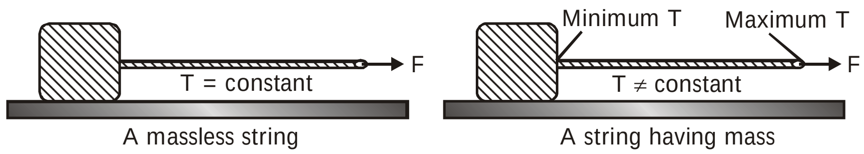


Fig.9 Tensions in the string

- The direction of tension at body (or a point) is always outward along the string i.e., away from the body along the string. A tension always have pulling action.

Free body diagrams

A system diagram is a sketch of all the objects involved in a situation. A free-body diagram (FBD) is a drawing in which only the object being analyzed is drawn, with arrows showing all the forces acting on the object.

(1) Free body diagrams represent all forces acting on one object.

(2) Forces that the object exerts on other objects do not appear in free body diagrams because they have no effect on the motion of the object itself.

(3) In drawing a free body diagram, you can represent the object as a single dot or a simplified shape the object.

(4) In FBD each force acting on the object is represented with an arrow. The arrow's direction shows the direction of the force and the arrow's relative length provides information about the magnitude of the force.

(5) Forces that have the same magnitude should be sketched with approximately the same length, forces that are larger should be longer, and smaller forces should be shorter.

(6) In case of objects in motion, the direction of acceleration should be made on the FBD in the direction of greater force (or net force).

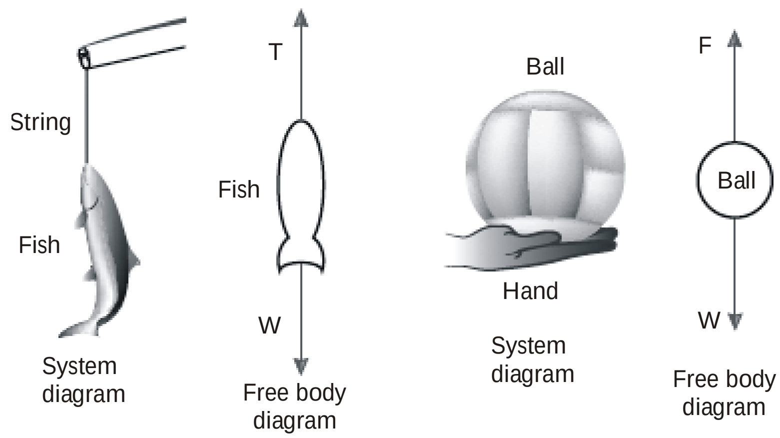


Fig.10 Making free body diagrams

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Motion of bodies connected by strings

Let us consider two bodies m_1 and m_2 placed on horizontal frictionless plane connected by a massless string. Let the mass m_1 is pulled by a force F. As a result the whole system moves in the direction of applied force with an acceleration a. Let the tension in the string be T (see fig.11).

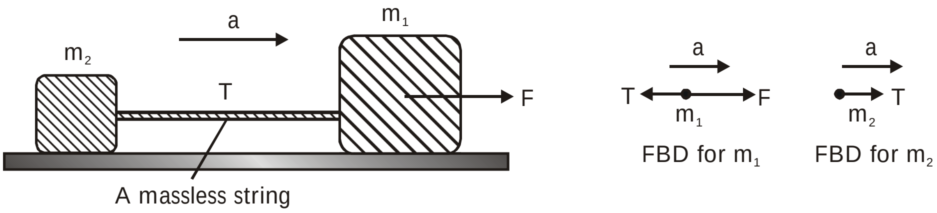


Fig.11 Motion of bodies connected by strings

For mass m_1, $F - T = m_1 a$ ----- (1) [F is greater force as it is in the direction of acceleration a]

For mass m_2, $T = m_2 a$ ----- (2) [Here, T is the only force acting on m_2]

(1) + (2) $\Rightarrow$ $(F - T) + T = m_1a + m_2a$

or $F = (m_1 + m_2)a$ or $\boxed{a = \dfrac{F}{m_1 + m_2}}$ From (2), we have, $T = m_2a$ $\therefore$ $\boxed{T = \dfrac{m_2F}{m_1 + m_2}}$

Motion of bodies connected by string passing over a light pulley (Atwood's Machine)

Let us consider two masses m_1 and m_2 passing over a light pulley connected through a string (see fig.12). The term 'light pulley' means the mass of pulley is neglected, it is assumed to be massless. Since the two bodies are connected with each other, both move with same acceleration a. Let $m_2 > m_1$ then, m_2 will go downwards while m_1 will go upwards.

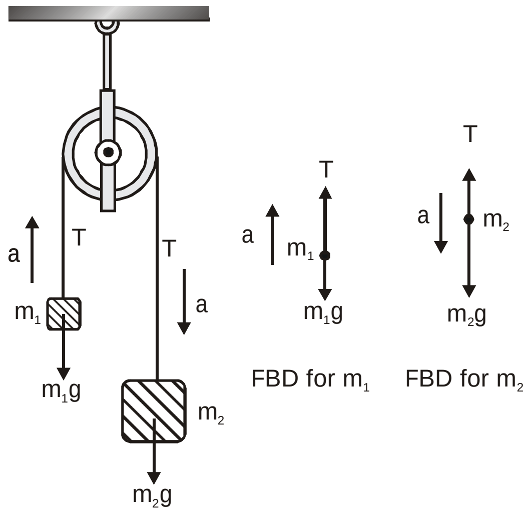


Fig.12 Motion of masses connected by a pulley

For m_1, $T - m_1g = m_1a$ ----- (1)

[Here, $T > m_1g$, as T is in the direction of acceleration a]

For m_2, $m_2g - T = m_2a$ ----- (2)

[Here, $m_2 g > T$, as m_2g is in the direction of acceleration a]

(1) + (2) $\Rightarrow$ $(T - m_1g) + (m_2g - T) = m_1a + m_2a$

or $(m_2 - m_1)g = (m_2 + m_1)a$

or $\boxed{a = \dfrac{(m_2 - m_1)g}{(m_2 + m_1)}}$

(Since $a \neq g$, two bodies are not free falling bodies.)

Putting the value of a in eq.(1), we get, $\boxed{T = \dfrac{2m_1m_2g}{m_1 + m_2}}$

Motion of bodies in contact

Let two bodies of masses m_1 and m_2 respectively are placed side by side touching each other. A push force 'F' is applied on m_1 such that both the bodies start moving together with an acceleration 'a'. Since both the bodies are touching each other there is a pair of action reaction force between them at place of their contact. These forces are called normal contact forces (see fig.13) and obviously they are equal in magnitude but opposite in direction (Newton's third law).

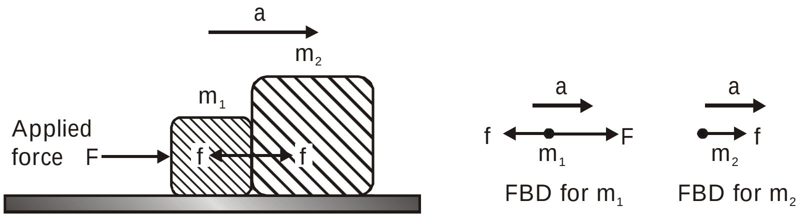


Fig.13 Motion of bodies in contact

For mass m_1, $F - f = m_1 a$ ----- (1) [F is greater force as it is in the direction of acceleration a]

For mass m_2, $f = m_2 a$ ----- (2) [Here, f is the only force acting on m_2]

(1) + (2) $\Rightarrow$ $(F - f) + f = m_1 a + m_2 a$

or $F = (m_1 + m_2)a$ or $\boxed{a = \dfrac{F}{m_1 + m_2}}$ From (2), we have, $f = m_2 a$ $\therefore$ $\boxed{f = \dfrac{m_2 F}{m_1 + m_2}}$

Weight of an object in a lift

A weighing machine measures the normal force not the 'true weight'. Thus, if the normal force changes, the weighing machine does not give reading of true weight, it gives a reading of normal force which we can be called '**apparent weight**' of the object.

Let us consider a girl standing in a lift.

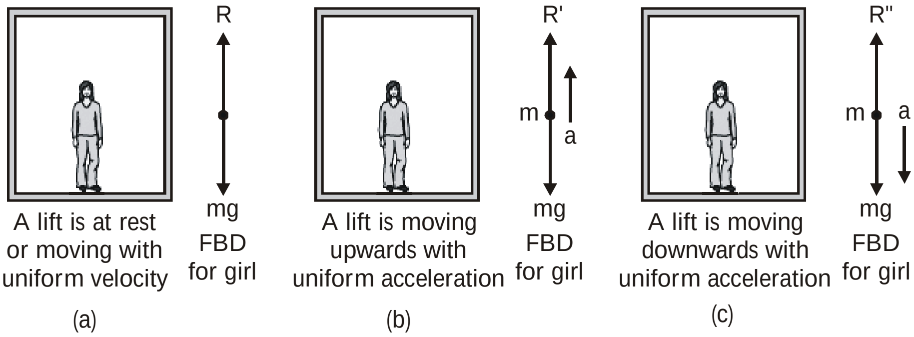


Fig.14 Weight of an object in a lift

(1) When the lift is at rest or in uniform motion, net acceleration of the system is zero [see fig.14(a)]. Thus, net force on it is zero.

$\therefore$ Net force, $F_{net} = mg - R = 0$ or $R = mg$

The R represents the apparent weight, i.e., **W = R = mg** [Apparent weight = true weight]

(2) When the lift is moving up with uniform acceleration a [see fig.14(b)]. Thus, net force on it is not zero.

$\therefore$ Net force, $F_{net} = R' - mg = ma$ [R' is greater force as it is in the direction of acceleration a]

or $R' = ma + mg = m(a + g)$

The R' represents the apparent weight, i.e., **W' = R' = m(a + g)** [Apparent weight > true weight]

(3) When the lift is moving down with uniform acceleration a [see fig.14(c)]. Thus, net force on it is not zero.

$\therefore$ $mg - R'' = ma$ [mg is greater force as it is in the direction of acceleration a]

or $R'' = mg - ma = m(g - a)$

The R'' represents the apparent weight, i.e., **W'' = R'' = m(g − a)** [Apparent weight < true weight]

Suppose the rope of the lift breaks, then it will fall freely under gravity i.e., a = g. In this situation, apparent weight, $W'' = R'' = m(g - g) = 0$. That is, the weighing machine will read zero weight.

2.7 Friction

It is a force that opposes the movement between two surfaces in contact. Some important points related to friction are :

(1) The magnitude of the friction force depends on the types of surfaces in contact. The frictional force is usually larger on the rough surfaces and smaller on the smooth surfaces. Friction depends on both the surfaces that are in contact, therefore, the value of friction is different for different pairs of surfaces.

(2) Friction is always parallel to the surface in contact.

(3) If an object is allowed to move on a surface then, more the distance travelled by the object on the surface, less will be the friction between them and vice-versa.

(4) Friction is caused by the irregularities on the two surfaces in contact.

(5) There are many kinds of friction that exist in different media :

(i) **Static friction :** It exists when two surfaces try to move across each other but not enough force is applied to cause motion.

(ii) **Sliding friction :** It exists when two surfaces slide across each other.

(iii) **Rolling friction :** It exists when one object rolls over another object.

(iv) **Air friction (air resistance) :** It exists when air moves around an object.

(v) **Viscous friction :** It exists when objects move through water or other liquids.

(6) Force of friction increases if the two surfaces are pressed harder. The greater the force pressing the two surfaces together, the greater will be the force of friction between them.

(7) Friction increases with weight. For a heavy object, the weight is quite large, therefore, the push force (pressing force) between the object and the floor is also large. Thus, the friction force between them is large.

(8) For hard contact surfaces, the force of friction does not depend on the 'area of contact' between the two surfaces. But, it is not true if the surfaces are wet, or if they are soft. Rubber is soft as compared to the surface of a road. The friction between rubber and surface of road also depends on how much rubber is contacting with the surface of road. Thus, wide tires (made of rubber) have more friction than narrow tires.

Static friction

It is the force exerted on an object at rest that prevents the object from sliding.

- The direction of static friction is opposite to the applied force. Also, it acts in a direction opposite to the direction in which an object tends to move.
- The maximum value of static friction is called the **starting friction** or **limiting friction**. It is the amount of force that must be overcome to start a stationary object moving.

The law of static friction may be written as, $\boxed{f_s \le \mu_s N}$

Where, μ_s = coefficient of static friction, depends only on the nature of surfaces in contact ;
N = normal force (or normal reaction).

Limiting (maximum) value of static friction is given by, $\mathbf{f_L = \mu_s N}$. If the applied force F exceeds f_L , the body begins to slide on the surface.

- If applied force F is less than f_L, then, $\mathbf{F = f_s}$ i.e., applied force is equal to the value of static friction and body will remain at rest.

Sliding friction (or kinetic friction)

It is the force exerted on an object in motion that opposes the motion of the object as it slides on another object.

- Sliding or kinetic friction is smaller than the limiting value of static friction. This is because it takes more force to break the interlocking between two surfaces than it does to keep them sliding once they are already moving.
- Kinetic friction, like static friction, is also found to be independent of the area of contact. Further, it is nearly independent of the velocity of the body.

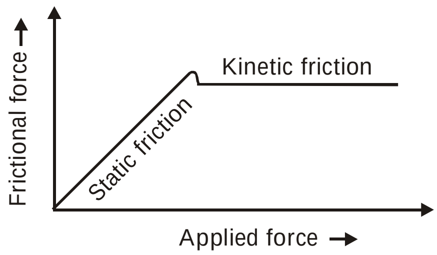


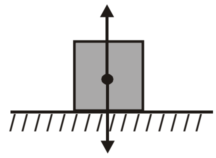
Fig.15 Variation of friction with applied force

The law of kinetic friction may be written as, $\boxed{f_k = \mu_k N}$

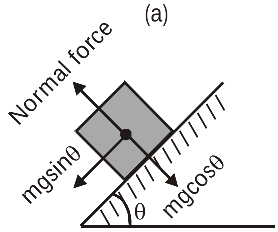
Where μ_k is the coefficient of kinetic friction, depends only on the nature of surfaces in contact.

$\boxed{\mu_s > \mu_k}$, μ_s or μ_k has not units as they are ratio of two forces.

- Normal force on a horizontal plane is, N = mg while normal force on an inclined plane is mgcosθ, where 'θ' is the angle made by the plane with the horizontal.
- Note that it is not motion, but relative motion that the frictional force opposes.



Normal force = weight = mg
(a)



Normal force = mgcosθ
(b)

Fig.16 Normal forces on horizontal and inclined plane

Angle of repose (θ)

If a body is placed on an inclined plane and if its angle of inclination is gradually increased, then at some angle of inclination θ the body will start just sliding down. This angle of the inclined plane at which the body just starts sliding is called angle of repose (θ).

From fig., we have, f_L = mgsinθ ----- (1)

R = mgcosθ ----- (2)

$$\frac{(1)}{(2)} \Rightarrow \frac{f_L}{R} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$$

or $\boxed{\tan\theta = \mu_s}$

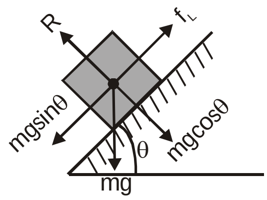


Fig.17 Angle of repose

(1) If angle of inclination (α) of the plane is less than angle of repose (θ), the body will remain at rest.

(2) If α = θ, then body will just slide i.e., will move uniformly.

(3) If α > θ, the body will accelerate downwards. The acceleration can be found by the fig.18.

F_{net} = mgsinα – f_k = mgsinα – μ_kR = mgsinα –$\mu_k$mgcosα

or ma = m(gsinα –$\mu_k$gcosα)

or **a = g(sinα –$\mu_k$cosα)**

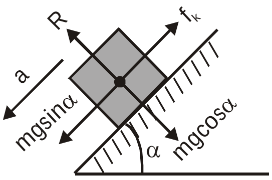


Fig.18

- If there is no friction, acceleration on the inclined plane, a = gsinα, where α is the angle made by the inclined plane with the horizontal.

Rolling friction

The rolling motion of the wheel is a combination of both spin (rotational) motion and linear (translational) motion. When one body rolls over the surface of another body, the resistance (opposition) to its motion is called the **rolling friction**.

- Rolling reduces the friction significantly. Since the rolling friction is smaller than the sliding friction, sliding is replaced in most machines by rolling by the use of ball bearings.
- Rolling friction increases with the deformation of tyre or wheel. Thus, rolling friction of a tyre or wheel made of rubber is more than a tyre or wheel made of iron. This is because the iron wheel deform negligibly while rubber tyre deform significantly.

CHAPTER 3 Gravitation

Gravitation is the weakest force in nature. It is negligible in the interactions of tiny particles, and thus plays no role in molecules, atoms, and nuclei. The gravitational attraction between objects of ordinary size, such as the gravitational force exerted by a building on a car, is too small to be noticed. When we consider very large objects, such as stars, planets, and satellites (moons), gravitation is of primary importance. The gravitational force exerted by the earth on us and on the objects around us is a fundamental part of our experience. It is gravitation that binds us to the earth and keeps the earth and the other planets on course within the solar system. The gravitational force plays an important role in the life history of stars and in the behaviour of galaxies.

3.1 The Newtonian gravitation

Sir Isaac Newton did not discover gravitation, its effects have been known throughout human existence. But he was the first one to understand the broader significance of gravitation. Newton discovered that 'gravitation is universal, it is not restricted to earth only', as other physicists of his time assumed.

Newton's universal law of gravitation

'Every object in the universe attracts every other object with a force which is proportional to the product of their masses and inversely proportional to the square of the distance between them. The force is along the line joining the centres of two objects'.

Let us consider two masses m_1 and m_2 lying at a separation distance r. Let the force of attraction between two objects be F. According to the universal law of gravitation,

$$F \propto m_1 m_2$$

$$F \propto \frac{1}{r^2}$$

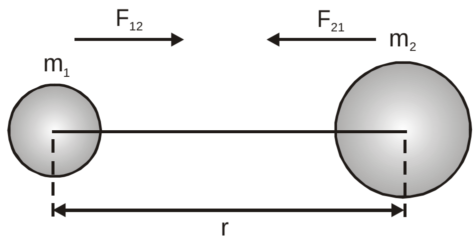


Fig.1 Newton's law of gravitation

Combining both, we get, $F \propto \frac{m_1 m_2}{r^2}$

or $\boxed{F = \frac{Gm_1 m_2}{r^2}}$

Where, G is the constant of proportionality and is called the universal gravitation constant.

- Universal gravitation constant is the magnitude of the force (in newton) between a pair of 1 kg masses that are kept 1 meter apart.

Unit of universal gravitation constant : According to the universal law of gravitation,

$$F = \frac{Gm_1 m_2}{r^2} \quad \text{or} \quad G = \frac{Fr^2}{m_1 m_2}$$

Thus, S.I. unit of $G = \frac{\text{Newton} \times (\text{meter})^2}{\text{kg} \times \text{kg}} = \frac{Nm^2}{kg^2} = Nm^2kg^{-2}$

- The value of the constant G is so small that it could not be determined by Newton or his contemporary experimentalists. It was determined by Henry Cavendish for the first time (1798) about 100 years later. Its accepted value today is 6.673×10^{-11} Nm^2kg^{-2}. It is because of the smallness of G that the gravitational force due to ordinary objects is not felt by us.

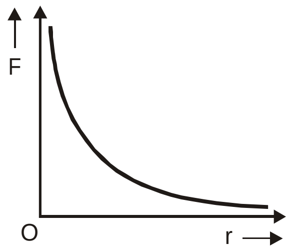


Fig.2 Dependence of gravitational force (F) on separation distance (r).

Characteristics of gravitational force :

(1) It is a universal force of attraction.

(2) It acts along the line joining the centres of each mass.

(3) It acts equally on each mass, i.e., it obeys Newton's third law i.e., $F_{12} = -F_{21}$

(4) It is weaker if the masses are further apart. It acts in an inverse square manner, i.e., $F \propto \frac{1}{r^2}$, where 'r' is the distance between the centres of the masses.

(5) It depends directly on the mass of each body involved, i.e., $F \propto m_1$ and $F \propto m_2$.

(6) It is a long range force i.e., its influence is extending to very large distances.

(7) It does not depend on the medium present between the two masses.

(8) The Newton's universal law of gravitation successfully explained several phenomena which were believed to be unconnected :

(i) The force that binds us to the earth. (ii) The motion of the moon around the earth.

(iii) The motion of planets around the Sun. (iv) The tides due to the moon and the Sun.

Why moon does not fall on earth directly ?

The motion of moon is just like the motion of an object in circular motion. The velocity of the moon is directed tangent to the circle at every point along its path. The acceleration of moon is directed towards the center of the circle i.e., towards the earth (the central body) around which it is orbiting. This acceleration is caused by a centripetal force which is supplied by the gravitational force between the earth and the moon. If this force were absent, the moon in motion would continue in motion at the same speed and in a direction tangential to the circular path and would have escaped away from the earth. If the moon had no tangential velocity, it would have fallen on earth due to gravitation. Thus, it is the tangential velocity and the gravitational force that are perpendicular to each other and keep the moon to fall around the earth without actually falling into it.

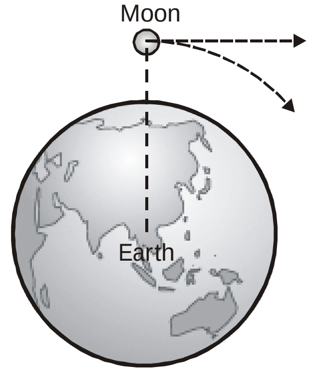


Fig.3 The Earth-Moon system

NUMERICAL CHALLENGE 3.1

Find the distance between a 0.300 kg billiard ball and a 0.400 kg billiard ball if the magnitude of the gravitational force between them is 8.80×10^{-11} N. Take, $G = 6.6 \times 10^{-11}$ Nm^2kg^{-2}.

Solution

Given, $m_1 = 0.3$ kg ; $m_2 = 0.4$ kg ; $F = 8.80 \times 10^{-11}$ N ; r = ?

According to Newton's law of gravitation, $F = \dfrac{Gm_1m_2}{r^2}$

or $r^2 = \dfrac{Gm_1m_2}{F} = \dfrac{(6.6\times10^{-11})(0.3)(0.4)}{8.8\times10^{-11}} = 0.09$

or r = **0.3 m**

3.2 Kepler's laws of planetary motion

Law of orbits : All planets move in elliptical orbits with the Sun situated at one of the foci of the ellipse (see fig.4). The closest point is P called the **perihelion** and the farthest point is A called the **aphelion**. The semimajor axis (R) is half the distance AP.

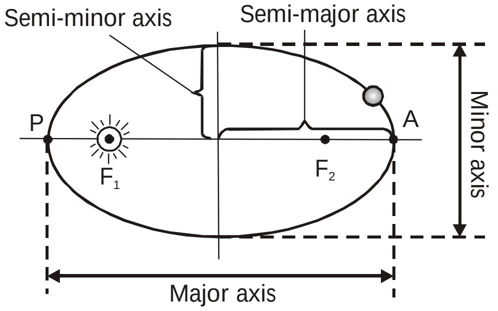


Fig.4 Law of orbits

Law of areas : The line that joins any planet to the sun sweeps (see fig.5) equal areas in equal intervals of time i.e., $\dfrac{\Delta A}{\Delta t} = \text{constant}$. This means, the planet moves faster when it is nearer to the Sun and it moves slower when it is farther from the Sun i.e, ($v \propto 1/r$).

For example, $\boxed{\dfrac{v_A}{v_P} = \dfrac{r_P}{r_A}}$. Since $r_A > r_P$, $v_A < v_P$.

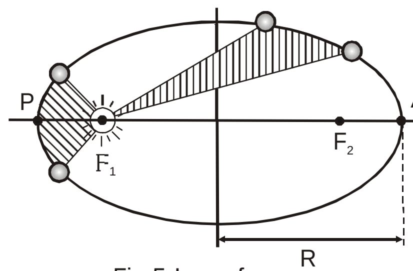


Fig.5 Law of areas

Law of periods : The square of the time period of revolution of a planet is proportional to the cube of the semi-major axis (mean distance) of the ellipse traced out by the planet i.e.,

$T^2 \propto R^3$ or $\dfrac{T^2}{R^3} = \text{constant}$. Also, $\left(\dfrac{T_1}{T_2}\right)^2 = \left(\dfrac{R_1}{R_2}\right)^3$

3.3 Acceleration due to gravity

The constant acceleration of a freely falling body is called the acceleration due to gravity. It is the acceleration of an object in free fall that results from the influence of Earth's gravity. Its magnitude is denoted with the letter g.

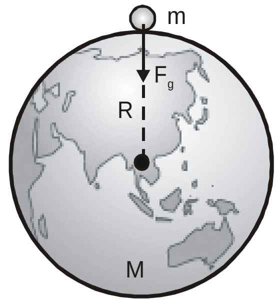


Fig.6 Acceleration due to gravity

Acceleration due to gravity at the surface of earth

Let us consider an object of mass m placed on the surface of Earth. Let the mass of Earth be M and radius of earth be R. The gravitational force on the object due to Earth is given by,

$$F_g = \frac{GmM}{R^2} \quad ...(1)$$

Let this force produces an acceleration 'a' in the object, then,

F_g = ma ...(2)

From eq.(1) and eq.(2), we get,

$$ma = \frac{GmM}{R^2}$$

$$\text{or } a = \frac{GM}{R^2}$$

This acceleration is called acceleration due to gravity and it is denoted by g i.e., a = g.

$$\therefore \quad \boxed{g = \frac{GM}{R^2}}$$

- The acceleration due to gravity g for any planet is (i) directly proportional to the mass of the planet (ii) inversely proportional to the square of the radius of the planet.

Acceleration due to gravity (g) on earth is 9.8 ms^{-2}. In Cgs system, value of g is 980 cm/s^2. In fps system, value of g is 32 ft/s^2.

- Among the planets, value of 'g' is maximum for Jupiter, $g_{jupiter}$ = 26 m/s^2.
- For two planets 1 and 2, ratio of their acceleration due to gravity, $\frac{g_2}{g_1} = \frac{M_2R_1^2}{M_1R_2^2}$

NUMERICAL CHALLENGE 3.2

The radius of the Earth shrinks by 10 %. By how much percent the acceleration due to gravity on the Earth's surface would (mass remaining constant) change ?

Solution

Initial radius = r ; mass of Earth remains constant = M

New radius, $r' = r - \frac{10}{100}r = \frac{90}{100}r = 0.9r$

Initial value of acceleration due to gravity, $g = \frac{GM}{r^2}$

New value of acceleration due to gravity, $g' = \frac{GM}{(r')^2} = \frac{GM}{(0.9r)^2} = \frac{1}{0.81}\left(\frac{GM}{r^2}\right) = \frac{1}{0.81}g$

Percentage change in the value of acceleration due to gravity is given by,

$$\frac{g'-g}{g} \times 100 = \frac{(1/0.81)g - g}{g} \times 100$$

$= \left(\frac{1}{0.81} - 1\right) \times 100 = +\frac{0.19}{0.81} \times 100$ = +23.45 % Positive sign shows that there is an increase in the value of g. The value of g increases by **23.45%**.

3.4 Factors affecting acceleration due to gravity

Shape of Earth

Our earth is not perfectly spherical. The radius of earth at poles (R_P) is slightly smaller than the radius of earth at equator (R_E).

$$g_P = \frac{GM}{R_P^2} \text{ --- (1) } \quad \text{and} \quad g_E = \frac{GM}{R_E^2} \text{ --- (2)}$$

$$\frac{(1)}{(2)} \Rightarrow \frac{g_P}{g_E} = \frac{GM / R_P^2}{GM / R_E^2} \quad \text{or} \quad \frac{g_P}{g_E} = \frac{R_E^2}{R_P^2} \text{ --- (3)}$$

Since, $R_E > R_P$, therefore, $g_P > g_E$.

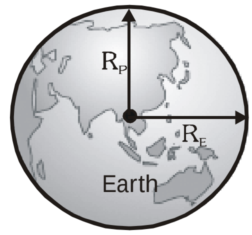


Fig.7 Shape of the Earth is not perfectly spherical, $R_P > R_E$

Rotation of Earth

Rotation of earth also affects the value of acceleration due to gravity at place on the surface of earth. Because of rotation, an object experiences a centrifugal force acting away from the axis of rotation which varies from place to place on the Earth. This centrifugal force is maximum at equator and minimum (zero) at the poles. As a result, value of g at equator is minimum and value of g at poles is maximum. Thus, because of rotation, $g_P > g_E$.

- As we move from a place on equator to a place on pole, value of g increases. In other words, if latitude angle increases from 0° (equator) to 90° (poles), g also increases.
- If rotation of earth stops, value of g will increase at the equator while it will remain unchanged at the poles.

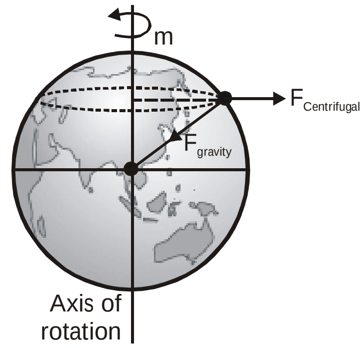


Fig.8 Rotation of Earth affects acceleration due to gravity

■ Considering the shape of the Earth and its rotation, we can conclude that acceleration due to gravity on the surface of earth is maximum at the poles and minimum at the equator.

Height above the surface of earth

$$g_A = \frac{GM}{(R+h)^2} = \frac{gR^2}{(R+h)^2} \quad \text{(see fig.9)}$$

- For h << R, i.e., a point very near the surface,

$$g_A = g\left(1 - \frac{2h}{R}\right)$$

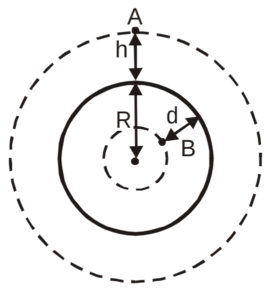


Fig.9 Effect of height(or depth) above (or below) the surface of Earth

Depth below the surface of earth

$$g_B = g\left(1 - \frac{d}{R}\right) \quad \text{(see fig.9)}$$

- At the centre of Earth, d = R thus, value of g at the centre is,

$$g_{Centre} = g\left(1 - \frac{R}{R}\right) = g(1-1) = 0$$

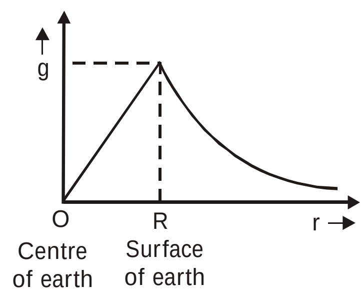


Fig.10 Variation g with distance from the centre of Earth

■ Acceleration due to gravity on the moon is one sixth of the acceleration due to gravity on the earth i.e.,

$$\frac{g_m}{g_e} = \frac{1}{6}$$

Thus, weight of the object on the moon = (1/6) x its weight on the earth.

3.5 Gravitational potential energy

Gravitational potential energy between two masses m_1 and m_2 having distance r between them is,

$$U = -\frac{Gm_1m_2}{r}$$

- Gravitational potential energy is always negative, the negative sign shows its attractive nature.

Gravitational potential energy of a system of n particles can be written as

$$U = -G\sum_{\substack{i\neq j \\ \text{All pairs}}} \frac{m_i m_j}{r_{ij}}$$

For example, for a three particle system (see fig.11),

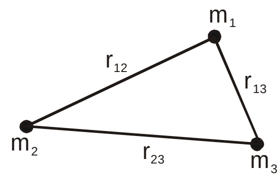


Fig.11 Gravitational potential energy of three particle system

$$U = -\frac{Gm_1m_2}{r_{12}} - \frac{Gm_2m_3}{r_{23}} - \frac{Gm_1m_3}{r_{13}}$$

- Gravitational potential energy of an object located at a height h above the surface of Earth is given by,

$$U = -\frac{GMm}{(R+h)}$$ Where, M = mass of Earth ; m = mass of object ; R = radius of earth.

- Gravitational potential energy of a particle located on the surface of earth,

$$U = -\frac{GMm}{R}$$

3.6 Orbital velocity

The speed of a satellite, spacecraft, or other body travelling in an orbit around the earth is called **orbital velocity**.

$$v_O = \sqrt{\frac{GM}{(R+h)}} = \sqrt{\frac{gR^2}{(R+h)}}$$ (see fig.12)

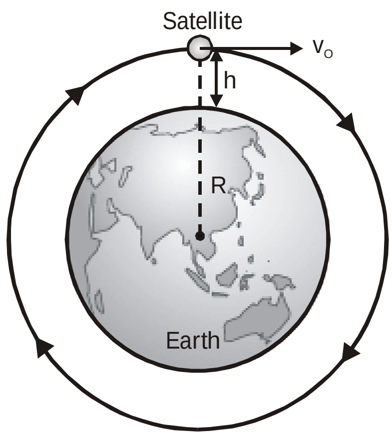


Fig.12 Orbital velocity of a satellite

- For a satellite orbiting quite near to the earth,

$$v_O = \sqrt{\frac{GM}{R}} = \sqrt{gR} \approx 8 \text{ km/s}$$

- Time period of a satellite orbiting around the earth in a circular path,

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

- Time period of a satellite orbiting very near to the surface of earth in a circular path,

$$T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}} \approx 85 \text{ minutes}$$

Geostationary orbit

It is an orbit of the earth made by an artificial satellite with a period exactly equal to the earth's period of rotation on its axis, i.e., 24 hours. If the orbit lies in the equatorial plane and is circular, the satellite will appear to be stationary. This is called a stationary orbit (or geostationary orbit) and it occurs at an altitude of 35800 km. Most communication satellites are in stationary orbits, with three or more spaced round the orbit to give worldwide coverage. Such satellites are called **geostationary satellites**.

Polar satellites

These are low altitude satellites (h is nearly 500 to 800 km), but they go around the poles of the earth in a north-south direction whereas the earth rotates around its axis in an east-west direction. Their time period is around 100 minutes and they cross any latitude many times a day.

3.7 Escape speed (or velocity)

The minimum speed needed by an object like space vehicle, rocket, etc., to escape from the gravitational field of the earth, moon, or other celestial body is called escape speed or velocity (v_e).

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

Some important points related to escape velocity are :

(1) Escape velocity is independent of the mass of the object projected from the Earth. For example, a spacecraft has the same escape speed as a molecule.

(2) Escape velocity is independent of the direction of the velocity.

(3) Escape velocity for earth is about 11.2 km/s. Escape velocity for moon is about 2.3 km/s, nearly five times smaller than that of earth. Among the planets, escape velocity is maximum for Jupiter, it is 59.5 km/s.

CHAPTER 4 Fluid mechanics

Fluids play a vital role in many aspects of everyday life. We drink them, breathe them, and swim in them; they circulate through our bodies, they control our weather, airplanes fly through them, and ships float in them. A **fluid** is any substance that can flow; we use the term for both liquids and gases. Usually, gases are easily compressed while liquids are quite incompressible. We begin our study with **fluid statics**, the study of fluids at rest (in equilibrium). The key concepts include density, pressure and buoyancy. **Fluid dynamics** is the study of fluids in motion. Fluid dynamics is much more complex and indeed is one of the most complex branches of mechanics.

4.1 Fluid statics

The branch of physics dealing with the properties of fluids at rest is known as **fluid statics** or **hydrostatics**.

Fluid pressure

The pressure of a fluid at a point inside it is defined to be the magnitude of the normal force (or thrust) exerted by the fluid on a unit area about that point.

$$\boxed{\text{Pressure} = \frac{\text{Thrust}}{\text{Area}} = \frac{\text{Force}}{\text{Area}}}$$

SI unit of pressure : 1 Pa = 1 N/m^2

A common unit of pressure is the atmosphere (atm), i.e. the pressure exerted by the atmosphere at sea level,

1 atm = 1.013 $\times$ 10^5 Pa ; 1 Bar = 10^5 Pa ; 1 atm = 760 mm of Hg = 76 cm of Hg

Another common unit of pressure is pounds/inch^2 (lb/in.^{2}), also called '**psi**'. We are accustomed to the '30 – 35 psi' pressure within our car's tyres.

1 atm = 14.7 psi ; 1 psi $\approx$ 6895 Pa ; 1 mm of Hg = 1.934 $\times$ 10^{-2} psi

- Pressure is a scalar quantity. Always remember, it is the component of the force normal (perpendicular) to the area under consideration for calculating pressure, not the force vector.

Blood pressure in human body is also measured in 'mm of Hg'. Pressure of flowing blood in major arteries is approx. 120 mm of Hg, when heart is contracted to its smallest size (systolic pressure). When the heart expands to its largest size, the pressure is about 80 mm of Hg (diastolic pressure).

Pascal's law

Fluids can exert pressure on the base and walls of the container in which they are enclosed.

- The French scientist Blaise Pascal observed that 'the pressure in a fluid at rest is the same at all points if they are at the same height'. This is called '**Pascal's law**'.

Fluid pressure acts in all directions, not just the direction of the applied force. When you inflate a car tyre, you are increasing the pressure in the tyre. This force acts up, down, and sideways in all directions inside the tyre.

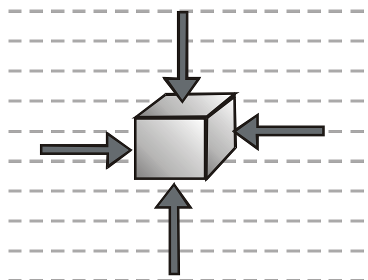


Fig.1 Fluid pressure acts perpendicular to the surface of the object

- The fluid pressure at any point on the object is perpendicular to the surface of the object at that point (see fig.1).

'Pressure applied to any part of an enclosed fluid at rest is transmitted in all directions equally to every portion of fluid and the walls of the containing vessel.' This is an another statement of **Pascal's law** and this property is used in hydraulic press ; hydraulic lift ; hydraulic brakes in cars, trucks.

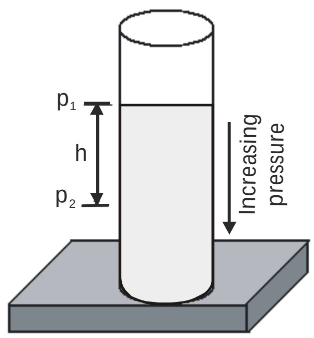


Fig.2 Pressure inside fluid increases with depth

If p_1 is the pressure on the surface of liquid (see fig.2) and p_2 is the pressure at a point within the liquid at a depth h, then, their pressure difference $(p_2 - p_1)$ is given by,

$$\Delta p = p_2 - p_1 = \rho g h$$

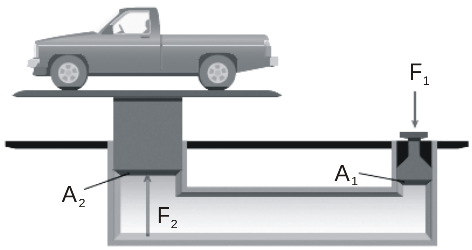


Fig.3 A hydraulic lift

The pressure is the same on both sides of the enclosed fluid, allowing a small force to lift a heavy object.

Pressure, $P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$ or $F_2 = \left(\frac{A_2}{A_1}\right)F_1$

Since, $A_2 >> A_1$, $F_2 >> F_1$.

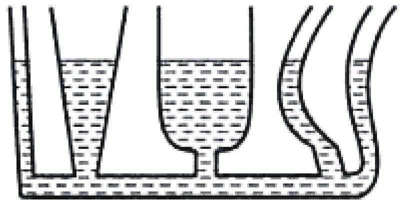


Fig.4 Hydrostatic paradox : Pressure at the bottom of each section of the vessel is same

- The pressure depends only on the height of the column of fluid above the surface where you measure the pressure. It does not depend on the area of the surface in contact or the shape of the liquid column. The greater the height of the column of fluid above a surface, the greater the pressure exerted by the fluid on the surface.
- Thickness of wall of the dam gradually increases as the depth increases. This is because pressure increases with the depth and to withstand great pressure, the thickness of the wall should be more.

The total force on a dam in which water is filled to a height H behind a dam of width W is given by (see fig.5),

$$\boxed{F = \frac{1}{2}\rho g W H^2}$$

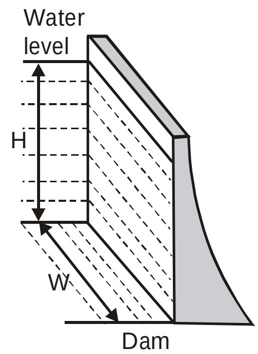


Fig.5

Absolute pressure

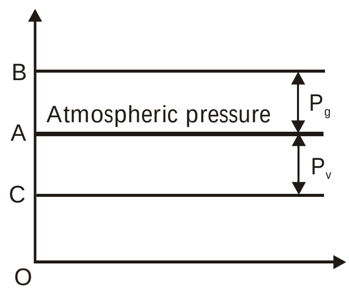
When pressure is measured above zero pascal (absolute zero or complete vacuum), it is called absolute pressure.

Gauge pressure (P_g)

When the pressure is measured above the atmospheric pressure, it is called gauge pressure.

Absolute pressure, OB = OA + AB

or $P_{absolute} = P_{atm} + P_g$



O represents 0 pascal (absolute vacuum)
AB = Gauge pressure = P_g
AC = Vacuum pressure = P_v
OA = Atmospheric pressure = P_a
Fig.6

- All pressure gauges read zero when open to atmosphere. They read the pressure difference between fluid pressure and the atmospheric pressure. It is measured by a 'pressure gauge'.

Vacuum pressure (P_v)

It is the pressure of a fluid below the atmospheric pressure. Its value is the amount by which it is below the atmospheric pressure. It is measured by a 'vacuum gauge'.

Absolute pressure, OC = OA – AC

or $P_{absolute} = P_{atm} - P_v$

Equilibrium of two immiscible liquids in a U tube

Let two immiscible liquids of different densities be poured into the two limbs of a U tube (see fig.). Suppose that, when the liquids are at rest, D is their surface of separation and A and C are their free surfaces. The horizontal plane through D intersects the other limb at E.

At equilibrium, the pressures at D and E will be equal. The pressure at D is (see fig.7) $P + \rho_1 g h_1$ and that at E is $P + \rho_2 g h_2$. Here P is the atmospheric pressure. Therefore,

$P + \rho_1 g h_1 = P + \rho_2 g h_2$

or $\boxed{\dfrac{h_1}{h_2} = \dfrac{\rho_2}{\rho_1}}$

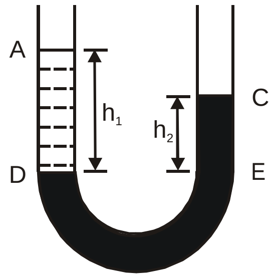


Fig.7

That is, at equilibrium, the heights of the two liquid columns above the common surface of contact are in the inverse ratio of their densities. The height of the heavier liquid will be smaller.

Note that the height of the liquid column does not depend on the cross-sectional area of the limb of the U tube. That is, the above equation will also hold if the two limbs have unequal diameters.

NUMERICAL CHALLENGE 4.1

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross-section and a radius of 5.00 cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0 cm. What force must the compressed air exert to lift a car weighing 13,275 N ? What air pressure produces this force ?

Solution

Area of small piston, $A_1 = \pi r_1^2 = \pi(5)^2 = 25\pi\ cm^2$; area of large piston, $A_2 = \pi r_2^2 = \pi(15)^2 = 225\pi\ cm^2$; force on large piston, F_2 = weight of the car = 13,275 N ; force on small piston, F_1 = ?

According to Pascal's law, the pressure is transmitted equally in all direction within the fluid, thus,

Pressure, $P = \dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$ or $F_1 = \left(\dfrac{A_1}{A_2}\right)F_2 = \left(\dfrac{25\pi}{225\pi}\right)(13275)$ = **1475 N**

The air pressure that produces this force is,

$P = \dfrac{F_1}{A_1} = \dfrac{1475\ N}{25\pi \times 10^{-4}\ m^2}$ = **1.878×10^5 Pa**

NUMERICAL CHALLENGE 4.2

In a huge oil tanker, salt water has flooded an oil tank to a depth of 5.00 m. On top of the water is a layer of oil 8.00 m deep (see fig.8). The oil has a density of 700 kg/m³. Find the pressure at the bottom of the tank. (Take 1020 kg/m³ as the density of salt water, air pressure as 1.01×10^5 Pa, g = 10 m/s²)

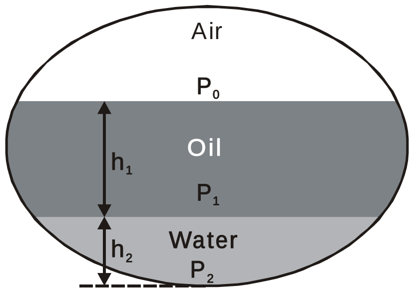


Fig.8 Numerical challenge 5.2

Solution

Given, height of layer of oil, h_1 = 8 m ; density of oil, ρ_1 = 700 kg/m³; height of layer of water, h_2 = 5 m ; density of water, ρ_2 = 1020 kg/m³; air pressure, $P_0 = 1.01 \times 10^5$ Pa.

Pressure at the bottom of the oil layer, $P_1 = P_0 + \rho g h_1$

or $P_1 = 1.01 \times 10^5 + (700\ )(10)(8)$

$= 1.01 \times 10^5 + 0.56 \times 10^5$ Pa

= **1.57×10^5 Pa**

Pressure at the bottom of the water layer, $P_2 = P_1 + \rho g h_2$

or $P_1 = 1.57 \times 10^5 + (1020\ )(10)(5)$

$= 1.57 \times 10^5 + 0.51 \times 10^5$ Pa

= **2.08×10^5 Pa**

4.2 Measurement of pressure

Atmospheric pressure

Atmospheric pressure is the pressure exerted on a surface by the weight of the atmosphere above the surface.

The air that surrounds Earth is called **atmosphere**. The layer of gases closest to Earth's surface is called the **troposphere**. The troposphere is between 8 and 18 kilometers thick. The troposphere contains 99% of the air in the atmosphere. The air is densest in this layer.

As the height above Earth increases, the number of particles of gas in the layers of the atmosphere decreases. The air gradually thins off into space. The highest layer, which is called the exosphere, ends at about 700 kilometers above Earth's surface. In this layer, negligibly small number of gas particles are present.

If you hold your hand out in front of you, Earth's atmosphere exerts a downward force on your hand due to the weight of the atmosphere above it.

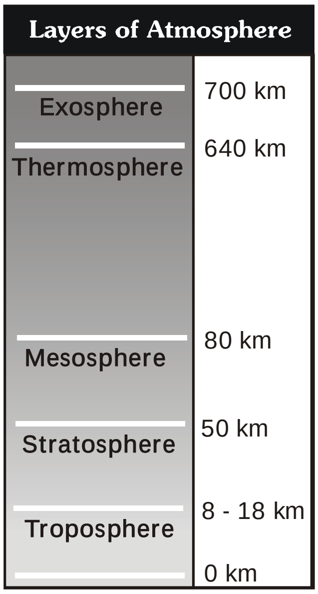


Fig.9 The atmosphere forms five layers of gases around Earth.

Altitude and air pressure

The particles of gas press on Earth's surface and on everything they surround. The force put on a given area by the weight of the air above it is called **air pressure** or **atmospheric pressure**. As you go higher in the atmosphere, the height of air column above you decreases. Thus, the weight of air above you decreases. Hence, the air pressure above you decreases. **Air pressure decreases with higher altitude.**

Mercury barometer

It is an instrument used to find the atmospheric pressure at any place. It consists of an evacuated glass tube put in a reservoir of mercury. Atmospheric pressure pushes mercury up in the tube. The mercury reaches a height where the pressure at the bottom of the column of mercury balances the pressure of the atmosphere.

Using formula, $P = \rho g h$ **or** $h = P/\rho g$, we can find the height of mercury column in the glass tube, which is,

760 mm of Hg = 76 cm of Hg = 1 atm

- Mercury is used in barometer because its density is high, thus, height of mercury column will be low ($h \propto 1/\rho$). If we use water in the barometer, then height of water will be 10.33 metre which is impractical.

  Another unit of pressure is 'Torr'. **1 torr = 1 mm of Hg**
- A mercury barometer measures 'absolute pressure' as the pressure inside the tube is zero i.e., complete vacuum.

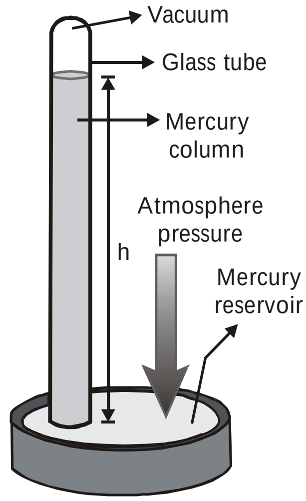


Fig.10 Mercury barometer

Manometers

Measuring pressure usually involves the use of liquid columns in vertical or inclined tubes. Pressure measuring devices based on this technique are called **manometers**. The mercury barometer is also an example of one type of manometer, but there are many other designs possible. Three common types of manometers include the piezometer tube, the U-tube manometer, and the inclined-tube manometer.

Piezometer tube

The simplest type of manometer consists of a vertical tube, open at the top, and attached to the container in which the pressure is to be measured (see fig.11). Since manometers involve columns of fluids at rest, the fundamental equation describing their use is $P = P_0 + \rho g h$ which gives the pressure at any elevation within a homogeneous fluid in terms of a reference pressure (P_0) and the vertical distance h between P and P_0.

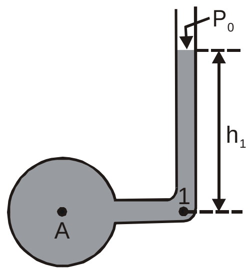


Fig.11 A piezometer tube

Remember that in a fluid at rest, pressure will increase as we move downward and will decrease as we move upward. Application of this equation to the piezometer tube of Fig.11 indicates that the pressure P_A can be determined by a measurement of h_1 through the relationship

$$P_A = P_1 = P_0 + \rho g h_1$$

(Point 1 and point A within the container are at the same horizontal level, thus, $P_A = P_1$)

Here, ρ is the density of the liquid in the container. Since the tube is open at the top, the pressure P_0 can be set equal to zero. In that case $P_A = \rho g h_1$, this is basically the 'gauge pressure', with the height h_1 measured from the upper surface to point 1.

- Although the piezometer tube is a very simple and accurate pressure measuring device, it has several disadvantages. It is only suitable if the pressure in the container is greater than atmospheric pressure (otherwise air would be sucked into the system), and the pressure to be measured must be relatively small so the required height of the column is reasonable. Also, the fluid in the container in which the pressure is to be measured must be a liquid rather than a gas.
- An important device whose operation is based upon the principle used in a piezometric tube is ' sphygmomanometer', the traditional instrument used to measure blood pressure.

Sphygmomanometer

Blood pressure is measured with a sphygmomanometer (see fig.12). The oldest kind of sphygmomanometer consists of a mercury manometer on one side attached to a closed bag—the cuff. The cuff is wrapped around the upper arm at the level of the heart. A rubber bulb forces air into a cuff and simultaneously into a manometer. The manometer measures the gauge pressure of the air in the cuff. At first, the pressure in the cuff is higher than the systolic pressure—the maximum pressure in the brachial artery that occurs when the heart contracts. The cuff pressure squeezes the artery closed and no blood flows into the forearm. A valve on the cuff is then opened to allow air to escape slowly. The measurer listens with a stethoscope to the artery at a point just below the cuff. When the cuff pressure decreases to just below the systolic pressure, a little amount of blood flows past the constriction in the artery with each heartbeat. The sound of turbulent blood flow past the constriction can be heard through the stethoscope. The manometer is calibrated to read the pressure in millimeters of mercury, and the value obtained is about 120 mm for a normal heart. Values of 130 mm or above are considered high, and medication to lower the blood pressure is often prescribed for such patients. As air continues to escape from the cuff, the sound of blood flowing through the constriction in the artery continues to be heard. When the pressure in the cuff reaches the diastolic pressure in the artery—the minimum pressure that occurs when the heart muscle is relaxed—there is no longer a constriction in the artery, so the pulsing sounds cease. At this point, continuous sounds of blood flow are heard. In the normal heart, this transition occurs at about 80 mm of mercury, and values above 90 require medical intervention. Blood pressure readings are usually expressed as the ratio of the systolic pressure to the diastolic pressure, which is 120/80 for a healthy heart.

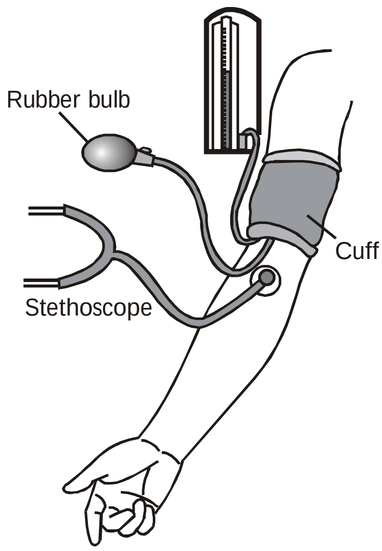


Fig.12 A sphygmomanometer measures blood pressure.

U-tube manometer

To overcome the difficulties noted in piezometric tube, another type of manometer which is widely used consists of a tube formed into the shape of a U (see fig.13). The fluid in the manometer is called the gauge fluid. One commonly used U-tube manometer consists of mercury as fluid. The pressure at point A and point 1 are the same, and as we move from point 1 to point 2, the pressure will increase by $\rho_1 g h_1$, ρ_1 is the density of fluid inside the vessel. The pressure at point 2 is equal to the pressure at point 3, since the pressures at same horizontal levels in a continuous mass of fluid at rest must be the same. Note that we could not simply "jump across" from point 1 to a point at the same horizontal level in the right-hand tube since these would not be points within the same continuous mass of fluid. With the pressure at point 3 specified, we now move to the open end where the pressure is P_0.

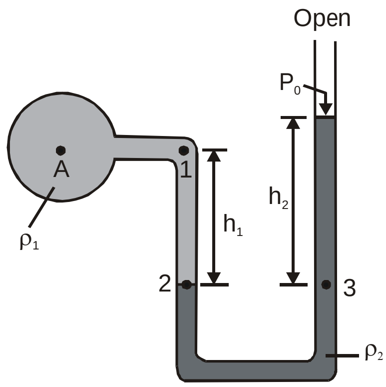


Fig.13 A U-tube manometer

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As we move vertically upward the pressure decreases by an amount $\rho_2 gh_2$, where ρ_2 is the density of gauge fluid. In equation form these various steps can be expressed as,

$P_A + \rho_1 gh_1 - \rho_2 gh_2 = P_0$

or $P_A = P_0 + \rho_2 gh_2 - \rho_1 gh_1$ ----- (1) (This is absolute pressure)

If we put P_0 as zero, we get,

$P_A = \rho_2 gh_2 - \rho_1 gh_1$ ------ (2) (This is gauge pressure)

- A major advantage of the U-tube manometer lies in the fact that the gauge fluid can be different from the fluid in the container in which the pressure is to be determined. The fluid in vessel A can be either a liquid or a gas. If A does contain a gas, the contribution of the gas column $\rho_1 gh_1$ is almost always negligible so that $P_A = P_2$, and in this instance eq.(1) becomes,

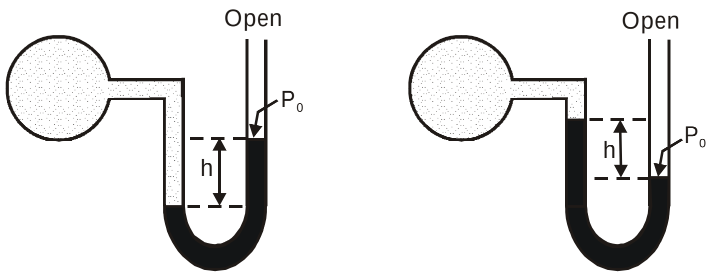
  $P_A = P_0 + \rho_2 gh_2$

  Similarly, eq.(2), becomes, $P_A = \rho_2 gh_2$

  Thus, from the above equation we can conclude that, for a given pressure, the height h_2 is governed by the density ρ_2 of the gauge fluid used in the manometer.

- If the pressure is large, then a heavy gauge fluid, such as mercury, can be used and a reasonable column height (not too long) can still be maintained. Alternatively, if the pressure is small, a lighter gauge fluid, such as water, can be used so that a relatively large column height (which can be easily read) can be achieved.

  The fig.14 shown below consists of a gas inside the vessel, thus, the contribution of the gas column is negligible. Fig.14(a) shows 'gauge pressure', a pressure greater than the atmospheric pressure. Fig.14(b) shows 'vacuum pressure', a pressure below the atmospheric pressure.



(a)
Gauge pressure = ρgh
$P_{asbolute} = P_0 + \rho gh$

(b)
Vacuum pressure = ρgh
$P_{asbolute} = P_0 - \rho gh$

P_0 is the atmospheric pressure.

Fig.14 A U-tube manometer consisting of gas inside the given vessel

- **Aneroid barometer** is also a pressure gauge that is used to measure the pressure inside a fluid.

NUMERICAL CHALLENGE 4.3

A manometer is attached to a container of gas to determine its pressure. Before the container is attached, both sides of the manometer are open to the atmosphere. After the container is attached, the mercury on the side attached to the gas container rises 12 cm above its previous level (see fig.15). What is the absolute pressure of the gas in Pa ? Atmospheric pressure, $P_0 = 1.01 \times 10^5$ Pa ; g = 9.8 m/s^2 ; density of mercury, r = 13600 kg/m^3.

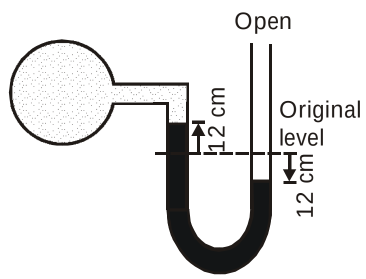


Fig.15 Numerical challenge 5.3

Solution

The mercury column is higher on the side connected to the container of gas, so we know that the pressure of the enclosed gas is lower than atmospheric pressure i.e., it is vacuum pressure. We need to find the difference in levels of the mercury columns on the two sides. It is not 12 cm! If one side went up by 12 cm, then the other side has gone down by 12 cm, since the same volume of mercury is contained in the manometer. Thus, the difference in the mercury levels is 24 cm i.e., h = 24 cm =0.24 m

Absolute pressure, $P_{absolute} = P_0 - \rho gh = 1.01 \times 10^5 - 13600 \times 9.8 \times 0.24$

$= 1.01 \times 10^5 - 0.32 \times 10^5 = \mathbf{6.9 \times 10^4\ Pa}$

NUMERICAL CHALLENGE 4.4

A woman's systolic blood pressure when resting is 160 mm Hg. What is this pressure in,

(a) Pa (b) lb/in^2 (psi) (c) atm (d) torr ?

Solution

(a) We know that, density of mercury, $\rho = 13600\ kg/m^3$; $g = 9.8\ m/s^2$.

Given, height of mercury column, h = 160 mm = 0.160 m

Pressure, $P = \rho gh = 13600 \times 9.8 \times 0.160 = \mathbf{2.132 \times 10^4\ Pa}$

(b) We know that, 1 mm of Hg = 1.934×10^{-2} psi

or 160 mm of Hg = $160 \times 1.934 \times 10^{-2}$ psi = **3.0944 psi**

(c) We know that, 760 mm of Hg = 1 atm

or 1 mm of Hg = (1/760) atm

or 160 mm of Hg = $(1/760) \times 160$ atm = **0.2105 atm**

(d) 1 mm of Hg = 1 torr

Thus, pressure = 160 mm of Hg = **160 torr**

NUMERICAL CHALLENGE 4.5

When a mercury manometer is connected to a gas main, the mercury stands 40.0 cm higher in the tube that is open to the air than in the tube connected to the gas main. A barometer at the same location reads 74.0 cm of Hg. Determine the absolute pressure of the gas in cm of Hg.

Solution

Since, the mercury stands higher in the tube that is open to the air, the pressure of the gas is 'gauge pressure', a pressure above the atmospheric pressure [refer fig.14(a)].

Thus, absolute pressure, $P_{absolute} = P_{atmosphere} + P_{gauge}$ = 74 cm of Hg + 40 cm of Hg = **114 cm Hg**

NUMERICAL CHALLENGE 4.6

A closed tank contains compressed air and oil ($\rho_{oil} = 900\ kg/m^3$) as is shown in fig.16. A U-tube manometer using mercury ($\rho_{Hg} = 13600\ kg/m^3$) is connected to the tank as shown. The column heights are $h_1 = 0.9$ m, $h_2 = 0.15$ m, h3 = 0.225 m. Determine the pressure reading in pascal of the gauge.

Solution

Following the general procedure of starting at one end of the manometer system and move to the other end, let us start at the air–oil interface in the tank and proceed to the open end where the pressure (P_0) is taken zero as we have to find gauge pressure. The pressure at level 1 is given by,

$P_1 = P_{air} + \rho_{oil}\ g\ (h_1 + h_2)$ ----- (1)

This pressure is equal to the pressure at level 2, since these two points are at the same horizontal level in a homogeneous fluid at rest. That is,

$P_1 = P_2 = P_{air} + \rho_{oil}\ g\ (h_1 + h_2)$ ----- (2)

As we move from level 2 to the open end, the pressure must decrease by $\rho_{Hg}\ g\ h_3$ and at the open end the pressure (P_0) is taken zero. Thus, the manometer equation can be expressed as,

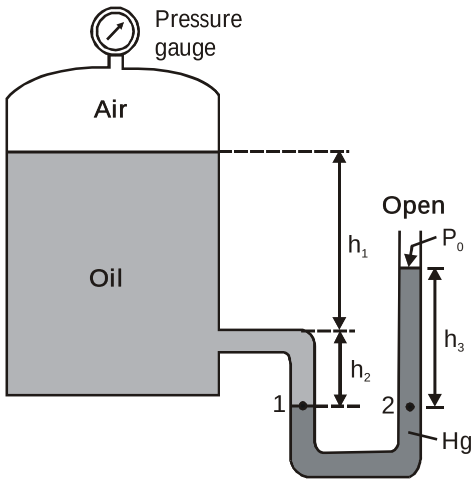


Fig.16 Numerical challenge 5.6

$P_2 - \rho_{Hg}\ g\ h_3 = P_0$

or $P_{air} + \rho_{oil}\ g\ (h_1 + h_2) - \rho_{Hg}\ g\ h_3 = 0$

or $P_{air} = \rho_{Hg}\ g\ h_3 - \rho_{oil}\ g\ (h_1 + h_2)$

$= 13600 \times 9.8 \times 0.225 - 900 \times 9.8 \times (0.9 + 0.15) = 2.9988 \times 10^4 - 0.9261 \times 10^4$

$= \mathbf{2.0727 \times 10^4\ Pa}$

4.3 Buoyancy

The tendency for an immersed body to be lifted up in a fluid, due to an upward force that acts opposite to the action of gravity is called buoyancy.

The buoyant force

It is an upward force that is exerted by a fluid on any object immersed partly or wholly in the fluid.

Archimedes' principle

According to Archimedes' principle, 'any object completely or partially submerged in a fluid experiences an upward buoyant force equal in magnitude to the weight of the fluid displaced by the object'.

Apparent weight

Because of an upward force acting on a body immersed in a fluid, either wholly or partially, there occur an apparent loss in weight of the body. The net weight of an object immersed in a fluid is called 'apparent weight'.

Sinking and floating

The buoyant force pushes an object in a fluid upward, but gravity pulls the object downward.

- If the weight of the object is greater than the buoyant force, the net force on the object is downward and it sinks (see fig.16)

Let an object of density ρ_S be immersed in a liquid of density ρ_L, W be the weight of object in air, F_B be the buoyant force.

- If $\rho_S > \rho_L$, the object will sink to the bottom (see fig.17).

Apparent weight, $W' = W - F_B$

$= M_S g - M_L g = (M_S - M_L)\, g = (\rho_S V_S - \rho_L V_L) g$

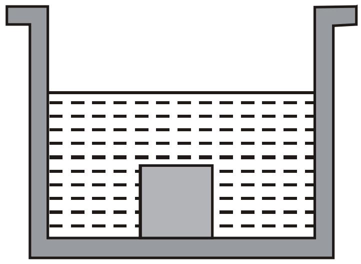
$= (\rho_S V_S - \rho_L V_S) g$

Here, $V_S = V_L$ because the object is completely immersed in water.

$$\boxed{W' = (\rho_S - \rho_L) V_S g}$$

Now, $W' = \left(1 - \dfrac{\rho_L}{\rho_S}\right)\rho_S V_S g$

Also, $\boxed{W' = W\left(1 - \dfrac{\rho_L}{\rho_s}\right)}$ $\left(\because \rho_S V_S g = W = \text{weight of object in air}\right)$

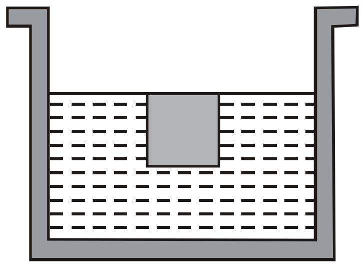


$\rho_S > \rho_L$
Fig.17 The body sinks to the bottom

- If the buoyant force is equal to the object's weight, the forces are balanced and the object floats.

If $\rho_S = \rho_L$, apparent weight = 0 i.e.,
weight of the body in air = buoyant force
or $W = F_B$ or $M_S g = M_L g$ or $M_S = M_L$
or $\rho_S V_S = \rho_L V_L$ or $\mathbf{V_s = V_L}$

This means, the body will just float or remain hanging at whatever height it is left inside the liquid (see fig.18).



$\rho_S = \rho_L$
Fig.18 The body just floats on liquid i.e, it is completely immersed in liquid

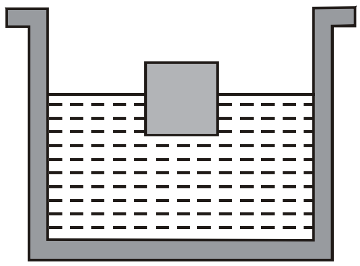
- If $\rho_S < \rho_L$, apparent weight = 0

i.e., weight of the body in air = buoyant force
or $W = F_B$ or $M_S g = M_L g$ or $M_S = M_L$

or $\rho_S V_S = \rho_L V_L$ or $\boxed{V_L = \dfrac{\rho_S V_S}{\rho_L}}$

Clearly, the volume (V_L) of liquid displaced is less than the total volume (V_S) of the body as $\rho_S < \rho_L$. This means the body will float but it is immersed partly in the liquid (see fig.19).

- The fraction of floating object inside a liquid, $\dfrac{V_L}{V_S} = \dfrac{\rho_S}{\rho_L}$



$\rho_S < \rho_L$
Fig.19 The body floats on liquid, it is partly immersed in liquid

- The fraction of floating object outside a liquid, $\frac{V_O}{V_S} = \frac{\rho_L - \rho_S}{\rho_L}$
- Buoyant force is the loss of weight of an object when it is immersed in a liquid.

$$\boxed{F_B = W_1 - W_2}$$

Where, W_1 is the weight of an object in air and W_2 is its weight (apparent weight) when it is completely immersed in the liquid.

NUMERICAL CHALLENGE 4.7

A plastic sphere of radius 0.04 m and mass 0.01 kg is immersed in a container of water. It is attached to a string that is attached to the bottom of the container [see fig.20(a)]. Find the tension in the string.

(Given, density of water = 1000 kg/m^3, g = 9.8 m/s^2).

Solution

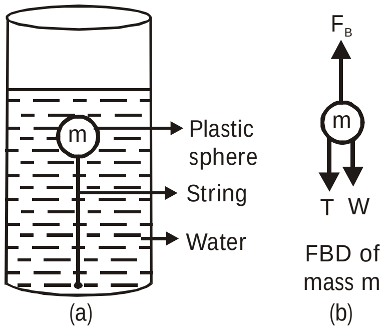


Fig.20 Numerical challenge 5.7

The buoyant force (F_B) acts upwards on the plastic sphere which is balanced by the tension (T) developed in the string and weight (W) of the sphere [see fig.20(b)], i.e.,

$F_B = T + W$

or $T = F_B - W = \rho_w V_w\, g - m_s g = (\rho_w V_w - m_s)g$----- (1)

V_w = volume of water displaced = volume of the sphere

($\because$ Sphere is completely immersed in water)

or $V_w = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.04)^3 = 2.6 \times 10^{-4}$ m^3;

mass of sphere, m_s = 0.01 kg

Now, $T = (\rho_w V_w - m_s)g = (1000 \times 2.6 \times 10^{-4} - 0.01) \times 9.8$

$= (0.26 - 0.01) \times 9.8 =$ **2.45 N**

NUMERICAL CHALLENGE 4.8

A boat is 4.0 m wide and 6.0 m long. When a heavy object is placed in it, the boat sinks 4.00 cm in the water. What is the weight of the object ? Take, density of water = 1000 kg/m^3.

Solution

Length, l = 6 m ; width, b = 4 m; depth, h = 4 cm = 0.04 m.

Volume of boat inside the water = volume of liquid displaced

or $V = l \times b \times h = 6 \times 4 \times 0.04 =$ **0.96 m³**

Due to the weight of the object, the boat further moves down in water, thus an extra buoyant force acts on the boat which must be equal to the weight of the object.

Thus, weight of the object, W = Extra buoyant force acting on the object = $\rho_w Vg = 1000 \times 0.96 \times 9.8$

= **9408 N**

4.4 Relative density (or specific gravity)

Relative density is given by, R.D = $\dfrac{\text{Density of object}}{\text{Density of water}} = \dfrac{\rho}{\rho_w}$

Also, $\boxed{\text{R.D.} = \dfrac{W_1}{W_1 - W_2}}$

Where, W_1 = weight of object in air ; W_2 = weight of object in water.

Hydrometer

A hydrometer is an instrument that measures the relative density or density of a liquid. When the hydrometer is placed in a liquid, it sinks to a certain depth. The depth to which it sinks depends on the density of the liquid, as shown in fig.21. The lower the density of the liquid, the deeper the hydrometer sinks.

- **Lactometer** is also a specially designed hydrometer used to measure the relative density of milk and hence testing its purity. The relative density of milk is nearly 1.03. On adding water to milk, the resulting mixture has relative density less than 1.03. Smaller the relative density, more will the amount of water added to the milk and hence lesser will be the purity of the milk.

Melting of ice that is floating on water

When a piece of ice floating on water in a beaker completely changes (melts) to liquid state, the level of water in the beaker remains unchanged.

Let m = mass of ice ; ρ = density of water ; V = volume of water displaced

$W = F_B$ or $mg = \rho_w Vg$

or $m = 1 \times V$ ($\rho_w = 1\ g/cm^3$) or $\mathbf{m = V}$ --- (1)

Now, volume V' of water formed on melting,

$V' = m/\rho = m/1$ or $\mathbf{V' = m}$ --- (2)

From (1) & (2), we get, **V' = V** , i.e., there is no change in the volume of the contents of the beaker. Hence, level of water will not change.

- Let us consider an ice cube containing a lead in it floating on water in a beaker. As ice melts, lead sinks to the bottom and the level of water in the glass falls. Let M = mass of ice cube, m = mass of lead piece. Now, weight of floating body is equal to buoyant force exerted by the liquid.

i.e., $\mathbf{W = F_B}$ or $(M + m)\,g = \rho_w Vg$ or $(M + m) = 1 \times V$ ($\rho_w = 1\ g/cm^3$)

or $\mathbf{M + m = V}$ --- (1)

On melting, the new volume, $V' = \dfrac{M}{\rho} + \dfrac{m}{\rho'} = \dfrac{M}{1} + \dfrac{m}{\rho'}$ or $\mathbf{V' = M + (m/\rho_{Lead})}$ --- (2) [$\rho_{Lead} > 1$]

Now, clearly m/ρ_{Lead} will be less than m, thus using (1) & (2), **V' < V**. Hence, level of water falls in the beaker.

4.5 *Equilibrium of floating bodies

For a floating body in equilibrium, there are two necessary conditions :

(1) The weight of the body W must be equal to the buoyant force F_B.

(2) The centre of gravity G of the body and the centre of buoyancy B must lie in the same vertical line [see fig.21(a)].

Centre of gravity : The centre of gravity of an object is the point at which the weight may be considered to act.

Centre of buoyancy : The centre of buoyancy of the object is located at the centre of gravity of the volume of the displaced liquid. It is the point through which the upward buoyant force seems to act.

***** for additional knowledge

If a floating body is slightly displaced from its equilibrium position by applying an external force, it gets slightly tilted. By tilting the floating body, the shape of the liquid displaced changes, thus, its initial centre of buoyancy B shifts to a new position B' towards the leaning side [see fig.21(b)]. The vertical line through the new position B' meets the line joining G and B (the initial vertical line) at a point M. This point is called 'meta centre'.

In tilted position, the weight W and buoyant force FB do not act in the same vertical line [see fig.21(b)]. This produces a rotation that may or may not restore the body to its equilibrium position.

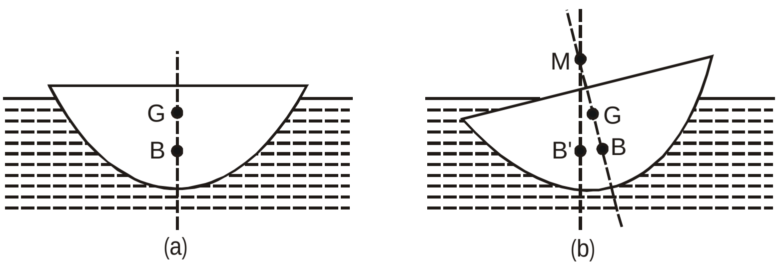


Fig.21 Equilibrium of a floating body

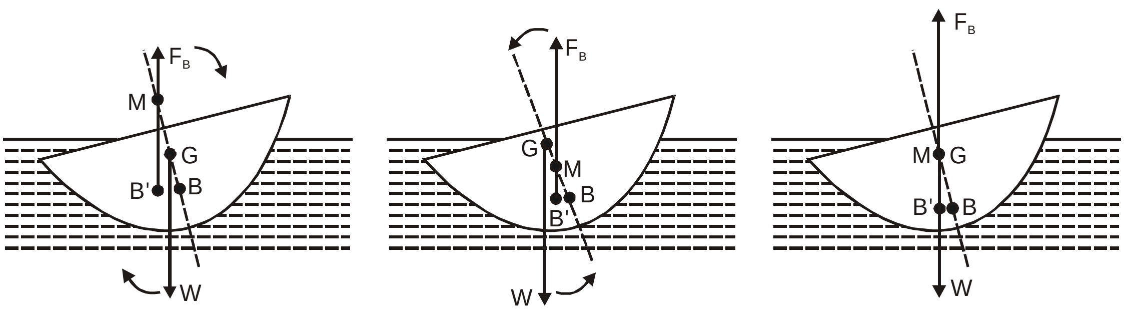
Depending on the position of the meta centre, the floating body may have three types of equilibrium :

(1) Stable equilibrium (2) Unstable equilibrium (3) Neutral equilibrium

Stable equilibrium : If the centre of gravity G of the body lies below the meta centre M [see fig.22(a)], the equilibrium is stable. This means on tilting the floating body, it has the tendency to get back to its original untilted position. This is possible when the body is heavily loaded at its bottom.

Unstable equilibrium : If the centre of gravity G of the body lies above the meta centre M [see fig.22(b)], the equilibrium is unstable. This means on tilting the floating body, it has the tendency to go further away from its original untilted position. This is possible when the body is heavily loaded at its top.

Neutral equilibrium : If the meta centre M coincides with the centre of gravity G of the body [see fig.22(c)], the equilibrium is neutral. This means on tilting the floating body, it has no tendency to go towards or go away from its original untilted position. That is, on tilting the body remains floating in the tilted position. This is possible when the mass of the body is distributed uniformly over the entire volume.



(a) Stable equilibrium (b) Unstable equilibrium (c) Neutral equilibrium

Fig.22 Different types of equilibrium for a floating body

Important Notes

CHAPTER 5 Work, energy, power

In common usage, the word 'work' means any physical or mental exertion. But in physics, work has a distinctly different meaning. Let us consider the following situations : (1) A student holds a heavy chair at arm's length for several minutes. (2) A student carries a bucket of water along a horizontal path while walking at constant velocity. (3) A student applying force against a wall. (4) A student studying whole day to prepare for examinations. It might surprise you to know that in all the above situations, no work is done according to the definition of work in physics., even though effort is required in all cases.

5.1 The concept of work

In physics, the word 'work' has a definite and precise meaning. Work is not done on an object unless the object is moved with the action of a force. The application of a force alone does not constitute work. For example, when a student holds the chair in his hand, he exerts a force to support the chair. But, work is not done on the chair as the chair does not move.

- Two important conditions that must satisfied for work to be done are : (i) a force should act on an object (ii) the object must be displaced. If any one of the above conditions does not exist, work is not done. This is the concept of work that we use in science.

Mathematical definition of work

A constant force is applied in the direction of the displacement of an object : Let a constant force, F acts on an object. Let the object be displaced through a distance, s in the direction of the force (see fig.1(a)). Let W be the work done. Here, we define work to be equal to '**the product of the force and displacement**'.

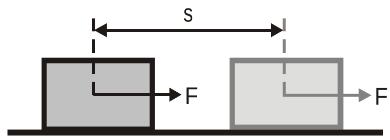


Fig.1(a) Work done by a constant force acting in the direction of displacement

Work done = force × displacement

$\boxed{W = F \times s}$

A constant force is applied at a certain angle with the direction of the displacement of an object : When the force on an object and the object's displacement are in different directions, the work done on the object is given by,

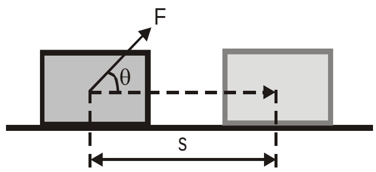


Fig.1(b) Work done by a constant force acting a an angle with the direction of displacement

$\boxed{W = F \times s \times \cos\theta}$

Where, the angle between the force and the direction of the displacement is θ (see fig.1(b)).

Here, we define work to be equal to '**the force multiplied by the displacement multiplied by the cosine of the angle between them**'.

- Work is a scalar quantity, it has only magnitude and no direction.

Unit of work : S.I. unit : Joule 1 Joule = 1 newton × metre or **1 J = 1 N m**

C.G.S. unit : Erg 1 erg = 1 dyne × centimetre or **1 erg = 1 Dyne cm**

1 Joule = 10^7 ergs

Definition of 1 joule : 1 J is the amount of work done on an object when a force of 1 N displaces it by 1 m along the line of action of the force.

Some important points related to work

(1) If θ = 0°, then cos 0° = 1 and W = F × s.

(2) If θ = 90°, then, W= 0 because cos 90° = 0. So, no work is done on a bucket being carried by a girl walking horizontally. The upward force exerted by the girl to support the bucket is perpendicular to the displacement of the bucket, which results in no work done on the bucket.

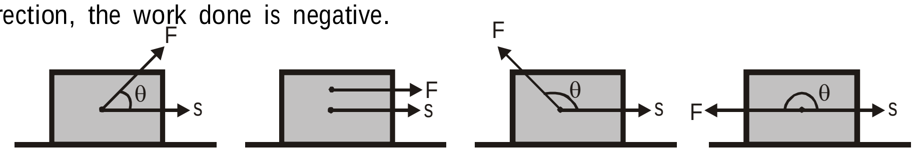
(3) If θ = 180°, then cos 180° = –1 and W = – F × s.

Concept of negative and positive work

The work done by a force can be either positive or negative.

Whenever angle (θ) between the force and the displacement is acute, i.e., 0° < θ < 90°, the work done is positive. Also, when angle (θ) between the force and displacement is zero, i.e., force and displacement are in same direction, the work done is positive.

Whenever angle (θ) between the force and the displacement is obtuse, i.e., 90° < θ < 180°, the work done is negative. Also, when angle (θ) between the force and displacement is 180°, i.e., force and displacement are in opposite direction, the work done is negative.



θ is acute angle
Work is positive
(a)

θ is zero
Work is positive
(b)

θ is obtuse angle
Work is negative
(c)

θ is 180°
Work is negative
(d)

Fig.2 Concept of positive and negative work

- Area under the force (F) - displacement (s) graph gives the work done on an object or a system.
- An artificial satellite is moving around the Earth in a circular path under the influence of centripetal force provided by the gravitational force between them. Centripetal force (F) is always perpendicular to the displacement (s) of the particle moving along a circular path. That is, the angle (θ) between them is 90°.

  Work done, $W = F\ s \cos\theta = F\ s \cos 90° = 0$

  Thus, work done by this centripetal force by the force is zero.

  - Work done by the centripetal force is always zero because it is always perpendicular to the displacement. For example, if an electron moves around a nucleus in a circular path due to centripetal force provided by the electric force between them, the work done by this force is zero.

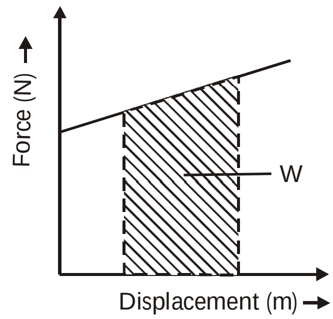


Fig.3 Area under force-displacement graph gives work done.

5.2 Work done by applied force against gravity

If an object is lifted up to a certain height (see fig.5), definitely, a work is done by the applied force. The applied force must be equal to the weight (= mg) of the object.

This work done is given by, $\mathbf{W = F \times s = mgh}$

Where, m = mass of object ; g = acceleration due to gravity ; h = height.

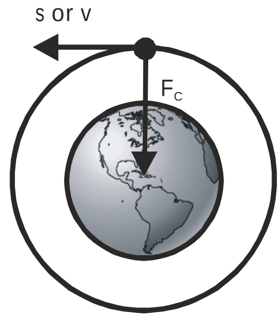


Fig.4 Work done by the centripetal force is zero

- Whenever a person holds an object in his hands or supports an object over his head, he is always applying a force in upward direction.
  - When a person lifts a body from the ground i.e., displaces it in upward direction, the work done by him is positive (see fig.) as force and displacement are in same direction. When a person puts an object from a certain height to the ground i.e., displaces it in downward direction, the work done by him is negative (see fig.) as force and displacement are in opposite direction.
  - When a person lifts a body from the ground i.e., displaces it in upward direction, the work done by force of gravity is negative (see fig.6) as force of gravity and displacement are in opposite direction. When a person puts an object from a certain height to the ground i.e., displaces it in downward direction, the work done by the force of gravity is positive (see fig.6) as force of gravity and displacement are in same direction.

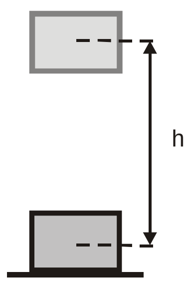


Fig.5 Work done against gravity

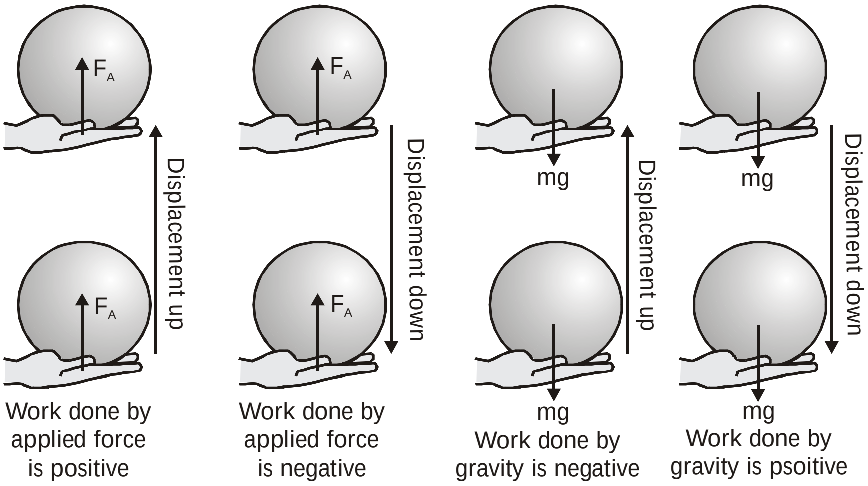


Fig.6 Work done by the applied force and gravity when an object is raised or lowered

- The work done against gravity depends on the difference in vertical heights of the initial and final positions of the object and not on the path along which the object is moved. This is because force of gravity is a conservative force. Fig.7 shows a case where a block is raised from position A to B by taking two different paths. Let the height AB = h. In both the situations the work done on the object is 'mgh'.

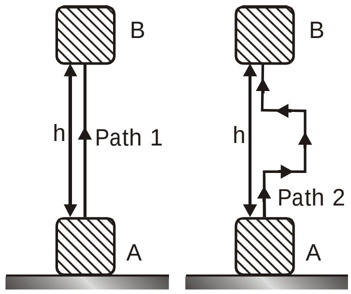


Fig.7 Work done against gravity depends only on initial position and final position

NUMERICAL CHALLENGE 5.1

1. A force of 6 N is applied on an object at an angle of 60° with the horizontal. Calculate the work done in moving the object by 2 m in the horizontal direction.

Solution

Given, force, F = 6 N ; angle between force and displacement, $\theta = 60°$; displacement, s = 2 m.

We know that work done, $W = F\, s \cos\theta = 6 \times 2 \cos 60° = 6 \times 2 \times (1/2) =$ **6 J**

2. A person lifts 5 kg potatoes from the ground floor to a height of 4 m to bring it to first floor. Calculate the work done.

Solution

Given, mass, m = 5 kg ; displacement, h = 4 m ; let us take, acceleration due to gravity, g = 9.8 m/s^2

Since the potatoes are lifted, work is being done against gravity. Therefore, we can write,

Work done = mgh = $49 \times 4 =$ **196 J**

3. A bag of grains of mass 2 kg is lifted through a height of 5 m. (a) How much work is done by the lift force ? (b) How much work is done by the force of gravity ?

Solution

Given, mass, m = 2 kg ; displacement, s = h = 5 m

(a) Here, work done by the lift force is positive as both lift force and the displacement are in same direction (upward direction).

Thus, work done by the lift force, $W = + mgh = + 2 \times 9.8 \times 5 =$ **98 J**

(b) Here, work done by the force of gravity is negative as the force of gravity acts in downward direction while the displacement is in upward direction i.e., both are in opposite directions.

Thus, work done by the force of gravity, $W = - mgh = - 2 \times 9.8 \times 5 =$ **– 98 J**

NUMERICAL CHALLENGE 5.2

1. Calculate the work done by applying a force of 20 N to a 0.4 kg box as it slides along a frictionless surface from rest to 10 m/s in 0.2 s.

Solution

Given, force, F = 20 N ; mass, m = 0.4 kg ; displacement, s = ? ; initial velocity, u = 0 ;

final velocity, v = 10 m/s ; time, t = 0.2 s ; Work done, W = ?

Now, displacement, $s = \left(\frac{v+u}{2}\right)t = \left(\frac{10+0}{2}\right)\times 0.2$ = 1 m

Work done, W = F × s = 20 × 1 = **20 J**

2. Calculate the work done on a cyclist if a braking force of 40 N [backward] slows the cyclist from 20 m/s to 15 m/s in 2.0 s.

Solution

Given, force, F = 40 N ; displacement, s = ? ; initial velocity, u = 20 m/s ; final velocity, v = 15 m/s ;

time, t = 2 s ; Work done, W = ?

Since, the displacement and force are in opposite direction, angle between them is 180° i.e., θ =180°.

Now, displacement, $s = \left(\frac{v+u}{2}\right)t = \left(\frac{15+20}{2}\right)\times 2$ = 35 m

W = F s cos θ = 40 × 35 × cos 180° = 40 × 35 × (–1) = **– 1400 J** (cos 180° = –1)

3. Calculate the work done in graph shown in fig.8.

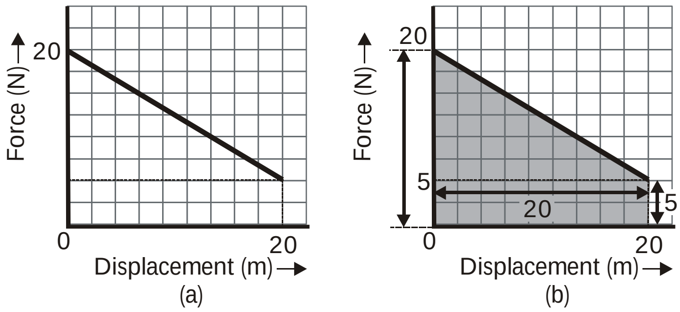


Fig.8 Numerical challenge 5.2

Solution

We know that area under the Force- displacement graph gives the total work done. Using fig.8(b), we can find the area under the given graph.

Work done, W = area of trapezium shown in fig.8(b)

$$= \frac{1}{2}(\text{Sum of the parallel sides}) \times (\text{distance between them}) = \frac{1}{2}(20+5)\times(20) = \mathbf{250\ J}$$

4. A truck pulls a 3000 kg car from rest with a horizontal force of 5000 N. The truck and car accelerate at 2.5 m/s^2 for 5.0 s to reach the speed of 45 km/h. How much work is done by the truck ?

Solution

Given, force, F = 5000 N ; displacement, s = ? ; acceleration, a = 2.5 m/s^2 ; initial velocity, u = 0 ;

final velocity, v = 45 km/h = (5/18) × 45 = 12.5 m/s ; Work done, W = ?

$v^2 - u^2 = 2as$

or $(12.5)^2 - (0)^2 = 2 \times 2.5 \times s$

or $s = \frac{12.5\times 12.5}{2\times 2.5}$ = 31.25 m

Work done, W = Fs cos θ = Fs cos 0° = 5000 × 31.25 = 156250 J = **156.25 kJ**

5.3 Energy

Without light that comes to us from the Sun, life on Earth would not exist. With the light energy, plants can grow and the oceans and atmosphere can maintain temperature ranges that support life. Although energy is difficult to define comprehensively, a simple definition is that **energy is the capacity to do work**. Thus, when you think of energy, think of what work is involved.

- An object that possesses energy can exert a force on another object. When this happens, energy is transferred from first object to the second object. The second object may move as it receives energy and therefore do some work. Thus, the first object had a capacity to do work. This implies that any object that possesses energy can do work.

Unit of energy :

S.I. unit : Since, energy is the capacity to do work, its unit is same as that of work, that is, joule (J). 1 J is the energy required to do 1 joule of work. Sometimes a larger unit of energy called kilo joule (kJ) is used, 1 kJ = 1000 J.

C.G.S. unit : Erg $\quad$ 1 J = 10^7 ergs

Forms of energy

The world we live in provides energy in many different forms. The various forms include potential energy, kinetic energy, heat energy, chemical energy, electrical energy and light energy.

Mechanical energy

The capacity to do mechanical work is called mechanical energy. Mechanical energy can be of two types : (1) Kinetic energy (2) Potential energy

- The sum of the gravitational potential energy and the kinetic energy is called mechanical energy.

Kinetic energy

This is the energy a body has due to its movement. To give a body KE, work must be done on the body. The amount of work done will be equal to the increase in KE.

- Kinetic energy is the energy associated with an object in motion.
- Kinetic energy possessed by an object of mass, m and moving with a uniform velocity, v is

$$\boxed{E_k = \frac{1}{2}mv^2}$$

- For a given mass, kinetic energy $E_K \propto v^2$. That is, more the value of v, more will be the kinetic energy. Also, if we double the velocity, the kinetic energy becomes four times, if we triple the velocity, the kinetic energy becomes nine times, and so on.
- For a given velocity, $E_K \propto m$. That is, more the mass, more will be the kinetic energy of a body. If a car and a truck are moving with same velocity, the truck possesses more kinetic energy than that of car because the truck has more mass than that of the car.

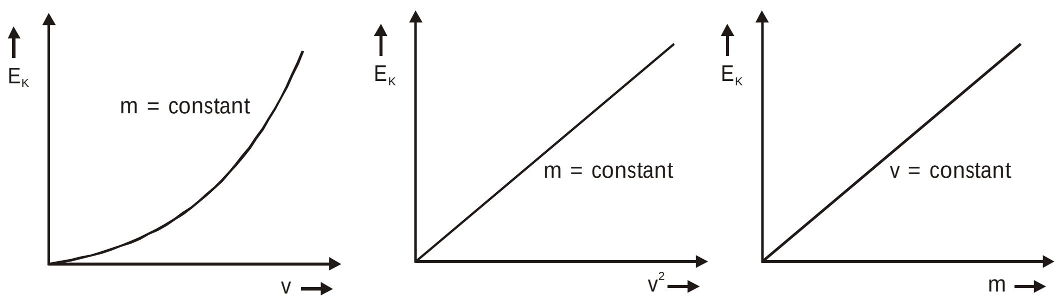


Fig.9 Graphs relating kinetic energy, velocity and mass.

- ### Relationship between momentum and kinetic energy

Momentum, p = mv ----- (1)

Kinetic energy, $E_K = \frac{1}{2}mv^2 = \frac{1}{2}mv^2 \times \frac{m}{m} = \frac{1}{2}\frac{(mv)^2}{m}$ ----- (2)

From (1) & (2), we get, $\boxed{E_K = \frac{p^2}{2m}}$ Also, $\boxed{p = \sqrt{2mE_K}}$

For a given momentum, kinetic energy is inversely proportional to mass ($E_K \propto 1/m$). This means smaller the mass, more will be the kinetic energy and vice-versa. For a given kinetic energy, momentum is directly proportional to the square root of mass ($p \propto \sqrt{m}$). This means heavier body will have more momentum and vice-versa. For a given mass, momentum is directly proportional to the square root of kinetic energy ($p \propto \sqrt{E_K}$). This means more the kinetic energy, more will be the momentum and vice-versa. (see fig. 10)

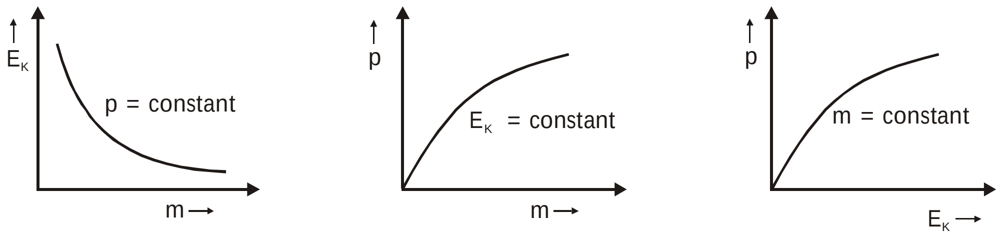


Fig.10 Graphs related to momentum, kinetic energy and mass.

Potential energy

The energy possessed by an object due to its position or configuration is called 'potential energy'.

- Potential energy is associated with an object that has the potential to move because of its position or configuration.

Gravitational potential energy

The energy associated with an object due to the object's position relative to a gravitational source is called gravitational potential energy.

- Gravitational potential energy is energy due to an object's position in a gravitational field. Imagine an egg falling off a table. As it falls, it gains kinetic energy. But, where does the egg's kinetic energy come from ? It comes from the gravitational potential energy that is associated with the egg's initial position on the table relative to the floor.

  We know that, the work done on the object against gravity is W = mgh. This work done is the energy gained by the object. This is the potential energy (E_p) of the object. That is,

  $$\boxed{E_p = W = mgh}$$

  The above formula actually represents, change in potential energy Δ PE = ($U_f - U_i$). Assuming initial potential energy (U_i) as zero and final potential energy (U_f) = E_p, we get, $\mathbf{E_p = mgh}$.

- If in a problem, several masses are involved at different vertical positions, then you can assume the potential energy of the mass at the lowest position as zero and you find the potential energies of other masses with respect to the mass at lowest position.

Elastic potential energy

Suppose a spring is placed on a tabletop and it is fixed at one end.

Now, push a block on the spring, compressing the spring, and then release the block. The block slides across the tabletop. The kinetic energy of the block came from the stored energy in the compressed spring (see fig.11). This potential energy is called **elastic potential energy**.

- Elastic potential energy is stored in any compressed or stretched object, such as a spring or the stretched strings of a tennis racket or guitar.

The length of a spring when no external forces are acting on it is called the **relaxed length** of the spring. When an external force compresses or stretches the spring, elastic potential energy is stored in the spring. The amount of energy depends on the distance the spring is compressed or stretched from its relaxed length.

The elastic potential energy stored in a spring is given by,

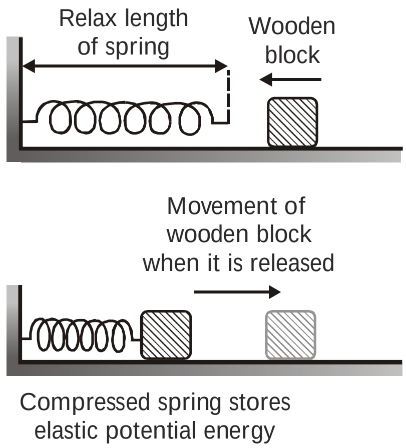


Fig.11 Elastic potential energy of a spring

$$\boxed{E_p = \frac{1}{2}Kx^2}$$

Where, K = spring constant or force constant and x = distance compressed or stretched from the relaxed position of a spring.

Some important points relate to work and energy

(1) Net work done on a particle or a system of particles is given by,

$$W_{net} = W_c + W_{nc} + W_{ext}$$

Where, W_c = work done by conservative forces ; W_{nc} = work done by the non conservative forces ; W_{ext} = work done by external or applied forces.

- Work done by conservative forces like elastic forces, gravitational forces is given by,

  W_{nc} = – change in potential energy = – Δ PE = – $(U_f - U_i)$ = $U_i - U_f$

  Where, U_f = final potential energy ; U_i = initial potential energy

- Work done by non conservative forces like frictional forces, air resistance, etc. is always negative as they are always opposite to displacement.

(2) Net work done by all the forces i.e., the work done by the unbalanced force is always equal to change in kinetic energy.

$W_{net} = \Delta KE = K_f - K_i$ or $\boxed{W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2}$ This is called **work-energy theorem**.

or $W_c + W_{nc} + W_{ext} = K_f - K_i$

(3) **Work done by the spring :** Since elastic forces are conservative forces, the work done by the spring is given by,

$W = -\Delta PE = -(U_f - U_i) = U_i - U_f$ or $\boxed{W = \frac{1}{2}Kx_i^2 - \frac{1}{2}Kx_f^2}$

(4) If non conservative forces are absent, W_{nc} =0. Then,

$W_c + W_{ext} = K_f - K_i$ or $U_i - U_f + W_{ext} = K_f - K_i$

or $\mathbf{W_{ext} = (K_f + U_f) - (K_i + U_i) = E_f - E_i}$

Where, E_f = final mechanical energy ; E_i = initial mechanical energy

(5) If non conservative forces are absent, W_{nc} =0 and if no external forces are acting, W_{ext} = 0. Then,

$W_c = K_f - K_i$

or $U_i - U_f = K_f - K_i$

or $U_f + K_f = U_i + K_i$ or $\mathbf{E_f = E_i}$

Where, E_f = final mechanical energy ; E_i = initial mechanical energy

Thus, in the absence of conservative forces and external forces, total mechanical energy remains conserved or constant.

(6) If a body is lifted with certain acceleration to reach height h, then work done by the external force is given by,

$W_c + W_{nc} + W_{ext} = K_f - K_i$

or $W_c + (0) + W_{ext} = K_f - K_i$ $[W_{nc} = 0]$

or $(U_i - U_f) + W_{ext} = K_f - K_i$

or $(0 - U_f) + W_{ext} = K_f - K_i$ [Assuming, U_i = 0]

or $W_{ext} = K_f - K_i + U_f$ or $\boxed{W_{ext} = \left(\frac{1}{2}mv^2 - \frac{1}{2}mu^2\right) + mgh}$

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NUMERICAL CHALLENGE 5.3

1. A 1400 kg car accelerates from 36 km/h to 54 km/h.

(a) How much work is done on the car ?

(b) If the car then brakes to a stop, how much work is done on it ?

Solution

(a) Given, mass of car, m = 1400 kg ; initial velocity, u = 36 km/h = 36 x (5/18) = 10 m/s ;

final velocity, v = 54 km/h = 54 x (5/18) = 15 m/s

According to work-energy theorem, work done, W = Change in kinetic energy

$$W=\frac{1}{2}mv^2-\frac{1}{2}mu^2=\frac{1}{2}m(v^2-u^2)=\frac{1}{2}\times1400\times[(15)^2-(10)^2]=\frac{1}{2}\times1400\times[225-100]$$

or $W=\frac{1}{2}\times1400\times125$ = **87500 J = 8.75 x 10^4 J**

(b) If the car is then stopped, then its final velocity becomes zero.

Now, initial velocity, u = 54 km/h = 15 m/s ; final velocity, v = 0

Work done, $W=\frac{1}{2}mv^2-\frac{1}{2}mu^2=\frac{1}{2}m(v^2-u^2)=\frac{1}{2}\times1400\times[(0)^2-(15)^2]=\frac{1}{2}\times1400\times[0-225]$

or $W=\frac{1}{2}\times1400\times[-225]$ = **– 157500 J = – 1.575 x 10^5 J**

2. A mass of 2 kg is attached to a light spring of force constant K =100 N/m. Calculate the work done by an external force in stretching the spring by 10 cm from its unstretched position.

Solution

Given, K = 100 N/m ; x_i = 0 ; x_f = 10 cm = 0.10 m

Work done by the external force = – work done by the spring = $-\left(\frac{1}{2}Kx_i^2-\frac{1}{2}Kx_f^2\right)=\left(\frac{1}{2}Kx_f^2-\frac{1}{2}Kx_i^2\right)$

or W_{ext} = $\frac{1}{2}K\left(x_f^2-x_i^2\right)=\frac{1}{2}K\left(x_f^2-x_i^2\right)=\frac{1}{2}\times100\times[(0.1)^2-(0)^2]=\frac{1}{2}\times100\times0.01$ = **0.5 J**

5.4 Conservation of energy

Energy appears in many forms, such as heat, motion, height, pressure, electricity, and chemical bonds between atoms.

Energy transformations

Energy can be converted from one form to another form in different systems, machines or devices. Systems change as energy flows from one part of the system to another. Parts of the system may speed up, slow down, get warmer or colder, etc. Each change transfers energy or transforms energy from one form to another. For example, friction transforms energy of motion to energy of heat. A bow and arrow transform potential energy in a stretched bow into energy of motion (i.e., kinetic energy) of an arrow.

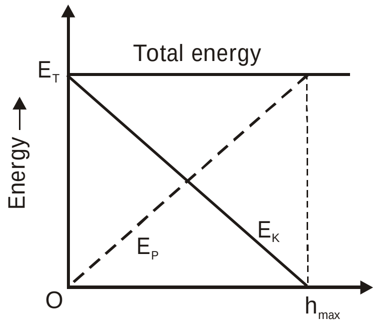
Law of conservation of energy

Energy can never be created or destroyed, just converted from one form into another. This is called **the law of conservation of energy**.

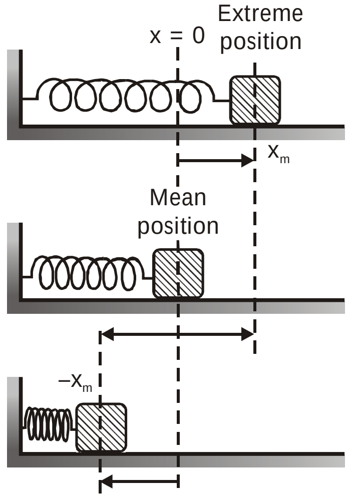
- The law of conservation of energy is one of the most important laws in physics. It applies to all forms of energy.

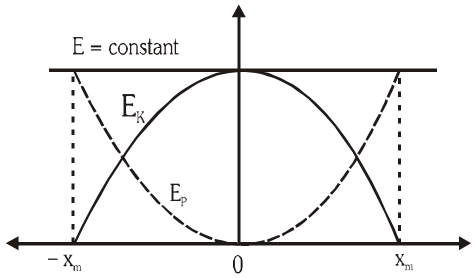
Energy has to come from somewhere : The law of conservation of energy tells us energy cannot be created from nothing. If energy increases somewhere, it must decrease somewhere else. The key to understanding how systems change is to trace the flow of energy. Once we know how energy flows and transforms, we have a good understanding of how a system works. For example, when we use energy to drive a car, that energy comes from chemical energy stored in petrol. As we use the energy, the amount left in the form of petrol decreases.

- When a body is dropped from a certain height under gravity then, in the absence of any non conservative forces like air resistance, the total mechanical energy of the body remains constant.
- When a spring fixed at one end and attached with other end is stretched or compressed on a frictionless surface and then allowed to release, it oscillates about its equilibrium position. But its total mechanical energy remains constant.



E_K = kinetic energy
E_P = potential energy
Energy changes during free fall





E_K = kinetic energy
E_P = potential energy
Energy changes of a vibrating spring

Fig.12 Conservation of energy during free fall and in a spring-mass system

NUMERICAL CHALLENGE 5.4

In fig.13, a frictionless metal block of mass 5.0 kg slides at a speed of 6.0 m/s into a fixed spring bumper with a spring constant of 720 N/m. What is the maximum compression of the block ?

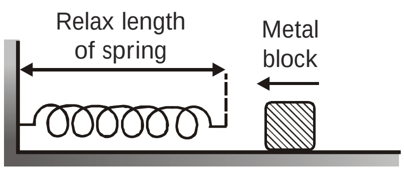


Fig.13 Numerical challenge 6.5

Solution

Given, mass, m = 5 kg ; speed, v = 6 m/s,

spring constant, K = 720 N/m

At maximum compression, the total kinetic energy will be converted to potential energy, i.e., $E_K = E_P$

or $\frac{1}{2}mv^2 = \frac{1}{2}Kx^2$ or $mv^2 = Kx^2$ or $x = v\sqrt{\frac{m}{K}} = 6 \times \sqrt{\frac{5}{720}} = 6 \times \sqrt{\frac{1}{144}} = \frac{6}{12}$ = **0.5 m**

5.5 Power

The engine in an old school bus could, over a long period of time, do as much work as jet engines do when a jet takes off. However, the school bus engine could not begin to do work fast enough to make a jet lift off. In this and many other applications, the rate at which work is done is more critical than the amount of work done.

- Power is the rate at which work is done. Power can also be defined as the rate at which energy is transferred.

$$P = \frac{\text{Work done}}{\text{time taken}} = \frac{W}{t}$$

SI unit of power : Watt (W) 1 Watt = 1 joule/second

or **1 W = 1 J s^{-1}**

Definition of 1 watt : If 1 joule work is done per second by a device or a machine then the power of that device or machine is 1 watt.

- We know that, $P = \frac{\text{Work done}}{\text{time taken}} = \frac{W}{t} = \frac{F \times s}{t} = F \times \left(\frac{s}{t}\right) = F \times v$

NUMERICAL CHALLENGE 5.5

1. A car with a mass of 1.50×10^3 kg starts from rest and accelerates to a speed of 18.0 m/s in 12.0 s. Assume that the force of resistance remains constant at 400.0 N during this time. What is the power developed by the car's engine ?

Solution

Given, mass, m = 1.50×10^3 kg ; initial velocity, u = 0 ; final velocity, v = 18 m/s ; time, t = 12 s ;

force of resistance, F_r = 400 N

Now, from first equation of motion,

$v = u + at$ or $18 = 0 + a(12)$ or $a = (18/12) = 1.5\ m/s^2$

Net force, $F_n = ma = 1.50 \times 10^3 \times 1.5 = 2250$ N

Also, net force, F_n = Force developed by car's engine – Force of resistance

or $F_n = F_c - F_r$ or $F_c = F_n + F_r = 2250\text{ N} + 400\text{ N} = 2650$ N

Displacement, $s = \left(\frac{v+u}{2}\right)t = \left(\frac{18+0}{2}\right) \times 12 = 108$ m

Work done by car's engine, $W = F \times s = 2650 \times 108$ J

Power developed by car's engine, $P = \frac{W}{t} = \frac{2650 \times 108}{12}$ = 23850 W = **2.385×10^4 W**

2. A man exerts a force of 200 N in pulling a cart at a constant speed of 16 m/s. Calculate the power spent by man.

Solution

Given, force, F = 200 N ; speed, v = 16 m/s

Power, $P = F \times v = 200 \times 16$ = **3200 Watt**

3. Calculate the power of an engine required to lift 10^5 kg of coal per hour from a mine 360 m deep. (g = 10 m/s²).

Solution

Given, mass, m = 10^5 kg ; t = 1 hr = 3600 s ; h = 360 m ; g = 10 m/s²

Since, work is done against gravity, thus, work done, W = mgh

Power of the engine, $P = \frac{W}{t} = \frac{mgh}{t} = \frac{10^5 \times 10 \times 360}{3600}$ = **10^5 W**

Commercial unit of energy

The unit joule is an extremely small unit, it is inconvenient to express large quantities of energy in terms of joule. We use a bigger unit of energy called kilowatt hour (kWh). It is called commercial unit of energy.

Definition of 1 kWh : If a machine or a device of power 1 kW or 1000 W is used continuously for one hour, it will consume 1 kW h of energy. Thus, 1 kWh is the energy used in one hour at the rate of 1000 W (or 1 kW).

$1\text{ kW h} = 1\text{ kW} \times 1\text{ h} = 1000\text{ W} \times 3600\text{ s} = 3600000$ J or **1 kWh = 3.6×10^6 J.**

CHAPTER 6 Sound

From our earliest years, we become accustomed to a great variety of sounds : our mother's voice, a telephone ringing, a kitten purring, a piano being played, a siren, a jet engine roaring, a rifle shot. Some of these sounds are pleasant to the ear and some are not. Sounds are a form of energy produced by rapidly vibrating objects. We hear sounds because this energy stimulates the auditory nerve in the human ear.

In the 18th century, philosophers and scientists debated the question, "If a tree falls in the forest and no one is there to hear it, will there be sound?" "Of course there will," said the scientists, "because the crash of the tree is a vibrating source that sends out sound waves through the ground and the air." To them, sound was the motion of the particles in a medium caused by a vibrating object.

"Of course not," said the philosophers, "because no observer is present." To them, sound was a personal sensation that existed only in the mind of the observer. This debate could never be resolved because one group was defining sound objectively in terms of its cause, and the other was defining it subjectively in terms of its effects on the human ear and brain. In physics, we study the transmission of sound objectively, leaving the subjective interpretation of the effects of sound waves on the human ear and brain to the philosophers.

The ears of most young people respond to sound frequencies of between 20 Hz and 20 000 Hz. Frequencies of less than 20 Hz are referred to as **infrasonic** and those higher than 20 000 Hz are called **ultrasonic**.

6.1 The speed of sound

Sound seems to move very quickly. However, during a thunderstorm, you see the lightning before you hear the thunder it causes. Light travels extremely fast (3.0×10^8 m/s in vacuum or air); sound travels much more slowly. Accurate measurements of the speed of sound in air have been made at various temperatures and air pressures. At normal atmospheric pressure and 0°C, the speed of sound in air is 332 m/s. If the air pressure remains constant, the speed of sound increases as the temperature increases. For every rise in temperature of 1°C, the speed of sound in air increases by 0.59 m/s. The speed of sound in air at normal atmospheric pressure can be calculated using the equation,

$$v = v_0\left(1+\frac{t}{273}\right)^{1/2}$$

Where, v_0 = speed of sound at 0 °C = 332 m/s ; t is temperature in °C.

For t << 273°C, an approximate formula is,

v = (332 + 0.6 t) m/s

- If the temperature drops below 0°C, the speed of sound in air is less than 332 m/s.

NUMERICAL CHALLENGE 6.1

Calculate the speed of sound in air when the temperature is 16°C.

Solution

Give, t = 16 °C v = ?

v = (332 + 0.6 t) m/s

= ($332 + 0.6 \times 16$) m/s

= 341.6 m/s

Speed of sound in different media

Speed of sound is different in different media. Sound waves can travel through solids, liquids and gases. The speed of sound is fastest in solids, faster in liquids, and slowest in gases. Speed of sound waves depends on the nature of material (or medium). As a sound wave travels through a material, the particles in the material collide with each other. In a solid, molecules are closer together than in liquids or gases, so collisions between molecules occur more rapidly than in liquids or gases. Thus, the speed of sound is fastest in solids, where molecules are closest together, and slowest in gases, where molecules are farthest apart.

Factors affecting speed of sound

(1) **Effect of density :** Higher the density, lesser will be the velocity and vice-versa.

$v \propto \sqrt{\frac{1}{\rho}}$ Also, $\frac{v_2}{v_1} = \sqrt{\frac{\rho_1}{\rho_2}}$

For example, under similar condition, $v_{H_2} > v_{O_2}$. This is because $\rho_{H_2} < \rho_{O_2}$.

(2) **Effect of humidity :** Density of water vapours is less than dry air at same pressure and temperature. Thus, density of moist air is less than that of dry air.

Now, $v \propto \sqrt{\frac{1}{\rho}}$

Therefore, speed of sound increases as humidity increases.

(3) **Effect of temperature :** As the temperature increases, speed of sound increases and vice-versa.

Since, $v \propto \sqrt{T}$ Where. T is temperature in Kelvin.

Also, $\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$

(4) **Effect of pressure :** There is no effect on the speed of sound by changing the pressure. It is true in every case whether the temperature is constant or not.

(5) **Effect of wind :** If wind speed and speed of sound are in same directions, they are added together i.e., speed of sound increases. If wind speed and speed of sound are in opposite directions, the net speed of sound is the difference of them i.e., speed of sound decreases.

NUMERICAL CHALLENGE 6.2

1. A pistol is used at the starting line to begin a 500 m race along a straight track. At the finish line, a puff of smoke is seen and 1.5 s later the sound is heard. What is the speed of the sound ?

Solution

Time taken by the light to reach our eyes is extremely small as it travels very fast thus, we can neglect the time taken by the light to reach our eyes.

Thus, time taken (t) by the sound to reach the observer is 1.5 s. Also, pistol is fired at the finish line, thus, the distance travelled (s) to reach the starting line is 500 m.

Now, $v = \frac{\text{distance}}{\text{time taken}} = \frac{s}{t} = \frac{500}{1.5}$ = **333.3 m/s**

2. A vibrating 400 Hz tuning fork is placed in fresh water. What is the frequency in hertz and the wavelength in metres (a) within the water at 25°C ? (b) when the sound waves move into the air at 25°C ?

Take, speed of sound in water at 25 °C = 1500 m/s and speed of sound in air at 25 °C = 345 m/s.

Solution

(a) Frequency of any wave does not change with change in medium. Thus, a 400 Hz tuning fork will produce a sound of frequency 400 Hz i.e,

frequency, ν = 400 Hz

Since, speed of wave changes from one medium to the another, the wavelength of wave also changes from medium to medium.

Given, speed of sound in water, v = 1500 m/s

We know that speed of wave = frequency x wavelength

or $v = \nu\lambda$

or wavelength, $\lambda = \frac{v}{\nu} = \frac{1500}{400}$ = **3.75 m**

(b) Frequency of wave produced, ν = frequency of tuning fork = 400 Hz

wavelength, $\lambda = \frac{v}{\nu} = \frac{345}{400}$ = **0.8625 m** (Given, speed of sound in air, v = 345 m/s)

6.2 The intensity of sound

There is a difference in loudness between a soft whisper and the roar of nearby thunder. However, the loudness of a sound you hear is a subjective evaluation that depends on several factors, including the objective quantity known as **intensity**. Frequency, wavelength, and speed are all properties of sound that can be measured accurately. Sound intensity is more difficult to measure because the amount of energy involved is small in comparison with other forms of energy. For example, the thermal energy equivalent of the sound energy emitted over a 90 minute period by a crowd of 50000 people is only enough to heat one cup of coffee! Sounds audible to humans can vary in intensity from the quietest whisper (10^{-12} W/m^2) to a level that is painful to the ear (10 W/m^2)—a difference of a factor of 10^{13}.

One unit used to measure the intensity level of sound is the bel (B), named after Alexander Graham Bell. The decibel (dB) is more common than the bel (1 dB = 10^{-1} B). On the decibel scale, 0 dB is the threshold of hearing (10^{-12} W/m^2). The scale is not linear, but is a logarithmic scale.

- Every change of 10 units on the decibel scale represents a tenfold effect on the intensity level. For example, a sound 10 times more intense than 0 dB is 10 dB, a sound 100 times more intense than 0 dB is 20 dB, and a sound 1000 times more intense than 0 dB is 30 dB. The level of sound that is painful to the human ear (130 dB) is 10^{13} times more intense than the level at the threshold of hearing. Some common sound intensity levels for are listed in Table 6.1.

Table 6.1 Some common sound intensity levels

S.No.	Source of sound	Loudness (in dB)	Intensity (W/m^2)
1.	Threshold of hearing	0	10^{-12}
2.	Normal breathing	10	$10^1 \times 10^{-12} = 10^{-11}$
3.	Soft whisper (at 5m)	30	$10^3 \times 10^{-12} = 10^{-9}$
4.	Normal conversation	60	$10^6 \times 10^{-12} = 10^{-6}$
5.	Busy traffic	70	$10^7 \times 10^{-12} = 10^{-5}$
6.	Average factory	80	$10^8 \times 10^{-12} = 10^{-4}$
7.	Threshold of pain	130	$10^{13} \times 10^{-12} = 10^1$

- The loudness of the sounds humans perceive relates to the intensity of the sound. However, the two measures are not the same because the human ear does not respond to all frequencies equally. Average human ear is most sensitive to sound frequencies between about 1000 Hz and 5000 Hz. Lower frequencies must have a higher sound level or intensity to be heard. Also, loudness is a measure of the response of the ear to the sound. Intensity is an objective property of the sound wave — in fact, it is related to the square of the wave amplitude, and does not depend on the particular characteristics of a person's ears. Loudness, on the other hand, is a subjective property of the sound that depends on the human ear, the sensitivity of the ear to the frequency of the sound, and the distance from the source of the sound.

Loudness can be considered as the intensity of an audible sound. If there are two sounds of equal intensity, one is audible and another is inaudible, then, our ears will hear the audible sound as a loud sound while the inaudible sound will not be detected by our ears.

NUMERICAL CHALLENGE 6.3

Normal breathing has an intensity level of 10 dB. What would the intensity level be if a sound were
(a) 10 times more intense ? (b) 100 times more intense ?

Solution

Intensity level in decibel = 10 dB thus, intensity in W/m^2 = $10^1 \times 10^{-12} = 10^{-11}$

(a) If a sound is 10 times more intense, its intensity in W/m^2 = $10 \times 10^{-11} = 10^{-10} = 10^2 \times 10^{-12}$
Thus, intensity level in dB = 20

(b) If a sound is 100 times more intense, its intensity in W/m^2 = $100 \times 10^{-11} = 10^{-9} = 10^3 \times 10^{-12}$
Thus, intensity level in dB = 30

- Intensity of sound (or any wave) is proportional to the square of the amplitude of the sound (or wave).

$$I \propto A^2$$

- Loudness or intensity level (β) is measured in decibels (dB)

$$\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$$

Where, I = intensity of sound ; $I_0 = 10^{-12}$ Watt/m^2 called threshold of hearing (minimum intensity that is just audible)

- For $I = I_0$, $\beta = 0$. Human ear is sensitive to the sound intensity (loudness) ranging from **0 - 180 dB.**

NUMERICAL CHALLENGE 6.4

1. A normal conversation involves sound intensities of about 3.0×10^{-6} W/m^2. What is the decibel level for this intensity ?

Solution

Given, $I_0 = 10^{-12}$ W/m^2 (the threshold of hearing) ; $I = 3.0 \times 10^{-6}$ W/m^2

Intensity level in dB, $\beta = 10\log_{10}\left(\frac{I}{I_0}\right) = 10\log_{10}\left(\frac{3\times10^{-6}}{10^{-12}}\right) = 10\log_{10}\left(3\times10^{6}\right) = 10[\log_{10}3 + \log_{10}10^6]$

or $\beta = 10[0.477 + 6\log_{10}10] = 10[0.477+6] = 10(6.477) =$ **64.77 dB**

2. A bell is rung at a sound intensity of 70 dB. A trumpet is blown at an intensity level that is greater by a factor of 10^3. What is the intensity level of the trumpet ?

Solution

Given, β = 70 dB

Now, $\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$ or $70 = 10\log_{10}\left(\frac{I}{I_0}\right)$ or $7 = \log_{10}\left(\frac{I}{I_0}\right)$

or $\left(\frac{I}{I_0}\right) = 10^7$ or $I = I_0 \times 10^7 = 10^{-12} \times 10^7 = 10^{-5}$ W/m^2

Now, intensity level (I') of trumpet is 10^3 times more than the intensity level of bell i.e.,
$I' = 10^3 \times 10^{-5} = 10^{-2}$

Now, intensity level of trumpet in dB,

$$\beta' = 10\log_{10}\left(\frac{I'}{I_0}\right) = 10\log_{10}\left(\frac{10^{-2}}{10^{-12}}\right) = 10\log_{10}\left(10^{10}\right) = 10\times10\log_{10}10 = \mathbf{100\ dB}$$

- Intensity of sound wave varies inversely with the square of the distance from the source ($I \propto 1/r^2$) i.e, as the distance increases the intensity of sound decreases.

$$\boxed{\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}}$$

NUMERICAL CHALLENGE 6.5

A helicopter hovers overhead during an air show, causing sound waves to emanate uniformly. If the first listener is 700 m away and the second listener is 1000 m away, by how much has the intensity level of the sound decreased when it reaches listener 2 ?

Solution

Let I_1 be the intensity of sound reached at listener 1 and I_2 be the intensity of sound reached at listener 2. Given, r_1 = 700 m ; r_2 = 1000 m.

$$\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2} \quad \text{or} \quad I_2 = \left(\frac{r_1}{r_2}\right)^2 I_1 = \left(\frac{700}{1000}\right)^2 I_1 = 0.49 I_1$$

Thus, the intensity level is almost one half by the time it reaches listener 2.

6.3 The reflection of sound waves

Just as a mirror reflects light, when sound waves radiating out from a source strike a rigid obstacle, the angle of reflection of the sound waves equals the angle of incidence.

Echoes

Echoes are produced when sound is reflected by a hard surface, such as a wall or cliff. An echo can be heard distinctly only if the time interval between the original sound and the reflected sound is greater than 0.1 s. The distance between the observer and the reflecting surface must be greater than 17 m for an echo to be heard (Fig. 1).

- The echo-sounder is a device that uses sound reflection to measure the depth of the sea. Similar equipment is used in the fishing industry to locate schools of fish. More sophisticated equipment of the same type is used by the armed forces to locate submarines. All such devices are called **SONAR** (SOund NAvigation and Ranging) devices.

Echolocation

Dolphins and orca whales rely on the production and reflection of sound to navigate, communicate, and hunt in dark waters. The location of an object using reflected sound is called **echolocation**. Both animals produce clicks, whistles, and other sounds that vary in intensity, frequency, and pattern. Lower frequency sounds (0.5 – 50 kHz) probably function mainly for social communication, while higher frequencies (40–150 kHz) are probably used for echolocation.

Most bats use echolocation for navigation in the dark and for finding food. The bat can identify an object by the echo and can even tell the size, shape, and texture of a small insect. If the bat detects a prey, it will generally fly toward the source of the echo, continually emitting high frequency pulses until it reaches its target and scoops the insect up into its wing membranes and into its mouth.

Let a person shout loudly and he hears an echo of his sound after a time 't' that is reflected from a wall or hard surface. Let the distance between the source (the person) and the reflecting surface (the wall) be 's' (see fig. 1). Then, the total distance travelled by the sound to reach again to the listener (the person) will be '2s'. Let 'v' be the speed of sound.

Now, speed, $v = \frac{\text{total distance travelled}}{\text{time taken}} = \frac{2s}{t}$

or total distance travelled, $2s = v \times t$ ----- (1)

Distance between the source and the listener, $s = \frac{v \times t}{2}$ ----- (2)

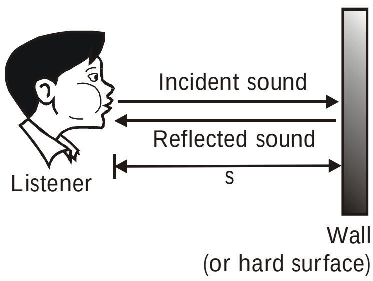


Fig.1 Hearing an echo

Let us take the speed of sound in air to be 344 m/s. The sound must go to the obstacle and reach back the ear of the listener on reflection after 0.1 s i.e., t = 0.1 s.

Hence, the total distance covered by the sound from the point of generation to the reflecting surface and back is,

$2s = v \times t = (344) \times 0.1 =$ **34.4 m**.

The distance between the observer and the reflecting surface is,

$s = \frac{v \times t}{2} = \frac{344 \times 0.1}{2} =$ **17.2 m**, for an echo to be heard.

NUMERICAL CHALLENGE 6.6

A ship is anchored where the depth of water is 120 m. An ultrasonic signal sent to the bottom of the lake returns in 0.16 s. What is the speed of sound in water ?

Solution

Given, depth of water, s = 120 m ; total time of taken by ultrasonic signal, t = 0.16 s

Speed, $v = \dfrac{\text{total distance travelled}}{\text{time taken}} = \dfrac{2s}{t} = \dfrac{2\times120}{0.16} = \mathbf{1500\ m/s}$

NUMERICAL CHALLENGE 6.7

1. A boy standing in front of a wall at a distance of 17 m produces 10 claps per second. He notices that the sound of his clapping coincides with the echo. The echo is heard only once when clapping is stopped. Calculate the speed of sound.

Solution

Let s be the distance of wall from the boy. To hear the echo, sound has to travel a total distance 2s i.e.,

total distance travelled by the sound = 2 x 17 = 34 m.

Since 10 claps are produced in one second, therefore each clap is produced after (1/10)s which is equal to the time taken for echo to be heard (given in the question that echo coincides with the sound of its clapping) thus,

t = (1/10) s

Speed, $v = \dfrac{\text{total distance travelled}}{\text{time taken}} = \dfrac{2s}{t} = \dfrac{2\times17}{(1/10)} = 340\ m/s$

2. A listener standing between two cliffs fires a gun. He hears the first echo after 1 second and the next after 2 more seconds. Find (a) his distance from the nearer cliff and (b) the distance between the two cliffs.

Take, speed of sound = 330 m/s.

Solution

(a) Let the listener be at a distance s_1 from the nearer cliff 1 and s_2 be his distance from the farther cliff 2 (see fig.2). The first echo will be heard when sound is reflected from the nearer cliff 1. Let t_1 be the time taken by the sound to get back to the listener (i.e., time for first echo). Given, t_1 = 1 s

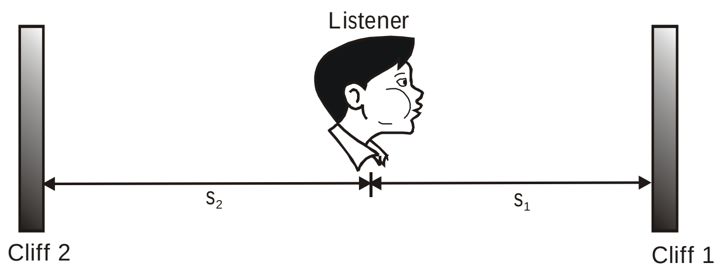


Fig.2 Numerical challenge 6.7 (2)

$\therefore\ s_1 = \dfrac{v\times t}{2} = \dfrac{330\times1}{2} = \mathbf{165\ m}$

(b) After the hearing of first echo, second echo is heard after 2 more seconds this means, the time taken t_2 by the sound to get reflected from the farther cliff = 1 s + 2 s = 3 s i.e., t_2 = 3 s.

$\therefore\ s_2 = \dfrac{v\times t}{2} = \dfrac{330\times3}{2} = \mathbf{495\ m}$

Total distance between the two cliffs = $s_1 + s_2$ = 165 + 495 = **660 m**

3. A pilot of an aeroplane travelling horizontally at 198 km/hr fires a gun and hears the echo from the ground after an interval of 3 seconds. If the velocity of sound is 330 m/s, find the height of the aeroplane from the ground.

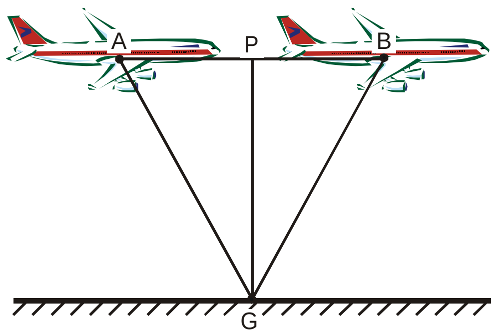


Fig.3 Numerical challenge 6.7 (3)

Solution

Let initially, the aeroplane is at position A when the is gun fired and the pilot hears the echo at position B (see fig.3). The sound first strikes the ground at point G, then reflected and finally reaches at B. Let height, GP = h ; AG = GB =x.

Total distance travelled by sound, s = AG + GB = x + x = 2x

Speed of sound, $v = \frac{s}{t} = \frac{2x}{t}$ or $x = \frac{v \times t}{2} = \frac{330 \times 3}{2}$ = 495 m

AB = velocity of aeroplane x time = 198 x (5/18) x 3 = 165 m

Now, AP = AB/2 = 165/2 = 82.5 m

By Pythagorus theorem, $GP = h = \sqrt{AG^2 - AP^2} = \sqrt{(495)^2 - (82.5)^2}$ = **488 m**

6.4 Free, damped and forced vibrations

Free vibrations (or free oscillations)

When a body is suspended at one point and if it is displaced slightly from its equilibrium position (or mean position), it starts vibrating or oscillating about its mean position. These are called the free or natural vibrations of the body. The time period of vibration depends on the shape and size i.e., the structure of the body and is called its free or natural period. The frequency of the freely vibrating body is called its natural frequency. Every body has its own natural frequency of vibration.

- The vibrations or oscillations of a body with constant amplitude and constant frequency are called the free vibrations.

Some examples of free vibrations are :

(1) Motion of the bob of a simple pendulum when it is displaced slightly from its mean position.

(2) A load suspended from a spring when pulled and released, starts vibrating with a period determined the hardness of the spring and the mass of the load.

(3) A tuning fork is struck against a hard rubber pad starts vibrating with its natural frequency.

(4) When we strike the keys of a piano, various strings are set in vibration at their natural frequencies.

Nature of free vibrations : In free vibrations, the restoring force on the vibrating system is directly proportional to its displacement from its mean position. The force is maximum when it is at the extreme ends of vibration and is zero at its mean position. The restoring force is always directed towards the mean position. The amplitude of a freely vibrating body should remain constant. The amplitude will remain constant only if there is no surrounding (resistive) medium. Once a body starts vibrating, it should continue with the same amplitude and same frequency forever. Free vibrations can occur only in vacuum, therefore these vibrations cannot be realised in practice.

Damped vibrations

In most systems, resistive forces, such as friction, air resistance are present and they retard the motion of the system. Consequently, the mechanical energy of the system decreases with time, and such oscillations are called **damped oscillations**.

- The periodic vibrations of decreasing amplitude are called **damped vibrations**.

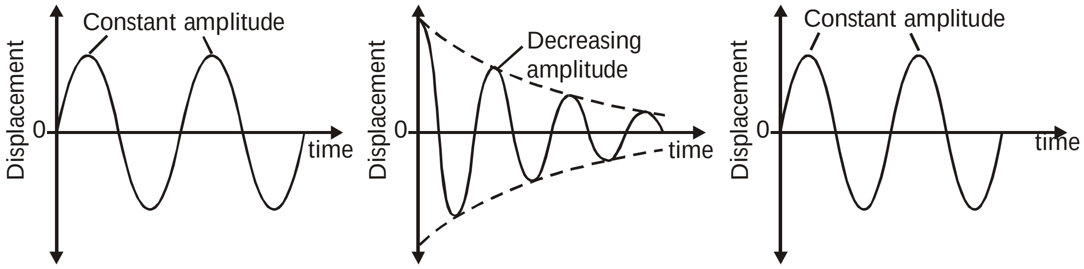
The damping occurs due to frictional force exerted by the surrounding medium. At any instant the frictional force is proportional to the velocity of the vibrating body. Due to the frictional force, the vibrating system loses energy continuously and hence its amplitude decreases gradually. After some time the total energy of the vibrating system is dissipated to the surroundings and eventually, the vibrating system comes to rest.

- Motion of a simple pendulum in air, vibration of a tuning fork in air, a spring block system oscillating on a rough surface are some examples of damped vibrations.

Forced oscillations

We have seen that the mechanical energy of a damped oscillator decreases with time as a result of the resistive force like friction. It is possible to compensate for this energy decrease by applying an external force that does positive work on the system. Such an oscillator then undergoes **forced oscillations.** At any instant, energy can be transferred into the system by an applied force that acts in the direction of motion of the oscillator. For example, a child on a swing can be kept in motion by appropriately timed "pushes." The amplitude of motion remains constant if the energy input per cycle of motion exactly equals the decrease in mechanical energy in each cycle that results from resistive forces.

- The vibrations that take place under the influence of an external periodic force are called the **forced vibrations**. When an external periodic force is applied, the body do not vibrate with its own natural frequency, but it gradually acquires the frequency of applied force (driving force). The amplitude of force vibration remains constant with time but its magnitude depends on the frequency of applied force. If the frequency of applied force is quite different from the natural frequency of vibrating body, then the amplitude of vibration is quite small. But if the frequency of the external force is exactly equal to the natural frequency of the vibrating body, the amplitude of vibration is very large.



(a) Free vibration : frequency of vibration is equal to natural frequency of the vibrating body

(b) Damped vibration : amplitude of vibration decreases gradually with time

(c) Forced vibration : frequency of vibration may be different from natural frequency of the vibrating body

Fig.4 Graphs showing free vibration, damped vibration and force vibration

6.5 Resonance

Every object has a natural frequency at which it will vibrate. To keep a child moving on a swing, we must push the child with the same frequency as the natural frequency of the swing. When a large truck passes your house, you may have noticed that the windows rattle. These are examples of a phenomenon called resonance, which is the response of an object that is free to vibrate to a periodic force with the same frequency as the natural frequency of the object. We also call this phenomenon **mechanical resonance** because there is physical contact between the periodic force and the vibrating object.

- Resonance is a special case of forced oscillations. The phenomenon of dramatic increase in amplitude when the driving force is close to the natural frequency of the oscillator is called **resonance**.
- Resonance can be demonstrated with a series of pendulums suspended from a stretched string (see fig.5). When A is set in vibration, E begins to vibrate in time with it. Although B, C, and D may begin to vibrate, they do not continue to vibrate nor do they vibrate as much. When B is set in vibration, D begins to vibrate in sympathy, but A, C, and E vibrate intermittently and only a little. The pairs A and E, and B and D each have the same lengths and, thus, have the same natural frequencies. They are connected to the same support, so the energy of A, for example, is transferred along the supporting string to E, causing it to vibrate. This occurs only if E is free to vibrate. The periodic vibratory force exerted by one pendulum moves through the supporting string to the other pendulums, but only the pendulum with the same natural frequency begins to vibrate in resonance. When an object vibrates in resonance with another, it is called a **sympathetic vibration**.

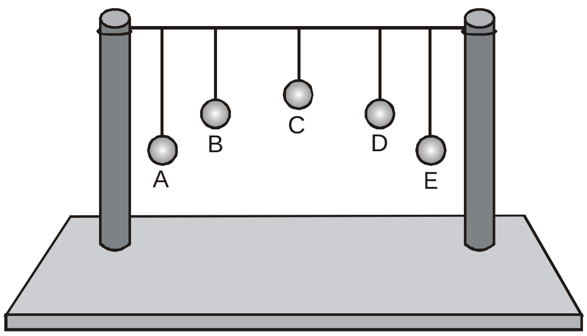


Fig.5 A series of pendulums suspended from a string used to show resonance.

- Another demonstration of resonance can be done by using two identical tuning forks mounted on two separate identical sound boxes (see fig.6). If tuning fork A is set into vibration, the other fork B also starts vibrating and a loud sound is heard. The vibrations produced in second fork B are due to resonance. Vibrating tuning fork A sets the vibrations in the air column of its sound box. These vibrations in the air column of its sound box. These vibrations are communicated to the sound box of the fork B. The air column of sound box of fork B starts vibrating due to resonance. Since the frequency of these vibrations is the same as the natural frequency of the fork B, the fork B also starts vibrating due to resonance.

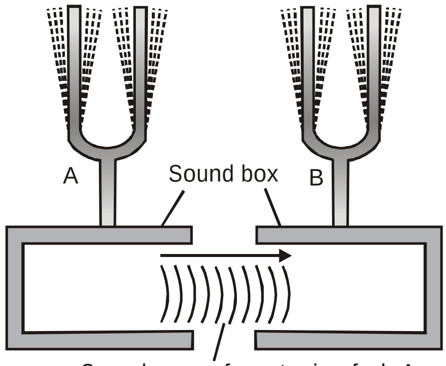


Fig.6 Resonance with tuning forks

Applications of resonance

(1) Mechanical resonance must be taken into account when designing bridges, airplane propellers, helicopter rotor blades, turbines for steam generators and jet engines, plumbing systems, and many other types of equipment. A dangerous resonant condition may result if this is not done. For example, in 1940 the Tacoma Narrows suspension bridge in Washington State collapsed when wind caused the bridge to vibrate. In 1841, a troop of British soldiers marched in step across a bridge, which created a periodic force that set the bridge in resonant vibration and caused the bridge to collapse. If an opera singer sings a note with the same natural frequency as that of a wineglass, the glass will begin to vibrate in resonance. If the sound has a high enough intensity, the wineglass could vibrate with an amplitude large enough that it shatters.

(2) The human body also has resonant frequencies. Experiments have shown that the entire body has a mechanical resonant frequency of about 6 Hz, of the head between 13 Hz and 20 Hz, and of the eyes between 35 Hz and 75 Hz. Large amplitude vibrations at any of these frequencies could irritate or even damage parts of the body. In transportation and road construction occupations, efforts are made to reduce the effects of mechanical vibrations on the human body.

(3) Even very large structures, like towers and skyscrapers, can resonate, whether the external source of energy is as small as a gust of wind or as large as an earthquake. Engineers must consider specific structural features to minimize damage by earthquakes.

(4) Radio and television provide another example of resonance. When you tune a radio or television, you are actually adjusting the frequency of vibration of particles in the receiver so that they resonate with the frequency of a particular signal from a radio or television station.

Stringed instruments

Stringed instruments, like a guitar produce music by making strings vibrate. Different methods are used to make the strings vibrate—guitar strings are plucked, piano strings are struck, and a bow is slid across violin strings. The strings are made of metallic wires.

The pitch (frequency) of the note depends on the length, diameter, and tension of the string. If the string is shorter, narrower, or tighter, the pitch increases. For example, pressing down on a vibrating guitar string shortens its length and produces a note with a higher pitch. Similarly, the thinner guitar strings produce a higher pitch than the thicker strings.

6.6 Mach Number

When aircraft or any object gets close to or goes faster than the speed of sound, a different unit is often used to describe its speed. It's called the **Mach number**, named after Ernst Mach, a prominent physicist from the late 1800s. Mach 1 is defined as the speed of sound at a given air temperature.

The Mach number of a source of sound is the ratio of the speed of the object to the speed of sound in air at that location.

$$\text{Mach number} = \frac{\text{Speed of object}}{\text{Speed of sound}}$$

- If mach number is less than one, this means objects are travelling at speeds less than the speed of sound in air. Such speeds are called have **subsonic** speeds.

  If mach number is equal to one, this means objects are travelling at speeds equal to the speed of sound in air. such speeds are called **sonic** speeds. Sonic speed is also called Mach 1.

  If mach number is greater than one, this means objects are travelling at speeds more than the speed of sound in air. Such speeds are called **supersonic** speeds. Speeds greater than Mach 1 are supersonic. Speeds for supersonic aircraft, such as the Concorde and fighter aircraft, are given in terms of Mach number rather than km/hr.

NUMERICAL CHALLENGE 6.8

The speed of sound an altitude of 10 km is approximately 1060 km/h. What is the Mach number of an aircraft flying at an altitude of 10 km with a speed of 1800 km/h ?

Solution

$$\text{Mach number} = \frac{\text{Speed of object}}{\text{Speed of sound}} = \frac{1800}{1060} = \mathbf{1.7}$$

The Mach number of an aircraft flying at 1800 km/h at an altitude of 10 km is **1.7**.

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