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THEORY AND METHODOLOGY

Quantifying bullwhip effect in a closed loop supply chain

Rupesh Kumar Pati · Prem Vrat · Pradeep Kumar

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Abstract The issue of sustainability has attracted attention towards closing the traditional supply chain through different reprocessing options. This paper develops an analytical expression for measuring the bullwhip effect in a six echelon closed loop supply chain for recycling of products like paper, plastic. A first order auto regressive end customer demand is assumed with each supply chain participant employing an order-up-to (OUT) policy and Minimum Mean Square Error (MMSE) forecasting scheme. The model assists the closed loop supply chain entities in anticipating the downstream demand and suggests them to carefully select the value of auto regressive parameter so as to avoid any order-process instability in the closed loop supply chain. Stability analysis helps in determining the combination of degree of segregation at source, yield and auto regressive parameter for maintaining a stable system. Sensitivity analysis of replenishment lead-time combination could be utilized by management for designing an optimal recycling-distribution system, under the condition of constant accumulated lead-time. Further, the segregation analysis reveals that increase in the degree of segregation at the source reduces the bullwhip effect in the closed loop supply chain.

Keywords Environmental issues · Variance amplification · Closed loop supply chain · Recycling

R. K. Pati (✉)
Indian Institute of Management, Kozhikode, Kerala 673570, India
e-mail: rupesh_pati@iimk.ac.in

P. Vrat
Management Development Institute, Gurgaon, Haryana 122007, India
e-mail: premvrat@mdi.ac.in

P. Kumar
Indian Institute of Technology, Mechanical and Industrial Engineering Department, Roorkee 247667 Uttaranchal, India
e-mail: kumarfme@gmail.com

1 Introduction

In the present times, there is tremendous pressure on the ecosystem due to rapid depletion of natural resources and increased environmental pollution. This has forced many countries to formulate their economic development strategies emphasizing the issue of sustainability with strict laws for "product-take-back" after their useful life. While, forward logistics involves material flow from manufacturer to the end user, reverse logistics involves return product flows from the user, followed by re-processing (through recycling, cannibalizing, remanufacturing, refurbishing, repairing and/or reusing) the product's into a useable form [31]. Guide and Van Wassenhove [15] coined the term "Closed-Loop Supply Chain (CLSC)" which refers to this complete loop—from the customer, back to the plant for the re-processing operations, and then once again back to the customer.

A closed loop supply chain is vital for sustainable business operations; hence the concept has received increased attention in recent past. This interest is driven by economic factors, stringent environmental regulations, shortage of natural resources and customer pressure on manufacturer to adopt "product-take-back" policies. In light of these challenges, the concept of the CLSC becomes an attractive alternative for optimizing end-of-life processing activities of products [23]. Kumar and Malegeant [24] have summarized the benefits associated with CLSC for the customer, business and the environment due to potential value creation opportunities.

A typical CLSC involves collection, inspection/separation, reprocessing, disposal and redistribution activities. Guide and Van Wassenhove [16] observed that products and their CLSC's often differ with respect to some features and parameters such as product acquisition, returns volume, return timing and quality, test, sort and grade, reconditioning, distribution and selling. Some degree of uncertainty is likely to be associated with these critical stages.

As markets tend to be more customer-oriented, the uncertainty associated with end customer demand and its consequences on the supply chain have become an important subject for research. The bullwhip effect (or variance amplification effect) is an outcome of this uncertainty. The bullwhip effect occurs when the demand variability propagates upstream in an amplified form. Lee et al. [25, 26] summarized the negative impacts as well as the possible causes of this phenomenon. They also proposed methods to reduce the impact of bullwhip effect. But, all these observations were limited to the bullwhip effect in the forward supply chain only.

Despite a growing body of literature covering topics related to forward logistics (e.g. [18, 33]) and reverse logistics management [9, 14, 21], our understanding of CLSC related issues remains scanty [5, 29]. This paper is an attempt to enrich the literature on CLSC by statistically investigating the bullwhip effect across a single product CLSC with product recycling as the only feasible reprocessing option. The demand for the raw material in the CLSC (i.e. end of life recyclable product) is influenced by the proportion of disposable waste (non-relevant waste) moving between echelons. The higher the segregation of non-recyclable waste at the initial stage, the lower is the demand in subsequent stages of product flow. Also, demand for recyclable waste by the manufacturer depends upon the quality of waste being collected for recycling i.e. higher the yield (one of the indicators of quality) of the recyclable waste at the manufacturer lower is the demand for raw material.

Extending the three echelon forward supply chain structure proposed by Hosoda and Disney [19], a sequential six echelon CLSC structure has been investigated. In the CLSC structure, the first three echelons are members of forward supply chain and the rest are part of reverse supply chain. It has been assumed that demand at the first echelon i.e. retailer stage follows First order Auto-Regressive [AR(1)] process. The Order-Up-To (OUT) policy is a standard ordering algorithm in many ERP systems and is used to achieve customer service, as well as, balance inventory and capacity investments [12]. Hence, it is assumed that each of the CLSC participants adopt the OUT policy with Minimum Mean Square Error (MMSE) forecasting scheme. Luong [27] suggested that with AR(p) demand process the forecasting technique that minimizes the expected mean-square forecast error will help to predict future demand value with as little error as possible, and hence, reduce the bullwhip effect. In contrast to Hosoda and Disney [19], the present contribution accounts for increased number of echelons and takes into account the impact of degree of segregation (in terms of Ψ i.e. proportion of relevant recyclable waste), the quality of recyclable waste (in terms of yield fraction (X)) and effect of lead-time between echelons on the variance amplification across the CLSC in recycling environment. This, paper seeks to find answer to the following issues of CLSC:

- Impact of providing an improved segregation system on the bullwhip effect of CLSC?
- Effect of quality of recyclable waste supplied to manufacturer on bullwhip effect?
- To what extent lead-time between entities affects demand variability across the CLSC?
- Can the upper entities of CLSC anticipate the demand pattern if customer follows AR(1) demand, and the entities use OUT inventory policy with MMSE forecasting scheme?

2 Literature review

In recent past, inclusion of product utilization phase (e.g. service, maintenance) and end-of-life phase (e.g. product recovery, refurbishing, recycling) have transformed traditional supply chains to CLSC's. There are numerous examples and cases available of products being reprocessed (e.g. [17, 24, 31, 32]. Ensuring smooth flows of returned goods through the supply chain and the extraction of maximum residual value from these products represent serious challenges for manufacturers [17]. This smooth flow depends on the extent of demand variability across the CLSC's.

In recent past, the interest in the exploration and measurement of the bullwhip effect has grown up, because the amplification of orders influences the upstream echelons costs, inventory, reliability and other important business processes. The numerical estimation of the demand variability for an echelons was suggested by Chen et al. [3] and is represented as a ratio of variance of demand (σ_D^2) and variance of order (σ_O^2). The ratio σ_O^2/σ_D^2 was also employed to quantify the bullwhip for various purposes by many researchers such as Dejonckheere et al. [6], Zhang [34], Zhou and Disney [36].

Dynamics of the supply chains is largely influenced by its information sharing capabilities. The propagation of demand variability across the traditional supply chain with different combinations of demand processes, forecasting schemes under order-up-to (OUT) inventory policies has been studied extensively by researchers in the recent past. Majority of the research has focused on the simple two-stage serial supply chain with the retailer facing AR(1) demand process. Lee et al. [25] and Chen et al. [3] quantified the bullwhip effect by assuming that the downstream retailer uses the Exponential Smoothening (ES) and Moving Average (MA) method respectively, to forecast lead-time demand. They had similar observation i.e. bullwhip effect gets reduced as the demand forecasts gets smoothened. Zhang [34] compared the impact of forecasting methods (i.e. MMSE, MA and ES) on the bullwhip effect and suggested that MMSE method is optimal when the demand model is known to be an AR(1) process and remains stable, because it leads to the lowest inventory cost. If the demand structure is not well specified or shifts over time, the MA or ES method may perform better as they are more flexible and adapt better to the changing structures of the demand. Chen et al. [3, 4] and Zhang [34] observed that bullwhip effect can be reduced by reducing the lead-time and centralizing the demand information, regardless of the forecasting method employed. Luong [27] developed a measure of bullwhip effect and subsequently investigated the effect of autoregressive coefficient and lead time. It was seen that with increase in lead time, the value of auto regressive coefficient at which the bullwhip effect reaches its maximum value also increases and approaches one. Luong and Phien [28] further extended this and measured the bullwhip by assuming AR(2) demand process. The numerical results for the AR(2) demand process show that autoregressive coefficients and lead-time have a large impact on the bullwhip effect. When both first-order and second-order autoregressive coefficients are positive, the bullwhip effect exists and it will increase as lead-time goes up. However, for the other value ranges of autoregressive coefficients, the behavior of bullwhip effect is very complicated. In contrast, Hosoda and Disney [19] analyzed a three echelon supply chain model with AR(1) demand process and MMSE forecasting scheme to obtain exact analytical expressions for bullwhip and net inventory variance at each echelon. Bullwhip was found to be determined by accumulated lead-time from the customer and the local replenishment lead-time.

Baganha and Cohen [1] modeled a more complex model with an AR(p) demand and a multi-echelon manufacturing and distribution system with OUT replenishment policy. It was found that a centralized distribution system reduces demand variability and has a stabilizing effect. In another observation, Graves [13] quantified the bullwhip effect for integrated moving average (IMA) demand process of order (0,1,1).

Recently, using a control systems engineering approach, Disney et al. [7] quantified the bullwhip effect and the net inventory variance in a single-echelon supply chain with OUT replenishment policy in cases of the identically and independently distributed, AR(1), MA(1) and ARMA(1,1) demand process. Gaalman and Disney [10] and Gaalman and Disney [11] studied the bullwhip effect produced by the OUT replenishment policy reacting to a stochastic ARMA(1,1) and ARMA(2,2) demand processes respectively. Duc et al. [8] also quantified the impact of the bullwhip effect for a simple two-stage supply chain with one supplier and one retailer assuming that the retailer employs a base stock inventory policy, and with ARMA(1,1) demand. They investigated the effects of the autoregressive coefficient, the moving average parameter, and the lead time on the bullwhip effect.

Most of the literatures available are concerned with quantification and subsequent analysis of bullwhip for the traditional supply chain. Very limited attempt has been made to quantify and understand bullwhip in CLSC. Tang and Naim [30] indicated that the greater the degree of information transparency, the greater the robustness of the hybrid system. Huang and Wang [20], Zhou and Disney [36] and Zhou et al. [35] highlight the effect of remanufacturing lead-time and the return rate on the inventory variance and bullwhip produced by the ordering policy. These studies were performed to investigate the effect of information distortion in a CLSC under hybrid manufacturing/remanufacturing system using systems dynamics methodology and control theory approach. This paper is an attempt to statistically understand the bullwhip effect in the CLSC for another reprocessing option i.e. recycling of any product.

3 Demand process

Consider a retailer's single item multi-period inventory problem. The retailer orders and replenishes its stock from a wholesaler on a fixed time interval to supply customer demand. There is a delay of l_1 periods between ordering and receiving of the goods (i.e. lead time is l_1). To simplify the analysis, excess inventory is assumed to be returned without cost. It is assumed that the retailer faces a first order auto regressive [AR(1)] demand model. The AR(1) demand process assumption is common when the autocorrelation exists among demand process. Many researchers have employed this assumption as previously mentioned [refer section 2]. The formulation of the demand process is given by:

$$D_t = \mu + \rho D_{t-1} + \varepsilon_t, \quad |\rho| < 1 \tag{1}$$

where D_t is the demand during period t; μ is a constant that determines the mean of the demand, ε_t is an i.i.d. white noise process being normally distributed random error with mean 0 and variance σ_ε^2; and ρ is the auto regressive (AR) parameter. The assumption of $|\rho|<1$ assures that the demand process is covariance stationary. We may set $\mu=0$ without loss of generality, thus the long term mean of the demand rate is zero [i.e. $E(D_t)=0$]. This has the advantage of not having an initial transient response. Therefore, the general expression for the variance that retailer experiences due to the AR(1) demand pattern from the customer(C) is given by:

$$\sigma_C^2 = \frac{\sigma_\varepsilon^2}{1-\rho^2} \tag{2}$$

4 Closed loop supply chain model

The Closed Loop Supply Chain (CLSC) with recycling as a reprocessing option under consideration has a Reverse Chain as a continuation of the Forward Chain, focusing on the trend that maintains the independent operation and management of both chains. The CLSC entities considered are retailer, wholesaler, manufacturer, supplier, segregator and the dealer (Fig. 1). The demand for a specific recycled product is made by the customer to the retailer, who orders for the required item to the wholesaler stage. The wholesaler's stock is replenished by the manufacturer, who in turn replenishes its stock of required raw material (i.e. recyclable waste) from the supplier. The dealer collects the raw material (consisting of mixture of relevant and non-relevant waste) from the source (or consumer) and replenishes the stock at the segregator stage. The segregator stage performs the function of segregating/ separating the desired recyclable raw material (relevant waste) from the materials received, and forwards it to the supplier stage. Thus, it is assumed that the dealer doesn't order for the waste product from the source for recycling but instead collects it from them. Hence, order variability by the dealer stage has not been accounted for, in the model formulation. In the paper, the customer turns into a consumer after purchased product reaches its end-of-life stage.

The sequence of events in any period at any echelon is as follows: the order placed earlier is received, and the demand is fulfilled at the beginning of the period, the inventory level is reviewed, and ordering decision is made at the end of the period. A periodic review policy is assumed in model formulation with each echelon using OUT replenishment policy and MMSE forecasting scheme. But, a specific length of the review period has not been assumed. All the results are consistent irrespective of the unit of review period adopted (day, week, month, etc.).

4.1 Retailer's and wholesaler's ordering policy model (Refer [19])

The wholesaler stage follows ARMA(1,1) demand process (i.e. order process from the retailer stage), with auto regressive (AR), moving average (MA) parameters as

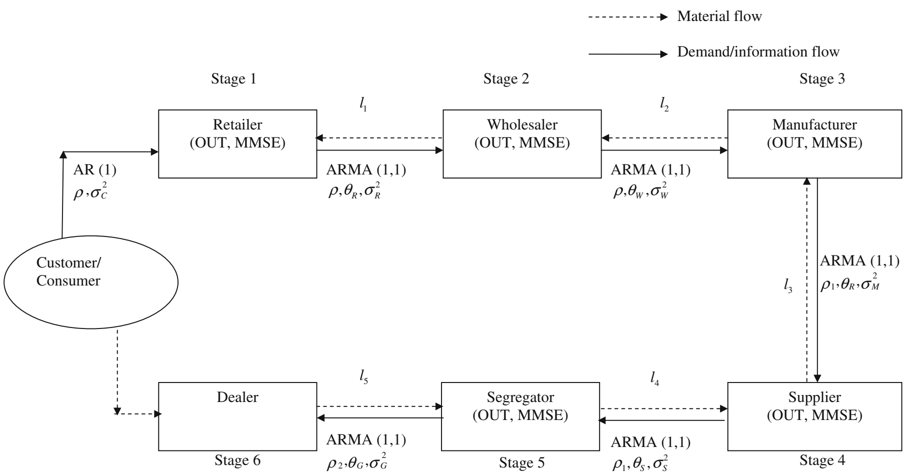


Fig. 1 Market demand transition for closed loop supply chain under study

ρ,θ_R respectively. The demand variance is given by σ_R^2, where $\sigma_{\varepsilon,R}^2$ represents variance of the error term.

$$\text{Bullwhip ratio at the retailer stage} = (BW)_R = \frac{\sigma_R^2}{\sigma_C^2} \tag{3}$$

Where, $\sigma_R^2 = \left(\frac{1+\theta_R^2-2\rho\theta_R}{1-\rho^2}\right)\sigma_{\varepsilon,R}^2$; $\sigma_{\varepsilon,R}^2 = (1+\rho\Omega_1)^2\sigma_\varepsilon^2 = \lambda^2\sigma_\varepsilon^2$; $\theta_R = \frac{\rho\Omega_1}{1+\rho\Omega_1} = \frac{\eta}{\lambda}$; and $\Omega_1 = \frac{1-\rho^{l_1}}{1-\rho}$, where l_1=fixed lead-time for wholesaler to fulfill an order.

Similarly, the manufacturer/recycling unit follows ARMA(1,1) demand process (i.e. order process from the wholesaler stage) with AR, MA parameters as ρ,θ_W respectively. The demand variance is given by σ_W^2, where $\sigma_{\varepsilon,W}^2$ is the variance of the error term.

$$\text{Bullwhip ratio at the wholesaler stage} = (BW)_W = \frac{\sigma_W^2}{\sigma_C^2} \tag{4}$$

Where, $\sigma_W^2 = \left(\frac{1+\theta_W^2-2\rho\theta_W}{1-\rho^2}\right)\sigma_{\varepsilon,W}^2$; $\theta_W = \frac{\eta+\rho\Omega_2\lambda-\Omega_2\eta}{\lambda+\rho\Omega_2\lambda-\Omega_2\eta} = \frac{\alpha}{\beta}$; $\sigma_{\varepsilon,W}^2 = \beta^2\sigma_\varepsilon^2$; and $\Omega_2 = \frac{1-\rho^{l_2}}{1-\rho}$, where l_2 is the fixed lead-time for the manufacturer to fulfill an order.

Hosoda and Disney [19] observed that, if the demand pattern faced by the retailer is AR(1) process, then the order process faced by the upper stages of the forward chain i.e. wholesaler and manufacturer's stages follows an ARMA(1,1) process.

4.2 Manufacturer's ordering policy

Manufacturer converts the segregated recyclable raw material supplied by the supplier stage into the desirable product, with fraction yield (X) signifying the quality of the relevant recyclable waste. Lower the yield fraction, poorer the quality of recyclable waste. With an OUT replenishment policy, the order placed by the manufacturer (M_t) at the end of time period 't' to the supplier stage for recyclable raw material is expressed as:

$$M_t = \frac{1}{X}\left[W_t + (S_{t,3} - S_{t-1,3})\right] \tag{5}$$

Where, $W_{t+1} = \rho W_t + \beta\varepsilon_{t+1} - \alpha\varepsilon_t$ (Refer [19]) and

$$S_{t,3} = \widehat{W}_t^{l_3} + z_3\widehat{\sigma}_{l_3} \tag{6}$$

- W_t ordered quantity in period 't' to be delivered in period $t+l_2$ (where l_2 is the fixed lead-time for the manufacturer to fulfill an order).
- $S_{t,3}$ Manufacturer's OUT inventory level of finished product at time period 't' and is equal to sum of lead-time demand forecast ($\widehat{W}_t^{l_3}$) from wholesaler and the safety stock ($z_3\widehat{\sigma}_{l_3}$).
- $\widehat{W}_t^{l_3}$ Conditional estimate of total demand by wholesaler to manufacturer over the lead time l_3 and is based on $\tau_{t,W} = \{W_t, W_{t-1}, W_{t-2}, ...\}$ i.e. history of wholesaler's demand observed up to period't'.

$\widehat{\sigma}_{l_3}$ Conditional estimate of standard deviation of demand by wholesaler to manufacturer over the lead time l_3

X Fraction yield (amount of finished product produced per unit of recyclable product)

Service level (z) is implicitly related to various costs parameters and is given by [34]:

$$z_3 = \phi^{-1}\left(\frac{b_3 - c_3(1 - r_3)}{h_3 + b_3}\right)$$

Where, $\phi()$ is a standard normal distribution function; b_3=unit shortage cost for backorders at manufacturer stage; c_3=unit cost of purchase at manufacturer stage; h_3=unit holding cost at manufacturer stage; and r_3=Discounting rate at the manufacturer stage to minimize the discounted expected cost over an infinite planning horizon for optimal inventory policy.

The above mentioned OUT inventory policy allows M_t to be negative, in which case we assume that the excess inventory is returned without penalty as commonly assumed e.g. Lee et al. [25], Chen et al. [3, 4]. Also, it has been noted that using conditional expectation, as the forecasting mechanism within the OUT policy will minimize the total inventory related cost over time [22]. As MMSE scheme is provided by conditional expectation [2, 19] therefore:

$$\begin{aligned}\widehat{W}_t^{l_3} &= E\left(\sum_{i=1}^{l_3} W_{t+i}/\tau_{t,W}\right) = \frac{(1-\rho^{l_3})}{(1-\rho)}\widehat{W}_{t+1} = \Omega_3\widehat{W}_{t+1} \text{ and } \widehat{\sigma}_{l_3}^2 \\ &= Var\left(W_t^{l_3} - \widehat{W}_t^{l_3}\right)\end{aligned} \tag{7}$$

Where $\Omega_3 = \frac{1-\rho^{l_3}}{1-\rho}$, $\widehat{W}_{t+1} = E(W_{t+1}/W_t, \varepsilon_t) = \rho W_t - \alpha\varepsilon_t$, $W_t^{l_3} = \sum_{\pi=1}^{l_3} W_{t+\pi}$ is total wholesaler's order during lead-time l_3.

From Eqs. (5–7) and as the standard deviation of the lead-time forecast error ($\widehat{\sigma}_{l_3}$) is constant over the time, the manufacturer's stage anticipated order to supplier stage based on anticipated demand from the wholesaler stage at time period't' is expressed as

$$\begin{aligned}M_t &= \frac{1}{X}\left[W_t + \left(\widehat{W}_t^{l_3} - \widehat{W}_{t-1}^{l_3}\right)\right] = \frac{1}{X}\left[W_t + \Omega_3\left(\widehat{W}_{t+1} - \widehat{W}_t\right)\right] \\ &= \frac{1}{X}[W_t + \Omega_3(\rho W_t - \alpha\varepsilon_t - \rho W_{t-1} + \alpha\varepsilon_{t-1})]\end{aligned} \tag{8}$$

Using Eqs. (6) and (8), supplier anticipates that the manufacturer's order quantity for period (t+1) is:

$$\begin{aligned}M_{t+1} &= \tfrac{1}{X}[W_{t+1} + \Omega_3(\rho W_{t+1} - \alpha\varepsilon_{t+1} - \rho W_t + \alpha\varepsilon_t)] \\ &= \tfrac{1}{X}\{\rho[W_t + \Omega_3(\rho W_t - \alpha\varepsilon_t - \rho W_{t-1} + \alpha\varepsilon_{t-1})] + (\beta + \rho\Omega_3\beta - \Omega_3\alpha)\varepsilon_{t+1} - (\alpha + \rho\Omega_3\beta - \Omega_3\alpha)\varepsilon_t\} \\ &= \tfrac{1}{X}[\rho M_t + (\beta + \rho\Omega_3\beta - \Omega_3\alpha)\varepsilon_{t+1} - (\alpha + \rho\Omega_3\beta - \Omega_3\alpha)\varepsilon_t] \\ \therefore M_{t+1} &= \rho_1 M_t + \delta\varepsilon_{t+1} - \gamma\varepsilon_t\end{aligned} \tag{9}$$

Where $\rho_1 = \frac{\rho}{X}$; $\gamma = \frac{\alpha+\rho\Omega_3\beta-\Omega_3\alpha}{X}$ and $\delta = \frac{\beta+\rho\Omega_3\beta-\Omega_3\alpha}{X}$

Equation (9) represents a scaled ARMA (1, 1) process as

$$M_{t+1} = \rho_1 M_t + \varepsilon_{t+1,M} - \theta_M \varepsilon_{t,M} \tag{10}$$

$$\text{Where, } \theta_M = \frac{\gamma}{\delta} \text{ and } \varepsilon_{t,M} = \delta \varepsilon_t$$

Thus, the supplier stage follows ARMA(1,1) demand process (i.e. order process from manufacturer stage), with AR and MA parameters as ρ_1, θ_M respectively. The demand variance is given by σ_M^2, where $\sigma_{\varepsilon,M}^2$ represents the variance of the error term.

$$\sigma_M^2 = \left(\frac{1 + \theta_M^2 - 2\rho_1\theta_M}{1 - \rho_1^2}\right)\sigma_{\varepsilon,M}^2,\ \sigma_{\varepsilon,M}^2 = \delta^2\sigma_\varepsilon^2 \tag{11}$$

Thus, the effect of variance amplification at the manufacturer stage is measured by: $(BW)_M = \frac{\sigma_M^2}{\sigma_C^2}$

4.3 Supplier's ordering policy

With an OUT inventory policy the order placed by the supplier $(Su)_t$ at the end of time period't' can be expressed as:

$$(Su)_t = M_t + \left(S_{t,4} - S_{t-1,4}\right) \tag{12}$$

$$\text{Where, } S_{t,4} = \widehat{M}_t^{l_4} + z_4\widehat{\sigma}_{l_4} \tag{13}$$

$S_{t,3}$=Supplier's OUT level for segregated relevant recyclable product at time 't' and is equal to sum of forecast lead-time demand ($\widehat{M}_t^{l_4}$) from manufacturer and the safety stock ($z_4\widehat{\sigma}_{l_4}$).

$\widehat{M}_t^{l_4}$=Conditional estimate of total demand for the segregated relevant recyclable product by manufacturer stage from the supplier stage over the lead time l_4 and is based on $\tau_{t,M} = \{M_t, M_{t-1}, M_{t-2}, ...\}$ i.e. history of manufacturer's demand observed up to period 't'.

$\widehat{\sigma}_{l_4}$=Conditional estimate of standard deviation of total demand for the segregated relevant recyclable product by manufacturer stage to supplier stage over the lead time l_4,$z_4 = \phi^{-1}\left(\frac{b_4 - c_4(1-r_4)}{h_4+b_4}\right)$ Where, $\phi()$ is a standard normal distribution function; b_4=unit shortage cost for backorders at supplier stage; c_4=unit cost of purchase at supplier stage; h_4=unit holding cost at supplier stage; and r_4=Discounting rate at the supplier stage to minimize the discounted expected cost over an infinite planning horizon for optimal inventory policy.

Considering the MMSE scheme:

$$\widehat{M}_t^{l_4} = E\left(\sum_{i=1}^{l_4} M_{t+i}/\tau_{t,M}\right) = \frac{\left(1 - {\rho_1}^{l_4}\right)}{(1 - \rho_1)}\widehat{M}_{t+1} = \Omega_4\widehat{M}_{t+1} \text{ and } \widehat{\sigma}_{l_4}^2$$

$$= Var\left(M_t^{l_4} - \widehat{M}_t^{l_4}\right) \tag{14}$$

Where $\Omega_4 = \frac{1-\rho_1^{l_4}}{1-\rho_1}$, $\widehat{M}_{t+1} = E(M_{t+1}/M_t, \varepsilon_t) = \rho_1 M_t - \gamma\varepsilon_t$, $M_t^{l_4} = \sum_{\pi=1}^{l_4} M_{t+\pi}$ is total order by the manufacturer during lead-time l_4.

From Eqs. (12–14) and as the standard deviation of the lead-time forecast error $(\widehat{\sigma}_{l_4})$ is constant over the time, the supplier stage anticipated order to segregator stage based on anticipated demand from the manufacturer at time period‘t’ is expressed as

$$\begin{aligned}(Su)_t &= M_t + \left(\widehat{M}_t^{l_4} - \widehat{M}_{t-1}^{l_4}\right) = M_t + \Omega_4\left(\widehat{M}_{t+1} - \widehat{M}_t\right)\\ &= M_t + \Omega_4(\rho_1 M_t - \gamma\varepsilon_t - \rho_1 M_{t-1} + \gamma\varepsilon_{t-1})\end{aligned} \tag{15}$$

Using Eqs. (9) and (15), segregator anticipates that the supplier’s order quantity for period (t+1) is:

$$\begin{aligned}(Su)_{t+1} &= M_{t+1} + \Omega_4(\rho_1 M_{t+1} - \gamma\varepsilon_{t+1} - \rho_1 M_t + \gamma\varepsilon_t)\\ &= \rho_1[M_t + \Omega_4(\rho_1 M_t - \gamma\varepsilon_t - \rho_1 M_{t-1} + \gamma\varepsilon_{t-1})] + (\delta + \rho_1\Omega_4\delta - \Omega_3\gamma)\varepsilon_{t+1} - (\gamma + \rho_1\Omega_4\delta - \Omega_3\gamma)\varepsilon_t\\ &= \rho_1(Su)_t + (\delta + \rho_1\Omega_4\delta - \Omega_3\gamma)\varepsilon_{t+1} - (\gamma + \rho_1\Omega_4\delta - \Omega_3\gamma)\varepsilon_t\\ \therefore (Su)_{t+1} &= \rho_1(Su)_t + \Xi\varepsilon_{t+1} - \Theta\varepsilon_t\end{aligned} \tag{16}$$

Where $\Xi = \delta + \rho_1\Omega_4\delta - \Omega_4\gamma$ and $\Theta = \gamma + \rho_1\Omega_4\delta - \Omega_4\gamma$

Equation (16) represents a scaled ARMA (1, 1) process as

$$(Su)_{t+1} = \rho_1(Su)_t + \varepsilon_{t+1,S} - \theta_S\varepsilon_{t,S} \tag{17}$$

$$\text{Where, } \theta_S = \frac{\Theta}{\Xi} \text{ and } \varepsilon_{t,S} = \Xi\varepsilon_t$$

Thus, the segregator stage follows ARMA(1,1) demand process (i.e. order process from the supplier stage), with AR and MA parameters as ρ_1,θ_S respectively. The demand variance is given by σ_S^2, where $\sigma_{\varepsilon,S}^2$ represents the variance of the error term.

$$\sigma_S^2 = \left(\frac{1 + \theta_S^2 - 2\rho_1\theta_S}{1 - \rho_1^2}\right)\sigma_{\varepsilon,S}^2,\ \sigma_{\varepsilon,S}^2 = \Xi^2\sigma_\varepsilon^2 \tag{18}$$

Thus, the variance amplification ratio i.e. bullwhip ratio at the supplier stage is given by:

$$(BW)_S = \frac{\sigma_S^2}{\sigma_C^2}$$

4.4 Segregator’s ordering policy

This is a critical stage in the reverse supply chain for recycling activity. It is at this stage that the mixture of relevant and non-relevant waste (received from dealer) is segregated, so that the desired recyclable waste could be sent to manufacturer for recycling. It is assumed that the mixture collected from the source by the dealer and passed on to the segregator stage consists of a fixed proportion of non-relevant waste i.e. 1-Ψ. With an OUT inventory policy the order placed by the segregator (G_t) at the end of time period ‘t’ can be expressed as:

$$G_t = \frac{1}{\Psi}\left[(Su)_t + \left(S_{t,5} - S_{t-1,5}\right)\right] \tag{19}$$

$$\text{Where, } S_{t,5} = \left(\widehat{S}u\right)_t^{l_5} + z_5\widehat{\sigma}_{l_5} \tag{20}$$

$S_{t,5}$ Segregator's OUT inventory level of recovered waste product at time period 't' and is equal to sum of forecast lead-time demand ($\left(\widehat{S}u\right)_t^{l_5}$) from manufacturer and the safety stock ($z_5\widehat{\sigma}_{l_5}$).

$\left(\widehat{S}u\right)_t^{l_5}$ Conditional estimate of total demand by supplier to segregator over the lead time l_5 and is based on $\tau_{t,S} = \left\{(Su)_t, (Su)_{t-1}, (Su)_{t-2}, \ldots\right\}$ i.e. history of supplier's demand observed up to period 't'.

$\widehat{\sigma}_{l_5}$ Conditional estimate of standard deviation of demand by supplier to segregator over the lead time l_5

Ψ Proportion of relevant recyclable waste from the initial waste recovered.

$z_5 = \phi^{-1}\left(\frac{b_5 - c_5(1-r_5)}{h_5+b_5}\right)$ Where, $\phi()$ is a standard normal distribution function; b_5=unit shortage cost for backorders at segregator stage; c_5=unit cost of purchase at segregator stage; h_5=unit holding cost at segregator; and r_5=discounting rate at the segregator stage to minimize the expected discounted cost over an infinite planning horizon for optimal inventory policy.

For MMSE scheme:

$$\begin{aligned} \left(\widehat{S}u\right)_t^{l_5} &= E\left(\sum_{i=1}^{l_5}(Su)_{t+i}/\tau_{t,S}\right) = \frac{\left(1-\rho_1^{l_5}\right)}{(1-\rho_1)}\left(\widehat{S}u\right)_{t+1} \\ &= \Omega_5\left(\widehat{S}u\right)_{t+1} \text{ and } \widehat{\sigma}^2_{l_5} = Var\left((Su)_t^{l_5} - \left(\widehat{S}u\right)_t^{l_5}\right) \end{aligned} \tag{21}$$

Where $\Omega_5 = \frac{1-\rho_1^{l_5}}{1-\rho_1}$, $\left(\widehat{S}u\right)_{t+1} = E\left((Su)_{t+1}/(Su)_t, \varepsilon_t\right) = \rho_1(Su)_t - \Theta\varepsilon_t$, $(Su)_t^{l_5} = \sum_{\pi=1}^{l_5}(Su)_{t+\pi}$ is total order by the supplier during lead-time l_5.

From Eqs. (19–21) and as $\widehat{\sigma}_{l_5}$ is constant over the time, the segregator stage's anticipated order to dealer stage based on anticipated demand from the supplier stage at time period't' is expressed as

$$\begin{aligned} G_t &= \frac{1}{\Psi}\left[(Su)_t + \left(\left(\widehat{S}u\right)_t^{l_5} - \left(\widehat{S}u\right)_{t-1}^{l_5}\right)\right] \\ &= \frac{1}{\Psi}\left[(Su)_t + \Omega_5\left(\left(\widehat{S}u\right)_{t+1} - \left(\widehat{S}u\right)_t\right)\right] \\ &= \frac{1}{\Psi}\left\{(Su)_t + \Omega_5\left[\rho_1(Su)_t - \Theta\varepsilon_t - \rho_1(Su)_{t-1} + \Theta\varepsilon_{t-1}\right]\right\} \end{aligned} \tag{22}$$

Incorporating Eq. (16) in Eq. (22) the dealer anticipates that the segregator's order quantity for period (t+1) is:

$$\begin{aligned} G_{t+1} &= \tfrac{1}{\Psi}\left\{(Su)_{t+1} + \Omega_5\left[\rho_1(Su)_{t+1} - \Theta\varepsilon_{t+1} - \rho_1(Su)_t + \Theta\varepsilon_t\right]\right\} \\ &= \tfrac{1}{\Psi}\left\{\rho_1\left[(Su)_t + \Omega_5\left(\rho_1(Su)_t - \Theta\varepsilon_t - \rho_1(Su)_{t-1} + \Theta\varepsilon_{t-1}\right)\right] + (\Xi + \rho_1\Omega_5\Xi - \Omega_5\Theta)\varepsilon_{t+1} - (\Theta + \rho_1\Omega_5\Xi - \Omega_5\Theta)\varepsilon_t\right\} \\ &= \tfrac{1}{\Psi}\left\{\rho_1 G_t + (\Xi + \rho_1\Omega_5\Xi - \Omega_5\Theta)\varepsilon_{t+1} - (\Theta + \rho_1\Omega_5\Xi - \Omega_5\Theta)\varepsilon_t\right\} \\ \therefore G_{t+1} &= \rho_2 G_t + \varphi\varepsilon_{t+1} - \varphi\varepsilon_t \end{aligned} \tag{23}$$

Where $\rho_2 = \frac{\rho_1}{\Psi}$; $\varphi = \frac{(\Xi + \rho_1\Omega_5\Xi - \Omega_5\Theta)}{\Psi}$ and $\chi = \frac{(\Theta + \rho_1\Omega_5\Xi - \Omega_5\Theta)}{\Psi}$

Equation (23) represents a scaled ARMA (1, 1) process as

$$G_{t+1} = \rho_2 G_t + \varepsilon_{t+1,G} - \theta_G \varepsilon_{t,G} \text{ Where, } \theta_G = \frac{\chi}{\varphi} \text{ and } \varepsilon_{t,G} = \varphi \varepsilon_t \tag{24}$$

Thus, the dealer stage follows ARMA(1,1) demand process (i.e. order process from the segregator stage), with AR and MA parameters as ρ_2,θ_G respectively. The demand variance is given by σ_G^2, where $\sigma_{\varepsilon,G}^2$ represents the variance of the error term.

$$\sigma_G^2 = \left(\frac{1 + \theta_G^2 - 2\rho_2\theta_G}{1 - \rho_2^2}\right)\sigma_{\varepsilon,G}^2,\ \sigma_{\varepsilon,G}^2 = \varphi^2\sigma_\varepsilon^2 \tag{25}$$

The bullwhip ratio observed at the segregator stage can be statistically measured as follows:

$$(BW)_G = \frac{\sigma_G^2}{\sigma_C^2}$$

From these statistical models (section 4.1–4.4), it can be seen that apart from the actual demand at the retailer stage being AR(1) process, all other order processes at various echelons have scaled ARMA(1,1) pattern. Furthermore, all the echelons discussed can be expressed in terms of parameter of the market demand process (ρ). Therefore, all the echelons from retailer to dealers have complete information of the market demand process with MMSE scheme. Figure 1 summarizes how the original AR(1) demand process is changed by the OUT policy with the MMSE scheme as it proceeds up the closed loop supply chain for recycling.

5 Analysis of results obtained from the model

This section examines the sensitivity of the model parameters like auto regressive parameter (ρ), fraction segregation at source (Ψ) and lead-time combination on the performance of CLSC system. The analysis is performed by considering various combinations of replenishment lead-times between successive echelons, with the sum of all downstream lead-time from retailer to dealer being fixed at 25.

As discussed in section 4, the variance amplification is measured with the help of Bullwhip ratio (BW). The bullwhip ratios at each echelon have been calculated using the variance amplification models with two different combinations of lead-times, under the condition that the AR parameters for market demand process is in range of $|\rho|<1$. The two lead-time combinations considered are (7,6,5,4,3) and (3,4,5,6,7), where the numbers indicate the upstream replenishment lead-time between successive echelons/stages starting from retailer stage. Figures 2 and 3 demonstrate the impact of the value of auto regressive parameter (ρ) on the bullwhip ratios at each stage for the situation with 100% segregation of recyclable relevant waste at the source (Ψ=100%) and 100% yield at manufacturer stage (X=100%). With Ψ, X=100%, the present model becomes similar to the one by Hosoda and Disney [19] with increased number of echelons. Hence, similar pattern of variance amplification can be observed.

- In case of $\rho>0$, bullwhip ratio is almost identical when auto regressive parameter (ρ) is relatively small (less than 0.3). The bullwhip effect does not occur when $\rho<0$.

- $(BW)_G$ is not affected by the values of l_1, l_2, l_3, l_4 or l_5 and has same shape under the constraints of constant accumulated lead-time in CLSC $(l_1 + l_2 + l_3 + l_4 + l_5)$,
- The condition of $(BW)_R \leq (BW)_W \leq (BW)_M \leq (BW)_S \leq (BW)_G$ is observed for the two different lead time combinations. This also verifies that the variance amplification occurs as we move upstream in a closed loop supply chain.

In practice, it is nearly impossible to have a CLSC with Ψ, X=100% i.e. entire volumes of waste collected by the dealer in the reverse chain are of relevant and recyclable quality. Hence, for a more realistic situation, Figs. 4 and 5 reveal the effect of AR parameter (ρ) on CLSC for X=78% and Ψ=95%. Figure 4 is for the lead-time combination of (7,6,5,4,3) while the Fig. 5 is for (3,4,5,6,7). The nature and shape of the figures are the same as discussed for X=100% and Ψ=100%. It could be seen that the bullwhip ratio increases significantly with decrease in yield fraction at the manufacturer and segregation at source. With decrease in yield at manufacturer stage, there will be an increase in demand for recyclable item from the higher echelons so as to fulfill lower echelon's demand. Hence, increased demand order leads to higher variance amplification. Similar intuitive interpretation can be given for decreased degree of segregation at source. Existence of wave form/unstable band range from 0.70≤ρ≤0.80 for the manufacturer, supplier and segregator stage could also be observed in Figs. 4 and 5. The transition of auto regressive (AR) parameter for an ARMA process from $\rho_1, \rho_2 \leq 1$ to $\rho_1, \rho_2 > 1$ for 0.70≤ρ≤0.80 leads to a waveform representing instability in bullwhip ratios at manufacturer (ρ_1), supplier (ρ_1) and segregator (ρ_2) stages. This violates the covariance stationarity condition (i.e.$|\rho_1|, |\rho_2| < 1$) at manufacturer, supplier and segregator stages in the above mentioned range.

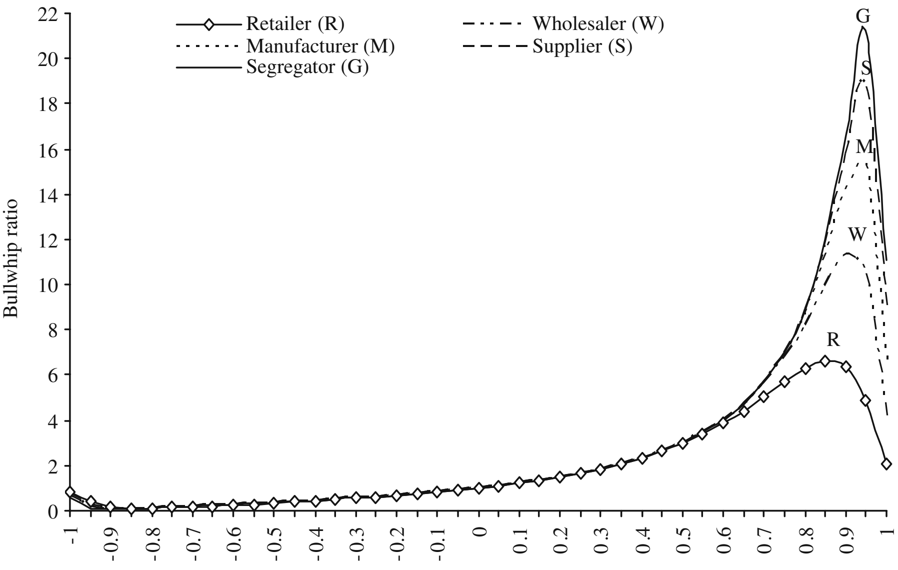


Fig. 2 Effect of AR parameter (ρ) on bullwhip of CLSC system, with X=100%, Ψ=100%, $l_1 = 7, l_2 = 6, l_3 = 5, l_4 = 4, l_5 = 3$

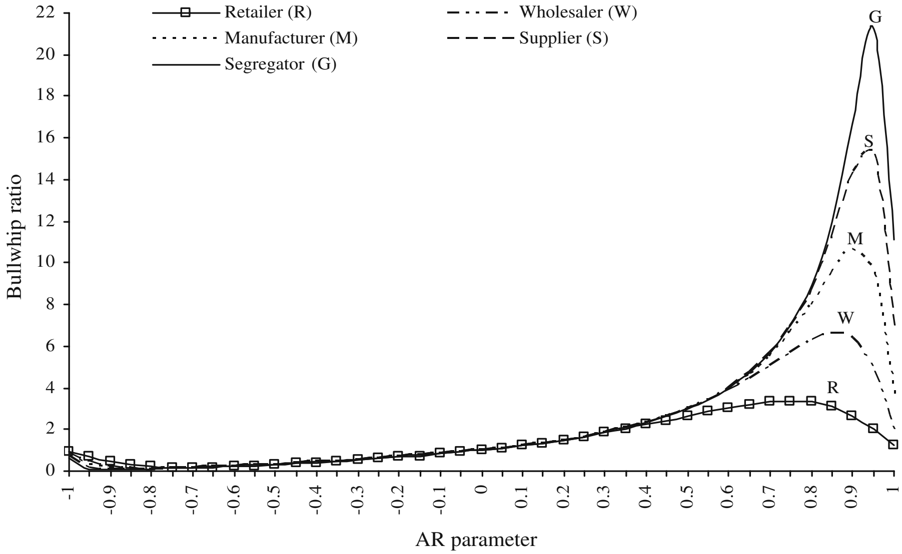


Fig. 3 Effect of AR parameter (ρ) on bullwhip of CLSC system, with X=100%, Ψ=100% $l_1 = 3, l_2 = 4, l_3 = 5, l_4 = 6, l_5 = 7$

Figure 6(a, b) depict the effect of decreased degree of segregation at the source on the bullwhip effect in the CLSC measured at the segregator stage for the lead-time combination (7,6,5,4,3). Along the X-axis the amount of segregation at the source is varied from 95% to 60% and its effect on the variance amplification is studied along

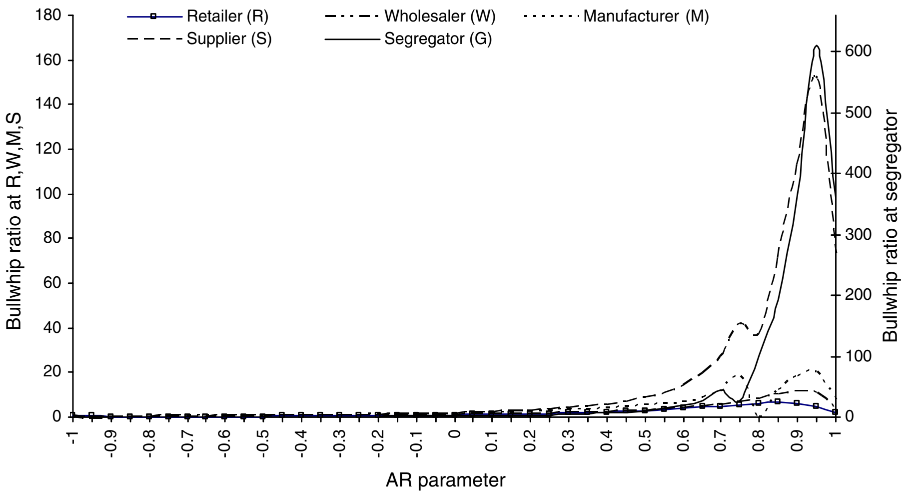


Fig. 4 Effect of AR parameter (ρ) on bullwhip of CLSC system, with X=78%, Ψ=95% $l_1 = 7, l_2 = 6, l_3 = 5, l_4 = 4, l_5 = 3$

Y-axis, for yield of 78% at the manufacturer stage. The following inferences could be drawn from the figures:

- Bullwhip effect does not occur when $\rho<-0.50$,
- For the case of $\rho\geq-0.50$, bullwhip ratio increases with the decrease in segregation at source. The ranges of $-0.50\leq\rho\leq -0.70$ and $0.50\leq\rho\leq0.70$ are exceptions to this trend. In the above mentioned range, there is a tendency of waveform or instability in the curve. For example, wave form of the curve for $\rho=0.6$ is observed when segregation at source decreases from 85% to 75%. After this stage, further decrease in segregation (Ψ) increases the bullwhip ratio. Similarly, curves for $\rho = 0.5, 0.7, -0.5, -0.6, -0.7$ can be explained. The presence of wave form can be attributed to violation of covariance stationarity condition at the segregator stage i.e. auto regressive (AR) parameter for an ARMA process changes from $\rho_2\leq1$ to $\rho_2>1$.
- With decrease in degree of segregation at the source (Ψ), width of instable band range in terms of market demand auto regressive parameter (ρ) increases.

To avoid the instability in the system, stability analysis [Fig. 7(a–d)] was performed. The stability analysis determines the relationship between the degree of segregation at the source (Ψ) and yield at the manufacturer (X) for different market demand parameter (ρ). Four lead-time combinations with different total supply chain replenishment lead-times were considered for stability analysis i.e. (7,6,5,4,3), (5,5,5,5,5), (6,5,4,3,2) and (5,4,3,2,1). For a particular lead-time combination each point in Fig. 7(a–d) denotes the minimum degree of segregation at source (Ψ), for a particular combination of ρ and X to avoid instability in the

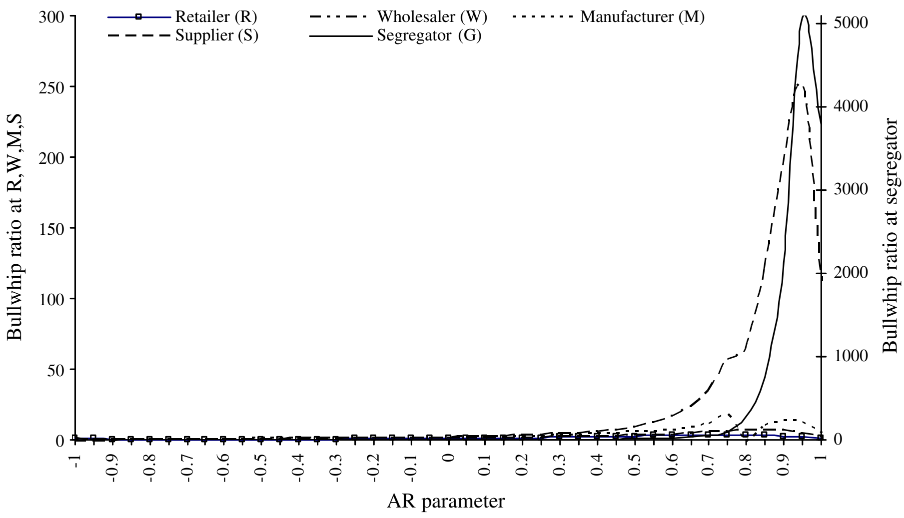


Fig. 5 Effect of AR parameter (ρ) on bullwhip of CLSC system, with X=78%, Ψ=95% $l_1 = 3, l_2 = 4, l_3 = 5, l_4 = 6, l_5 = 7$

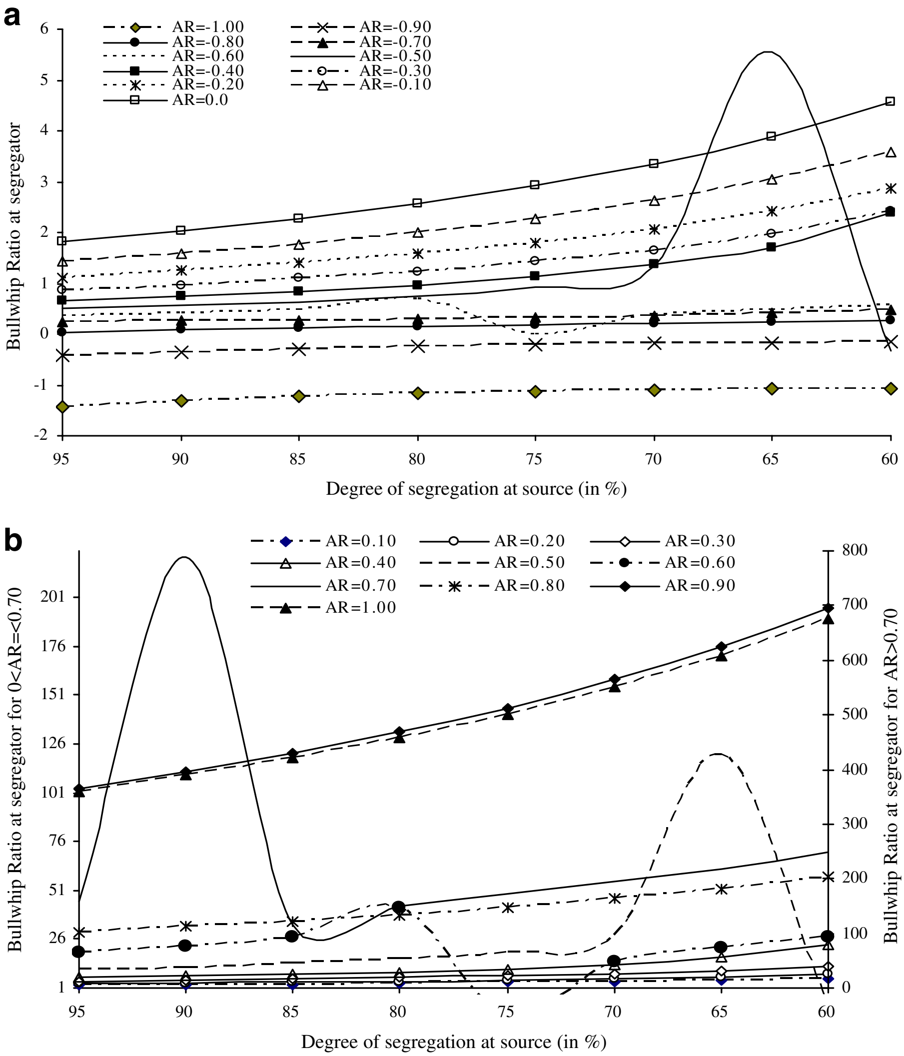


Fig. 6 **a** Effect of segregation at source (Ψ) on bullwhip ratio for CLSC system measured at segregator stage for AR≤0 and X=78%, where AR is initial market demand parameter (ρ) and lead-time combination (7,6,5,4,3). **b** Effect of segregation at source (Ψ) on bullwhip ratio for CLSC system measured at segregator stage for AR>0, X=78%. Where AR is initial market demand parameter (ρ) and lead-time combination (7,6,5,4,3)

system. With further decrease in degree of segregation at source the system gets unstable. The following inference can be made from Fig. 7(a–d):

- The presence of instability region is not affected by the total lead-time of the system. It keeps the same shape under the constraint that number of echelons/stages and operations performed by each echelon are fixed.

- For a particular lead-time combination, the steepness of the individual curve increases with increase in market demand parameter (ρ) on both sides (i.e. positive as well as negative ρ).
- Unstable band range of AR parameter with varying degree of segregation at source can be determined for a fixed yield (X) at the manufacturer stage.

Figure 8 demonstrates the impact of different lead-time combinations on the bullwhip effect of the closed loop supply chain network for recycling, while keeping the total lead time across the CLSC system fixed (at 25units) and under

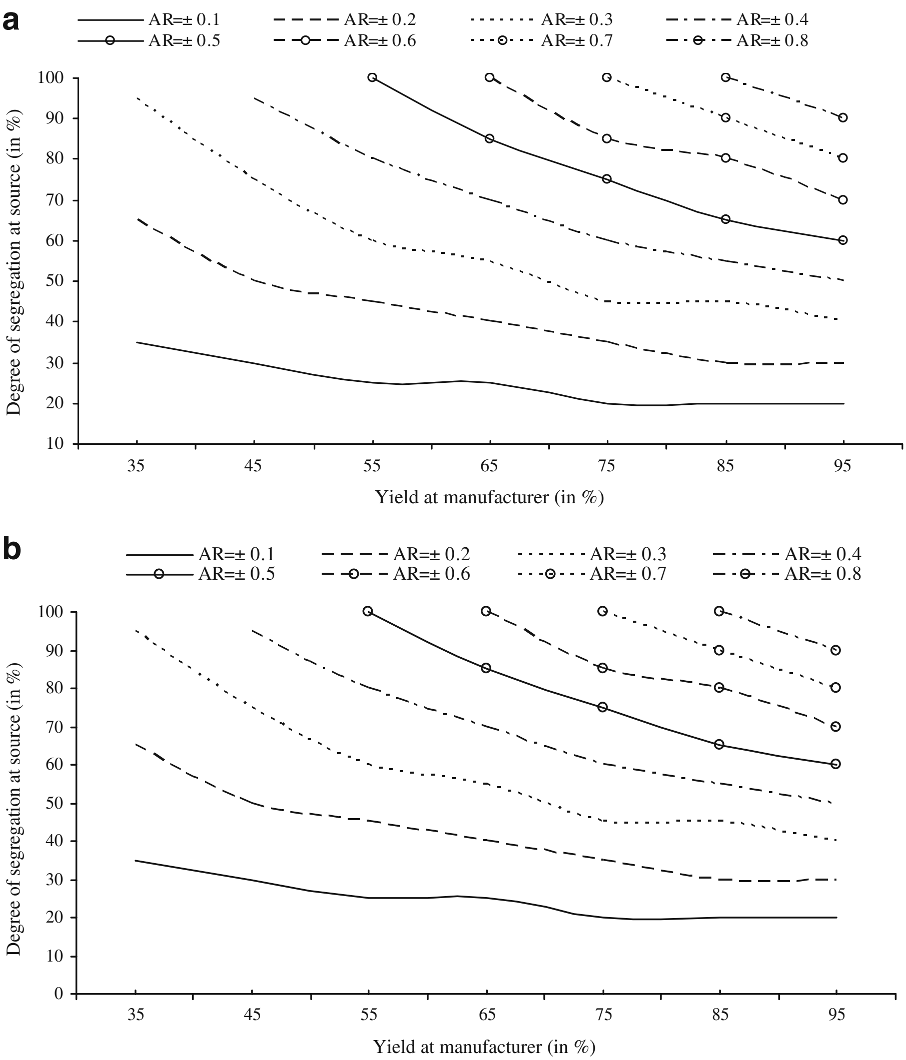
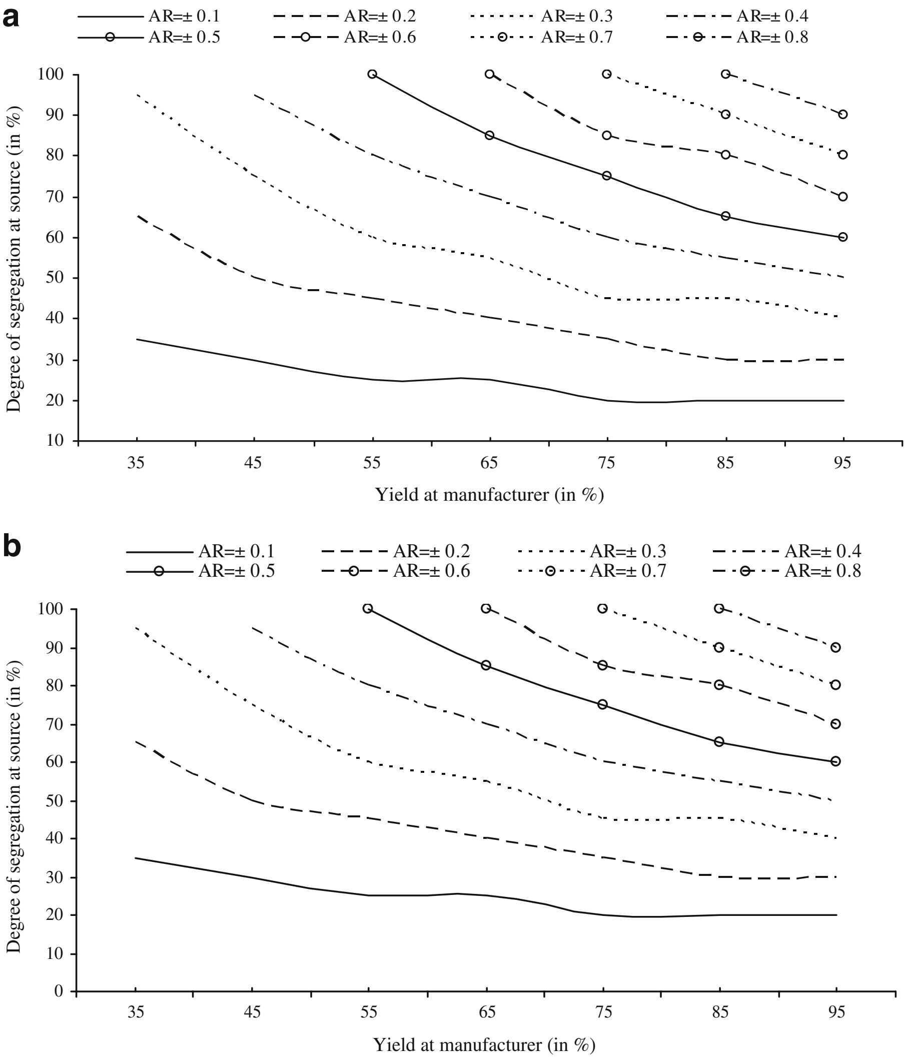


Fig. 7 **a** Stability analysis in CLSC for lead-time combination (7,6,5,4,3) and total CLSC replenishment lead-time is 25 unit. **b** Stability analysis in CLSC for lead-time combination (5,5,5,5,5) and total CLSC replenishment lead-time is 25 unit. **c** Stability analysis in CLSC for lead-time combination (6,5,4,3,2) and total CLSC replenishment lead-time is 20 unit. **d** Stability analysis in CLSC for lead-time combination (5,4,3,2,1) and total CLSC replenishment lead-time is 15 unit

Fig. 7 (continued)



the condition of X=78% and Ψ=95%. The following observations could be made from the lead time analysis:

- Bullwhip effect does not occur when $\rho<-0.50$.
- In case of $-0.50 \leq \rho \leq 0.70$, bullwhip ratio is almost same for all combinations of leadtimes.
- For higher value of market demand process parameter ($\rho>0.70$), the lead-time combination having lower lead-time between successive echelons at end of the CLSC has lower variance amplification. Hence, the CLSC system should be

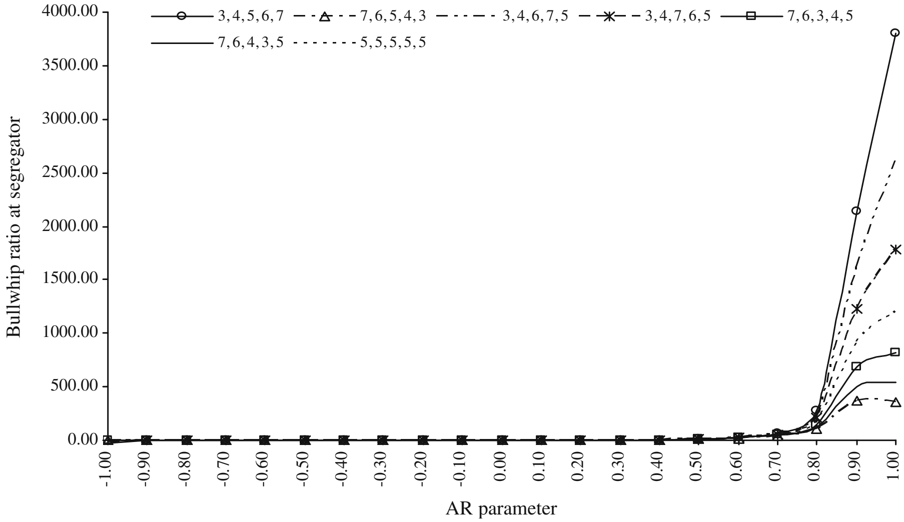


Fig. 8 Effect of lead-time combination on bullwhip ratio Of CLSC system measured at segregator stage. Note in all combination $l_1+l_2+l_3+l_4+l_5=25$ and X=78%, Ψ=95%

designed in such a way that the initial replenishment lead-time should be greater than lead-time at later stages.

Figure 9 depicts the effect of different combinations of lead-times in the CLSC with recycling, for ρ=0.85, X=78% and Ψ=95%. It represents the variance amplification of the market demand when it passes from retailer to the segregator stage and it is again indicated that the CLSC system with lower replenishment lead-time at beginning and higher lead time at the end stages results in greater variance

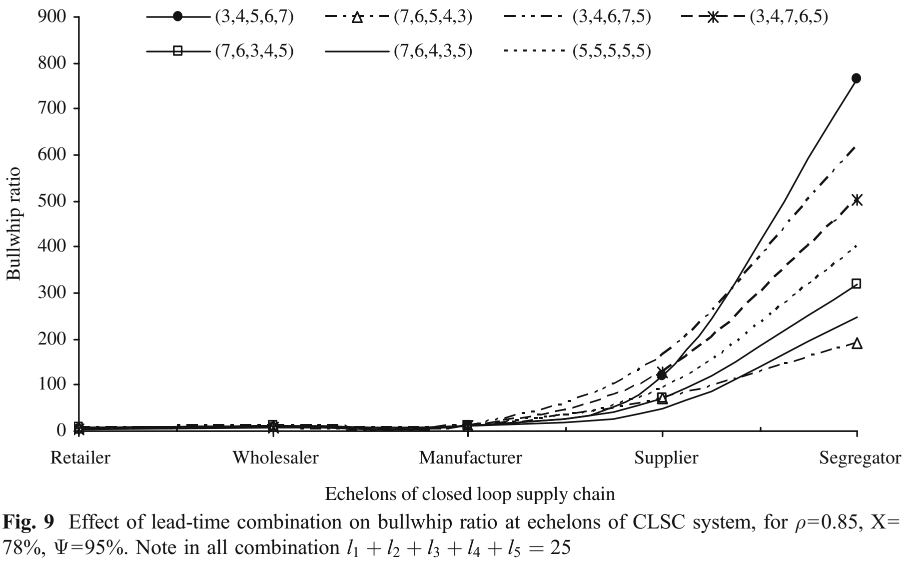


Fig. 9 Effect of lead-time combination on bullwhip ratio at echelons of CLSC system, for ρ=0.85, X= 78%, Ψ=95%. Note in all combination $l_1 + l_2 + l_3 + l_4 + l_5 = 25$

amplification/Bullwhip Ratio (BW). Hence, order of lead-time combination with increasing variance amplification effect at the segregator stage is:

$$(BW)_{7,6,5,4,3} < (BW)_{7,6,4,3,5} < (BW)_{7,6,3,4,5} < (BW)_{5,5,5,5,5} < (BW)_{3,4,7,6,5} < (BW)_{3,4,6,7,5} < (BW)_{3,4,5,6,7}$$

6 Summary of major findings and managerial implications

A statistical method has been considered to investigate the variance amplification effect in a multi echelon CLSC for recycling of products like paper, plastic. The members of the CLSC system include retailer, wholesaler, manufacturer, supplier, segregator, and dealer. The customer (turns to consumer after end-of-life of product) acts as an intermediate link between the forward and reverse chain. The dealer collects mixture of non-relevant and relevant recyclable waste product to start the reverse supply chain process. By implications the dealer does not order from the source (or consumer), hence, no variance amplification has been considered at the dealer stage. A first order auto regressive [AR(1)] stationary demand process has been assumed at the retailer and each supply chain participant adopts OUT policy with MMSE forecasting scheme. The presence of CLSC parameters representing the proportion of non-relevant waste collected from consumer, and quality of input relevant waste represented in terms of its yield adds a new dimension to the multi-echelon traditional supply chain. The major findings in the specified CLSC are:

- The AR demand process is transformed into scaled ARMA(1,1) process as it moves up the CLSC (Fig. 1), as in the case of forward supply chain studied by Hosoda and Disney [19]. Hence, the presence of reverse chain with segregation (represented through Ψ) and quality issues (represented through X) at segregator and manufacturer stages respectively, does not affect the demand process propagation across the CLSC.
- The value of the auto regressive parameter (ρ) remains the same up to the wholesaler stage and then it is changed at each echelon depending on the yield at manufacturer and degree of segregation at source. Where as, the value of the MA parameters for the scaled ARMA(1,1) process changes at each echelon and is a function of its replenishment lead-time, downstream lead-time, AR parameter for the echelon in question.
- As the ordering process contains complete information of market demand, the upstream supply chain participants may exploit an ARMA(1,1) model to estimate both AR and MA parameters to create MMSE forecast. Then with the knowledge of accumulated lead times and the demand process, each participant may estimate the quantity of demand over the lead-time.
- The scenario of Ψ, X=100% (Figs. 2 and 3) is rarely possible for recycling of products like plastic, paper. Analysis of CLSC with reduced segregation at source (95%) and yield (78%) revealed that the variance amplification is affected by lead-time combination between echelons (Figs. 4 and 5).

- Another interesting feature was the presence of waveform or instability band width in the variance amplification curves for different upstream echelons with change in ρ (Figs. 4 and 5). This pattern was also observed when Ψ was decreased from 95% to 60% for varying ρ. This phenomenon was attributed to the violation of stationarity condition at the respective echelon i.e. AR parameter for an ARMA process changes from $\rho_1, \rho_2 \leq 1$ to $\rho_1, \rho_2 > 1$. The management must carefully select value of ρ so as to avoid any order process instability in the CLSC. The management can avoid the unstable region by selecting the proper combination of X, Ψ and ρ as depicted by Fig. 7(a–d).
- The decrease in segregation at source led to an increase in bullwhip ratio. Hence, the management must formulate efficient collection systems and strategies for increased segregation at the source i.e. the dealer stage.
- Lead-time analysis (Figs. 8 and 9) helps in selection of appropriate lead-time combination that result's in the lowest possible variance amplification in the CLSC system. This could be utilized by the management for designing an efficient distribution system for recycling, where the replenishment lead-time should be allocated in the decreasing order starting from the highest for retailer to the lowest for the segregator stage. The intuitive justification of this could be that at the later stages larger quantities of materials are required to be transported in between the echelons (due to effect of X and Ψ) hence lower lead-time will reduce the anticipated demand and orders in these stages.

The findings in this paper relate to a specific CLSC system under the assumptions made. We are yet to determine whether this system is optimal for a given set of cost functions. The present work can be further extended by quantifying the bullwhip effect in a CLSC system in terms of amplification ratios of the variance of net inventory levels. The impact of various other market demand processes like ARMA (1,1), IMA(0,1,1), MA(0,0,1) etc. on the bullwhip effect of closed loop supply chain is also worth investigating further.

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