Solving a System of Equations Algebraically

- comparison method
- substitution method
- elimination method

Solve by comparison:

With the comparison method, you can solve a system of equations if they are both equal to the same variable or algebraic expression.

Solve by comparison:

$2x + 3y = 13$ (-2)
$-4x + y = -5$

$\begin{cases} -4x - 6y = -26 \\ -4x + y = -5 \end{cases}$ Equivalent

$-4x = 6y - 26$
$-4x = -y - 5$

$6y - 26 = -y - 5$
$6y + y = -5 + 26$
$7y = 21$
$y = 3$

$-4x = -y - 5$
$-4x = -3 - 5$
$-4x = -8$
$x = 2$
$\therefore (x, y) = (2, 3)$

Solve by comparison:

$3x + 5y = 11$ (4)
$4x - 2y = -20$ (3)

$12x + 20y = 44$ $\quad 12x = -20y + 44$
$12x - 6y = -60$ $\quad 12x = 6y - 60$

$-20y + 44 = 6y - 60$
$-20y - 6y = -60 - 44$
$-26y = -104$
$y = 4$

$12x = 6y - 60$
$12x = 6(4) - 60$
$12x = 24 - 60$
$12x = -36$
$x = -3$

$\therefore (x, y) = (-3, 4)$

Solve by comparison:

$2x + 3y = 5$

$3x - 4y = -18$

Solve by comparison:

$8x - 3y = 6$

$6x + 12y = -24$

Solve by comparison:

$x - 3y = -7$

$3x - 2y = -7$

Solve by Substitution

The substitution method involves solving an equation for a variable and substituting it into the other equation.

Solve by Substitution

$2x + 3y = 13$

$-4x + y = -5 \longrightarrow y = 4x - 5$

$2x + 3(4x - 5) = 13$

$2x + 12x - 15 = 13$

$14x = 28$

$x = 2$

$y = 4(2) - 5$

$y = 8 - 5$

$y = 3$

$\therefore (x, y) = (2, 3)$

Solve by Substitution

$3x + 2y = 2$

$x - 4y = 3 \longrightarrow x = 4y + 3$

$3(4y + 3) + 2y = 2$

$12y + 9 + 2y = 2$

$14y = -7$

$y = -\frac{1}{2}$

$x = 4\left(-\frac{1}{2}\right) + 3$

$x = -2 + 3$

$x = 1$

$\therefore (x, y) = \left(1, -\frac{1}{2}\right)$

Solve by Substitution

$x + 3y = 2$ $\longrightarrow$ $x = -3y + 2$

$2x + 3y = 13$

$2(-3y+2) + 3y = 13$

$-6y + 4 + 3y = 13$

$-3y = 9$

$y = -3$

$x = -3(-3) + 2$

$= 9 + 2$

$= 11$

$\therefore (x, y) = (11, -3)$

Solve by Substitution

$3x + 5y = 11$

$4x - 2y = -20$ $\longrightarrow$ $-2y = -4x - 20$

$y = 2x + 10$

$3x + 5(2x+10) = 11$

$3x + 10x + 50 = 11$

$13x = -39$

$x = -3$

$y = 2(-3) + 10$

$y = -6 + 10$

$y = 4$

$\therefore (x, y) = (-3, 4)$

Solve by Substitution

$4x + 2y = -2$
$2x - 3y = 1$

Solve by Elimination (Multiplication/Addition)

The elimination method involves obtaining an equivalent system of equations such that, when the two equations are added together, one variable is eliminated.

Then substitution is used to obtain the value of the second missing variable.

Solve by Elimination

$2x + 3y = 5$
$3x - 3y = 10$

$5x = 15$

$x = 3$

$2(3) + 3y = 5$

$6 + 3y = 5$

$3y = -1$

$y = -1/3$

$\therefore (x, y) = (3, -1/3)$

Solve by Elimination

$2x + 3y = 6$
$x + 2y = 5 \quad (-2)$

$2x + 3y = 6$
$-2x - 4y = -10$

$-1y = -4$

$y = 4$

$x + 2y = 5$

$x + 2(4) = 5$

$x + 8 = 5$

$x = -3$

$\therefore (x, y) = (-3, 4)$

Solve by Elimination

$2x - 3y = 22$
$4x + y = 2 \quad (3)$

$2x - 3y = 22$
$12x + 3y = 6$
$14x = 28$
$x = 2$

$4x + y = 2$
$4(2) + y = 2$
$8 + y = 2$
$y = -6$

$\therefore (x, y) = (2, -6)$

Solve by Elimination

$3x + 2y = 1 \quad (-2)$
$2x - 3y = 5 \quad (3)$

$-6x - 4y = -2$
$6x - 9y = 15$
$-13y = 13$
$y = -1$

$3x + 2(-1) = 1$
$3x - 2 = 1$
$3x = 3$
$x = 1$
$\therefore (x, y) = (1, -1)$

Solve by Elimination

$2x + 3y = 5$ (-3)

$3x - 4y = -18$ (2)

$-6x - 9y = -15$

$6x - 8y = -36$

$-17y = -51$

$y = 3$

$2x + 3(3) = 5$

$2x + 9 = 5$

$2x = -4$

$x = -2$

$\therefore (x, y) = (-2, 3)$

Solve by Elimination

$2x + 5y = 8$

$3x - y = 12$

Solve by Elimination

$8x - 3y = 6$

$6x + 12y = -24$