EE 413 Wireless Communications

Term Project

Simulation Study: Comparison of Selection Diversity and Maximal Ratio Combining

Utku Gürsoy / 26572

23.01.2023

Fall 2022/2023

Table of Contents

Introduction

Fading is a phenomenon that occurs in wireless communication systems where the signal strength of the transmitted signal varies over time and space. In Rayleigh fading, the signal strength can vary significantly over a short period of time, making it difficult to maintain a stable communication link. Selection diversity can be used to overcome the effects of Rayleigh fading by selecting the best signal from multiple available signals. Diversity is a technique that uses multiple antennas or receivers to improve the performance of a wireless communication system. Diversity can be used to improve the reliability of the communication link by providing a more robust signal. There are different types of diversity, such as space diversity, polarization diversity, and frequency diversity. Selection diversity and maximal ratio combining (MRC) are techniques used in wireless communication systems to improve the reliability and robustness of the signal transmission. They both can be implemented in both the transmitter and receiver of a wireless communication system and can be used together to achieve even better performance. Selection diversity is a technique that involves selecting the best signal from multiple available signals. This is done by comparing the signal quality of the multiple signals and selecting the one with the highest signal-to-noise ratio (SNR). MRC is a technique that combines multiple signals in a way that maximizes the signal-to-noise ratio (SNR). In this project, these two methods are simulated via MATLAB and results are compared with each other.

Literature Review

In wireless communication channels, a signal can be scattered by many objects in its environment, such as buildings, trees, or other structures. These scatterings cause the signal to arrive at the receiver via multiple paths, with different amplitudes and phases. This phenomenon is known as multipath fading. Rayleigh fading is commonly used to model wireless communication channels because it is a simple and widely used model for the statistical behavior of the amplitude of a signal as it passes through a multipath fading channel. The Rayleigh distribution is a probability distribution for non-negative-valued random variables (see Appendix B), it describes the amplitude of a signal that has been scattered by many reflecting objects in a randomly varying environment, such as a wireless communication channel. This scattering causes the signal to arrive at the receiver via multiple paths, with different amplitudes and phases, leading to a phenomenon known as multipath fading [2]. The Rayleigh distribution is a good model for the amplitude of the received signal in a wireless communication channel because it is relatively simple to work with mathematically (Appendix C), and it captures the key features of multipath fading, such as the random variation of signal amplitude and the lack of a dominant path (NLoS, no-line-of-sight). Additionally, it is a good model for channels that have a large number of scatterers, such as wireless communication channels in urban environments.

Rayleigh fading is particularly useful in the context of diversity techniques because it allows for the modeling of the random variation of the signal amplitudes across multiple signal paths [2]. Diversity techniques, such as selection diversity and maximal ratio combining (MRC), are used to mitigate the effects of multipath fading by using multiple signal paths to increase the reliability of the communication link [2]. In selection diversity, the receiver selects the signal path with the highest signal-to-noise ratio (SNR) (see Appendix D) and discards the others. In MRC, the receiver combines the signals from multiple signal paths to increase the SNR (see Appendix E).

Outage probability is a measure of the reliability of a wireless communication system. It is the probability that the signal-to-noise ratio (SNR) at the receiver falls below a certain threshold, resulting in a failure of the communication link [2]. The outage probability is typically expressed as a function of the average SNR, which is a measure of the average signal power at the receiver divided by the average noise power. A lower outage probability indicates a more reliable communication system. Outage probability is usually calculated by generating a large number of random channel realizations and determining the fraction of realizations for which the SNR falls below the threshold. This is done by Monte Carlo simulation. Outage probability is an important performance metric for wireless communication systems because it provides a way to evaluate the reliability of the system under different operating conditions. For example, a system with a low outage probability can provide a reliable connection even in the presence of fading, while a system with a high outage probability may have difficulty maintaining a connection in the same conditions.

Methods

utkugursoy_ee413_project.m

```matlab
%% Parameters
M = [1 2 3 4 10 20]; % number of branches
N = 10^5; % number of samples
threshold = 1; % threshold for outage
snr_db = -40:1:10; % average SNR in dB
snr = 10.^(snr_db/10); % average SNR

%% Rayleigh Fading
h = sqrt(0.5)*(randn(N,max(M)) + 1i*randn(N,max(M))); % fading coefficients

%% Selection Diversity
figure;
%subplot(1,2,1);
outage_sel = zeros(1,length(snr));
for m = 1:length(M)
    for ii = 1:length(snr)
        g1 = max(abs(h(:,1:M(m))).^2,[],2)/snr(ii); % channel gain
        outage_sel(ii) = sum(g1<threshold)/N; % outage probability
    end
    semilogy(-snr_db, outage_sel, '-o'); hold on;
end
legend('M=1','M=2','M=3','M=4','M=10','M=20');
xlabel('10*log(γ¯/γ_{0})'); ylabel('Outage Probability (P_{out})');
title('Selection Diversity');

%% Maximal-Ratio Combining

figure;
%subplot(1,2,2);
outage_mrc = zeros(1,length(snr));
for m = 1:length(M)
    for ii = 1:length(snr)
        g2 = sum(abs(h(:,1:M(m))).^2,2)/snr(ii); % channel gain
        outage_mrc(ii) = sum(g2<threshold*M(m))/N; % outage probability
    end
    semilogy(-snr_db-15, outage_mrc, '-x'); hold on;
end

%% Plot results
legend('M=1','M=2','M=3','M=4','M=10','M=20');
xlabel('10*log(γ¯/γ_{0})'); ylabel('Outage Probability (P_{out})');
title('Maximal Ratio Combining');
```

(Figure I: MATLAB code)

Explanation of the code:

This code simulates the performance of selection diversity and maximal ratio combining (MRC) in a wireless communication system using MATLAB.

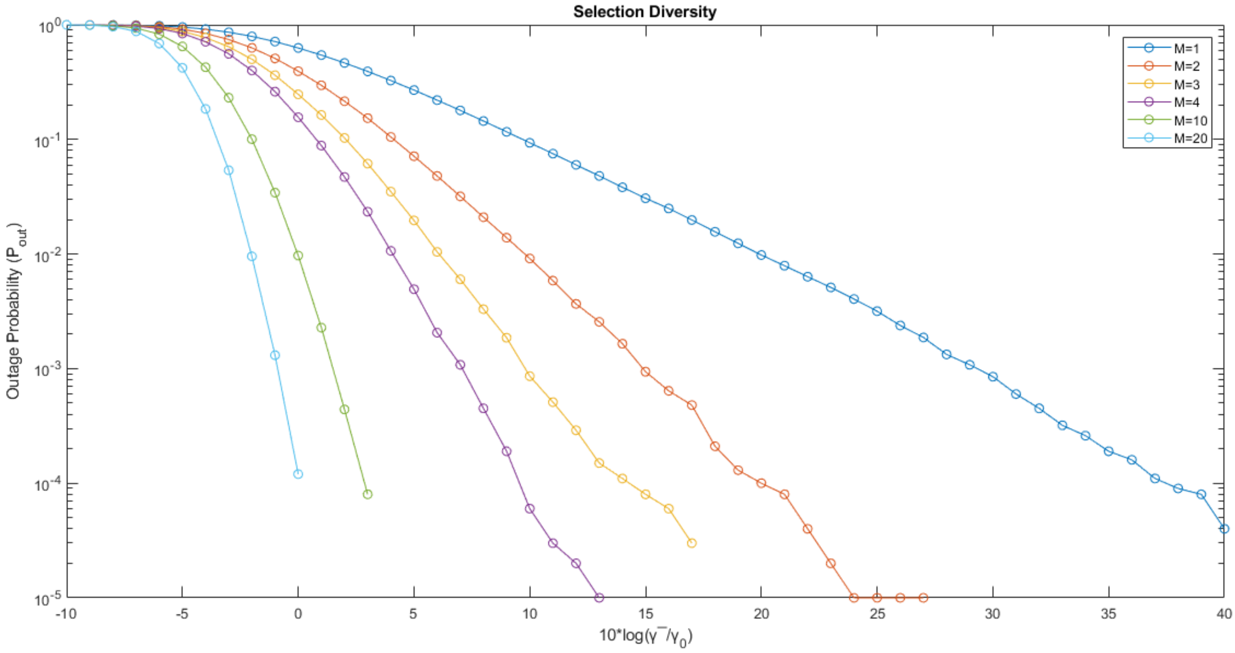
The first step of the code defines several parameters that are used in the simulation. The variable "M" is an array that contains the number of branches used in the diversity techniques. The variable "N" is the number of samples used in the simulation. The variable "threshold" is the threshold used to calculate the outage probability. The variable "snr_db" is an array that contains the average signal-to-noise ratio (SNR) in dB, and the variable "snr" is an array that contains the average SNR in linear units.

The next step of the code generates the fading coefficients using Rayleigh fading. This is done by using the "randn" function to generate random numbers with normal distribution, and then multiplying them by the square root of 0.5 to obtain the fading coefficients. The code then starts the simulation of selection diversity. A figure is created, and a loop is used to simulate the performance of selection diversity for different values of M. In each iteration of the loop, another loop is used to calculate the outage probability for different values of SNR. The variable "g1" is calculated as the maximum of the absolute square of the fading coefficients for each branch, divided by the SNR. The outage probability is calculated as the ratio of the number of samples where "g1" is less than the threshold, to the total number of samples. The results are plotted using the "semilogy" function, and the legend is updated for each iteration of the loop.

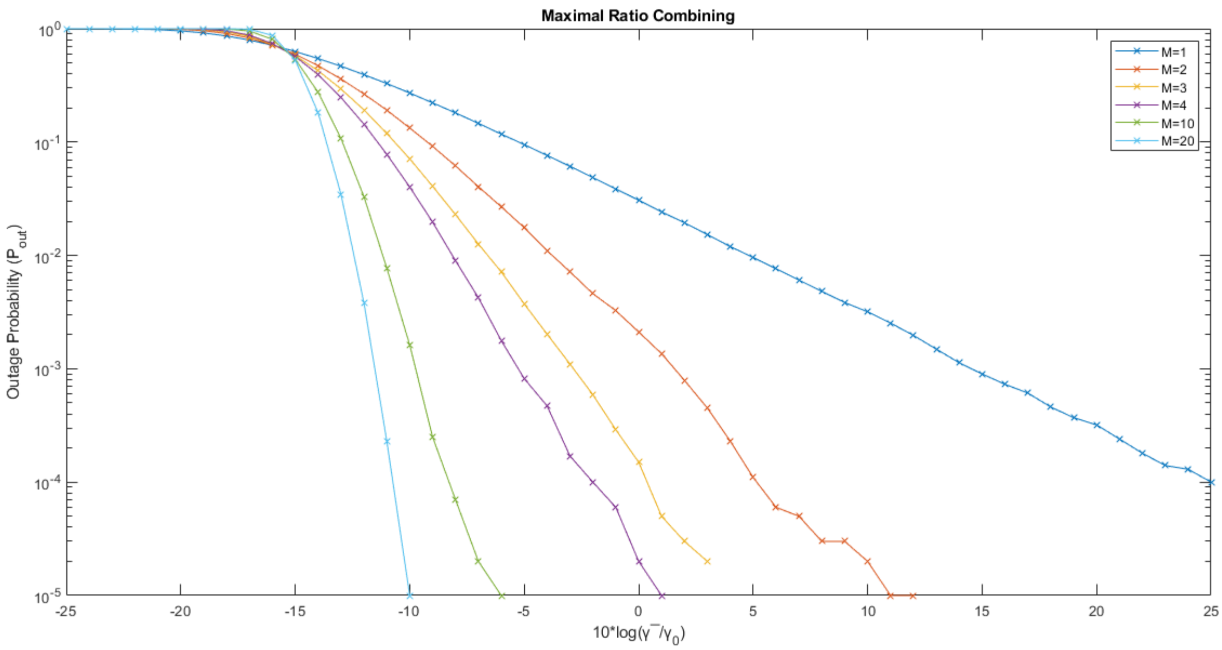
The code then starts the simulation of maximal ratio combining (MRC). Another figure is created, and a similar loop structure is used as for the selection diversity. The variable "g2" is calculated as the sum of the absolute square of the fading coefficients for each branch, divided by the SNR. The outage probability is calculated as the ratio of the number of samples where "g2" is less than the threshold multiplied by M, to the total number of samples. The results are plotted using the "semilogy" function, and the legend is updated for each iteration of the loop.

Finally, the code plots the results of the simulation in two separate figures, one for selection diversity and one for MRC. The x-axis represents the average SNR in dB, and the y-axis represents the outage probability. The code can be modified to change the parameters of the simulation such as the number of branches, the threshold, and the SNR.

Results



(Figure 1: MATLAB simulation of Selection Diversity with 10^5 samples)



(Figure 2: MATLAB simulation of Maximal Ratio Combining with 10^5 samples)

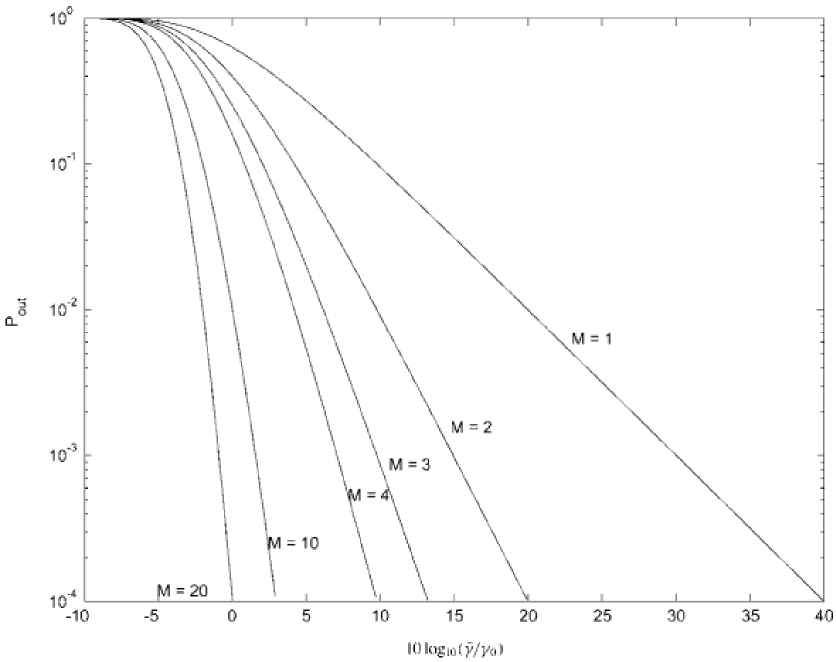


Figure 7.2: Outage probability of selection combining in Rayleigh fading.

(Figure 3: Plot of Selection Diversity from the textbook [1])

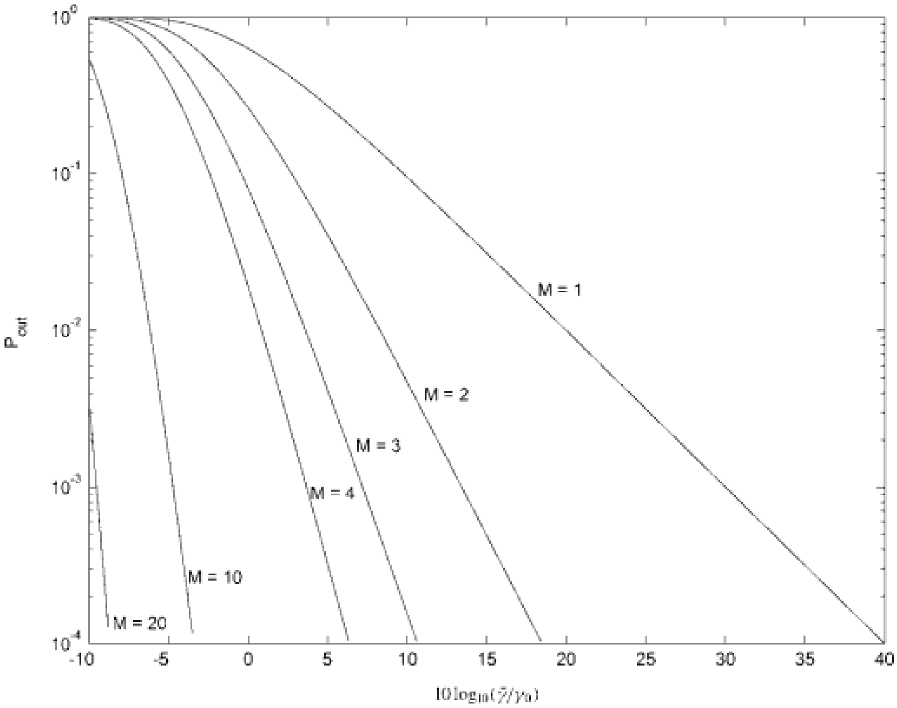
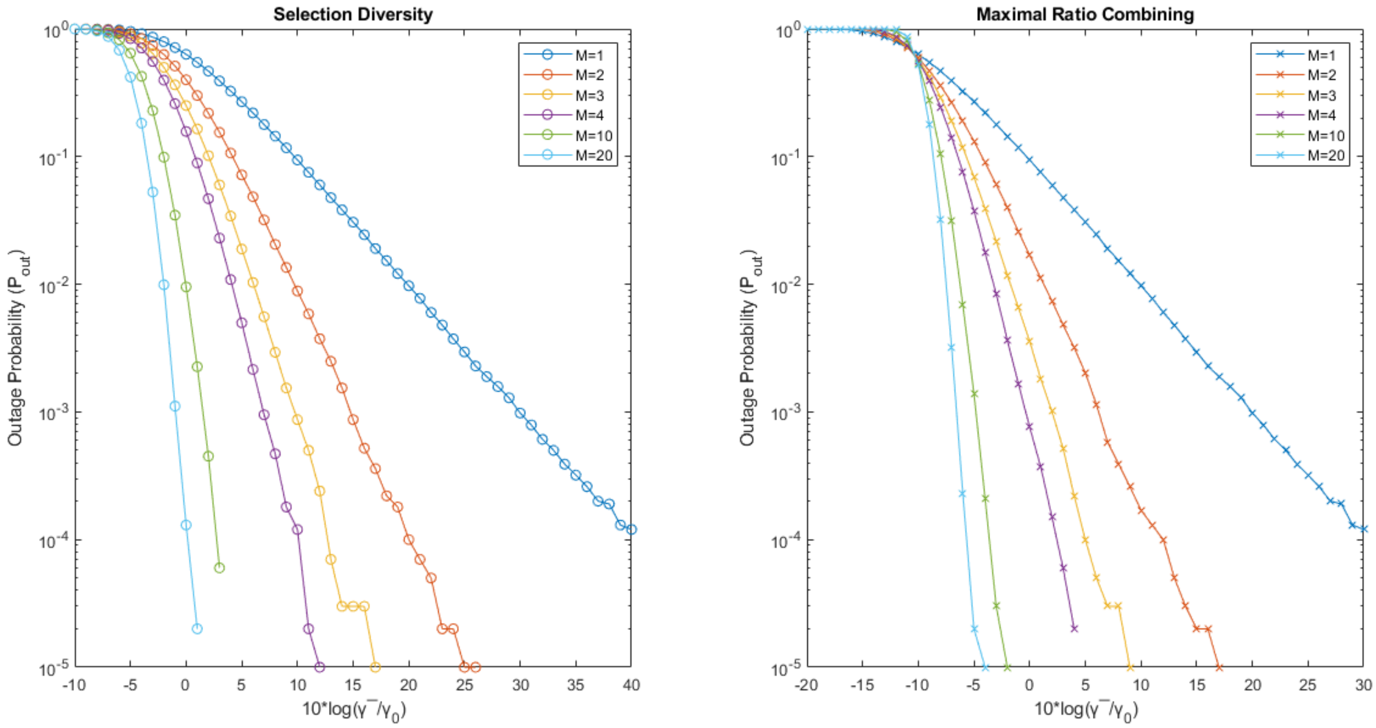
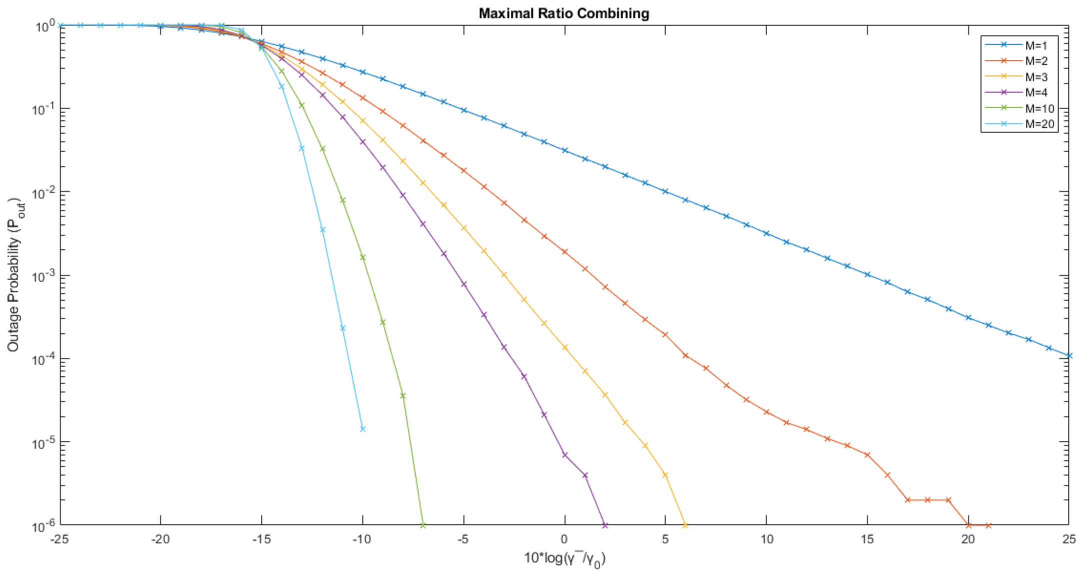


Figure 7.5: P_{out} for maximal-ratio combining with i.i.d. Rayleigh fading.

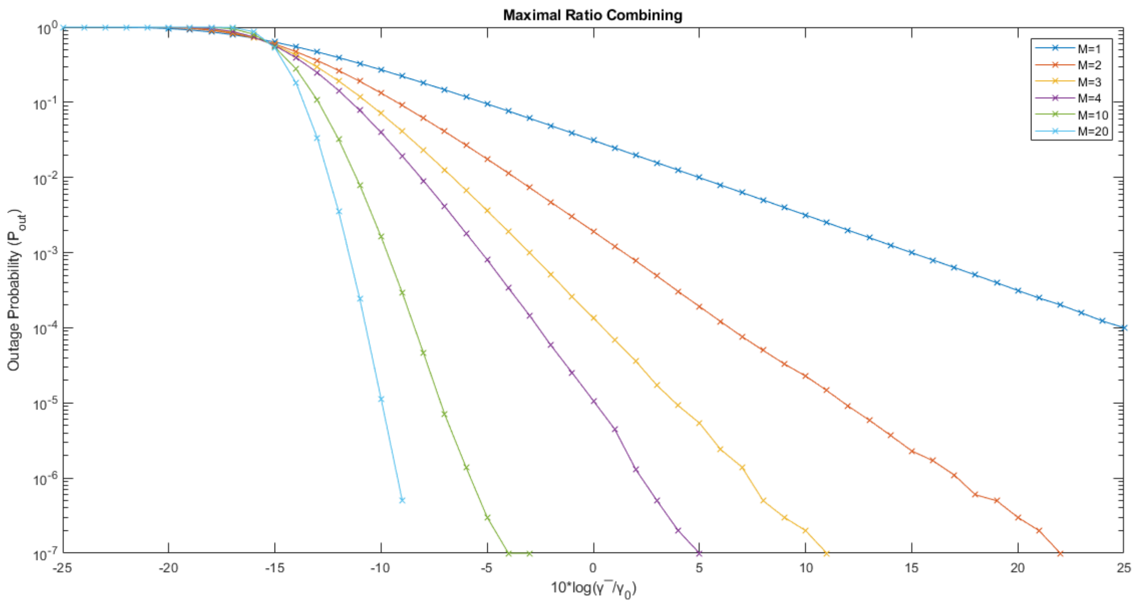
(Figure 4: Plot of Maximal Ratio Combining from the textbook [1])



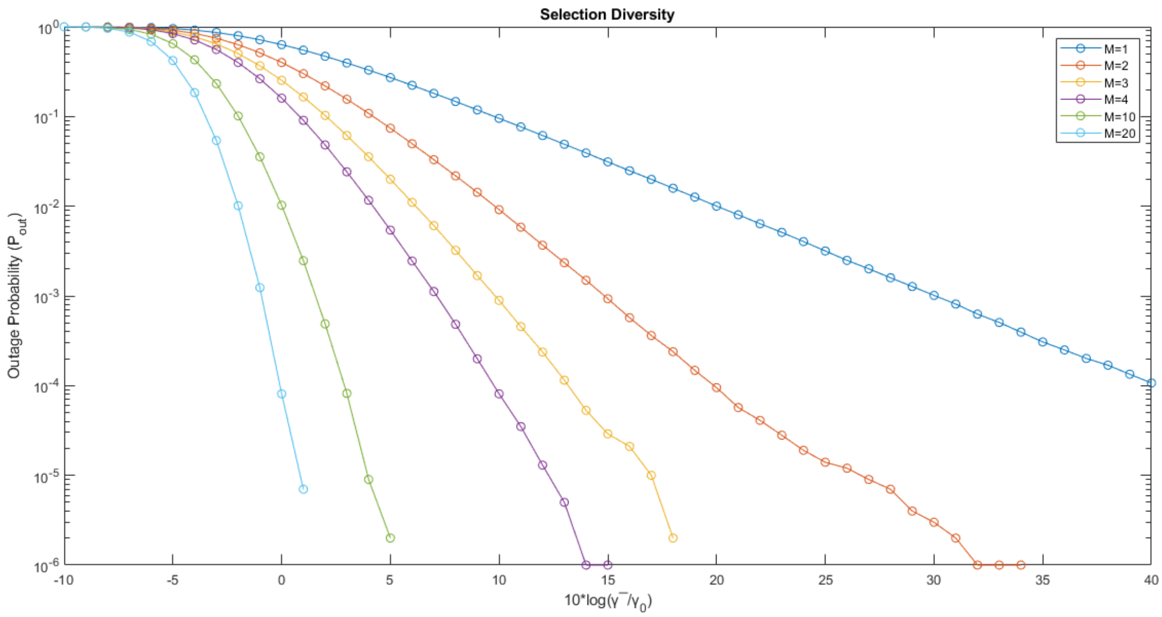
(Figure 5: Selection Diversity and Maximal Ratio Combining plotted together)



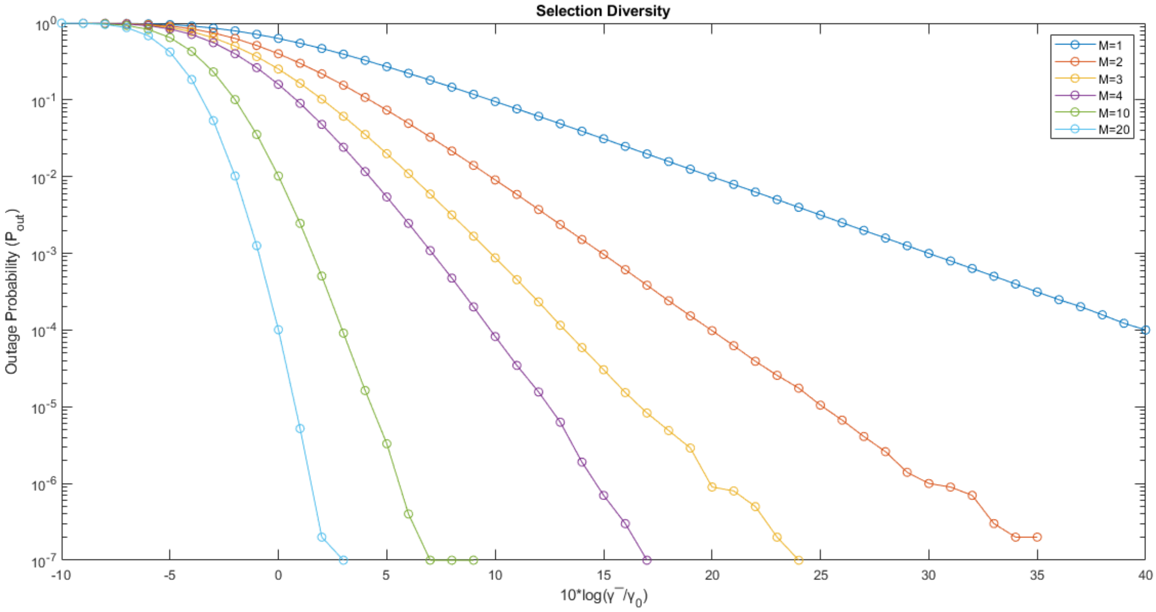
(Figure 6: Maximal Ratio Combining with 10^6 samples)



(Figure 7: Maximal Ratio Combining with 10^7 samples)



(Figure 8: Selection Diversity with 10^6 samples)



(Figure 9: Selection Diversity with 10^7 samples)

Discussion

Simulations are done via different number of transceiver antennas and one receiver antenna. Since there is a single receiver (not MIMO), M can also be said to be the number of channels between transmitter antenna TX and receiver antenna RX. For $M = 1$, there is a single channel between TX and RX and for $M = 2$ there are two channels, etc. As the number of channels increase, outage probability for the same signal-to-noise ratio (SNR) decreases, in all simulation cases. This shows that communication performance of two stations can significantly be improved using multiple antennas in TX side. There are two techniques examined in the scope of this project, which are selection diversity and maximal ratio combining. Selection diversity picks the channel with the greatest SNR available at given channels for the communication. This solution yields lower outage probability values for large number of M. According to results, 10^{-4} outage probability is achieved with the lowest $0\ dB$ SNR value when the $M = 20$ (Figure 9). On the other hand, maximal ratio combining targets maximizing the SNR at the receiver by summing the SNR values of individual channels at the receiver. In the best case 10^{-4} outage probability is achieved with the lowest $-10\ dB$ SNR when the $M = 20$ (Figure 7). This shows that maximal ratio combining technique yields better results compared with selection diversity. These simulation results are similar to the textbook results shown in Figures 4 and 5. Another important point is, even though the increasing the number of antennas improves the communication between TX and RX, after some value of M, adding more channels to the system does not provide any considerable improvement for the communication. Hence, antenna number seems converging at some point as M increases. This observation tells us that using antennas more than necessarily has no significant impact on the overall communication and the optimum number of antennas that give the best outage probability should be determined while designing a diversity system.

Sample size indicates the number of iterations. In order to achieve accurate outage probability results from the simulations, sample size should be appropriately chosen. Increasing sample size by an order of 10 yields more accurate results for the lower outage probability values both for Selection Diversity and Maximal Ratio Combining. In general, it is desired to detect outage for 100 times in a simulation. The probability of 100 outage occurs in a sample size of 10^x is roughly $100/10^x$. Hence, when sample size is 10^5 simulations give accurate results up until the outage probability of 10^{-3}. Similarly, when sample size is 10^6 simulations give accurate results up until the outage probability of 10^{-4} and when sample size is 10^7 simulations give accurate results up until the outage probability of 10^{-5}. Figures 1, 2, 6, 7, 8, 9 show the simulation results for both selection diversity (Figures 1, 8, 9) and maximal ratio combining (Figures 2, 6, 7). Note that in Figure 1 and Figure 2, where the sample sizes are 10^5 for both selection diversity and maximal ratio combining, respectively, the outage probability values below 10^{-3} start to deteriorate, i.e., curves no longer behave smoothly; this indicates the outage probability results below 10^{-3} are not reliable for these figures. In Figures 6, 8 sample sizes are 10^6 for both selection diversity and maximal ratio combining, respectively. Note that this time the outage probability values below 10^{-4} start to deteriorate; and similarly in Figures 7, 9 sample sizes are 10^7 for both selection diversity and maximal ratio combining, respectively, and the outage probability values below 10^{-5} start to deteriorate. Figure 3 and 4 show the outage probability plots which are based on the textbook page 210 and page 215 [1]. The obtained simulation results showed in Figure 7 and Figure 9 are very similar to the textbook results in Figure 3 and Figure 4 for both Selection Diversity and Maximal Ratio Combining, respectively.

Simulation runtime depends on the sample size, increasing the sample size also increases the runtime. Hence, running the simulations where the sample size is 10^7 require significantly more time compared to the simulations where the sample size is 10^6 and 10^5. Increasing the sample size also requires more memory for the computations. A simulation for the sample size of 10^8 is also tried, however running the simulation consumed all available memory in the computer and therefore results could not be obtained, see Appendix A.

Conclusion

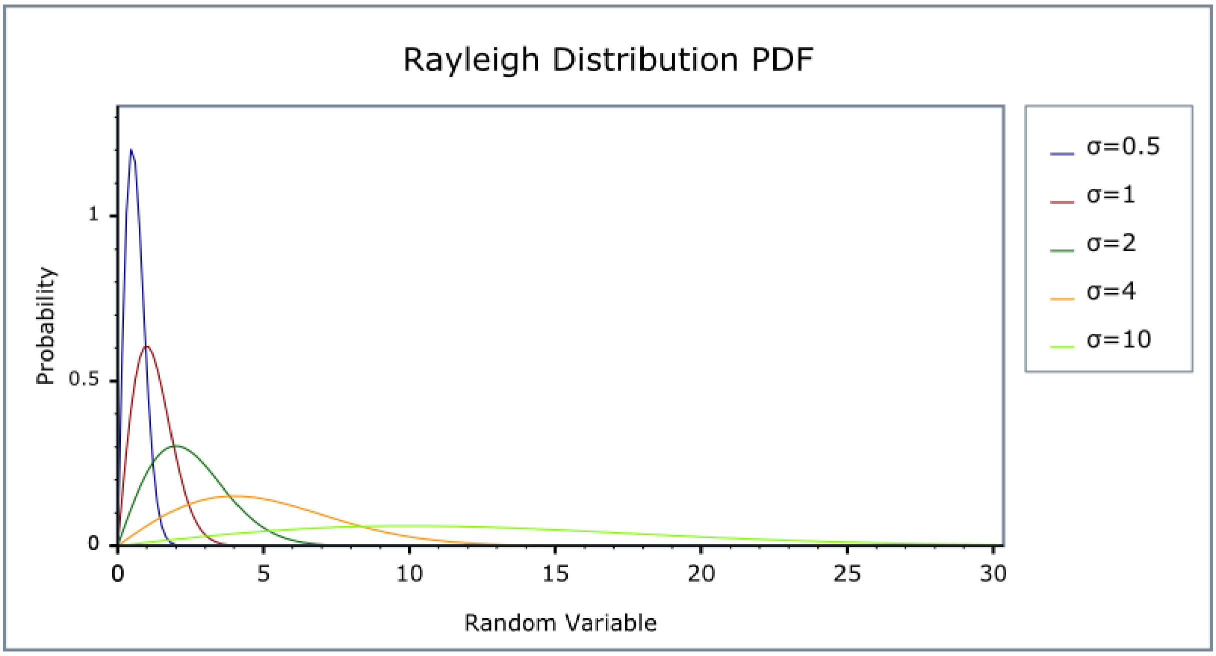
Rayleigh fading is used to model wireless communication channels with diversity because it is a simple and widely used model for the statistical behavior of the amplitude of a signal as it passes through a multipath fading channel. Outage probability is a measure of reliability in wireless communication systems. It is the likelihood of the signal-to-noise ratio falling below a threshold, leading to a failure of the communication link. It is usually presented as a function of average SNR and a lower probability means a more reliable system. It is an important metric for evaluating wireless communication systems. Selection Diversity and Maximal Ratio Combining are two techniques used in wireless communication to improve the reliability of the communication link by using multiple signal paths to transmit a signal. Selection Diversity selects the signal path with the best signal-to-noise ratio (SNR) and discards the others. Maximal Ratio Combining combines the signals from multiple signal paths to increase the SNR. The variable N in the MATLAB implementation represents the number of samples used in the simulation. A larger value of N will result in a more accurate estimate of the outage probability, as it increases the number of iterations of the simulation. However, it will also increase the computation time required to run the simulation.

Appendix

A) Simulation error when sample size is 10^8:

```
Requested 100000000x20 (29.8GB) array exceeds maximum array size preference (15.9GB). This might cause MATLAB to become unresponsive.
```

B) Rayleigh Distribution PDF



C) Rayleigh Distribution Formula

4. The probability density function of the Rayleigh distribution is

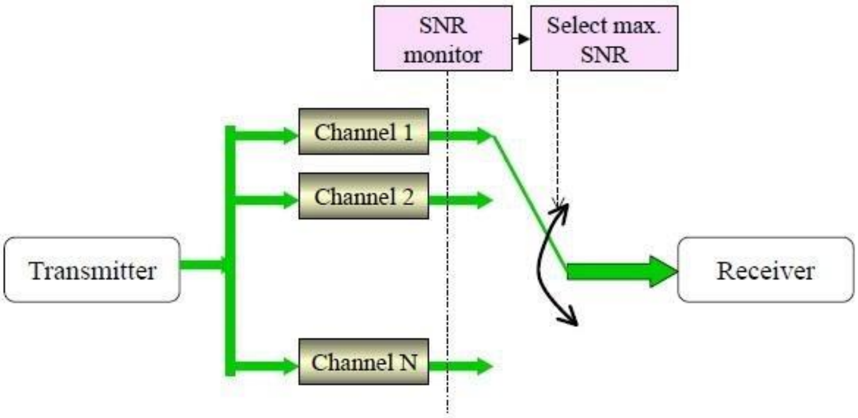
$$f(x) = \frac{x}{\theta^2} e^{-x^2/(2\theta^2)}, \quad x \geq 0,$$

where θ is a positive-valued parameter. It is known that the mean and variance of the Rayleigh distribution are

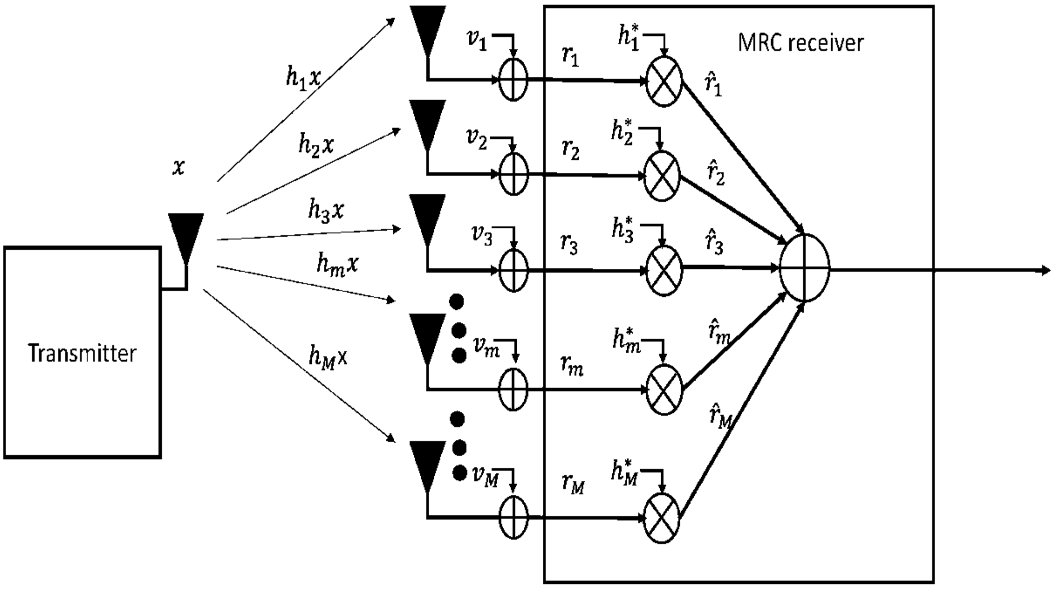
$$\mu = \theta\sqrt{\frac{\pi}{2}} \quad \text{and} \quad \sigma^2 = \theta^2 \frac{4-\pi}{2}.$$

Let $X_1, \ldots, X_n$ be a random sample from a Rayleigh distribution.

D) Selection Diversity System



E) Maximal Ratio Combining System



References

[1] Goldsmith, A. (2005). *Wireless Communications*. Cambridge University Press.

[2] Gürbüz, Ö. (2022). EE 413 *Wireless Communications Lecture Slides*, Sabancı University