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The Fungi Decomposition Rate Model

Summary

Question A of the Mathematical Modeling Competition of 2021 is A question about fungi. Starting from fungi decomposition of landing plants, we are required to build A mathematical model to describe the decomposition of ground litter and wood fiber through fungal activities in the presence of a variety of fungi. Provides model analysis and describes the interactions between different types of fungi. This includes predictions of the relative strengths and weaknesses of each species and of combinations of species that are likely to persist. Describe how the diversity of a fungal community within a system affects the overall efficiency of the system in decomposing litter on the ground. At the same time, we found many complex and effective and correct data, obtained our final mathematical formula through partial differential equations, and obtained models and images with tools such as MATLAB and Python, which were scientific and effective. The results clearly show that the decomposition rate of bacteria increases steadily at the beginning, and then after the density of species is saturated, the decomposition rate begins to decline rapidly.

Keywords: Fungi; Decomposition rate; MATLAB; Python; Differential equation

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Miao Fangran, Wang Xizhi, Wang Zhuo

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Question A of the Mathematical Modeling Competition of 2021 is A question about fungi. Starting from fungi decomposition of landing plants, we are required to build A mathematical model to describe the decomposition of ground litter and wood fiber through fungal activities in the presence of a variety of fungi. Provides model analysis and describes the interactions between different types of fungi. This includes predictions of the relative strengths and weaknesses of each species and of combinations of species that are likely to persist. Describe how the diversity of a fungal community within a system affects the overall efficiency of the system in decomposing litter on the ground. At the same time, we found many complex and effective and correct data, obtained our final mathematical formula through partial differential equations, and obtained models and images with tools such as MATLAB and Python, which were scientific and effective. The results clearly show that the decomposition rate of bacteria increases steadily at the beginning, and then after the density of species is saturated, the decomposition rate begins to decline rapidly.

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Contents

1 Introduction

1.1 Problem Background

Carbon cycle is a significant process in biosphere. It is the material circulation chain between the inorganic environment and organic organisms that keeps the balance of carbon dioxide in the atmosphere. The breakdown of organic material is a key component of the carbon cycle. It describes the exchange of carbon throughout the geochemical cycle, where the microorganisms play an important role. Fungi is a typical microorganism which is vital to decompose the plant material and woody fiber. It is significant to do meticulous study to fungus decomposition to the environment.

1.2 Literature Review

Fungal ecology is believed to play a crucial role in shaping Earths ecosystems because of fungis ability to recycle nutrients in the system as decomposers[1], provide resources as symbionts of the majority of land plants[2], and impact the success and dynamics of other organisms as pathogens[3, 4] It is significant to do meticulous study to fungus decomposition to the environment. By means of understanding the influence factor to the decomposition rate of the fungus thoroughly, scholar can apply it into other microorganism, and it has positive effect on ecosystem research. Recent research development have revealed that there are a considerable number of traits of fungi related to decomposition rate and certain traits link with each other[5]. In that paper, several characters of fungi were discussed in order to determine the influence of wood decomposition. Researchers studied numerous traits associated with each fungus and tried to determine the role of these traits in wood block decomposition. In particular, the researchers found that are better able to adapt to different ranges of water conditions also tend to decompose wood more slowly. Fungi that grow faster and outperform other fungi tend to decompose wood faster[5].

1.3 Our work

This paper will concentrate on two traits, the growth rate of fungus and the fungus tolerance to moisture. Based on statistic data of three fungi species, we match this data with a same form of formula by interpolation and matching, which can describe the relationship between the fungus growth rate, the fungus tolerance to moisture and the decomposition rate accurately with a certain range. Then this paper combines different species fungus together observe the composite effect of the decomposition to the ground litter. It assigns a value to fungus called ranking. This is the value to describe the competitive capacity of fungus, and we research the interaction between different species of fungus. To research how the fungus and environment have influence with each other, This paper introduces more variables with respect to the environment and revise the formula concerned with the decomposition rate. The effect of different environmental conditions on the decomposition rate of fungi were described more specifically. And the analysis of different species combination to the environment change indicates the importance of the biodiversity. So this paper finally analyze the importance of the biodiversity to the whole environment when different degree of variations occur. It will make some sense in the field of ecology.

2 Preparation of the Models

2.1 Assumptions

Here are our assumptions:

For a single species of fungus[5]:

- Under an ideal environment, which means that the temperature is perfect and the food is sufficient, the decomposition rate of fungi is mainly related to the extension rate and the moisture trade-off.
- The decomposition rate has a linear relationship with extension rate.
- The logarithm of decomposition rate has a linear relationship with moisture trade-off.
- If the environment is not so perfect, we introduce one parameter to adjust the decomposition rate—temperature.

With such condition, we can conclude two partial differential equations:

$$\begin{cases} \dfrac{\partial f}{\partial x} & = C_1 \\ \dfrac{\partial (\log f)}{\partial y} & = C_2 \end{cases} \tag{1}$$

Where f represents the decomposition rate, x represents extension rate and y represents the moisture trade-off. C_1, C_2 represents the linear slope between the independent variables and dependent variables.

Solve the equation, we acquire that:

$$f = Ax \cdot e^{By} \tag{2}$$

where A and B are constants, which varies with the type of fungi.

To adjust the decomposition rate with temperature, we introduce the Temperature function, which is $u(T)$. Thus:

$$f = u(T) \cdot Ax \cdot e^{By} \tag{3}$$

When it comes to multiple fungi, we put forward our fifth assumption:

- The decomposition rate will change when the number of fungus changes.

Then, we also introduce another function of one specific fungus' number:

$$f = v(P) \cdot u(T) \cdot Ax \cdot e^{By} \tag{4}$$

The last problem is that how to measure the decomposition rate. Martin Witkamp[6] measured it by calculating the emission of carbon dioxide. It reminded us of different measure method. And what we need to do is to uniform the standard. Thus, the last assumption is:

- Every fungus decomposition rate can be measured by the amount of carbon dioxide released, and can also be translated into the mass loss of the raw material.

After that, we re-write f in the form of dm/dt, where m represents mass and t represents time.

2.2 Data Collection

It is really hard to collect data for there are a lot of literature with different measurement indexes. Thanks to the paper from Daniel et al.[7], we get lots of useful data. The relationship between fungi's decomposition rate and moisture niche width and the relationship between the decomposition rate and temperature. Then uniform the standard—Express the rate of decomposition with the rate of mass loss.

Repeat such process with 5-6 papers, we finally get three fungi's data.

- For fungus 1, $A = 2.123, B = -0.4249$
- For fungus 2, $A = 1.877, B = 0.5152$
- For fungus 3, $A = 3.553, B = 0.7636$

For the correction factor, temperature and fungi's number, please check out in Appendix B for more information. We created correction factors for each species.

3 The Models

To build up the model, we figure out that it is easy to use object-oriented programming.

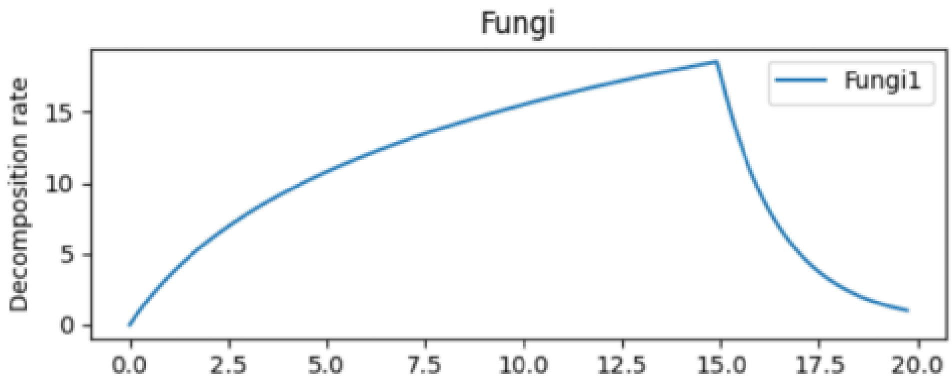
We firstly set different objects for fungi. The fungi itself have many attributes: the inherent coefficients A and B, the extension rate, the decomposition rate, the number of fungi. After setting the extension rate, we can get the decomposition rate and the number easily.

Then we set up the objects for environment: The environment contains the following attributes: the moisture, temperature and wood number which represents the content of lignocellulosic.

After the interaction of the fungi and environment, we can get the curve of the rate of decomposition.

3.1 Model 1

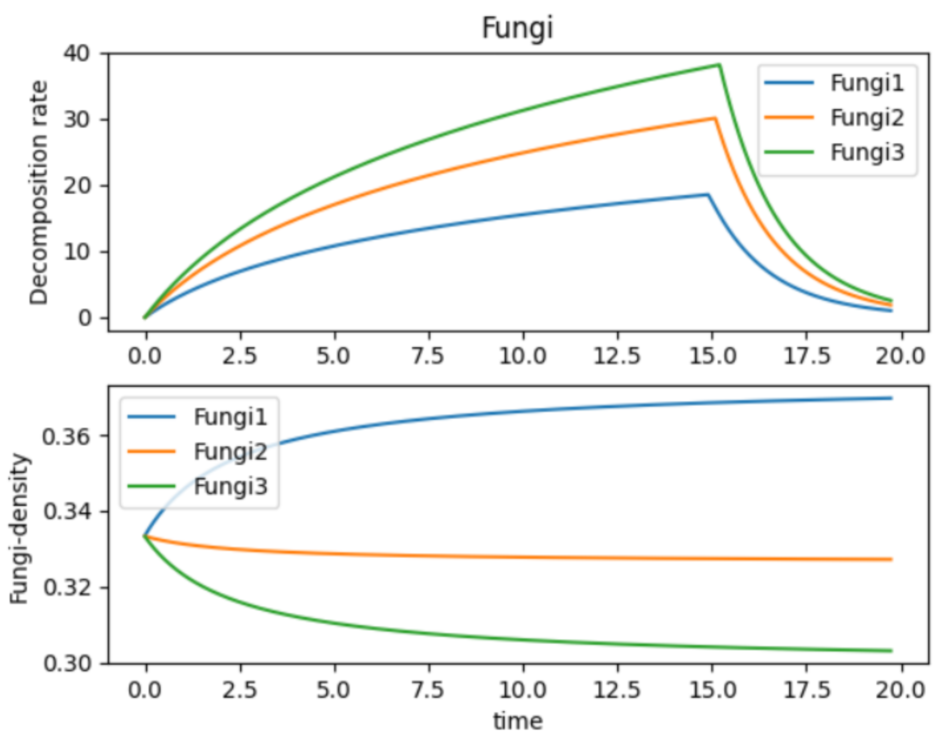
In this model, we try to figure out the decomposition rate of one specific fungus against time under proper temperature.



We can conclude that the decomposition rate will increase as time goes on, for the number of fungus is increasing. But when the number is too large, the content of lignocellulosic can't meet the fungus' requirement. Then, the decomposition rate decreases.

3.2 Model 2

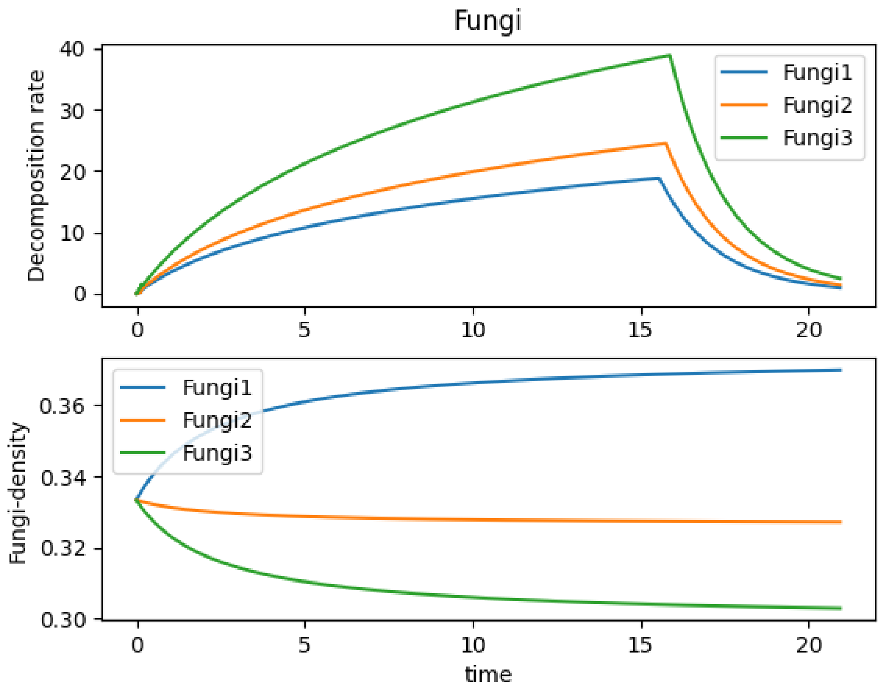
In this model, we try to figure out the interactions between the fungi.



We can conclude that almost all the fungi will experience the same process as model 1. But the difference is that their density varies greatly. Since in the competition within the population, one party may always become the dominant party, so that the number will occupy the majority.

3.3 Model 3

In this model, we try to figure out the interactions with the temperature.



We simulate the temperature from proper temperature to the low one and then to the high one. We can still see that fungus 2's decomposition rate decrease obviously. In the origin model, we set fungus 2 to be sensitive to the temperature while fungus 1 not.

3.4 Model 4

In this model, we want to figure out the outside environment, such as dried, wet environment.

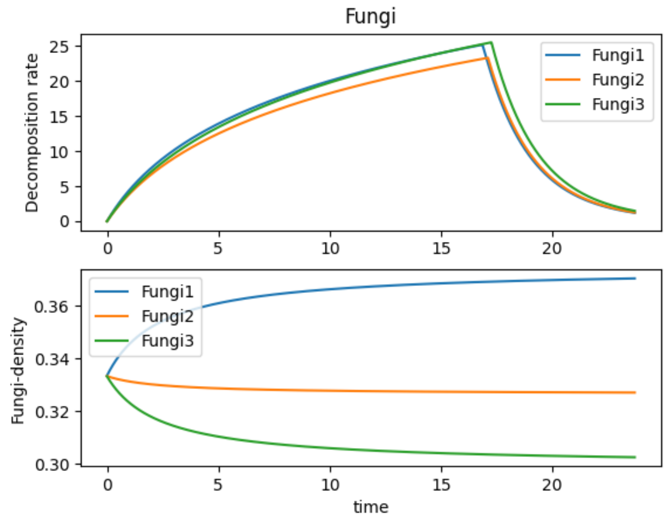


Figure 1: The extremely dried environment

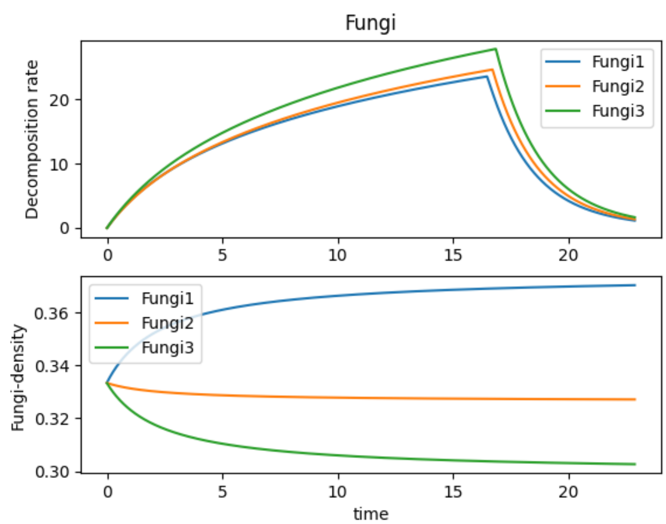


Figure 2: The dried environment

The decomposition rates become the same as the environment get drier. (Notice that the maximum of y-axis is 25)

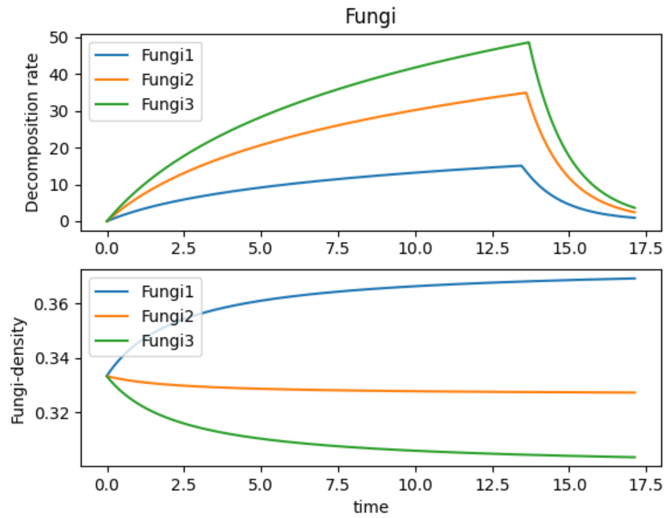


Figure 3: The extremely wet environment

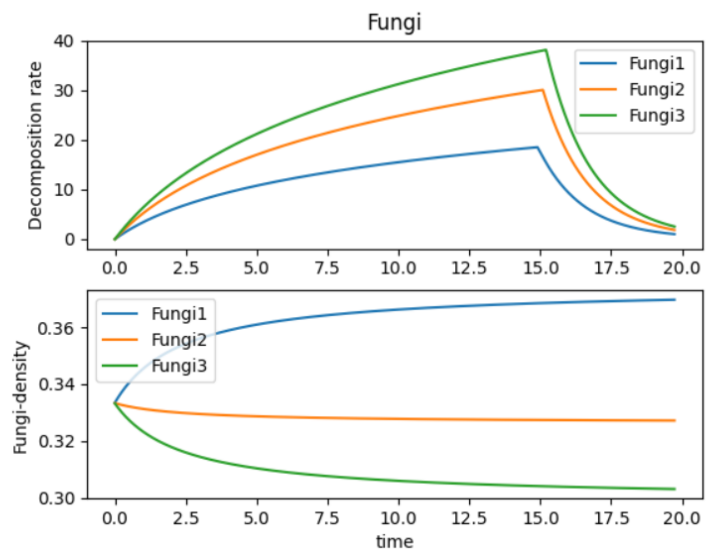


Figure 4: The wet environment

The decomposition rate become higher as the environment get wetter. (Notice that the maximum of y-axis is 50)

4 Strengths and weaknesses

4.1 Strengths

In this model, we set up two mutually related models, one is a single species, and the other is the community combination of multiple species. Not only can we clearly see the change in the rate of decomposition of individual species. There is also competition and dependence among multiple species, as well as changes in the rate of decomposition under community conditions. We can also clearly see that the decomposition rate of bacteria increases steadily at the beginning, and then after the density of species is saturated, the decomposition rate begins to decline rapidly.

At the same time, we found many complex and effective and correct data, obtained our final mathematical formula through partial differential equations, and obtained models and images with tools such as MATLAB and Python, which were scientific and effective.

4.2 Weaknesses

Honestly, there are lots of weaknesses:

- We can't consider the death rate of each fungus, which means they grow in an almost ideal environment.
- The environment has lots of factors that can influence the decomposition rate of fungi, we just consider the moisture and the temperature. For instance, the light intensity can influence.
- We were only looking at fungi that break down carbon, however, there are also fungi can break down nitrogen.
- The shape of the colony may also affect the decomposition rate.
- We only consider the inner influence within the single population—fungi. We can't consider other living things' influence.

References

[1] Mark D. Fricker, Luke L. M. Heaton, Nick S. Jones, and Lynne Boddy. The mycelium as a network. *Microbiol Spectr*, 5(3), 2017.

[2] Kathleen K. Treseder and Jay T. Lennon. Fungal traits that drive ecosystem dynamics on land. *Microbiol Mol Biol Rev*, 79(2):243–262, 2015.

[3] S. E Smith and D. J Read. Mycorrhizal symbiosis. *Quarterly Review of Biology*, 3(3):273–281, 2008.

[4] Maron, JL, Marler, Klironomos, JN, Cleveland, and CC. Soil fungal pathogens and the relationship between plant diversity and productivity. *ECOL LETT*, 2011,14(1)(-):36–41, 2011.

[5] Nicky Lustenhouwer, Daniel S. Maynard, Mark A. Bradford, Daniel L. Lindner, Brad Oberle, Amy E. Zanne, and Thomas W. Crowther. A trait-based understanding of wood decomposition by fungi. *Proceedings of the National Academy of Sciences*, 117(21):11551–11558, 2020.

[6] Martin Witkamp. Decomposition of leaf litter in relation to environment, microflora, and microbial respiration. *Ecology*, 47(2):194–201, 1966.

[7] Daniel S Maynard, Mark A Bradford, Kristofer R Covey, Daniel Lindner, Jessie Glaeser, Douglas A Talbert, Paul Joshua Tinker, Donald M Walker, and Thomas W Crowther. Consistent trade-offs in fungal trait expression across broad spatial scales. *Nature microbiology*, 4(5):846–853, 2019.

Appendices

Appendix A Introductory college level article

According to the above concrete discussion and contents of this paper. We can reach the following conclusions:

- Fungi is an important microorganism to decompose the plant material and woody Fiber
- The growth rate of fungus and the fungus tolerance to moisture are two typical traits which is relevant to the decomposition rate.
- Different species of fungus have different traits, and usually they have competitive relationship.
- Environment is inconstant. Mixed populations of different species fungi are more stable.

We have known that the fungus important role in ecosystem is participating in the circulation of matter as a decomposer, and it plays an important role in the circulation of the substances such as CO2 in nature. We have mentioned that fungi can decompose the woody fiber. According to the data, The essence of wood degradation is to degrade the cellulose and lignin in wood fiber. The main way of lignin biodegradation is to break the Ca-CB and B-O-4 bonds. In addition to the oxidation of the side chains, the aromatic rings will crack through methylation before being detached from the lignin macromolecules. In general, pure lignin is not the only carbon source of microorganisms, and lignin degradation microorganisms generally have the ability to decompose cellulose and hemicellulose. The decomposition of lignin always involves the decomposition of cellulose and hemicellulose. Wood rot fungi play an important role in the biodegradation of lignin. Wood rot fungi can be divided into white rot fungi, brown rot fungi and soft rot fungi. Among them, white rot fungi have the strongest effect and are the most widely studied. However, the decomposition and transformation of lignin by fungi in the actual composting system is not the result of the action of white rot fungi alone, and the effect of brown rot fungi, soft rot fungi and other low filamentous fungi should not be ignored. The degradation ability of lignin by brown rot bacteria and soft rot bacteria is weaker than that of cellulose degradation. They decompose cellulose in lignin matrix at first, and then degrade part of lignin slowly. Brown rot fungi, belonging to basidiomycota, can effectively erode cellulose in wood materials and degrade polysaccharides, but the ability to decompose lignin is weak, and the final degradation products contain brown lignin. Therefore, we have viewed the decomposition mechanism of fungi specifically. Then we can see the importance to the ecosystem, which the example is another species of fungi, Mycorrhizal fungi.

The energy obtained by mycorrhizal fungi is mainly provided by the symbiotic related plants for the photocontract, and in turn provides the soil source nutrients for their plant hosts. They can

- Involve carbon cycle, affects greenhouse gas emission and fixation
- Participate in the soil nutrient cycle and link the subterranean—above ground link

- Participate in phytoremediation to improve plant stress resistance
- Decompose plant litter and promote the formation of humid acid.

In conclusion, fungi is significant to The cyclic processes of an ecosystem. They provide a lot of biological resources and greatly increase biodiversity.

Appendix B Code

The class of **Fungi** and **environment** are as below:

```
import math


class Fungi1:  # Middle temperature most fast decomposition rate
    c1 = 2.123
    c2 = -0.4249

    def __init__(self, extension_rate, environment):
        self.extension_rate = extension_rate
        self.moisture = environment.moisture
        self.dr = 0.0  # decomposition rate
        self.number = 1.0  # The total number

    def set_number(self, time_interval):
        self.number = self.number + self.extension_rate * time_interval

    def set_dr(self, environment):
        self.dr = self.c1 * self.extension_rate * math.exp(self.c2 * self.moisture)

        self.dr = self.dr * math.log2(self.number)

        if 40 > environment.temperature > 25:
            self.dr = self.dr * 0.8
        elif 15 < environment.temperature <= 25:
            self.dr = self.dr * 1.0
        elif 10 <= environment.temperature < 15:
            self.dr = self.dr * 0.6
        elif 0 < environment.temperature < 10:
            self.dr = self.dr * 0.35
        elif 0 >= environment.temperature:
            self.dr = self.dr * 0.1
        elif environment.temperature > 40:
            self.dr = self.dr * 0.4

        if environment.wood_number < (self.number * 2):
            self.dr = self.dr * (environment.wood_number / (self.number * 2))


class Fungi2:  # High temperature most fast decomposition rate
    c1 = 1.877
    c2 = 0.5152

    def __init__(self, extension_rate, environment):
        self.extension_rate = extension_rate
        self.moisture = environment.moisture
        self.dr = 0.0  # decomposition rate
        self.number = 1.0  # The total number

    def set_number(self, time_interval):
        self.number = self.number + self.extension_rate * time_interval
```

```
    def set_dr(self, environment):
        self.dr = self.c1 * self.extension_rate * math.exp(self.c2 * self.moisture)

        self.dr = self.dr * math.log2(self.number)

        if 35 > environment.temperature > 25:
            self.dr = self.dr * 1.0
        elif 15 < environment.temperature <= 25:
            self.dr = self.dr * 0.85
        elif 10 <= environment.temperature < 15:
            self.dr = self.dr * 0.6
        elif 0 < environment.temperature < 10:
            self.dr = self.dr * 0.3
        elif 0 >= environment.temperature:
            self.dr = self.dr * 0.1
        elif environment.temperature > 35:
            self.dr = self.dr * 0.8

        if environment.wood_number < (self.number * 2):
            self.dr = self.dr * (environment.wood_number / (self.number * 2))


class Fungi3:  # Low temperature most fast decomposition rate
    c1 = 3.553
    c2 = 0.7636

    def __init__(self, extension_rate, environment):
        self.extension_rate = extension_rate
        self.moisture = environment.moisture
        self.dr = 0.0  # decomposition rate
        self.number = 1.0  # The total number

    def set_number(self, time_interval):
        self.number = self.number + self.extension_rate * time_interval

    def set_dr(self, environment):
        self.dr = self.c1 * self.extension_rate * math.exp(self.c2 * self.moisture)

        self.dr = self.dr * math.log2(self.number)

        if 40 > environment.temperature > 25:
            self.dr = self.dr * 0.65
        elif 15 < environment.temperature <= 25:
            self.dr = self.dr * 0.85
        elif 10 <= environment.temperature < 15:
            self.dr = self.dr * 1.0
        elif 0 < environment.temperature < 10:
            self.dr = self.dr * 0.8
        elif 0 >= environment.temperature:
            self.dr = self.dr * 0.3
        elif environment.temperature > 40:
            self.dr = self.dr * 0.4
```

```
        if environment.wood_number < (self.number * 2):
            self.dr = self.dr * (environment.wood_number / (self.number * 2))


class Environment:
    def __init__(self, moisture, temperature, wood_number):
        self.moisture = moisture
        self.temperature = temperature
        self.wood_number = wood_number
        self.fungi1_density = self.fungi2_density = self.fungi3_density = 1 / 3

    def set_moisture(self, new_moisture):
        self.moisture = new_moisture

    def set_temperature(self, new_temperature):
        self.temperature = new_temperature

    def set_fungi_density(self, fungi1, fungi2, fungi3):
        total = fungi1.number + fungi2.number + fungi3.number
        self.fungi1_density = fungi1.number / total
        self.fungi2_density = fungi2.number / total
        self.fungi3_density = fungi3.number / total

    def set_wood_number(self, fungi1, fungi2, fungi3, time_interval):
        self.wood_number = self.wood_number - fungi1.dr * time_interval - fungi2.dr * time_
```

The **main code** is as below:

```
from Fungi import Fungi1, Fungi2, Fungi3, Environment
import sys
import matplotlib.pyplot as plt
import numpy as np

if __name__ == '__main__':
    environment = Environment(0.61, 26, 100)
    fungi1 = Fungi1(4.71, environment)
    fungi2 = Fungi2(4.11, environment)
    fungi3 = Fungi3(3.77, environment)
    a = []
    b = []
    c = []
    d = []
    e = []
    f = []
    g = []
    t = 0
    for i in range(sys.maxsize ** 10):
        fungi1.set_dr(environment)
        fungi1.set_number(0.001)
        fungi2.set_dr(environment)
        fungi2.set_number(0.001)
        fungi3.set_dr(environment)
        fungi3.set_number(0.001)
        environment.set_fungi_density(fungi1, fungi2, fungi3)
```

```
        environment.set_wood_number(fungi1, fungi2, fungi3, 0.001)
        if environment.wood_number <= 1:
            break
        t = t + 1
        a.append(fungi1.dr)
        b.append(environment.fungi1_density)
        c.append(fungi2.dr)
        d.append(environment.fungi2_density)
        e.append(fungi3.dr)
        f.append(environment.fungi3_density)
        g.append(environment.wood_number)
    print(a)
    print(b)
    print(c)
    print(d)
    print(e)
    print(f)
    print(g)
    print(t)
    x = np.arange(0, t * 0.01-0.005, 0.01)
    plt.subplot(2, 1, 1)
    plt.plot(x, a)
    plt.plot(x, c)
    plt.plot(x, e)
    plt.ylabel('Decomposition rate')
    plt.title('Fungi')
    plt.legend(['Fungi1', 'Fungi2', 'Fungi3'])
    plt.subplot(2, 1, 2)
    plt.plot(x, b)
    plt.plot(x, d)
    plt.plot(x, f)
    plt.xlabel('time')
    plt.ylabel('Fungi-density')
    plt.legend(['Fungi1', 'Fungi2', 'Fungi3'])
    plt.show()
```