# Modeling and Optimal Operation of Distributed Energy Systems via Dynamic Programming

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Abstract—This paper presents a hierarchical optimization method for determining optimal operation of a distributed energy system which consists of a distribution network and customers with dispersed generation and storage systems. The optimal operation problem of a distributed energy system is divided into two subproblems; optimal operation of a dispersed energy system for a customer and optimal operation of power flow on distribution network. While Dynamic Programming (DP) is used for solving optimal operation of a dispersed energy system for a customer, successive approximation method is applied for obtaining optimal operation of power flow on distribution network. The practicability and generality of the developed hierarchical optimization method for determining optimal operation of a distributed energy system are evaluated through some simulations using some simple models of a distributed energy system consisting of a distribution network and customers with dispersed generation and storage systems.

Index Terms—Distributed Energy Systems, Dispersed Generation, Modeling, Simulation, Optimization

# I. INTRODUCTION

With the development of high performance dispersed generators such as solar photovoltaic generation, wind power generation, fuel cells and micro gas turbines, and the deregulation of the power market, a dramatic increase in customer-level dispersed generation is quite possible in the near future. The authors have used the term "super-distributed environment" for the situation in which dispersed generators hold the dominant position in a power system. Furthermore, we have used the term "super-distributed energy system" when dispersed generations such as fuel cells and micro gas turbines not only supply electric power but also heat energy.

With the advent and expansion of high-performance, small inexpensive computers, the concept of centralized computer systems based on mainframes has changed radically from building-oriented development to operation-oriented development, or in other words, from top-down design to bottom-up design, thus giving rise to modern distributed computing systems. Similarly, it appears likely that energy systems face a radical reform, namely, that the existing large-scale power plants will be replaced by thousands of small-scale dispersed generators.

Most previous studies of dispersed generators have assumed that the share of such generators in a power system will be small, and therefore, analytic methods such as stability analysis and power flow analysis have dealt mostly with centralized systems based on large-scale power plants. One cannot assert that such distributed energy systems will become the mainstream in the future. However, establishing essentially different analytical methods for distributed energy systems in terms of reliability, stability and economic performance, is an important issue of research in future energy systems.

This paper presents a hierarchical optimization method for determining optimal operation of a distributed energy system which consists of a distribution network and customers with dispersed generation and storage systems. The optimal operation problem of a distributed energy system is divided into two subproblems; optimal operation of a dispersed energy system for a customer and optimal operation of power flow on distribution network. While Dynamic Programming (DP) is used for solving optimal operation of a dispersed energy system for a customer, successive approximation method is applied for obtaining optimal operation of power flow on distribution network.

The practicability and generality of the developed hierarchical optimization method for determining optimal operation of a distributed energy system are evaluated through some simulations using some simple models of a distributed energy system consisting of a distribution network and customers with dispersed generation and storage systems.

This paper is organized as follows. In Section 2, modeling of a distributed energy system which consists of customers and distribution network is considered form the viewpoint of daily operation. Optimal operation of a distributed energy system which consists of a distribution network and customers with dispersed generation and storage systems is developed based on Dynamic Programming (DP) and successive approximation method in Section 3. In Section 4, some numerical simulations using typical distributed energy system are executed in order to verify the practicability and generality of the developed hierarchical optimization method.

#### II. MODELING OF DISTRIBUTED ENERGY SYSTEM

Because customer's behavior is complex, it is necessary to optimum operate the energy system in the distributed environment. Therefore, each component are modeled as follows.

# A. Modeling of customer

The customer who introduces dispersed generation can become not only a simple load but also a generator according

TABLE I COMPONENTS OF CUSTOMER

Component	State of Component	State of Model	Control
Load	Load	Static	Impossible
Photovoltaic	Generation	Static	Impossible
Wind power	Generation	Static	Impossible
Fuel cells	Generation	Static	Possible
Micro gas turbine	Generation	Static	Possible
Energy storage	Load or Generation	Dynamic	Possible

to the amount of power generation and the power consumption at the moment. Therefore, the customer of distributed energy system does behavior quite different from the customer of a present electric power system. In this paper, customer's model assumes the house of a standard family. Therefore, the customer is classified into three kinds; Load, Generator and Energy Storage, and is modeled. Table 1 shows the candidate of customer's component.

The load is a necessary, indispensable component because it need not examine the system if there is no demand for the electric power. Customer has electric consumption. A lot of parameters exist because each customer does various behavior when the electric power is consumed. Moreover, the restriction cannot be put on customer's electric power demand. The purpose of this is to consume the electric power freely as well as an existing electric power system.

Dispersed generation can be classified into two kinds of the one that it is possible to control and the one that it is not possible to control. And, the following features exist.

- Uncontrollable: Generators by natural energy of photovoltaic generation and wind power generation, etc, because power generation is influenced from meteorological conditions, the customer cannot control.
- Controllable: Generators such as fuel cells and micro gas turbines that customer can control. However, the output electric power is small though they can be controlled. Moreover, because the heat supply is done by using the rejection heat, a completely free control is not possible.

In this paper, it is assumed that demand is covered with dispersed generation. Moreover, the customer is assumed to be an ordinary family, and dispersed generation assumes the device that can be set up near the customer. In this paper, the photovoltaic generation is selected as dispersed generation that the customer owns. Because the power generation efficiency rose and an easy installation, the photovoltaic generation is widely used in the house.

Dispersed generations of the photovoltaic generation and wind generation, etc. have the fault that the amount of power generation is influenced by meteorological conditions. Then, we propose a steady power supply system that combines with those generation and energy storage. And, it is clarified that the amount of shortage of the electric power of the entire system is lowered by optimum operating dispersed generation

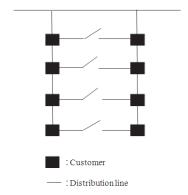
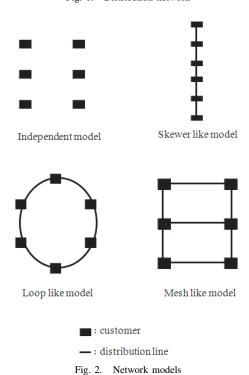


Fig. 1. Distribution network



and energy storage.

#### B. Modeling of network

In an existing electric power system, the direction of the electric power flows is one way and the interaction between customer don't exist. However, by the customers are connected mutually on the electric power network, the interchange of the surplus electricity and the shortage electric power becomes possible between customers. As a result, there is a possibility that the power supply reliability and the economy are improved.

A present electric power system is a mesh or a radial network. The customer is connected mutually in a radial distribution line in normal circumstances. The purpose of radially supplying electricity in normal circumstances is to specify the accident line. Figure 1 shows a present distribution network.

TABLE II LIST OF SYMBOLS

Symbol	Definition	unit
N	Number of customers	
T	Number of time zones	
$P_L$	Real power flow on distribution line	kW
$P_S$	Input and output of storage system	kW
$W_S$	Storage capacity of storage system	kWh
C	Network structure of distribution network	-
$P_{L_{\max}}$	Upper limit of transmitted power	kW
$P_{S\max}$	kw capacity of storage system	kW
$W_{S\max}$	kwh capacity of storage system	kWh
$P_D$	Demand pattern of customer	kW
$P_P$	Photovoltaic generation pattern	kW
$\eta_{in},~\eta_{out}$	Conversion efficiency of storage system	kW

In this paper, the flow of energy when customers that introduce dispersed generation are connected mutually on the electric power network is analyzed, and the utility of the proposal algorithm is verified. Moreover, the model of the electricity grid is assumed to four kinds; Independent model, Skewer like model, Loop like model, and Mesh like model. However, independent model do not exchange the electric power. However, the electric power is interchanged only from the customer of the neighboring house. Figure 2 shows the model of the network.

Moreover, there is an upper bound in the current that can be flowed in an actual network because it can burn off the electric wire by the over current. We thinks that t is desirable to optimal operation electric power, electric voltage and current. However, in this paper dealt with only exchange of electric voltage. The current is converted into the electric power because it is assumed that the voltage is constant. Therefore, the current limitation becomes a limitation of the electric power interchanged between customers.

#### C. Cooperative system and competitive system

Operation and control strategies of distributed energy systems can be divided into the cooperative system type where customers cooperate to realize some targets for the whole energy system, and the competitive system type where customers pursues own interests.

- 1) Cooperative system: In this model, electric power supply is considered an important part of the social infrastructure, and customers aim at reduction of equipment/operation cost and improvement of reliability of the entire systems.
- 2) Competitive system: Operation and control strategies of distributed energy systems can be divided into the cooperative system type where customers cooperate to realize some targets for the whole energy system, and the competitive system type where customers pursues own interests.

# III. OPTIMAL OPERATION FOR DISTRIBUTED ENERGY SYSTEMS

#### A. Optimal Operation Problem for Distributed Energy Systems

Operation of distributed energy systems is classified into "Cooperative system" and "Competitive system". "Cooperative system" is systems such that customers, an electric power company and so on operate systems to cooperate mutually toward the whole target realization, and "Competitive system" is systems that each of them pursues its profits.

In both the two operation models, there exist complicated interaction based on local information and global information because behaviors of customers are not independent, and this interaction will determine behavior of an energy system.

In this paper, we define the system form as important infrastructure in society for supplying electric energy, and investigate on "Cooperative system" which customers, an electric power company and so on aim at minimizing the insufficient quantity of the whole energy system.

Given N: a total number of customers, T: a number of time zones T,  $P_{Di}(t)$ :a consumed electricity of the i-th customer at time t,  $P_{Pi}(t)$ :a generated electricity of the i-th customer at time t,  $P_{Li}(t)$ :a real power flow on distribution line of the i-th customer at time t,  $P_{Si}(t)$ :a active power which flows through the power line linked to the i-th customer at time t,  $P_{Li\max}(t)$ :an input and output of energy storage of the i-th customer at time t,  $C = [c_{ij}], c_{ij} = 1$  or 0: a connection matrix, "Cooperative system" is formulated as the following minimization problem to minimize the shortage electricity on the whole.

$$\min_{P_S, P_L} J(P_S, P_L) = \sum_{t=1}^{T} \sum_{i=1}^{N} (P_{P_i}(t) + P_{S_i}(t) + \sum_{j \in C'_i} P_{L_{ij}}(t) - P_{D_i}(t))^2 \tag{1}$$

subj. to 
$$-P_{S_{i \max}} \le P_{S_i(t)} \le P_{S_{i \max}}$$
 (2)

$$-P_{Lij\max} \le P_{Lij}(t) \le P_{Lij\max} \quad (3)$$

$$P_{L\,ii} = -P_{L\,ii} \tag{4}$$

$$0 \le W_{Si}(t) \le W_{Si\,\text{max}} \tag{5}$$

$$W_{Si}(t) = \sum_{h=1}^{t} P_{Si}(h)$$
 (6)

where 
$$C_i'=\{j\mid c_{ij}=1, j=1,2,\cdots N\},\ i=1,2,\cdots,N;$$
  $t=1,2,\cdots,T.$ 

The distributed energy system in this research become the dynamic optimization problem because customers have energy storage. However, this system have the characteristic that static and dynamic properties are mixed because of optimal operation of the network after determining the optimal operation of each customer.

# B. Hierarchical Optimization

In this paper, we regard the optimal operation of distributed energy system as the following hierarchical optimization.

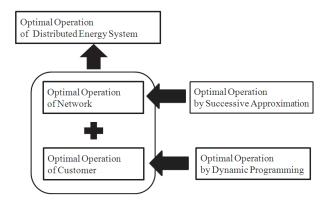


Fig. 3. General flow of proposed approach

- Optimal operation problem as operation of a energy storage for one customer
- Optimal operation problem as network after determining optimal operation each customer

and consider the result combining with the two solutions as optimal operation of network and customers. Figure 3 shows flow of the proposed method.

# C. Optimal Operation of Each Customer based on Dynamic Programming

1) Dynamic Programming: DP: Dynamic Programming (DP) has many advantages over a wide variety of control and dynamic optimization problems such as optimal operation of an energy storage system in an electric power system. DP is useful in solving a variety of problems and can greatly reduce the computational effort in finding optimal trajectories or control policies.

Behaviors of customers having storage system which change by past effect can be formulated as a dynamical optimization problem. Thus, we need to optimize the problem by considering of temporal changes of systems.

In this paper, Dynamic Programming (DP) is used to optimally operate energy storage of each customer. Dynamic programming is a optimization method in which a problem is solved by identifying a collection of subproblems and tackling them one by one, smallest first, using the answers to small problems to solve larger ones, until the whole lot of them is solved. Figure 4 shows optimal operation of storage system via dynamic programming.

2) Optimal Operation of Each Customer: In this paper, we consider operation of a strage equipment for a customer as an optimal operation problem. Given S: a number of states, S: a number of time zones T,  $P_{Di}(t)$ :a consumed electricity of the i-th customer at time t,  $P_{Pi}(t)$ :a generated electricity of the i-th customer at time t,  $P_{Li}(t)$ :a real power flow on distribution line of the i-th customer at time t,  $P_{Si}(t)$ :an input and output of energy storage of the i-th customer at time t, operation of storage system every time is resulted in the following shortest path problem by using DP.

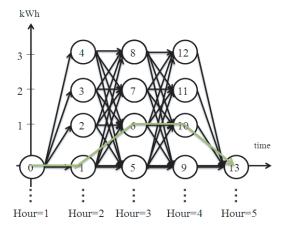


Fig. 4. Optimal operation of storage system via dynamic programming

$$\min_{P_S, P_L} J(P_S, P_L) = \sum_{t=1}^{T} \sum_{i=1}^{N} (P_{P_i}(t) + P_{S_i}(t) + \sum_{j \in C'_i} P_{L_{ij}}(t) - P_{D_i}(t))^2$$
(7)

subj. to 
$$-P_{Si\max} \le P_{Si}(t) \le P_{Si\max}$$
 (8)

$$P_{Lij}(t) = 0 (9)$$

$$P_{Lji} = -P_{Lij} \tag{10}$$

$$0 \le W_{Si}(t) \le W_{Si\,\text{max}} \tag{11}$$

$$W_{Si}(t) = \sum_{h=1}^{t} P_{Si}(h)$$
 (12)

where  $C'_i = \{j \mid c_{ij} = 1, j = 1, 2, \dots N\}, i = 1, 2, \dots, N; t = 1, 2, \dots, T.$ 

#### <Optimal Operation of Each Customer>

#### Step 0: [Preparation]

Define a number of states S, a number of time zones T, an input and output of storage system  $P_S$ , an initial storage capacity of storage system  $W_S$ , k = 1.

#### Step 1: [Calculation of Route]

Using a least squares method, calculate

$$\sum_{i=1}^{S} (P_{P_i}(t) - P_{D_i}(t) - P_S)^2$$
 (13)

#### Step 2: [Selection of Optimal Value of Each State]

As the input and output of the storage system expressing a number of Sth-state at t+1th-stage, using  $P_{t+1,j}$ , set the value of this state as  $f_{(P_{b(t+1,j)})}$ . Move to ith-state at tth-stage from jth-state at t+1th-stage and set the value of the state at that time  $f_{(P_{b(t,i)},P_{b(t+1,j)})}$ , calculate

$$f(P_{bt+1,j}) = \min_{i \in S_t} [f(P_{bt,i} + f(P_{bt,i}, P_{bt+1,j}))]. \quad (14)$$

**Step 3: [Termination Criterion]** 

if k = T, terminate, otherwise, k = k + 1 and go to Step 1.

# D. Optimal Operation of Network based on Successive Approximation

In this paper, we consider the optimal operation of network as an electric power equalization problem. Given N: a total number of states, S: a number of time zones T,  $P_{D_i}(t)$ :a consumed electricity of the i-th customer at time t,  $P_{Pi}(t)$ :a generated electricity of the i-th customer at time t,  $P_{Li}(t)$ :a active power which flows through the power line linked to the i-th customer at time t,  $P_{Si}(t)$ :an input and output of energy storage of the i-th customer at time t, the electric power equalization problem on t can be formulated as follows by successive approximation.

$$\min_{P_S, P_L} J(P_S, P_L) = \sum_{t=1}^{T} \sum_{i=1}^{N} (P_{P_i}(t) + P_{S_i}(t) + \sum_{j \in C_i'} P_{L_{ij}}(t) - P_{D_i}(t))^2 + \sum_{j \in C_i'} P_{L_{ij} \max} \le P_{L_{ij} \max}$$
subj. to
$$-P_{L_{ij} \max} \le P_{L_{ij}} \le P_{L_{ij} \max}$$
(16)

subj. to 
$$-P_{Lij \max} \le P_{Lij} \le P_{Lij \max}$$
 (16)

#### <Optimal Operation of Network>

#### Step 0:[Preparation]

Give an amount of consumption for every time of customers  $P_D(t)$ , a generation of electricity  $P_P(t)$ , a number of customers N, a storage capacity of storage system  $W_S$ , a loss by accumulation of electricity and electric discharge of energy storage  $\eta_{in}, \eta_{out}$ , a capacity restrictions of a power line  $P_L$ , minute quantity  $\Delta P$ , a maximum iteration number  $k_{\text{max}}$ .

# Step 1:[Calculation]

Calculate difference of the electric power generation for every time of each customer and an amount of consumption  $B(t) = P_P(t) - P_D(t)$ .

# Step 2:[Definition of network]

Define construction of network, set k = 1.

#### Step 3:[Optimal Operation Problem for Network]

Calculate an average Ave of whole network every time B(t). Flow B(t) of each customer for every time  $\Delta P$ by  $\Delta P$  until B(t) approaches Ave.

# Step 4:[Termination Criterion]

If  $P_L$  reach  $P_{L_{\text{max}}}$ , or  $k = k_{\text{max}}$ , then terminate. Otherwise, set k = k + 1, and go to Step 2.

# IV. NUMERICAL SIMULATION RESULTS USING TYPICAL DISTRIBUTED ENERGY SYSTEM

#### A. Numerical experimental condition

The numeric experiment is analyzed by the following requirements. All customers own dispersed generator of the same performance and the same capacity. The dispersed generator generates electricity from 7 o'clock to 18 o'clock, and the amount of power generation is 5units. Customer's load pattern is assumed to be two kinds (AM-type and PM-type). The

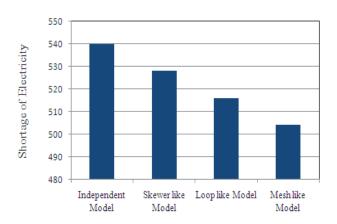


Fig. 5. Optimal operation of distribution network

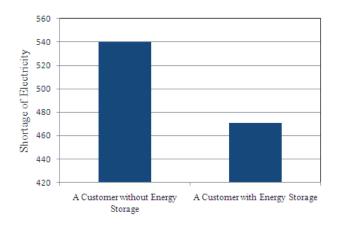


Fig. 6. Optimal operation of customer

power consumption of AM-type is caused between 0 o'clock and 12 o'clock. The power consumption of PM-type is caused between 12 o'clock and 24 o'clock. And, the consumption PD is ten units respectively. Where, the total of the customer Nis 6, the limit of the distribution line  $P_{Lmax}$  is 1, a maximum iteration number  $k_{\rm max}$  is 1000, minute quantity  $\Delta P$  is 0.1.

# B. Optimal operation of distribution network

Figure 5 shows the shortage in the network shown in Figure 2 during a day. Where, the customer of AM-type is 3, and the customer of PM-type is 3.

#### C. Optimal operation of customer

Figure 6 shows the shortage of the case to use the proposed optimum operation by the dynamic programming and the case without the operations. The model of the network is only independence. The number in the state S is 20. The efficiency of the converter of the energy storage is assumed to be 0.9.

## D. Optimal operation of customers and distribution

The optimum operation based on the dynamic programming and the optimum operation of the network based on successive



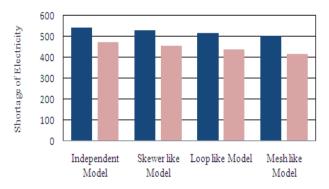


Fig. 7. Optimal operation of customers and distribution network

approximation is combined. The result of the proposal and the result of section B with the optimum operation of distribution network are compared in Figure 7.

#### E. Consideration of numeric experiment

In the optimum operation of distribution network in section B, because the customer can exchange the electric power by the network, the shortage becomes small more than an independent type. The order in which the shortage is small is a mesh type, a loop type, and a spit type. Therefore, the shortage becomes small according to becoming if the density of the network becomes high.

In optimal operation of customer based on DP in section C, each customer's shortage becomes small by the effect of the energy storage. In hierarchical optimization that combines the optimum operation of the customer and the optimum operation on the network in section D, the entire shortage on the network becomes small more than the case only with the optimum operation on the network. Therefore, the shortage becomes small according to becoming if the density of the network becomes high as well as section C.

The conclusion is as follows. The shortage of the case where the customer is connected on the network is smaller than that of the case where the customer is independent. Moreover, the shortage becomes small according to becoming if the density of the network becomes high. Therefore, the proposed hierarchical optimization that combines the optimum operation of the customer and the optimum operation on the network is the most effective in the shortage.

#### V. CONCLUSIONS

In this paper, we proposed the original models concerning with a customer, dispersed generation, a network and operation method, and constructed the algorithms. Furthermore, we executed the simulation in order to verify the model and algorithm and considered methods for evaluating numerical simulation results. From numerical experiments, optimal operation for each customer based on DP, one for network

based on successive approximation and one for combination of them were obtained. Consequently, we were able to propose effective algorithms which can evaluate not qualitatively but quantitatively even if target model or its parameters are changed, and indicate possibility for future expansion. The proposed algorisms can corresponds to the super distributed energy system by considering hundreds of cities whose one city is collection of hundreds of customers.

The algorisms constructed in this paper have high flexibility method which can apply detailed data although the quantitative evaluation incorporating random elements, such as a concrete numerical value and the weather, to a real system was not able to be carried out this time. We can consider it becomes a more realistic model by improving practicability from coarse approximation based on random output like a method giving fluctuation to the average of the power consumption of customers, and the generation of electricity of dispersed generation.

However, the verification about the validity of the model is insufficient. Furthermore, we need investigating the validity of the proposed model and higher detailed model. We have possibility to obtain good result by optimization at the upper stage that is operation optimization problem for whole network.

The purpose of the distributed energy system is to supply good electricity at a low price as much as possible without power stoppage after using it efficiently. In fact, the emphasis in performance optimization of power systems has been on economic operation only, using the so-called economic dispatching approach. Moreover, we need to verify economic load allotment which minimize cost concerning power generation and transportation of required electric power by dispersed generation which is one side for supplying electric power as cheaply as possible.

We think necessary to consideration of supplying of electric power without violating the needs for adequate reserve capacities while minimizing the cost of equipment. That is, on a construction of operation optimization problem, it is possible to supply electricity at low price efficiently by considering a equipment program optimization problem, such as network structure of a power line, upper limits of a power line and power generation capacity of photovoltaics, capacity restrictions of kW-kWh of storage system and so on.

#### REFERENCES

- T. Ishi and K. Yasuda, "The Basic Concept and Decentralized Autonomous Control of Super Distributed Energy Systems," IEEJ Trans, Vol. 123 - B, No. 8, pp. 907 - 917 (2003)
- [2] T. Ishi and K. Yasuda"Hierarchical Decentralized Autonomous Control in Super-distributed Energy Systems,"IEEJ Trans, Vol2, No.1, pp. 63-71, Jan. 2007. (in Japanese)
- [3] Y. Ohsawa, S. Kawamoto, N. Kimura and H.Tanaka, "Power System Engineering," Ohmsha(2001) (in Japanese)
- [4] H. Tamaki, H. Suwa and K. Yasuda, "system optimization," Ohmsha(2001) (in Japanese)
- [5] Atif S. Deds, "MODEN POWER SYSTEMS CONTROL AND OPERA-TION," Kluwer Academic Publishers
- [6] The Federation of Electric Power Companies of Japan website.[Online]. Available: http://www.fepc.or.jp/present/jigyou/japan/index.html
- [7] Public Relations Office Government of Japan website.[Online]. Available: http://www.gov-online.go.jp/pr/theme/taiyokohatsuden.hukyukakudai.html