Standard transfer function for Universal filter

$$\Rightarrow \frac{a_2 s^2 + a_1 s + a_0}{1 + \frac{s}{\omega_0 \cdot Q} + \frac{s^2}{\omega_0^2}}$$

→ By putting value of a_0, a_1, a_2 → all four T.F can be achieved.

L.P.F

$$\frac{V_{03}}{V_i} = \frac{+H_0}{1 + \frac{s}{\omega_0 \cdot Q} + \frac{s^2}{\omega_0^2}}$$

So, $\boxed{a_0 = H_0}$, $a_1 = 0$, $a_2 = 0$

H.P.F

$$\frac{V_{01}}{V_i} = \frac{\frac{H_0}{\omega_0^2} s^2}{1 + \frac{s}{\omega_0 \cdot Q} + \frac{s^2}{\omega_0^2}}$$

So, $\boxed{a_2 = \frac{H_0}{\omega_0^2}}$

B.P.F

$$\frac{V_{02}}{V_i} = \frac{-\frac{H_0}{\omega_0} s}{1 + \frac{s}{\omega_0 \cdot Q} + \frac{s^2}{\omega_0^2}}$$

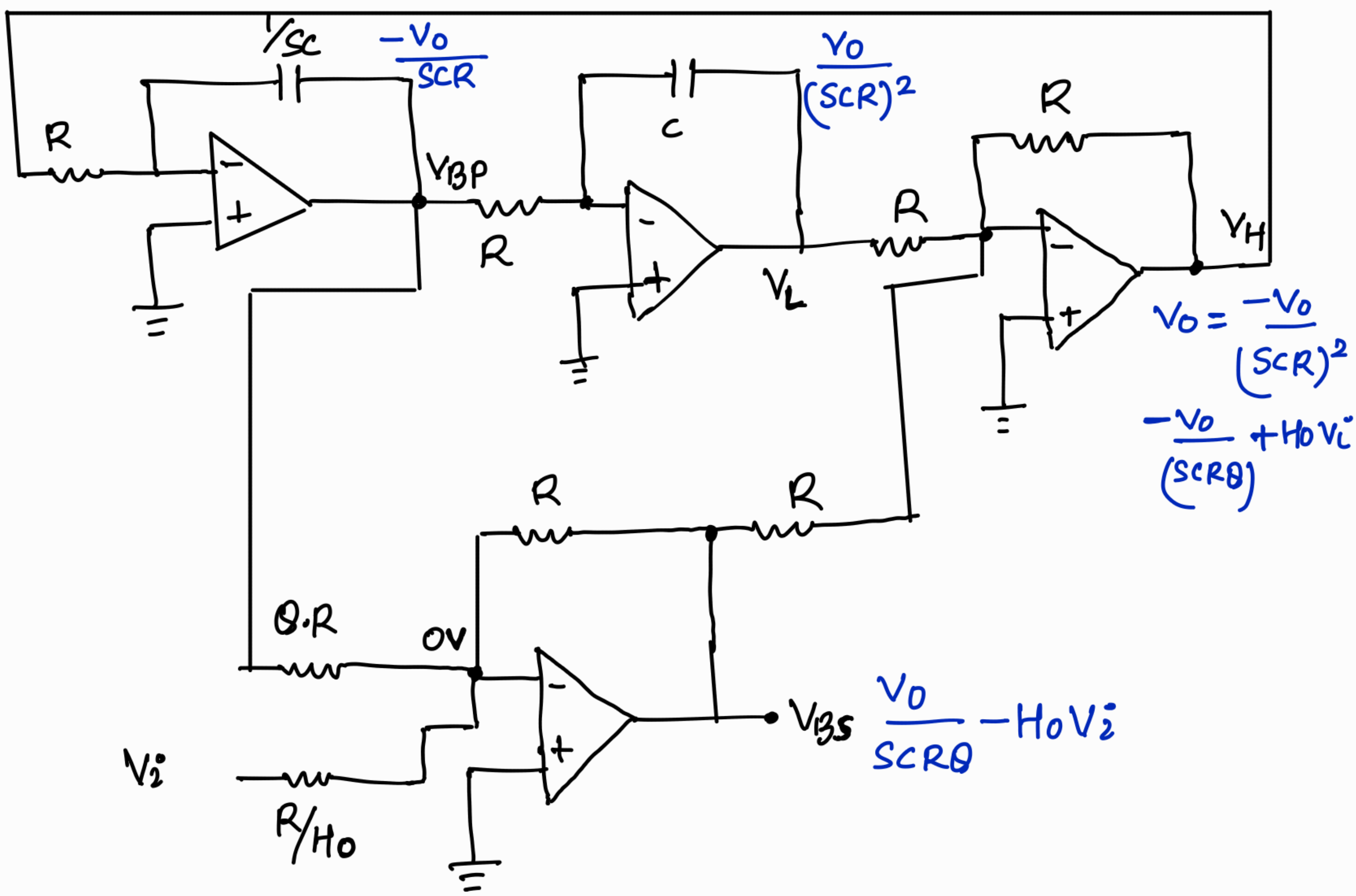
So, $\boxed{a_1 = -H_0/\omega_0}$

B.S.F

$$\frac{V_{o4}}{V_i} = \frac{H_o + \frac{H_o}{\omega_o^2} s^2}{1 + \frac{s}{\omega_o \cdot Q} + \frac{s^2}{\omega_o^2}}$$

So, $\boxed{a_0 = H_o}$ $\boxed{a_2 = H_o/\omega_o^2}$

for B.P.F, $\boxed{Q = \frac{f_o}{B.W}}$ $(B.W)_{3dB}$



$$V_o = \frac{-V_o}{(SCR)^2} - \frac{V_o}{(SCRQ)} + H_oV_i$$

$$V_o\left[1 + \frac{1}{(SCR)^2} + \frac{1}{SCRQ}\right] = H_oV_i$$

$$\frac{V_H}{V_i} = \frac{V_o}{V_i} = \frac{H_o}{1 + \frac{1}{(SCR)^2} + \frac{1}{(SCRQ)}}$$

$$\boxed{\frac{V_H}{V_i} = \frac{H_o (SCR)^2}{\left((SCR)^2 + \frac{SCR}{Q} + 1\right)}}$$

★. High pass filter gets integrated to form Bandpass.

$$\frac{V_{B.P}}{V_i} = \frac{H_o (SCR)}{\left((SCR)^2 + \frac{SCR}{Q} + 1\right)}$$

★. B.P filter gets integrated to form Low pass.

$$\frac{V_{L.P}}{V_i} = \frac{H_o}{\left((SCR)^2 + \frac{SCR}{Q} + 1\right)}$$

★. All pass is combination of B.P, H.P & L.P, such that location of zeroes is mirror image of location of poles.

★. $(V_{BSF})_{T.F} = (V_{L.P})_{T.F} + (V_{H.P})_{T.F}$

$$\frac{V_{BS}}{V_i} = \frac{\left[1 + \frac{s^2}{w_o^2}\right] H_o}{\left[1 + \frac{s}{w_o Q} + \frac{s^2}{w_o^2}\right]},$$

$$\boxed{w_o = \frac{1}{RC}}$$

Q = Quality factor,

Ability of Universal Active filter

→ To simulate any 2^{nd} order system with poles always on left half plane and zeroes anywhere on 's' plane.

Calculation for Q

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{\left.\frac{s}{\omega_0}\right|_p = \frac{1}{2Q} \pm \sqrt{1 - \frac{1}{4Q^2}}}$$

→ Poles of Universal filter,

for $\boxed{Q \leq \frac{1}{2}}$ $\left\{ Q = \frac{1}{2} \rightarrow \text{critical damping } (\varepsilon = 1) \right\}$

Underdamped	→ $\varepsilon < 1$
Critically damp.	→ $\varepsilon = 1$
Overdamped	→ $\varepsilon > 1$
Undamped	→ $\varepsilon = 0$

$Q < \frac{1}{2}$ → exponential, negative real axis.
→ Overdamped system

when Q is high → Equal to the natural frequency of system.

Ringing freq. = $\boxed{\omega = \omega_0\sqrt{1-\frac{1}{4Q^2}}}$,

→ No. of rings visible = value of Q.

→ Phase sensitivity is highest at $\omega = \omega_0$ = Resonance freq.

$$\boxed{\left.\frac{\partial \phi}{\partial \omega}\right|_{\omega=\omega_0} = \frac{2Q}{\omega_0}}$$

→ Phase detectors are used for tunning filters. (ω_0).

→ Higher the 'Q' → sharper the Notch.

→ Use of notch filter → To remove unwanted powerline frequency.

Phase → (0 to 2π)

0 to π → one pair of poles,

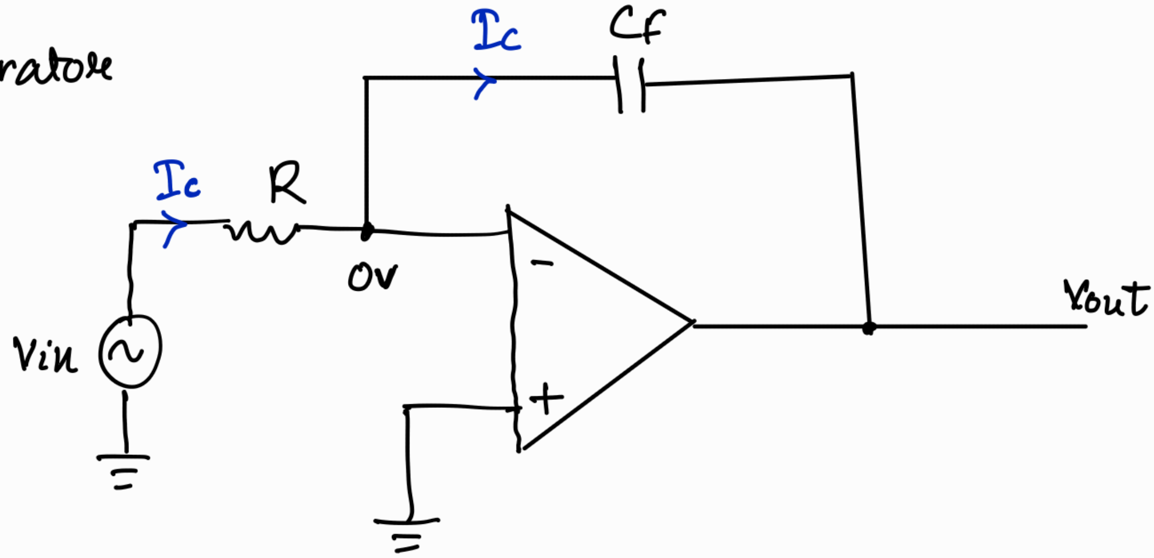
above π → occur due to zeroes,

→ Can be used as spectrum analyzer for periodic analyzer

fundamental frequency → Pickedup at Bandpass filter o/p.

Rest of harmonics occur @ Bandstop o/p.

Integrator

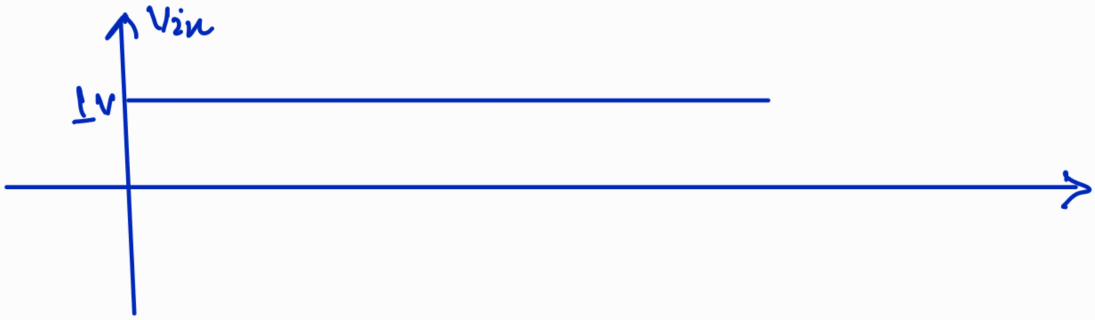


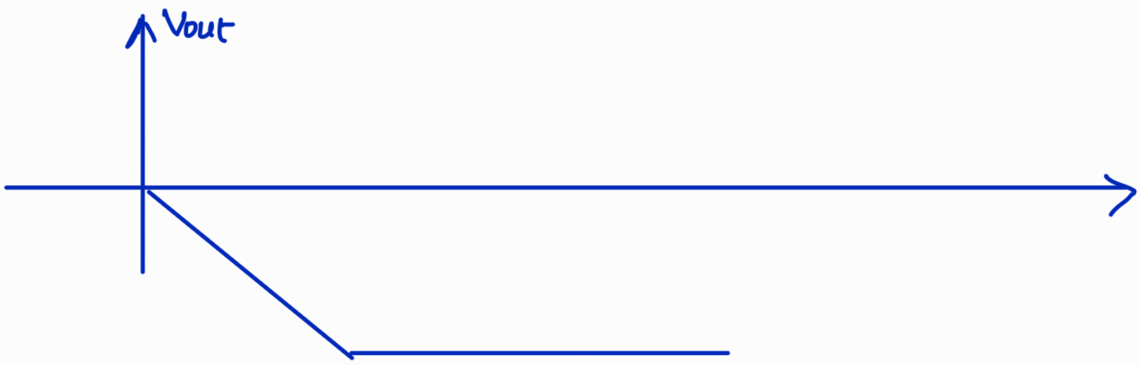
$$\frac{V_{in}-0}{R} = I_c \quad , \quad I_c = C_f\frac{dv}{dt} \quad , \quad Q = CV$$

Cap → not changing with time.

$$\frac{V_{in}}{R} = C_F \cdot \frac{dV_c}{dt} = C_F \times \frac{d}{dt}\left[0 - V_{out}\right]$$

$$\boxed{V_{out} = \frac{-1}{RC_f}\int V_{in}(t)\,dt} + V_{out}(0^+)$$

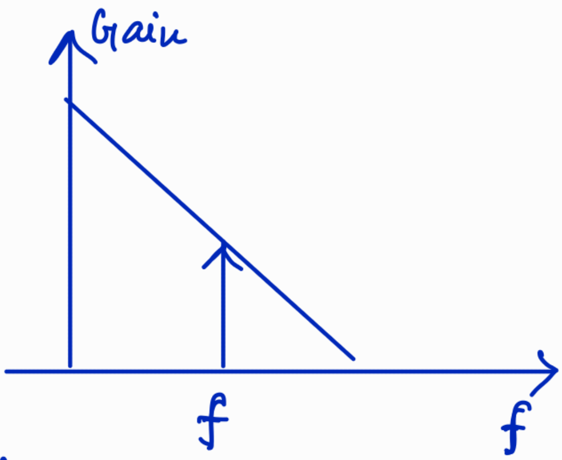




$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}, \quad V_{out} = \frac{-X_C}{R} \times V_{in}$$

$$A_V = \frac{-X_C}{R}$$

$$|A_V| = \frac{1}{2\pi R C_f \cdot f}$$

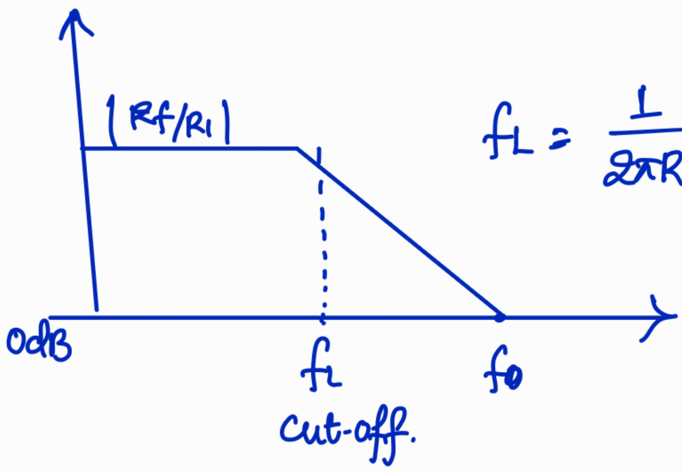


$f \uparrow \longrightarrow$ Gain $\downarrow$.

→ I/p offset voltage of practical opamp.

→ In practical → Put feedback resistor ‖ with C_F.

So, $A_V = -\frac{R_F}{R_1}$ at lower freq.



$$f_L = \frac{1}{2\pi R_f C_F}$$

$$f_0 = \frac{1}{2\pi R \cdot C_F}$$

For proper integraⁿ input freq. $(f_S) > 10 f_L$.

i/p freq. should lie b/w f_L & f_0.

Resistor Noise

$$\boxed{\overline{V_n^2} = 4 K_b T_a \cdot R \Delta f}$$

$\overline{V_n^2}$ → Voltage

T_a → absolute temp.

Δf → Bandwidth

K_b → Boltzmann's constant $(1.38 \times 10^{-23} J/^{o}K)$

Resistor Noise spectral density

$$\boxed{N_o = \sqrt{(\overline{V_n^2})/\Delta f} = \sqrt{(4 K_b T R)}}$$

Noise due of operational amplifier

$$\overline{V_n^2} = \int_{-\infty}^{\infty} S_{uu}(j\omega) \, |H(j\omega)|^2 \, df$$

S_{uu} = Noise spectral density of Noise source,

$$|H(j\omega)|^2 = \frac{1}{(1+2\pi R f c)^2}$$

$$N_o = \sqrt{\frac{\overline{V_n^2}}{df}} = \sqrt{\alpha \frac{KT}{c}}$$

α = order → $\alpha \uparrow$ → Noise $\uparrow$.

Total Noise → $\overline{V_n^2} = \sum_{u=1}^{n} \int_{-\infty}^{\infty} S_{uu}(j\omega) \, |H_u(j\omega)|^2 \, df$