Proof of the Gaussian Integral

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$$\int_{-\infty}^{\infty} e^{x^2} dx$$

This integral is not solvable with elementry functions, and therefore another procedure is needed

$$I = \int_{-\infty}^{\infty} e^{-x^2} dx$$

$$I^2 = \int_{-\infty}^{\infty} e^{-x^2} dx \cdot \int_{-\infty}^{\infty} e^{-y^2} dy$$

$$I^2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-x^2} e^{-y^2} dxdy$$

$$I^2 = \int_{0}^{2\pi} \int_{0}^{\infty} e^{-r^2} rdrd\theta$$

$$I^2 = \int_{0}^{2\pi} d\theta \cdot \int_{0}^{\infty} e^{-r^2} rdr$$

$$I^2 = 2\pi \cdot \int_{0}^{\infty} e^{-r^2} rdr$$

$$I^2 = 2\pi \cdot \frac{e^{-r^2}}{2}\bigg|_{0}^{\infty}$$

$$I^2 = 2\pi \cdot \frac{1}{2}$$

$$I^2 = \pi$$

$$I = \sqrt{\pi}$$

therefore:

$$\int_{-\infty}^{\infty} e^{x^2} dx = \sqrt{\pi}$$