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CV Assignment 3

(1.1) The epipoles are the right & left null space of E

so $E \cdot e = 0$ (right epipole)
$e'^T E = 0$ (left epipole)

$[t_x]Re = 0$ (as $E = [t_x]R$)

Now Re is a rotational transformation of the epipole taking e from left to right side, hence scalar multiple of the translation vector

$\boxed{\text{so } Re = \lambda t}$

So,

$[t_x] \cdot R \cdot e = 0$

$\underbrace{[t_x]\lambda t}$ $= 0$

essentially a cross product of t with t itself which should give 0. hence right epipole is a scalar multiple of translation vector

II $e'^T E = 0$

$e'^T [t_x] R = 0$

from the similar observation $e'^T = \lambda' t$

$\lambda' t [t_x] R = 0$

$\rightarrow$ cross product of same vectors is 0.

so left epipole $\boxed{e'^T = \lambda' t}$

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(1.2) Given, $R = I$

$t = [t_x, 0, 0]^T$

Now, $E = [t_x] R$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}}$$

Now let the corresponding points in two camera views as $p_1 = [x_1, y_1, 1]^T$ are $p_2 = [x_2, y_2, 1]^T$. The epipolar constrains tells

$$p_2^T E p_1 = 0$$

Hence

$$[x_2 \; y_2 \; 1] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = 0$$

$$y_2 \cancel{t_x} - x_2 \cancel{t_x} = 0$$

Hence

$$y_2 = x_2$$

This demonstrates that in the image plane, the y-coordinate of corresponding points is always the same.

(1.3) From the slide deck

let $R_{rect} = \begin{bmatrix} r_1^T \\ r_2^T \\ r_3^T \end{bmatrix}$ Given: epipole using SVD over E

$$r_1 = e_1 = \frac{T}{\|T\|}$$

$$r_2 = \frac{1}{\sqrt{T_x^2 + T_y^2}} \begin{bmatrix} -T_y & T_x & 0 \end{bmatrix}$$

$$r_3 = r_1 \times r_2$$

Now if ~~as~~ r_2 & r_3 are orthogonal

$$Rect\, e_1 = \begin{bmatrix} r_1^T e_1 \\ r_2^T e_1 \\ r_3\ e_1 \end{bmatrix} = \begin{bmatrix} \cancel{T} & 1 \\ \cancel{0} & 0 \\ \cancel{0} & 0 \end{bmatrix}$$

~~Rect T =~~

$$Rect \cdot \frac{T}{\|T\|} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{Rect = \begin{bmatrix} \|T\| \\ 0 \\ 0 \end{bmatrix} \cdot T^{-1}}$$

Ans