

Assignment-1

(12.13.5.5)

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12.)Question: Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

- 1) the youngest is a girl
- 2) at least one is a girl?

Solution: Let random variable $X_i = 0$, if it is a girl child and $X_i = 1$ if it is a boy.

Where $i = 1$ for first child and $i = 2$ for second child.

It is given that:

$$\Pr(X_i = 1) = \Pr(X_i = 0) \quad (1)$$

$$\Pr(X_i = 1) + \Pr(X_i = 0) = 1 \quad (2)$$

$$\implies \Pr(X_i = 0) = \Pr(X_i = 1) = 0.5 \quad (3)$$

We are supposed to find Probability of both girl child(for some given conditions)

$$\implies \Pr(X_1 = 0) \times \Pr(X_2 = 0) \quad (4)$$

This is because gender of each child is independent of another

Variables	Corresponding child	Probability
$X_1 = 0$	Younger Girl Child	$\frac{1}{2}$
$X_1 = 1$	Younger Boy Child	$\frac{1}{2}$
$X_2 = 0$	Elder Girl Child	$\frac{1}{2}$
$X_2 = 1$	Elder Boy Child	$\frac{1}{2}$

Probabilities	Values
$\Pr(X_1 + X_2 = 2)$	$\frac{1}{4}$
$\Pr(X_1 + X_2 = 1)$	$\frac{1}{2}$
$\Pr(X_1 + X_2 = 0)$	$\frac{1}{4}$

- 1) If the youngest child is girl:

By conditional Probability we have

$$\Pr((X_1 + X_2 = 0)|X_1 = 0) = \frac{\Pr(X_1 + X_2 = 0)}{\Pr(X_1 = 0)} \quad (5)$$

$$\Pr(X_1 + X_2 = 0) = \Pr(X_1 = 0) \times \Pr(X_2 = 0) \quad (6)$$

$$\implies \Pr(X_2 = 0) = 0.5 \quad (7)$$

- 2) If at least one of the child is girl:

By conditional Probability we have

$$\Pr(X_1 + X_2 = 0|(X_1 + X_2 \leq 1)) = \frac{\Pr(X_1 + X_2 = 0)}{\Pr(X_1 + X_2 \leq 1)} \quad (8)$$

$$\Pr(X_1 + X_2 \leq 1) = 1 - \Pr(X_1 + X_2 = 2) \quad (9)$$

$$= 1 - \frac{1}{4} = \frac{3}{4} \quad (10)$$

Probability of both are girl given that at least one is girl is

From (8)

$$\frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \quad (11)$$