

Assignment-1

(12.13.5.5)

Uttam Paharia
CS22BTECH11060

5.)Question: The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs

- 1) none
- 2) not more than one
- 3) more than one
- 4) at least one

will fuse after 150 days of use.

Solution: Let the **Probability** of i^{th} bulb to fuse after 150 days of use be $P(i)$.

$$P(i) = 0.05 \quad \forall i \in \{1, 2, 3, 4, 5\}$$

Probability of one bulb getting fused is independent of another

$$= 0.0225925$$

- 4) Probability that at least one bulb is fused is 1
- Probability that no bulb is fused:

$$= 1 - (0.95)^5 = 0.2262190625$$

- 1) Probability that none of the 5 bulbs fuses is:

$$\begin{aligned} P(\bar{1} \cap \bar{2} \cap \bar{3} \cap \bar{4} \cap \bar{5}) &= \prod_{i=1}^{i=5} (1 - P(i)) \\ &= (1 - 0.05)^5 = 0.95^5 \\ &= 0.7737809375 \end{aligned}$$

- 2) Probability that not more than one bulb fuses is same as exactly 0 bulb fuses or exactly one bulb fuses:

Probability of no bulb to fuse is: $(0.95)^5$

Probability of exactly one bulb to fuse is:

$$\binom{5}{1} \times P(1 \cap \bar{2} \cap \bar{3} \cap \bar{4} \cap \bar{5})$$

(choosing which bulb is defective)

$$= 5 \times (0.95)^4 \times 0.05 = 0.2036265625$$

- 3) Probability that more than one bulb will fuse will be:

= 1 - Probability that no bulb is fused - Probability that exactly one bulb is fused

$$= 1 - (0.95)^5 - 5 \times 0.05 \times (0.95)^4$$