Assignment-1
(12.13.5.5)

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12.)Question: Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

1) the youngest is a girl
2) at least one is a girl?

Solution: Let random variable $X = 0$,if it is a girl child and $X = 1$ if it is a boy.
It is given that:

$$\Pr(X = 1) = \Pr(X = 0) \tag{1}$$
$$\Pr(X = 1) + \Pr(X = 0) = 1 \tag{2}$$
$$\implies \Pr(X = 0) = \Pr(X = 1) = 0.5 \tag{3}$$

Let random variable $Y = 1$ for first child and $Y = 2$ for second child
We are supposed to find Probability of both girl child(for some given conditions)

$$\implies \Pr(X = 0, Y = 1) \times \Pr(X = 0, Y = 2) \tag{4}$$

This is because gender of each child is independent of another

1) If the youngest child is girl:

$$\Pr((X = 0)(Y = 1)) = 1 \tag{5}$$
$$\therefore \Pr(((X = 0, Y = 1)(X = 0, Y = 2))|(X = 0, Y = 1)) \tag{6}$$
$$= \Pr((X = 0, Y = 2)) = 0.5 \tag{7}$$

2) If at least one of the child is girl possible cases are as follows:

$$\Pr((X = 0, Y = 1)(X = 0, Y = 2)) \tag{8}$$
$$\Pr((X = 1, Y = 1)(X = 0, Y = 2)) \tag{9}$$
$$\Pr((X = 0, Y = 1)(X = 1, Y = 2)) \tag{10}$$

and all of the three case are equally likely as probability of a child to be boy or girl is same Let event

$$A = ((X = 0, Y = 1)(X = 0, Y = 2)) \tag{11}$$
$$B = ((X = 0, Y = 1) + (X = 0, Y = 2)) \tag{12}$$

$$\therefore \Pr(A|B) = \frac{1}{3}$$

Alternatively by conditional Probability we have

$$\Pr(P|Q) = \frac{\Pr(PQ)}{\Pr(Q)} \tag{13}$$
$$\implies \Pr(A|B) = \frac{\frac{1}{4}}{\frac{3}{4}}$$
$$= \frac{1}{3} \tag{14}$$