Assignment-1
(12.13.5.5)

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5.)Question: The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs

1) none
2) not more than one
3) more than one
4) at least one

will fuse after 150 days of use.

Solution: Let the **Probability** of i bulb to fuse after 150 days of use be $\Pr(i)$.
Probability of a bulb to fuse after 150 days is =0.05
Probability of one bulb getting fused is independent of another
Let

$$p = 0.05 \tag{1}$$
$$q = 1 - p = 0.95 \tag{2}$$

Let Cumulative Distributive Function be:

$$F_X(i) = Pr(X \leq i) \tag{3}$$
$$Pr(X = i) = {}^5C_i p^i q^{5-i} \tag{4}$$

$$\therefore F_X(i) = \sum_{r=0}^{i} {}^5C_r p^r q^{5-r} \tag{5}$$

$$F_X(0) = 0.95^5 = 0.7737809375 \tag{6}$$

$$F_X(1) = (0.95)^5 + {}^5C_1 \times (0.5) \times (0.95)^4 \tag{7}$$

$$F_X(2) = (0.95)^5 + {}^5C_1 \times (0.5) \times (0.95)^4 \tag{8}$$
$$+ {}^5C_2 \times (0.5)^2 \times (0.95)^3 \tag{9}$$

$$F_X(6) = 1 \tag{10}$$

1) Probability that none of the 5 bulbs fuses is:

$$Pr(X = 0) = F_X(0) \tag{11}$$

form (11)

$$= 0.95^5 \tag{12}$$
$$= 0.7737809375 \tag{13}$$

2) Probability that not more than one bulb fuses is same as exactly 0 bulb fuses or exactly one bulb fuses:

$$Pr(X \leq 1) = F_X(1) = 0.9774075 \tag{14}$$

3) Let A be event that none of the bulbs is fused and B be event that exactly one bulb is fused
Probability that more than one bulb will fuse will be:

$$Pr(X \leq 6) - Pr(X \leq 1) = F_X(6) - F_X(1) \tag{15}$$

form (15)

$$= 0.0225925 \tag{16}$$

4) Probability that at least one bulb is fused is

$$Pr(X \leq 6) - Pr(X \leq 0) = \quad = F_X(6) - F_X(0) \tag{17}$$

form (17)

$$= 1 - (0.95)^5 = 0.2262190625 \tag{18}$$