

Assignment-1

(12.13.5.5)

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5.)Question: The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs

- 1) none
- 2) not more than one
- 3) more than one
- 4) at least one

will fuse after 150 days of use.

Solution: Let the **Probability** of i bulb to fuse after 150 days of use be $\Pr(i)$.

Probability of a bulb to fuse after 150 days is $=0.05$
Probability of one bulb getting fused is independent of another

Let

$$p = 0.05 \quad (1)$$

$$q = 1 - p = 0.95 \quad (2)$$

Let X be a random variable

Let Cumulative Distributive Function be:

$$F_X(i) = \Pr(X \leq i) \quad (3)$$

$$\Pr(X = i) = {}^5C_i p^i q^{5-i} \quad (4)$$

$$\therefore F_X(i) = \sum_{r=0}^i {}^5C_r p^r q^{5-r} \quad (5)$$

$$F_X(0) = 0.95^5 = 0.7737809375 \quad (6)$$

$$F_X(1) = (0.95)^5 + {}^5C_1 \times (0.5) \times (0.95)^4 \quad (7)$$

$$F_X(2) = (0.95)^5 + {}^5C_1 \times (0.5) \times (0.95)^4 \quad (8)$$

$$+ {}^5C_2 \times (0.5)^2 \times (0.95)^3 \quad (9)$$

$$F_X(5) = 1 \quad (10)$$

1) Probability that none of the 5 bulbs fuses is:

$$\Pr(X = 0) = F_X(0) \quad (11)$$

from (11)

$$= 0.95^5 \quad (12)$$

$$= 0.7737809375 \quad (13)$$

2) Probability that not more than one bulb fuses is same as exactly 0 bulb fuses or exactly one bulb fuses:

$$\Pr(X \leq 1) = F_X(1) = 0.9774075 \quad (14)$$

3) Let A be event that none of the bulbs is fused and B be event that exactly one bulb is fused
Probability that more than one bulb will fuse will be:

$$\Pr(1 < X \leq 5) \quad (15)$$

$$= F_X(5) - F_X(1) \quad (16)$$

from (15)

$$= 0.0225925 \quad (17)$$

4) Probability that at least one bulb is fused is

$$\Pr(1 \leq X \leq 5) \quad (18)$$

$$= F_X(5) - F_X(0) \quad (19)$$

from (18)

$$= 1 - (0.95)^5 = 0.2262190625 \quad (20)$$