Assignment-1
(12.13.5.5)

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12.)Question: Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

1) the youngest is a girl
2) at least one is a girl?

Solution: Let random variable $X = 0$,if it is a girl child and $X = 1$ if it is a boy.
It is given that:

$$\Pr(X = 1) = \Pr(X = 0) \tag{1}$$

$$\Pr(X = 1) + \Pr(X = 0) = 1 \tag{2}$$

$$\implies \Pr(X = 0) = \Pr(X = 1) = 0.5 \tag{3}$$

Let random variable $i = 1$ for first child and $i = 2$ for second child
We are supposed to find Probability of both girl child(for some given conditions)

$$\implies \Pr(X_{i=1} = 0) \times \Pr(X_{i=2} = 0) \tag{4}$$

This is because gender of each child is independent of another

1) If the youngest child is girl:

$$\Pr(X_{i=1} = 0) = 1 \tag{5}$$

$$\therefore \Pr(((X_{i=1} = 0)(X_{i=2} = 0))|X_{i=1} = 0) \tag{6}$$

$$= \Pr(X_{i=2} = 0) = 0.5 \tag{7}$$

2) If at least one of the child is girl possible cases are as follows:

$$\Pr(X_{i=1} = 0, X_{i=2} = 0) \tag{8}$$

$$\Pr(X_{i=1} = 1, X_{i=2} = 0) \tag{9}$$

$$\Pr(X_{i=1} = 0, X_{i=2} = 1) \tag{10}$$

and all of the three case are equally likely as probability of a child to be boy or girl is same
Let event

$$A = \Pr((X_{i=1} = 0)(X_{i=2} = 0)) \tag{11}$$

$$B = (X_{i=1} = 0 + X_{i=2} = 0) \tag{12}$$

$$\therefore \Pr(A|B) = \frac{1}{3}$$

Alternatively by conditional Probability we have

$$\Pr(P|Q) = \frac{\Pr(PQ)}{\Pr(Q)} \tag{13}$$

$$\implies \Pr(A|B) = \frac{\frac{1}{4}}{\frac{3}{4}}$$

$$= \frac{1}{3} \tag{14}$$