Assignment-1
(12.13.5.5)

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12.)Question: Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

1) the youngest is a girl
2) at least one is a girl?

Solution: Let random variable $X_i = 0$, if it is a girl child and $X_i = 1$ if it is a boy.
Where $i = 1$ for first child and $i = 2$ for second child.
It is given that:

$$\Pr(X_i = 1) = \Pr(X_i = 0) \tag{1}$$

$$\Pr(X_i = 1) + \Pr(X_i = 0) = 1 \tag{2}$$

$$\implies \Pr(X_i = 0) = \Pr(X_i = 1) = 0.5 \tag{3}$$

We are supposed to find Probability of both girl child(for some given conditions)

$$\implies \Pr(X_1 = 0) \times \Pr(X_2 = 0) \tag{4}$$

This is because gender of each child is independent of another

1) If the youngest child is girl:
   By conditional Probability we have

$$\Pr(P|Q) = \frac{\Pr(PQ)}{\Pr(Q)} \tag{5}$$

$$\therefore \Pr((X_1 = 0, X_2 = 0)|X_1 = 0) = \frac{\Pr(X_1 = 0, X_2 = 0)}{\Pr(X_1 = 0)} \tag{6}$$

$$\Pr(X_1 = 0, X_2 = 0) = \Pr(X_1 = 0) \times \Pr(X_2 = 0) \tag{7}$$

$$\implies \Pr(X_2 = 0) = 0.5 \tag{8}$$

2) If at least one of the child is girl:
   By conditional Probability we have

$$\Pr(P|Q) = \frac{\Pr(PQ)}{\Pr(Q)} \tag{9}$$

$$\implies \Pr(X_1 = 0, X_2 = 0|((X_1 = 0) + (X_2 = 0))) = \frac{\Pr(X_1 = 0, X_2 = 0)}{\Pr(X_1 = 0 + X_2 = 0)} \tag{10}$$

$$\Pr(X_1 = 0 + X_2 = 0) = 1 - \Pr(X_1 = 1, X_2 = 1) \tag{11}$$

$$= 1 - \frac{1}{4} = \frac{3}{4} \tag{12}$$

Probability of both are girl given that at least one is girl is
From (**??**)

$$\frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \tag{13}$$