Assignment-1
(12.13.5.5)

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12.)Question: Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

1) the youngest is a girl
2) at least one is a girl?

Solution: Let random variable $X_i = 0$, if it is a girl child and $X_i = 1$ if it is a boy.
Where $i = 1$ for first child and $i = 2$ for second child.
It is given that:

$$\Pr(X_i = 1) = \Pr(X_i = 0) \tag{1}$$
$$\Pr(X_i = 1) + \Pr(X_i = 0) = 1 \tag{2}$$
$$\implies \Pr(X_i = 0) = \Pr(X_i = 1) = 0.5 \tag{3}$$

We are supposed to find Probability of both girl child(for some given conditions)

$$\implies \Pr(X_1 = 0) \times \Pr(X_2 = 0) \tag{4}$$

This is because gender of each child is independent of another

Variables	Corresponding Child	Probability
$X_1 = 0$	Younger girl child	$\frac{1}{2}$
$X_1 = 1$	Younger boy child	$\frac{1}{2}$
$X_2 = 0$	Elder girl child	$\frac{1}{2}$
$X_2 = 1$	Elder boy child	$\frac{1}{2}$

TABLE II
VARIABLES DECLARATION

By **Moment Generating function** we have :

$$MGF_{X_1}(s) = \frac{1}{2} \times e^{0s} + \frac{1}{2} \times e^{1s} \tag{5}$$
$$MGF_{X_2}(s) = \frac{1}{2} \times e^{0s} + \frac{1}{2} \times e^{1s} \tag{6}$$
$$MGF_{X_1+X_2}(s) = MGF_{X_1}(s) \times MGF_{X_2}(s) \tag{7}$$
$$\implies MGF_{X_1+X_2}(s) = \frac{1}{4} \times e^{0s} + \frac{1}{2} \times e^{1s} + \frac{1}{4} \times e^{2s} \tag{8}$$
$$\Pr(X_1 + X_2 = i) = \text{coefficient of } e^{is} \tag{9}$$

Probabilities	Values
$\Pr(X_1 + X_2 = 2)$	$\frac{1}{4}$
$\Pr(X_1 + X_2 = 1)$	$\frac{1}{2}$
$\Pr(X_1 + X_2 = 0)$	$\frac{1}{4}$

TABLE IV
SOME USEFUL PROBABILITIES

1) If the youngest child is girl:
   By conditional Probability we have

$$\Pr((X_1 + X_2 = 0)|X_1 = 0) = \frac{\Pr(X_1 + X_2 = 0)}{\Pr(X_1 = 0)} \tag{10}$$
$$\Pr(X_1 + X_2 = 0) = \Pr(X_1 = 0) \times \Pr(X_2 = 0) \tag{11}$$
$$\implies \Pr(X_2 = 0) = 0.5 \tag{12}$$

2) If at least one of the child is girl:
   By conditional Probability we have

$$\Pr\left(X_1 + X_2 = 0 | (X_1 + X_2 \leq 1)\right) = \frac{\Pr\left(X_1 + X_2 = 0\right)}{\Pr\left(X_1 + X_2 \leq 1\right)} \tag{13}$$

$$\Pr\left(X_1 + X_2 \leq 1\right) = 1 - \Pr\left(X_1 + X_2 = 2\right) \tag{14}$$

$$= 1 - \frac{1}{4} = \frac{3}{4} \tag{15}$$

Probability of both are girl given that at least one is girl is
From (13)

$$\frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \tag{16}$$