Assignment-1
(12.13.5.5)

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12.)Question: Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that
1) the youngest is a girl
2) at least one is a girl?

Solution: Let random variable $X_i = 0$, if it is a girl child and $X_i = 1$ if it is a boy.
Where $i = 1$ for first child and $i = 2$ for second child.
It is given that:

$$\Pr(X_i = 1) = \Pr(X_i = 0) \quad (1)$$

$$\Pr(X_i = 1) + \Pr(X_i = 0) = 1 \quad (2)$$

$$\implies \Pr(X_i = 0) = \Pr(X_i = 1) = 0.5 \quad (3)$$

We are supposed to find Probability of both girl child(for some given conditions)

$$\implies \Pr(X_1 = 0) \times \Pr(X_2 = 0) \quad (4)$$

This is because gender of each child is independent of another
1) If the youngest child is girl:

$$\Pr(X_1 = 0) = 1 \quad (5)$$

$$\therefore \Pr((X_1 = 0, X_2 = 0)|X_1 = 0) \quad (6)$$

$$= \Pr(X_2 = 0) = 0.5 \quad (7)$$

2) If at least one of the child is girl possible cases are as follows:

$$\Pr(X_1 = 0, X_2 = 0) \quad (8)$$

$$\Pr(X_1 = 1, X_2 = 0) \quad (9)$$

$$\Pr(X_1 = 0, X_2 = 1) \quad (10)$$

and all of the three case are equally likely as probability of a child to be boy or girl is same
Let event

$$A = \Pr(X_1 = 0, X_2 = 0) \quad (11)$$

$$B = (X_1 = 0 + X_2 = 0) \quad (12)$$

$$\therefore \Pr(A|B) = \frac{1}{3}$$

Alternatively by conditional Probability we have

$$\Pr(P|Q) = \frac{\Pr(PQ)}{\Pr(Q)} \quad (13)$$

$$\implies \Pr(A|B) = \frac{\frac{1}{4}}{\frac{3}{4}}$$

$$= \frac{1}{3} \quad (14)$$