

# Assignment-1

## (12.13.5.5)

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**5.)Question:** The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs

- 1) none
- 2) not more than one
- 3) more than one
- 4) at least one

will fuse after 150 days of use.

**Solution:** Let the **Probability** of  $i$  bulb to fuse after 150 days of use be  $\Pr(i)$ .

Probability of a bulb to fuse after 150 days is  $=0.05$   
Probability of one bulb getting fused is independent of another

Let

$$p = 0.05 \quad (1)$$

$$q = 1 - p = 0.95 \quad (2)$$

Let Cumulative Distributive Function be:

$$F_X(i) = \Pr(X \leq i) \quad (3)$$

$$F_X(i) = \sum_{r=0}^i {}^5C_r p^r q^{5-r} \quad (4)$$

$$F_X(0) = \Pr(X = 0) = \Pr(1'2'3'4'5') = \prod_{i=1}^{i=5} (1 - \Pr(i)) \quad (5)$$

$$= 0.95^5 = 0.7737809375 \quad (6)$$

$$F_X(1) = F_X(0) + \Pr(X = 1) \quad (7)$$

$$= (0.95)^5 + {}^5C_1 \times (0.5) \times (0.95)^4 \quad (8)$$

$$F_X(2) = F_X(1) + \Pr(X = 2) \quad (9)$$

$$= (0.95)^5 + {}^5C_1 \times (0.5) \times (0.95)^4 \quad (10)$$

$$+ {}^5C_2 \times (0.5)^2 \times (0.95)^3 \quad (11)$$

$$F_X(6) = 1 \quad (12)$$

- 1) Probability that none of the 5 bulbs fuses is:

$$F_X(0) \quad (13)$$

form (10)

$$= 0.95^5 \quad (14)$$

$$= 0.7737809375 \quad (15)$$

- 2) Probability that not more than one bulb fuses is same as exactly 0 bulb fuses or exactly one bulb fuses:

$$F_X(1) = 0.9774075 \quad (16)$$

- 3) Let A be event that none of the bulbs is fused and B be event that exactly one bulb is fused  
Probability that more than one bulb will fuse will be:

$$F_X(6) - F_X(1) \quad (17)$$

form (14)

$$= 0.0225925 \quad (18)$$

- 4) Probability that at least one bulb is fused is

$$F_X(6) - F_X(0) \quad (19)$$

form (16)

$$= 1 - (0.95)^5 = 0.2262190625 \quad (20)$$