Assignment-1
(12.13.5.5)

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5.)Question: The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs

1) none
2) not more than one
3) more than one
4) at least one

will fuse after 150 days of use.

Solution: Let the **Probability** of i^{th} bulb to fuse after 150 days of use be $P(i)$.
$P(i) = 0.05 \ \forall i \in \{1, 2, 3, 4, 5\}$
Probability of one bulb getting fused is independent of another

1) Probability that none of the 5 bulbs fuses is:

$$P(\bar{1} \cap \bar{2} \cap \bar{3} \cap \bar{4} \cap \bar{5}) = \prod_{i=1}^{i=5}(1 - P(i)) \quad (1)$$

using equation (1)

$$= (1 - 0.05)^5 = 0.95^5$$

$$= 0.7737809375$$

2) Probability that not more than one bulb fuses is same as exactly 0 bulb fuses or exactly one bulb fuses:
Probability of no bulb to fuse is:

$$(0.95)^5$$

Probability of exactly one bulb to fuse is:

$$\binom{5}{1} \times P(1 \cap \bar{2} \cap \bar{3} \cap \bar{4} \cap \bar{5}) \quad (2)$$

(choosing which bulb is defective)
using equation (2)

$$= 5 \times (0.95)^4 \times 0.05 = 0.2036265625$$

3) Let A be event that none of the bulbs is fused and B be event that exactly one bulb is fused

Probability that more than one bulb will fuse will be:

$$= 1 - P(A) - P(B) \quad (3)$$

using equation (3)

$$= 1 - (0.95)^5 - 5 \times 0.05 \times (0.95)^4$$

$$= 0.0225925$$

4) Probability that at least one bulb is fused is

$$1 - \text{Probability that no bulb is fused} \quad (4)$$

using equation (4)

$$= 1 - (0.95)^5 = 0.2262190625$$