AC CIRCUITS

Chapter at a Glance

Series AC circuits R-L, R-C, R-L-C circuits. Impedance, Reactance, Phasor diagram, Impedance Triangle, Power Factor, Average power, Apparent power, Reactive power, Power triangle (Numerical)

R L C Series Circuit

1. The three elements *R*, *L* and *C* as shown in figure below are connected in series across an ac supply of R.M.S. voltage.

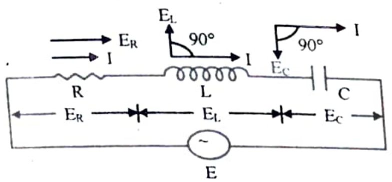


Fig. (a)

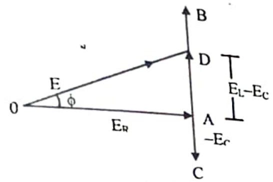


Fig. (b)

Assuming that: -

$E_R = IR =$ Voltage drop across '*R*' (in phase with I)

$E_L = IX_L =$ Voltage drop across '*L*' (leading I by $\pi/2$)

$E_C = IX_C =$ Voltage drop across '*C*' (lagging I by $\pi/2$)

X_L And X_C are the inductive and capacitive reactance

$X_L = 2\pi fL$ And $X_C = 1/\omega C = 1/2\pi fL$

Referring figure (b), $OD = \sqrt{(OA^2 + AD^2)}$

i.e. $$E = \sqrt{\left[(IR)^2 + (IX_L - IX_C)^2\right]} = I\sqrt{\left[R^2 + (X_L - X_C)^2\right]}$$

$$\therefore \quad I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{E}{\sqrt{R^2 + X^2}} = \frac{E}{Z}$$

The term $\sqrt{R^2 + (X_L - X_C)^2}$ is known as the impedance of the circuit

Phase angle ϕ is given by $\tan\phi = \dfrac{(X_L - X_C)}{R} = \dfrac{X}{R}$ = Net reactance / resistance.

Power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$

Hence, it is seen that if the equation of the applied voltage is $e = E_m \sin\omega t$.

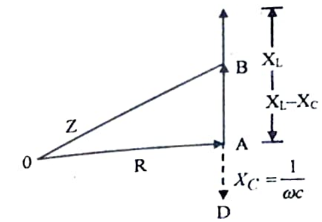


Fig. (c)

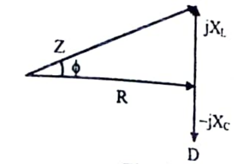


Fig. (d)

Then equation of the resulting current in an R-L-C circuit is given by $i = I_m \sin(\omega t \pm \phi)$.

The +ve sign is to be used when current leads i.e., when $X_C > X_L$ and the –ve sign is to be used when current lags i.e., when $X_L > X_C$

In general, the current lags or leads the supply voltage, by an angle ϕ such that $\tan\phi = \dfrac{X}{R}$.

Using symbolic notation [figure (d)], $Z = R + j(X_L + X_C)$

Its phase angle is $\phi = \tan^{-1}\left[(X_L - X_C)/R\right]$

$$Z = Z\angle\tan^{-1}(X_L - X_C)/R$$

If $\quad V = V\angle 0$, then $I = \dfrac{V}{Z}$

Power $(P) = VI\cos\phi$

Apparent Power:

Apparent power *(S)* is the power delivered to an electrical circuit. The measurement of apparent power is in volt amperes (VA).

$S = I^2Z = IE$

Where, S = apparent power (VA)
I = RMS current (A)
E = RMS voltage (V)
Z = impedance (Ω)

True Power:

True power *(P)* is the power consumed by the resistive loads in an electrical circuit. Equation below is a mathematical representation of true power. The measurement of true power is in watts.

$P = I^2R = EI\cos\theta$

where, P = true power (watts)
I = RMS current (A)
E = RMS voltage (V)
R = resistance (Ω)
θ = angle between E and I sine waves

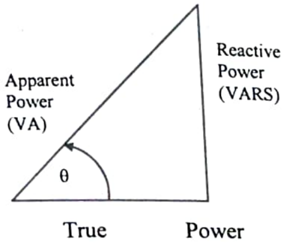
Reactive Power:

Reactive power (Q) is the power consumed in an AC circuit because of the expansion and collapse of magnetic (inductive) and electrostatic (capacitive) fields. Reactive power is expressed in volt-amperes-reactive (VAR). Equation below is a mathematical representation for reactive power.

$Q = I^2X = EI\sin\theta$

where, Q = reactive power (VAR)
I = RMS current (A)
X = net reactance (Ω)
E = RMS voltage (V)
θ = angle between E and I sine waves

Power Triangle:



Multiple Choice Type Questions

1.1. If the peak value of a sine wave is 100 volts, then its *rms* value will be [WBUT 2007]
a) 70.7 V b) 63.6 V c) 100 V d) 88 V
Answer: (a)

1.2. The power factor of a purely inductive circuit is [WBUT 2008, 2011, 2014]
a) zero b) one c) infinity d) 0.5
Answer: (a)

1.3. For an inductive circuit, current [WBUT 2008(EVEN)]
a) lags the voltage
b) leads the voltage
c) is in phase with the voltage
d) is independent of the voltage phase
Answer: (a)

1.4. A resistance of 8.0Ω and an inductive reactance of 6.0Ω will offer an impedance of [WBUT 2009(EVEN)]
a) 14 Ω b) 10 Ω c) 11 Ω d) none of these
Answer: (b)

1.5. If $e_1 = A\sin\omega t$ and $e_2 = B\sin(\omega t - \phi)$, then [WBUT 2009]
a) e_1 lags e_2 by ϕ b) e_2 lags e_1 by ϕ c) e_2 leads e_1 by ϕ d) e_1 is in phase with e_2
Answer: (b)

1.6. The form factor of a wave is 1. Its shape is [WBUT 2010, 2011, 2015]
a) sinusoidal b) triangular c) square d) sawtooth
Answer: (a)

1.7. Inductive reactance of a coil of inductance 0.2 H at 50 Hz is [WBUT 2011]
a) 62.8 Ω b) 628 Ω c) 0.2 Ω d) 20 Ω
Answer: (a)

1.8. In an electrical circuit, if the current lags the voltage by 60°, the circuit nature is [WBUT 2012]
a) $R-C$ b) $R-L$ c) LC d) none of these
Answer: (b)

1.9. If $E_1 = A\sin\omega t$ and $E_2 = A\sin(\omega t - \theta)$, then [WBUT 2012]
a) E_1 lags E_2
b) E_2 lags E_1
c) E_1 and E_2 are in phase
d) none of these
Answer: (b)

1.10. Time constant of LR circuit is given by [WBUT 2013]
a) L/R b) R/L c) 1/LR d) LR
Answer: (a)

1.11. A sinusoidal voltage is represented by $v = 141.4\sin(314.18t - 90°)$. The r.m.s. value of the voltage, its frequency and phase angle are respectively [WBUT 2013]
a) 141.42V, 314.16Hz, 90°
b) 100V, 50Hz, –90°
c) 87.92V, 60Hz, 90°
d) 200V, 56Hz, –90°
Answer: (b)

1.12. The admittance of a parallel circuit is $0.5\angle -30°$. The circuit is [WBUT 2016]
a) inductive b) capacitive c) resistive d) in resonance
Answer: (a)

1.13. In a three-phase star connected system, the relation between the phase and the line voltage is [WBUT 2006, 2008(EVEN)]
a) $V_p = V_L$ b) $V_p = \sqrt{3}\,V_L$ c) $V_p = \frac{V_L}{\sqrt{3}}$ d) $V_p = \frac{V_L}{3}$
Answer: (c)

1.14. In a 3 phase system, the *emfs* are [WBUT 2007, 2009(ODD)]
a) 30° apart b) 60° apart c) 90° apart d) 120° apart
Answer: (d)

1.15. The form factor of a current waveform is 1. The shape of the waveform is **[WBUT 2018]**

a) sinusoidal b) triangular c) square d) saw tooth

Answer: (c)

Hint: Form factor $= \frac{V_{rms}}{V_{avg}} = 1$, for square wave.

1.16. Two alternating currents are represented by $I_1 = \sin(\omega t - 30^\circ)$ **and** $I_2 = \sin(\omega t + 30^\circ)$ **[WBUT 2018]**

a) I_1 leads I_2 by 60° b) I_1 lags I_2 by 60°

c) I_2 leads I_1 by 30° d) I_2 lags I_2 by 30°

Answer: (b)

Short Answer Type Questions

2.1. Derive an expression of **[WBUT 2007]**
a) average
b) r.m.s value of a half-wave rectified voltage wave

OR,

Derive the expression of (i) average (ii) R.M.S. value of a half-wave rectified voltage wave. **[WBUT 2009]**

OR,

Deduce an expression of average and RMS value of a half wave rectified voltage wave. **[WBUT 2016]**

Answer:

Mathematical derivation: Average Value

$$V_{avg} = \frac{1}{2\pi}\int_0^\pi V_m \sin\omega t\, d(\omega t) = \frac{1}{2\pi}(-V_m\cos\omega t)_0^\pi = \frac{-V_m}{2\pi}(0-1-1) = \frac{1}{\pi}V_m = 0.319V_m$$

Similarly, $I_{avg} = \frac{1}{\pi}\int_0^\pi I_m \sin\omega t\, d(\omega t) = 0.319 I_m$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^\pi V_m^2 \sin^2\omega t\, d(\omega t)} = V_m\sqrt{\frac{1}{4\pi}\int_0^\pi (1-\cos 2\omega t)\, d(\omega t)}$$

$$= V_m\sqrt{\frac{1}{4\pi}\int_0^\pi d(\omega t) - \frac{1}{4\pi}\int_0^\pi \cos 2\omega t\, d(\omega t)} = V_m\sqrt{\frac{1}{4} - \frac{1}{8\pi}[\sin 2\omega t]_0^\pi}$$

$$= V_m/2 = 0.5V_m$$

Similarly, $I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^\pi I_m^2 \sin^2\omega t\, d(\omega t)} = 0.5 I_m$

2.2. An alternating voltage is represented by $v = 62.35\sin 323t$. Determine (a) the maximum value (b) r.m.s. value (c) average value (d) frequency of the wave (e) form factor. **[WBUT 2008]**

Answer:

$v = 62.35\sin 323t$

(a) the maximum value = 62.35 V

(b) r.m.s. value $= \frac{\text{maximum value}}{\sqrt{2}} = \frac{62.35}{\sqrt{2}}V = 44.088\text{V}$

(c) average value $= \frac{1}{\pi}\int_0^\pi v_m \sin\omega t\, d(\omega t) = \frac{v_m}{\pi}[-\cos\omega t]_0^\pi = \frac{2v_m}{\pi} = 0.637\times 62.35\text{V} = 39.7$

(d) $2\pi f = 323$

Frequency $f = \frac{323}{2.\pi}\text{ Hz} = 51.40\text{Hz}$

(e) Form factor $= \frac{\text{r.m.s. value}}{\text{mean value}} = \frac{44.088}{39.7} = 1.11$

2.3. A full wave rectified sinusoidal voltage is clipped at $\frac{1}{\sqrt{2}}$ of its maximum value. Calculate the average and *r.m.s.* value of such a voltage waveform. Also calculate the form factor and peak factor. **[WBUT 2008(EVEN)]**

Answer:

The average value of the rectified sinusoidal voltage

$$V_{avg} = \frac{1}{\pi}\int_0^\pi \frac{V_m}{V_2}\sin\omega t\, d(\omega t) = \frac{V_m}{\sqrt{2}\pi}\int_0^\pi \sin\omega t\, d(\omega t)$$

$$= \frac{V_m}{\sqrt{2}\pi}[-\cos\omega t]_0^\pi = \frac{-V_m}{\sqrt{2}\pi}[-1-1] = \frac{\sqrt{2}V_m}{\pi} = 0.45V_m$$

The r.m.s value of the rectified waveform

$$= \sqrt{\frac{1}{\pi}\int_0^\pi \left(V_m/\sqrt{2}\right)^2 \sin^2\omega t\, d(\omega t)}$$

$$= \frac{V_m}{\sqrt{2\pi}}\sqrt{\frac{1}{2}\int_0^\pi 2\sin^2\omega t\, d(\omega)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\int_0^\pi (1-\cos^2\omega t)\, d(\omega t)}$$

$$= \frac{V_m}{2\sqrt{\pi}}\sqrt{(\pi-0) + \left.\frac{\sin^2\omega t}{2}\right|_0^\pi} = \frac{V_m}{2} = 0.5V_m$$

Form factor $= \frac{\text{r.m.s value}}{\text{peak value}} = \frac{0.5V_m}{0.45V_m} = 1.11$

Peak factor $= \frac{\text{peak value}}{\text{r.m.s value}} = \frac{V_m}{0.5V_m} = 2$

2.4. Define R.M.S. value of alternating quantity & derive its expression for sinusoidal current. **[WBUT 2010(EVEN)]**

Answer:

Effective value of AC is the amount of AC that produces the same heating effect as an equal amount of DC. In simpler terms, one-ampere effective value of AC will produce the same amount of heat in a conductor, in a given time, as one ampere of DC. The heating effect of a given AC current is proportional to the square of the current. Effective value of AC can be calculated by squaring all the amplitudes of the sine wave over one period, taking the average of these values, and then taking the square root.
The effective value, being the root of the mean (average) square of the currents, is known as the root-mean-square, or RMS value.

Explanation (RMS value)

To understand the concept of effective value let us consider a sinusoidal current (I) wave (Figure 1).
The values of I are plotted on the upper curve, and the corresponding values of I^2 are plotted on the lower curve. The I^2 curve has twice the frequency of I and varies above and below a new axis. The new axis is the average of the I^2 values, and the square root of that value is the RMS, or effective value, of current.

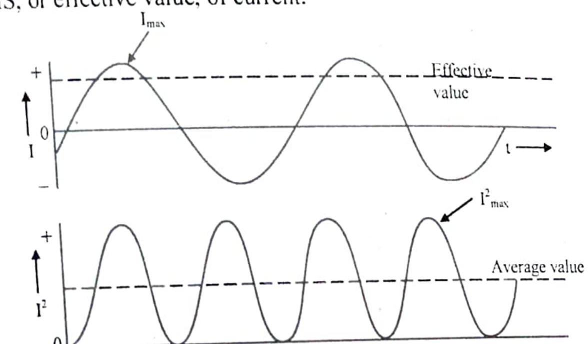


Fig: 1 Effective value of current

There are six basic equations that are used to convert a value of AC voltage or current to another value, as listed below.
The values of current (I) and voltage (E) that are normally encountered are assumed to be RMS values; therefore, no subscript is used.
Another useful value is the **average value** of the amplitude during the positive half of the cycle. The mathematical relationship between I_{avg}, I_{max}, and I_{rms} have been deduced below.
Mathematically,

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2 \omega t\, d(\omega t)} = V_m\sqrt{\frac{1}{4\pi}\int_0^{2\pi}(1-\cos 2\omega t)\, d(\omega t)}$$

$$= V_m\sqrt{\frac{1}{4\pi}\int_0^{2\pi} d(\omega t) - \frac{1}{4\pi}\int_0^{2\pi}\cos 2\omega t\, d(\omega t)} = V_m\sqrt{\frac{1}{2}-\frac{1}{8\pi}[\sin 2\omega t]_0^{2\pi}}$$

$$= V_m/\sqrt{2} = 0.707V_m$$

Similarly, $I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I_m^2 \sin^2 \omega t\, d(\omega t)} = 0.707\, I_m$

The **average value** of the amplitude of a sine wave is always calculated during the positive half of the cycle, since the average value evaluated over a complete cycle would be zero. The mathematical relationship between I_{avg} and I_{max} have been deduced below.

Mathematical derivation:

Average Value

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t\, d(\omega t) = \frac{1}{\pi}(-V_m\cos\omega t)_0^{\pi} = \frac{-V_m}{\pi}(0-1-1) = \frac{2}{\pi}V_m = 0.637V_m$$

Similarly, $I_{avg} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\omega t\, d(\omega t) = 0.637\, I_m$

Hence, $V_{av} = 0.637\, V_{max} = 0.90\, V_{rms}$
$I_{av} = 0.637\, I_{max} = 0.90\, I_{rms}$

2.5. At $t = 0$, the instantaneous value of a 50 Hz, sinusoidal current is 5 Amp and increases in magnitude further. Its R.M.S. value is 10 Amp.
a) Write the expression for its instantaneous value
b) Find the current at $t = 0.01$ and $t = 0.015$ sec
c) Sketch the waveform indicating these values. **[WBUT 2010]**

Answer:

The R.M.S. value of the current is 10 Amp.

Hence, the peak value $= \sqrt{2}\times 10A = 14.14A$

The expression of instantaneous value of current $i = I_m \sin(\omega t + \phi)$

The phase angle is leading since the value of function (current) is positive at t = 0.

$I_m = 14.14\ A$

At $t = 0$

$i = I_m \sin\phi = \sqrt{2}\times 10 \sin\phi = 5$

or, $\sin\phi = \frac{1}{2\sqrt{2}} = 0.35$

or, $\phi = \sin^{-1} 0.35 = 20.7°$

(a) Hence, the expression of instantaneous value of current can be written as

$$i = 14.14\sin(\omega t + 20.7°)$$

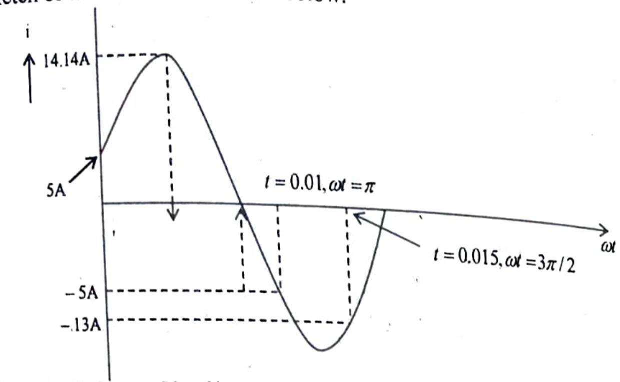
(b) The value of current at $t = 0.01$sec can be computed as follows:

$$i = 14.14\sin\left(2\pi\times 50\times 0.01\times\frac{180°}{\pi} + 20.7°\right) = 14.14 \sin 200.7° = -5A$$

Value of current at $t = 0.015\text{sec}$

$$= i = 14.14\sin(2\pi \times 50 \times 0.015 \times 180°/_{\pi} + 20.7°)A = -13.2A$$

(c) The sketch of the waveform is drawn below.



$\omega = 2\pi f$ rad/sec $= 2 \times \pi \times 50$ rad/sec $= 100\pi$ rad/sec

2.6. Derive a mathematical expression for r.m.s. value of a sinusoidal voltage $v = V_m \sin\omega t$. **[WBUT 2011]**

Answer:

The elementary AC generator (Figure 1) consists of a conductor or loop of wire in a magnetic field that is produced by an electromagnet. The two ends of the loop are connected to slip rings, and they are in contact with two brushes. When the loop rotates it cuts magnetic lines of force, first in one direction and then the other.

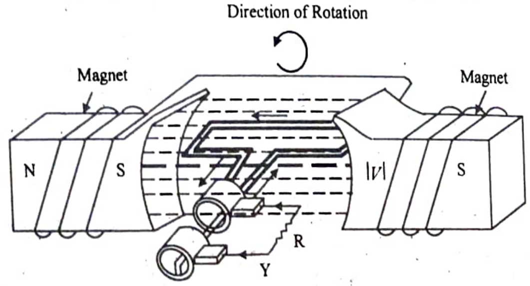


Fig: 1 Simple AC Generator

Development of a Sine-Wave Output

Step1:

At the instant the loop is in the vertical position (Figure 2, 0°), the coil sides are moving parallel to the field and do not cut magnetic lines of force. In this instant, there is no voltage induced in the loop.

Step2:

As the coil rotates in a counter-clockwise direction, the coil sides will cut the magnetic lines of force in opposite direction. The direction of the induced voltages depends on the direction of movement of the coil. The potential drop across resistor R will cause a current to flow through it.

Step3:

The voltage and hence current continues to increase until it reaches a maximum value when the coil is perpendicular to the magnetic lines of force (Figure 2, 90°) and is cutting the greatest number of magnetic lines of force.

Step4:

As the coil continues to turn, the voltage and current induced decrease until they reach zero, where the coil is again in the horizontal position (Figure 2, 180°).

Step5:

In the other half revolution, an equal voltage is produced except that the polarity is reversed (Figure 2, 270°, 360°). The current flow through R is in the opposite direction. The periodic reversal of polarity results in the generation of a voltage, as shown in Figure 2. The rotation of the coil through 360° results in an AC sine wave output.

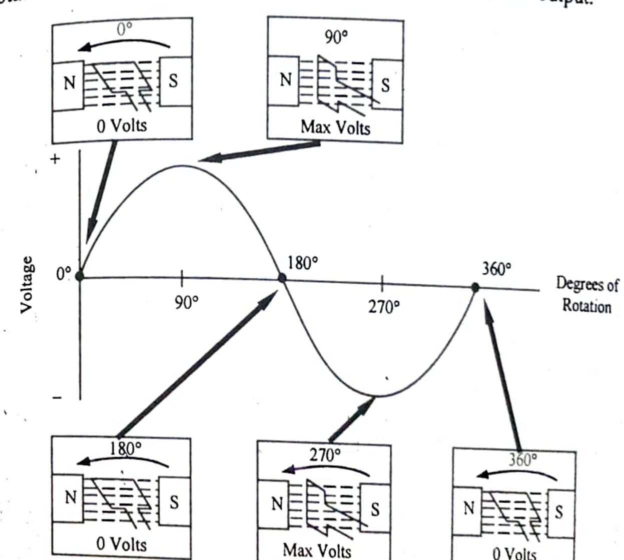


Fig: 2 Developing a Sine-Wave Voltage

AC generation is summarized below:

- A simple generator consists of a conductor loop turning in a magnetic field, cutting across the magnetic lines of force.
- The sine wave output is the result of one side of the generator loop cutting lines of force. In the first half turn of rotation this produces a positive current and in

the second half of rotation produces a negative current. This completes one cycle of AC generation.

- When a voltage is produced by an AC generator, the resulting current varies in step with the voltage. As the generator coil rotates from 0° to 360°, the output voltage goes through one complete cycle.
- In one cycle, the voltage increases from zero to E_{max} in one direction, decreases to zero, increases to E_{max} in the opposite direction (negative $E_{\cdot}$), and then decreases to zero again.
- The value of E_{max} occurs at 90° and is referred to as peak voltage.
- The time taken by the waveform to complete one cycle is called the *period*, and the number of cycles per second is called the frequency (measured in *hertz*).
- The output voltage of an AC generator is sinusoidal in nature, as depicted in the figure 2. It can be expressed mathematically as

$E = E_{max} \sin\theta$

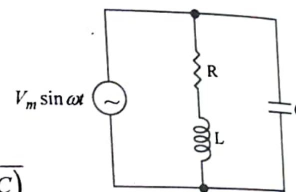
where, $\theta = \omega t$ and

$\omega = 2\pi f$, f being the frequency of the supply expressed in Hertz.

2.7. Derive an expression for the resonant frequency of a parallel circuit, one branch consisting of a coil of inductance *L* and a resistance *R* and the other branch of capacitance C. **[WBUT 2012, 2015]**

Answer:

The circuit diagram is drawn as given



$$\text{The load impedance} = \frac{\frac{-j}{\omega C}\cdot(R + j\omega L)}{\frac{-j}{\omega C} + R + j\omega L}$$

$$= \frac{\omega L - jR}{-j + R\omega C + j\omega^2 LC} = \frac{\omega L - jR}{R\omega C - j(1-\omega^2 LC)}$$

$$= \frac{(\omega L - jR)\left[R\omega C + j(1-\omega^2 LC)\right]}{\left[R\omega C - j(1-\omega LC)\right]}$$

$$= \frac{R\omega^2 LC - jR^2\omega C + j\omega L(1-\omega^2 LC) + R(1-\omega^2 LC)}{\left[R\omega C - j(1-\omega LC)\right]\left[R\omega C + j(1-\omega^2 LC)\right]}$$

$$= \frac{R\omega^2 LC - jR^2\omega C + j\omega L - j\omega^3 LC + R - R\omega^2 LC}{D}$$

For the impedance to be purely resistive,

$$-jR^2\omega C + j\omega L - j\omega^3 LC = 0$$

$$R^2C - L + \omega^2 LC = 0$$

$$\omega^2 = \frac{L - R^2C}{LC}$$

$$\text{Resonant frequency, } \omega_r = \sqrt{\frac{L - R^2C}{LC}}.$$

2.8. A two element series circuit consumes 700 V of power and has power factor of 0.707 leading when energized by a voltage source of waveform $v = 141\sin(314t + 30°)$. Find out the circuit elements. **[WBUT 2013]**

Answer:

Power consumed in a coil $= V_{rms} I_{rms} \cos\phi$

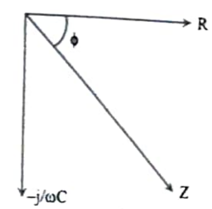
For this problem, $V_{rms} = \frac{V_{peak}}{V_2} V = \frac{141}{\sqrt{2}} V \simeq 100V$

As given, $V_{rms} I_{rms} \cos\phi = 700$

$I_{rms} = \frac{700}{100\times 0.707} A = 7\sqrt{2}\, A$

Impedance of the coil $= \frac{V_{rms}}{I_{rms}} = \frac{141}{\sqrt{2}} \times \frac{1}{7\sqrt{2}} \Omega = \frac{141}{14} \Omega = 10.04\Omega$

The power factor of the coil is 0.707 leading which means that the coil has a resistor and a capacitor. The phasor diagram for the impedance is drawn below.



$|z| = 10.04\Omega$

$\phi = \cos^{-1} 0.707 = 45°$

$R = 10.04\cos 45° = 10.04\times 0.707 = 7.1\Omega$

$X_C = \frac{1}{\omega C} = 10.04\times \sin 45°\Omega = 10.04\times 0.707 = 7.1\Omega$

$C = \frac{1}{\omega X_C} = \frac{1}{2\pi f\times 7.1} F = \frac{1}{314\times 7.1} F = 0.45\,\text{mF}$

So, the circuit elements are a 7.1Ω resistor and 0.45 mF capacitor.

2.9. Two impedances $Z_1 = (47.92 + j76.73)\Omega$ and $Z_2 = (10 - j5)\Omega$ are connected in parallel across a 200 volt, 50 Hz supply. Find the current through each impedance and total current. What is the phase difference angle of each branch current with respect to the applied voltage? **[WBUT 2014]**

Answer:

Two impedances $Z_1 = (47.92 + j76.73)\Omega$ and $Z_2 = (10 - j5)\Omega$ are connected in parallel across a 200V, 50Hz supply

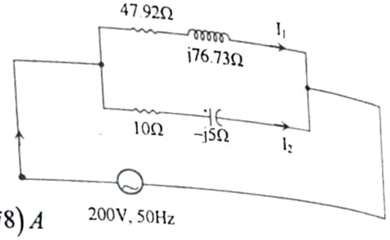
$Z_1 = (47.92 + j76.73)\Omega = 90.46\angle 58^\circ \Omega$

$Z_2 = (10 - j5)\Omega = 11.18\angle -26.56^\circ \Omega$

$\dot{I}_1 = \dfrac{200}{90.46\angle 58^\circ} A = 2.2\angle -58^\circ A$

$= (1.166 - j1.866) A$

$\dot{I}_2 = \dfrac{200}{11.18\angle -26.56^\circ} = 17.9\angle 26.56^\circ A = (16 + j8) A$



So current through $Z_1 = 2.2A$

and current through $Z_2 = 17.9A$

Total current $= (1.166 - j1.866 + 16 + j8) A = (17.166 + j6.134) A = 18.23\angle 20^\circ A$

Hence total current = 18.23A

Phase different of current voltage source and $Z_1 = 58^\circ$ and that between $Z_2 = 26.56^\circ A$

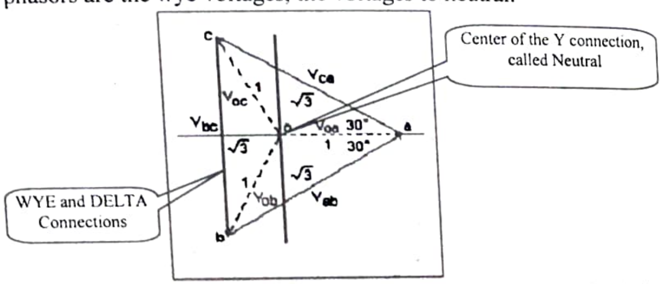
2.10. What is a three-phase balanced A.C. system? Show that, in a three-phase balanced a.c. circuit, the sum of current in the neutral is zero. [WBUT 2012]

Answer:

1st Part:

The key to understanding three-phase is to understand the phasor diagram for the voltages or currents. In the diagram, shown below, a, b and c represent the three lines, and o represents the neutral. The phasors are drawn by the solid lines bc, ca, ab are the line or delta voltages, the voltages between the wires.

The dotted phasors are the wye voltages, the voltages to neutral.



The center of the Y connection is, in a way, equidistant from each of the three line voltages, and will remain at a constant potential. It is called the ***neutral***.

They correspond to the two different ways a symmetrical load can be connected.

We imagine the vectors to be rotating anticlockwise with time with angular velocity $\omega = 2\pi f$, their projections on the horizontal axis representing the voltages as functions of time. e.g. V_{ab} is the voltage at point a relative to point b.

The same phasor diagram holds for the currents.

In this case, the line currents are the vectors, marked by the dotted lines and the vectors marked by the bold lines are the currents through a delta load.

The dotted and bold vectors

- Differ in phase by 30°, and
- In magnitude by a factor of $\sqrt{3}$, as is marked in the diagram.

2nd Part:

In the star connected system, R_1, Y_1 & B_1 and joined together to form a neutral point, thus reducing the total number of conductors to three. If the neutral wire is also taken out the system will be a 3-phase 4 wire system. Such interconnection of the three phase is called star connection and is designated by a symbol Y. The junction of the three wires is normally called star or neutral point. If i_R, i_Y & i_B are the instantaneous values of the currents in the three phases, then their sum should be equal to zero to satisfy the property of interconnection. These currents may be represented by the following equations for a 3 phase symmetrical balanced system.

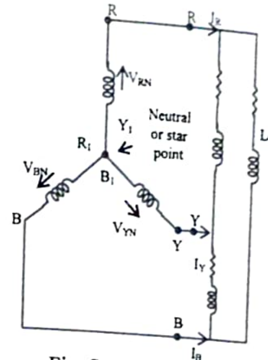


Fig: Star connected system

$$i_R = I_{max} \sin\theta$$
$$i_Y = I_{max} \sin(\theta - 120^\circ)$$
$$i_B = I_{max} \sin(\theta - 240^\circ)$$
$$i_R + i_Y + i_B = I_{max}[\sin\theta + \sin(\theta - 120^\circ) + \sin(\theta - 240^\circ) = 0$$

Thus in a balanced 3 phase start connected circuit, the sum of the instantaneous currents in the three phases is always zero on the current in the neutral conductor is zero at every instant.

2.11. Proof that for a balanced start connected supply system connected to a balanced star connected load, the current through the neutral wire is zero. [WBUT 2014]

Answer:

In case of star-connected system V_{RN}, V_{YN} and V_{YN} are the rms values of the phase voltages.

The voltages across the lines, V_{RY}, V_{YB} and V_{BR} are obtained from the phase voltages as shown below.

Line voltage across terminals R and Y,

$$V_{RY} = V_{RN} + V_{NY} = V_{RN} + (-V_{YN}) = V_{RN} - V_{YN} \text{ (Phasor difference)}$$

Similarly, line voltage across terminals Y and B, $V_{YB} = V_{YN} - V_{BN}$ and the line voltage across terminals B and R, $V_{BR} = V_{BN} - V_{RN}$. Hence the line voltages in a star-connected system are obtained by subtracting vectorially, the concerned phase voltages. Line voltages so obtained are equal and spaced 120°, apart, that is V_{YB} is lagging V_{RY} by 120°

and V_{BR} is lagging V_{YB} by 120°, thus maintaining the phase sequence of the system as RYB. The phase voltages V_{RN}, V_{YN} and V_{BN} have been represented in Fig. 1, assuming the phase sequence of the system as RYB and taking V_{RN} as the reference phasor. The line voltage V_{RY} is obtained by adding V_{RN} and $V_{NY}\left(-V_{NY}\right)$ vectorially. The other line voltages V_{YB} and V_{BR} are obtained in a similar manner. The complete phasor diagram showing phase and line voltages is shown in Fig. 1, in which the phase current is lagging its own phase voltage by an angle ϕ. Referring to Fig. 1, it is quite clear that the line voltages are 30° ahead of the phase voltages and are given by,

Line voltage $V_{RY} = V_{RN} - V_{YN} = 2V_{RN}\cos 30° = 2V_{ph} \times \sqrt{3}/2 = \sqrt{3}V_{ph}$

Hence, in a star-connected, 3-phase system,

Line voltage, $V_L = \sqrt{3}$ phase voltage $\left(V_{ph}\right)$ ... (1)

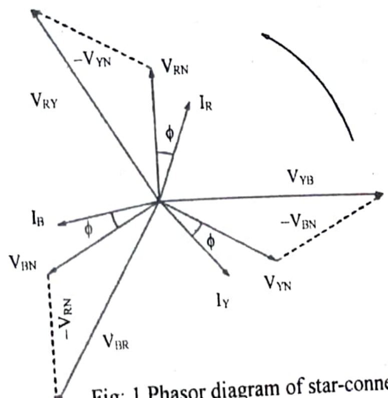


Fig: 1 Phasor diagram of star-connected circuit

It is quite obvious that the current in the line is equal to the phase current. Hence in a star connected 3-phase system,

Line current, I_L = phase current I_{ph} ... (2)

The angle between the line current and the corresponding the voltage is $(30° + \phi)$ in case of lagging loads and $(30° - \phi)$ in case of leading loads.

2.12. A star connected three-phase load draws a current of 15 A at a lagging power factor of 0.9 from a balanced 440 V, 50 Hz supply. Find the circuit elements in each phase of the elements connected in series. **[WBUT 2016]**

Answer:

The line to line voltage of the star-connected load is

$V_{LL} = 440$ V

$$V_{ph} = \frac{440}{\sqrt{3}} = 254 \text{ Volts}$$

It draws a current of 15A.

So, the per phase impedance $= \frac{254}{15}\Omega = 16.9\Omega \simeq 17\Omega$ The power factor is 0.9 lagging, so it can be inferred that there is a resistance and an inductance in series.

$Z = 17\Omega$.

Resistance $R = Z\cos\phi = 17 \times 0.9 = 15.3\Omega \simeq 15\Omega$.

Inductive reactance $X_L = \omega L = 17\sin\phi = 17 \times 0.44 = 7.37\Omega$

Inductance $L = \frac{X_L}{\omega} = \frac{7.37}{2 \times \pi \times 50}\text{H} = \frac{2.346}{100}\text{H} = 23.46\text{ mH}$

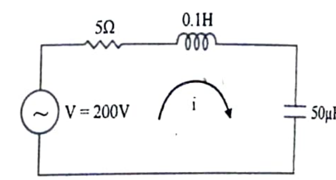
2.13. A coil having a resistance of 5Ω and inductance of 0.1H is connected in series with a 50μF capacitor. A sinusoidal voltage of 200V is applied to the circuit. At what frequency the current in the circuit will be maximum? Calculate this current and voltage across the capacitor at this frequency. **[WBUT 2018]**

Answer:

A coil of $R = 5\Omega$, $L = 0.1$ H and $C = 50\ \mu$F are in series

Applied voltage = 200 V

Let f be the frequency of the applied voltage



The applied voltage

$v = V_m \sin\omega t$

where, $\omega = 2\pi f$, the radian frequency

Considering $V_m = 200$ V

$$i = \frac{V_m}{|Z|}\sin(\omega t \pm \phi)$$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$R = 5\Omega$

$X_L = \omega L = 0.1\omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{\omega \times 50 \times 10^{-6}} = \frac{10^6}{50\omega}$$

Maximum current occurs at resonance during resonance, inductive reactance = Capacitive reactance and net reactance of the circuit is zero.

$$0.1\omega = \frac{10^6}{50\omega}$$

$$\omega^2 = \frac{10^6}{5}$$

$$\omega = \frac{10^3}{\sqrt{5}} = 447 \text{ rad/sec}$$

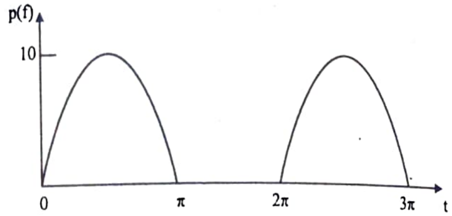
The current impedance at maximum frequency is resistive in nature.
The current at resonant frequency

$$I = \frac{V}{R} = \frac{200}{5} = 40 \text{ A}$$

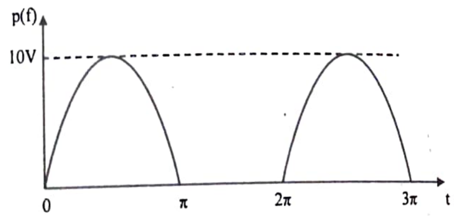
The voltage developed across the capacitor for maximum current condition (resonant condition)

$$= X_C \times 40 = \frac{\sqrt{5} \times 10^6}{10^3 \times 50} \times 40 = \frac{10^3}{\sqrt{5}} \times 4 = 447 \times 4V = 1788 \text{ V}$$

2.14. Find the average and rms voltage of the voltage waveform shown. What is the power dissipation across a 9Ω resistor. Supplied with voltage. **[WBUT 2018]**



Answer:



$$V_{avg} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin \omega t d(\omega t) = \frac{1}{2\pi}\int_0^{\pi} 10 \sin \omega t d(\omega t) = \frac{10}{2\pi}(-\cos \omega t)\Big|_0^{\pi}$$

$$= -\frac{10}{2\pi}[\cos \pi - \cos 0] = -\frac{10}{2\pi}[-1-1] = \frac{10}{\pi} = 3.18 \text{ V}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2 \omega t \, d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2 \omega t \, d(\omega t)}$$

$$= V_m \sqrt{\frac{1}{4}\pi \int_0^{\pi} 2\sin^2 \omega t d(\omega t)} = \frac{V_m}{2}\sqrt{\frac{1}{\pi}\int_0^{\pi}(1-\cos 2\omega t) d(\omega t)}$$

$$= \frac{V_m}{2}\sqrt{\frac{1}{\pi}\int_0^{\pi} d(\omega t) - \frac{1}{\pi}\int_0^{\pi}\cos 2\omega t \, d(\omega t)} = \frac{V_m}{2}\sqrt{\frac{\pi}{\pi} - \frac{1}{2\pi}(\sin 2\omega t)\Big|_0^{\pi}}$$

$$= \frac{V_m}{2}\sqrt{1 - \frac{1}{2\pi} \times 0} = \frac{V_m}{2} = \frac{10}{2} = 5 \text{ V}$$

Power dissipation across a 9Ω resistor

$$= \frac{V_{rms}^2}{R} = 2.778 \text{ W}$$

Long Answer Type Questions

3.1. Define phasor diagram, the phenomenon of resonance in a circuit containing an inductance, a capacitor and a resistor is series. **[WBUT 2005, 2009]**

OR,

What is resonance? Deduce the expression of frequency in a series RLC circuit at resonance. **[WBUT 2010, 2013, 2016]**

Answer:

Resonance may be described as the condition existing in any physical system when a fixed amplitude sinusoidal forcing function produces a response of maximum amplitude. The phenomenon of resonance occurs in electrical, mechanical, hydraulic, acoustic and many other physical systems. The following examples will serve to throw some light on the term.

The soldiers marching on a bridge are advised to break steps. This is done to prevent any undesirable vibrations which might occur due to the matching of the natural frequency of vibration of the bridge with that of the march of the soldiers.

or, for example, consider the case of an opera single who possesses the acumen of shattering crystal goblets by striking a particular note at a given frequency.

In each case, the phenomenon of resonance occurs through proper frequency matching. We shall however, restrict our focus to electrical systems only.

In view of study of resonance in electrical systems, the following points might be noted.

- Resonance occurs in an AC circuit when inductive reactance and capacitive reactance are equal to one another: $X_L = X_c$
- When this occurs, the total reactance, $X = X_L - X_c$ becomes zero and the impendence is totally resistive.
- Because inductive reactance and capacitive reactance are both dependent on frequency, it is possible to bring a circuit to resonance by adjusting the frequency of the applied voltage.
- Resonant frequency (f_{Res}) is the frequency at which resonance occurs, or where $X_L = X_c$. Equation below is the mathematical representation for resonant frequency. $f_{Res} = \dfrac{1}{2\pi\sqrt{LC}}$

  where f_{Res} = resonant frequency (Hz)
  L = inductance (H)
  C = capacitance (f)

Series Resonance

In a series R-C-L circuit, as in the Figure below, at resonance the net reactance of the circuit is zero, and the impedance is equal to the circuit resistance; therefore, the current output of a series resonant circuit is at a maximum value for that circuit and is determined by the value of the resistance. (Z = R)

At resonance, the impedance of the circuit as observed by the constant amplitude variable frequency sinusoidal forcing function is minimum since the reactive effect of the series inductance is nullified by the reactive effect of the capacitance. The input impedance of the source is assumed to be infinity.

The impedance of a network can be expressed as $Z = R + j\omega L - j/\omega C$

At resonance, $j\omega L = j/\omega C$

if the resonance frequency is designated by ω_0, then $\omega_0^2 = \dfrac{1}{LC} = \omega_n^2$

where ω_n is the natural frequency of oscillation of the circuit.

So, it can be concluded that resonance occurs in a series RLC circuit when the supply frequency is equal to the natural frequency of the network.

Since the nature of the impedance at resonance is purely resistive, it should be further noted that at resonance the supply voltage and source current are in phase.

Since $|Z| = R$; $\quad \phi = 0°$; $\quad |I| = \dfrac{|V|}{R}$,

i.e., response or current assumes maximum value at resonance large voltage magnification occurs across inductance and capacitance, which however cancel each other.

The expressions of V_L and V_C are as follows:

$$V_L = \frac{V_m}{R}\cdot\omega_0 L\sin(\omega t + 90°)$$

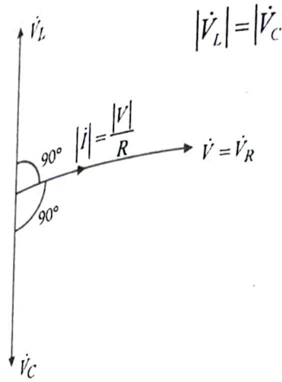
since voltage across inductance leads the current and thereby the supply voltage by 90°.

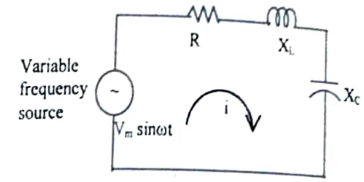
$$V_C = \frac{V_m}{R}\cdot\frac{1}{\omega_0 C}\sin(\omega t - 90°)$$

since voltage across capacitance lags the current and thereby the supply voltage by 90°.

The phasor diagram during resonant conditions with current as reference phasor can be drawn as,

$|\dot{V}_L| = |\dot{V}_C|$ at resonant frequency





Quality factor:

Quality Factor of a Series Resonant Circuit

The Q factor of an RLC series circuit can be defined in any of the following ways.

(i) It may be expressed as the voltage magnification that the circuit produces at resonance. During resonance current assumes maximum value, which is equal to V/R.

Voltage across inductance or capacitance equal to $I_{max} X_L = I_{max} X_c$.

Therefore, voltage magnification $= I_{max} X_L / V$

$= (I_{max} X_L)/(I_{max} R)$

$= X_L/R = \omega_r L/R = 1/\omega_r RC$

or, Q factor at resonance

$Q_0 = \omega_r L/R = 1/\omega_r RC$

(ii) Q Factor of a coil is the measure of its energy storage capability when it carries alternating current. It is a figure of merit of the coil. Mathematically, it is defined as $Q = 2\pi$ (max. energy stored / energy dissipated per cycle)

Since energy can be stored by either the inductor or the capacitor and dissipated by the resistor, Q might be expressed in terms of the instantaneous energy associated with each of the reactive elements and the average power dissipated in the resistor

$$Q = 2\pi\frac{[\omega_L(t) + \omega_C(t)]_{max}}{P_{avg}\cdot T}$$

where, P_{avg} is the average power lost in the resistor and T is the period of the sinusoidal frequency at which Q is being evaluated.

Consider the case of series resonance where the forcing function is $v = V_m \sin\omega t$ and the corresponding response at resonance is $i = \dfrac{V_m}{R}\sin\omega t$.

Energy stored by inductance $\omega_L(t) = \dfrac{1}{2}L_i^2 = \dfrac{1}{2}\cdot L\left(\dfrac{V_m}{R}\right)^2 \sin^2\omega_0 t$

The instantaneous energy stored by the capacitor is $\omega_C(t)=\frac{1}{2}C v_c^2$

$$=\frac{1}{2}C\left[\frac{1}{C}\int_0^t i(t)\,dt\right]^2=\frac{1}{2C}\frac{V_m^2}{R^2\cdot\omega_0^2}\cos^2\omega_0 t=\frac{1}{2R^2C}\cdot V_m^2\cdot LC\cos^2\omega_0 t=\frac{V_m^2\cdot L}{2R^2}\cos^2\omega_0 t$$

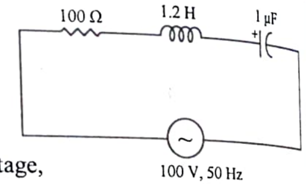
The total instantaneous energy $=\frac{1}{2}\frac{L}{R^2}\cdot V_m^2\left(\sin^2\omega_0 t+\cos^2\omega_0 t\right)$

energy dissipated in resistor $=\frac{1}{2}I_m^2R=\frac{1}{2}\frac{V_m^2}{R^2}\cdot R=\frac{1}{2}V_m^2/R$

Multiplying with one period $P_{avg}T=\frac{1}{2f_0}\frac{V_m^2}{R}$

3.2. A resistance of 100 ohms is connected with an inductance of 1.2 Henry and capacitance of microfarad in series. The combination is connected across 100 volts, 50 Hz supply.
Find. **[WBUT 2007]**
a) Current in the resistance
b) Voltage across the capacitance
c) Power consumed.
d) Draw phasor diagram.

Answer:
The circuit diagram is shown in the figure:



a) Current in the resistance $=\frac{V_m}{|z|}\sin(\omega t\pm\phi)$

Assuming that 100V is the r.m.s. value of supply voltage,

$$V_m=\sqrt{2}\times 100\text{V}=141.4\text{V}$$

$$|z|=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

$$\omega L=2\pi\times 50\times 1.2\,\Omega=120\pi\,\Omega$$

$$\frac{1}{\omega C}=\frac{1}{2\pi\times 50}\times\frac{1}{10^{-6}}\Omega=\frac{10^6}{100\pi}\Omega=\frac{10^4}{\pi}\Omega$$

$$|z|=\sqrt{(100)^2+\left(120\pi-\frac{10^4}{\pi}\right)^2}\,\Omega=\sqrt{(100)^2+(376.99-3183.1)^2}\,\Omega$$

$$=\sqrt{(100)^2+(2806.11)^2}\,\Omega=\sqrt{10^4+7874253.33}\,\Omega=2807.89\,\Omega$$

The impedance diagram is drawn: $\phi=-\tan^{-1}\frac{2806.11}{100}=-87.958°$

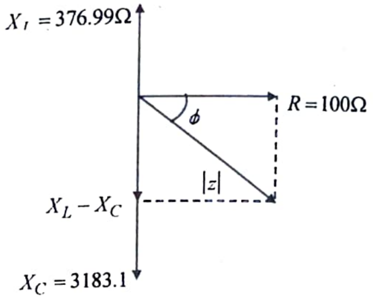
Current through resistance $=\frac{100\times\sqrt{2}}{2807.89}\sin(\omega t+87.958°)=0.050\sin(\omega t+87.958°)$

b) Voltage across the capacitance $=0.05\cdot X_C\sin(\omega t+87.958°-90°)$

$$=0.05\times 3183.1\sin(\omega t-2.042°)$$
$$=159.155\sin(\omega t-2.042°)$$

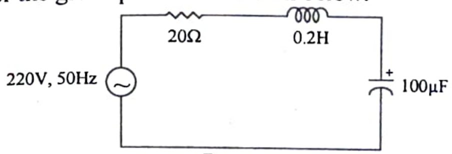
c) Power consumed $=I_{r.m.s}^2R=\left(\frac{100}{2807.89}\right)^2\cdot 100=0.1268\text{ W}$

d)



3.3. A resistance of 20Ω, an inductance of 0.2 H and a capacitance of 100 μF are connected in series across 220 V, 50 Hz. Determine the following
a) Impedance **b) Current**
c) Voltage across R, L, C **d) Power factor and angle of lag**
e) Power in Watts and VA. **[WBUT 2008(EVEN)]**

Answer:
The circuit diagram for the given problem is drawn below:



(a) Impedance $=R+j\omega L-\frac{j}{\omega C}=20+j\left[2\pi\times 50\times 0.2-\frac{1}{2\pi\times 50\times 100\times 10^{-6}}\right]$

$$=20+j\left[20\pi-\frac{100}{\pi}\right]=20+j\left[20\pi-\frac{100}{\pi}\right]$$

$$=20+j20\left[\pi-\frac{5}{\pi}\right]=20+j20[1.55]\Omega=20+j31\,\Omega$$

Impedance $=\sqrt{(20)^2+(31)^2}\tan^{-1}\frac{31}{20}=36.89\angle 57.17°\Omega$

(b) Current $=\frac{\text{voltage}}{\text{Impedance}}=\frac{220}{36.89}\angle-57.17°=5.96\angle-57.17°\text{A}$

(c) Voltage across resistance $=(5.96\times 20)$ V lagging the supply voltage by

119.2 V lagging the supply voltage by 57.17°

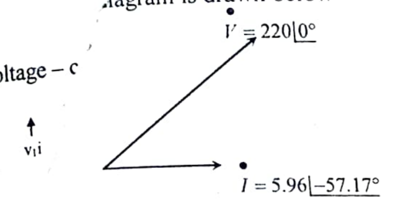
inductance $= 5.96 \times 0.2 \times 2\angle -57.17° + 90° \times \pi \times 50$

$= 374.48$ V by 32.83°

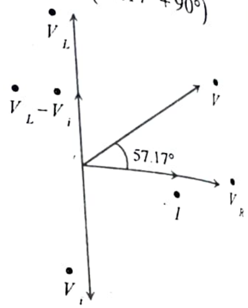
capacitance $= 5.96 \times \dfrac{1}{2\pi \times 50 \times 100 \times 10^{-6}} V = 5.96 \times \dfrac{100}{\pi} V$

$= 189.7$ V lagging the supply voltage by $(57.17° + 90°)$

The mathematical e... diagram is drawn below



The voltage – c



...e diagram is not drawn to seal)

...) Power factor $= \cos(57.17°)$ lag $= 0.542$ lag

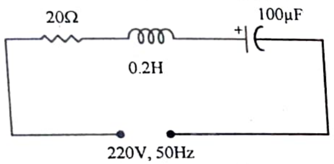
Angle of lag $= 57.17°$

(iii) (e) Power in watts $= VI\cos\phi = 220 \times 5.96 \times \cos 57.17°$ W $= 710.67$ W

Power in $VA = 220 \times 5.96$ VA $= 1311.2$ VA

3.4. A resistance of 20 Ω, an inductance of 0.2 H and a capacitance of 100 μF are connected in series across a 220 V, 50 Hz supply.
a) Determine **[WBUT 2008]**
i) impedance **ii) current** **iii) power factor** **iv) power consumed.**

Answer:



(i) Impedance $= R + j\omega L + \dfrac{1}{j\omega C} = R + j\left(\omega L - \dfrac{1}{\omega C}\right)$

$= 20 + j\left[2 \times \pi \times 50 \times 0.2 - \dfrac{1}{2\pi \times 50 \times 100 \times 10^{-6}}\right] = 20 + j\left[20\pi - \dfrac{100}{\pi}\right]$

$= 20 + j20\left[\dfrac{\pi^2 - 5}{\pi}\right] = 20 + j31 = 36.89\angle 57.17°$

Impedance $= 36.89\,\Omega$

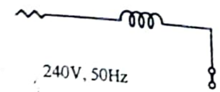
(ii) Current $= \dfrac{\text{Voltage}}{\text{impedance}} = 5.96$ A

(iii) Power factor $= \cos 44.24° = 0.542$ lagging

(iv) Power consumed $= VI\cos\phi = 220 \times 7.885 \times 0.715 = 710.67$ W

b) Draw the phasor diagram.

Answer:



The phasor diagram is drawn with current as the reference phasor.

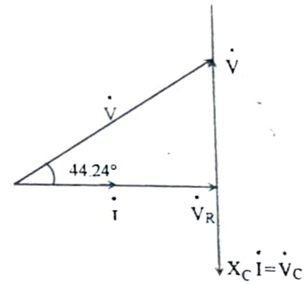
$\dot{I} = |I|\angle\phi = 5.96\angle 0°$

$\dot{V}_R = |V_R|\angle\phi = |I|\,R\angle 0°$

$\dot{V}_L = |V_L|\angle\phi = |I|\,X_L\angle 90°$

$\dot{V}_C = |V_C|\angle\phi = |I|\,X_C\angle -90°$

$\dot{V} = |V|\angle\phi = 220\angle 44.24°$



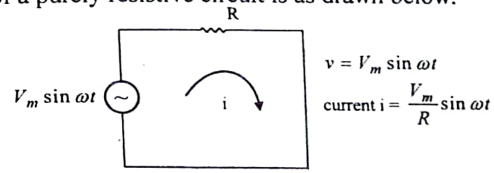
3.5. a) Draw the circuit diagram, waveform of voltage and current, phasor di... of (i) purely resistive circuit (ii) purely inductive circuit (iii) purely capacitive ci... supplied by sinusoidal voltage. **[WBUT 20...]**

OR,

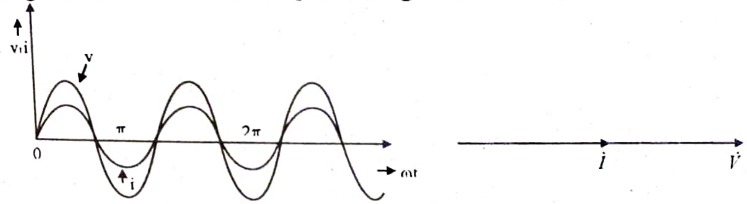
Prove that current in purely resistive circuit is in phase with applied A.C. voltage in and current in purely capacitive circuit leads applied voltage by 90° and draw their waveforms. **[WBUT 2010]**

Answer:

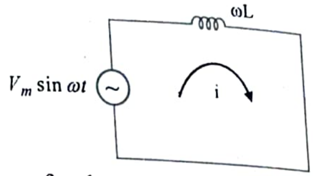
(i) Circuit diagram of a purely resistive circuit is as drawn below.



The voltage current waveforms and phasor diagram are drawn below.



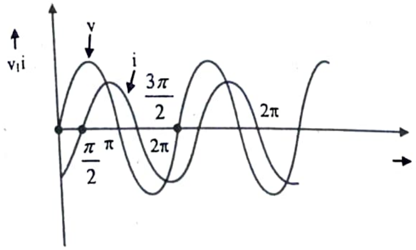
(ii) Circuit diagram of a purely inductive circuit supplied by sinusoidal sources is drawn below

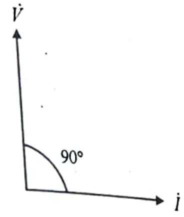


The mathematical expressions of and current are $v = V_m \sin \omega t$

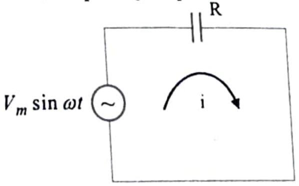
$$i = \frac{V_m}{\omega L}\sin(\omega t - 90°)$$

The voltage – current waveforms and phasor diagrams are drawn below.





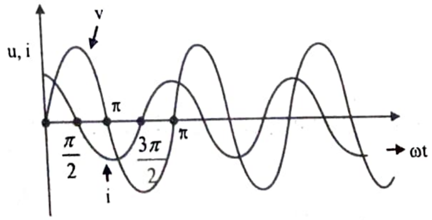
(ii) Circuit diagram of a purely capacitive circuit is as drawn below:

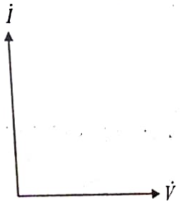


$v = V_m \sin \omega t$

current $i = V_m \omega C \sin(\omega t + 90°)$

The waveforms of voltage, current and the corresponding phasor diagrams are drawn below.





b) A coil takes a current of 2A when connected to a 240 V, 50 Hz sinusoidal supply and consumes 200 W. Calculate the resistance, impedance and inductance of the coil. **[WBUT 2009]**

Answer:

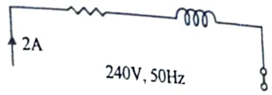
Supply voltage = 240V

Source current = 2A

Power consumed = 200W

Resistance of coil $R = \frac{\text{Power consumed}}{\text{Current Squared}} = \frac{200}{2\times 2}\Omega = 50\Omega$

Impedance (magnitude) of the circuit $= \frac{\text{voltage}}{\text{Current}} = \frac{240}{2}\Omega = 120\Omega$



Let L be the inductance of the coil.

Then ωL is the inductive reactance

From geometry, we know

$Z = \sqrt{R^2 + \omega^2 L^2}$

or, $(120)^2 = (50)^2 + \omega^2 L^2$

or, $(\omega L)^2 = 14400 - 2500 = 11{,}900$

or, $\omega L = 109.09\Omega$

or, $\omega = 2\pi f = 100\pi$ rad/sec

$L = \frac{109.09}{100\pi} = 0.347\text{ H} = 347\text{ mH}$

∴ Resistance $= 50\Omega$

Impedance $= 120\Omega$

Inductance = 347mH

3.6. a) A coil having resistance of 50Ω and inductance of 0.02H is connected in parallel with a capacitor of 25 μF across a 200 V, 50 Hz supply. Find the current in the coil and the capacitor. Also find total current taken from the supply and overall power factor. Draw a neat phasor diagram. **[WBUT 2010(EVEN)]**

b) Find the resultant current in the following form:

$i = i_m \sin(\omega t \pm \phi)$, **if the current at a node are** $i_1 = 5\sin\omega t$, $i_2 = 10\sin\left(\omega t - \frac{\pi}{6}\right)$, $i_3 = 5\cos\left(\omega t + \frac{\pi}{6}\right)$ **and** $i_4 = 10\sin\left(\omega t + \frac{3}{6}\right)$.

Answer:

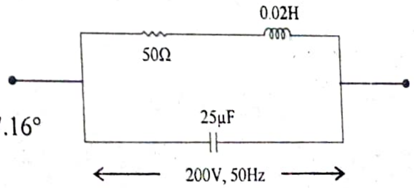
a) The impedance of the coil

$= (50 + j\omega \times 0.02)\Omega$

$\omega = 2\pi f = 2\pi \times 50$ rad/sec.

So, impedance of the coil

$= (50 + j\,2\pi)\Omega = (50 + j\,6.28)\Omega = 50.39\angle 7.16°$



Hence current in the coil,

$$I_L = \left(\frac{200}{50.39}\right)A = 3.97A$$

Lagging source voltage by 7.16°

Current in the capacitor, $\dot{I}_c = 200 \times 25 \times 10^{-6}\text{ A} = 5 \times 10^{-3}\text{ A}$

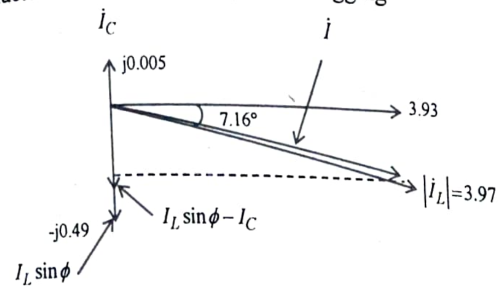
Leading the voltage by 90°

Total current taken form the supply $= \dot{I}_L + \dot{I}_c = 3.87\angle -7.16° + 5\times10^{-3}\angle 90°$

$$= 3.93 - j0.49 + j0.005 = 3.93 - j0.485$$

$$\dot{I} = 3.96\angle -7.04° \text{ A}$$

Total current taken from the supply is 3.96 A. Lagging the source voltage by 7.04°
Overall power factor $= \cos 7.04°$ lagging $= 0.99$ lagging



b) The four currents at a node are

$$i_1 = 5\sin\omega t\,; \qquad i_2 = 10\sin\left(\omega t - \frac{\pi}{6}\right);$$

$$i_3 = 5\cos\left(\omega t + \frac{\pi}{6}\right); \qquad i_4 = 10\sin\left(\omega t + \frac{\pi}{6}\right)$$

The currents can be expressed in phasor form as

$$\dot{I}_1 = 5\angle 0°\,; \qquad \dot{I}_2 = 10\angle -\frac{\pi}{6}$$

$$\dot{I}_3 = 5\angle\frac{2\pi}{3}\,; \qquad \dot{I}_4 = 10\angle\frac{\pi}{6}$$

Expressing the currents in the rectangular form from polar form we have

$$\dot{I}_1 = 5 + j0 = 5\,; \qquad \dot{I}_2 = 10\cos\frac{\pi}{6} - j10\sin\frac{\pi}{6} = 8.66 - j5\,;$$

$$\dot{I}_3 = 5\cos\frac{2\pi}{3} + j5\sin\frac{2\pi}{3} = -2.5 + j4.33\,; \qquad \dot{I}_4 = 10\cos\frac{\pi}{6} + j10\sin\frac{\pi}{6} = 8.66 + j5$$

To get the resultant current, we have to get the vector summation of the four currents

$$\dot{I} = \dot{I}_1 + \dot{I}_2 + \dot{I}_3 + \dot{I}_4 = 5 + j0 + 8.66 - j5 - 2.5 + j4.33 + 8.66 + j5$$

$$= 19.82 + j4.33 = 20.29\angle 12.3°$$

$$i = 20.29\sin(\omega t + 12.3°)$$

3.7. a) Explain what are meant by phase and phase difference of sinusoidal waves.
[WBUT 2010]

Answer:

Phase: Phase in sinusoidal waves is the fraction of a wave cycle which has elapsed relative to an arbitrary point.

Phase Difference:
A term more commonly used is **phase difference**. The phase difference can be used to describe and compare two different voltages or currents or a current and a voltage that have the same frequency, which pass through zero values in the same direction at different times.

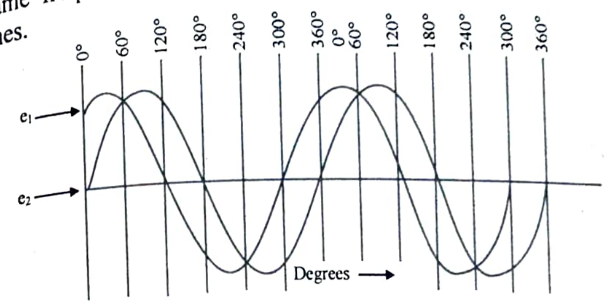


Fig: 1 Phase Relationship

If the phase difference between two currents, two voltages, or a voltage and a current is zero degrees, they are said to be "in-phase."
In the above diagram, e_2 is considered to be the reference voltage and the phase of e_1 is defined with respect to e_2. For example if
$e_2 = E_2\sin\omega t$, then $e_1 = E_2\sin(\omega t + \alpha)$ where α=60 electrical degrees since the zero crossing of e_2 occurs 60 electrical degrees before that of e_1 in the same direction.
The voltage e_1 is said to lead e_2 by 60 electrical degrees, or it can be said that e_2 lags e_1 by 60 electrical degrees.

b) A coil of resistance 30 Ω and inductance 320 mH is connected in parallel to a circuit consisting of 75 Ω in series with 150 µF capacitor. The circuit is connected to a 200 volt, 50 Hz supply. Determine supply current and circuit power factor.
[WBUT 2010]

Answer:
Resistance of the coil = 30Ω
Inductance of the coil = 320mH
Frequency of supply = 50 Hz

$$\omega = 2\pi f = 2\times\pi\times 50 \text{ rad/sec} = 100\pi \text{ rad/sec.}$$

Impedance of the coil

$$(30 + j\omega\times 320\times10^{-3})\Omega = (30 + j\times 100\pi\times 320\times 10^{-3})\Omega$$

$$= (30 + j\,100.53)\Omega = 105\angle 73.38°\Omega$$

Impedance of the capacitive circuit $= (75 - \frac{j}{\omega C})\Omega = \left(75 - \frac{j}{100\pi\times 150\times 10^{-6}}\right)\Omega$

$$= \left(75 - j\frac{1000}{15\pi}\right)\Omega = (75 - j\,21.22)\Omega$$

$$= 78\angle -15.8°\ \Omega$$

For a 200V, 50 Hz supply,

Current through the coil $= \frac{200\angle 0^\circ}{105\angle 73.38^\circ} A = 1.9\angle -73.38^\circ A = (0.53 - j1.82)A$

Current through the capacitive circuit $= \frac{200\angle 0^\circ}{78\angle -15.8^\circ} = 2.56\angle 15.8^\circ A = (2.46 + j0.7)A$

The source current $= (0.53 - j1.82 + 2.46 + j0.7)A$

$= (2.99 - j\,1.12)A = 4.24\angle -20.53^\circ A$

So the supply current $= 4.24A$

Supply p.f. $= \cos(20.53^\circ)$lagging $= 0.94$ lagging.

3.8. A circuit consists of series combination of elements as resistance of 6 Ω, inductance of 0.4 H and a variable capacitor across 100 V, 50 Hz supply. Calculate (i) value of capacitance at resonance, (ii) voltage drop across capacitor and (iii) Q factor of coil. [WBUT 2010, 2013]

Answer:

The given circuit parameters are

Resistance $R = 6\Omega$

Inductance $L = 0.4H$

Capacitance is variable

Supply voltage = 100V

Supply frequency = 50 Hz

$\omega = 2\pi f$ rad/sec $= 2\times\pi\times 50$ rad/sec $= 100\pi$ rad/sec

At resonance, $\omega L = \frac{1}{\omega C}$

$C = \frac{1}{\omega^2 L}$

(i) Capacitance at resonance

$$C = \frac{1}{(100\pi)^2 \times 0.4} F = \frac{100\times 10^{-6}}{\pi^2 \times 0.4} F = \frac{100}{\pi^2 \times 0.4} \mu F = 25.33\mu F$$

(ii) Current during resonance $= \frac{100}{6} A = 16.67A$

Voltage drop across the capacitance during resonance $= X_c I = \frac{1}{\omega C} I$

$$= \frac{10^6}{100\pi \times 25.33} \times 16.67$$

$$= \frac{10^4 \times 16.67}{25.33\times\pi} V = 2094.8V$$

iii) Q-factor of coil $= \frac{\omega L}{R} = \frac{100\pi \times 0.4}{6} = 20.9$

$$Q = \frac{1}{\omega_o RC} = \frac{10^6}{100\pi \times 6 \times 25.33} = 20.9$$

$$Q = \text{voltage magnification} = \frac{\text{Voltage across capacitance}}{\text{Supply voltage}} = \frac{2094.8}{100} = 20.9$$

3.9. a) Derive a mathematical expression for the average real power delivered by a single phase a.c. source with an e.m.f. of $e = \sqrt{2}E_m \sin\omega t$ when the source current is $i = \sqrt{2}I_m \sin(\omega t - \theta)$. [WBUT 2011]

b) Define power factor of an a.c. circuit. State the major disadvantages of poor power factor. [WBUT 2011]

OR,

Define power factor of an A.C. circuit. State the disadvantages associated with having a load power factor. [WBUT 2012]

Answer:

a) Consider the general case when the phase difference between voltage and current is ϕ. If the current lags the voltage by ϕ, the voltage and current may be written as

$v = \sqrt{2}V_m \sin\phi$, $i = \sqrt{2}I_m \sin(\phi - \theta)$ $\omega t = \phi$

Instantaneous power, $p = vi = 2V_m I_m \sin\theta \sin(\theta - \phi)$

$$= \frac{2V_m I_m}{2}\left[\cos\phi - \cos(2\theta - \phi)\right] \left[\because \sin\alpha \sin\beta = \frac{1}{2}\cos(\alpha-\beta) - \frac{1}{2}\cos(\alpha+\beta)\right]$$

Active Power $P = \frac{1}{2\pi}\int_0^{2\pi} p\,d\phi = \frac{2V_m I_m}{2\pi}\int_0^{2\pi}\frac{1}{2}\left[\cos\theta - \cos(2\phi - \theta)\right]d\phi$

$$= \frac{2V_m I_m}{4\pi}\int_0^{2\pi}\cos\theta\,d\phi - \frac{2V_m I_m}{4\pi}\int_0^{2\pi}\cos(2\phi - \theta)\,d\phi$$

$$= \frac{2V_m I_m}{4\pi}\cos\theta\left[\phi\right]_0^{2\pi} - \frac{2V_m I_m}{4\pi}\left[\frac{1}{2}\sin(2\phi - \theta)\right]_0^{2\pi}$$

(since $\cos\theta$ is constant for a given circuit)

$$= \frac{2(\sqrt{2}V)(\sqrt{2}I)}{4\pi}(\cos\theta)(2\pi) - \frac{2V_m I_m}{8\pi}\left[\sin(4\pi - \theta) - \sin(-\theta)\right]$$

$$= 2VI\cos\theta - \frac{2V_m I_m}{8\pi}(-\sin\theta + \sin\theta)$$

$\therefore \quad P = 2VI\cos\theta$

b) 1st Part:

Power factor (pf) is defined as the ratio between true power and apparent power. True power is the power consumed by an AC circuit, and reactive power is the power that is stored in an AC circuit. $\cos\theta$ is called the power factor (pf) of an AC circuit. It is the ratio of true power to apparent power, where θ is the phase angle between the applied voltage and current sine waves and also between P and S on a power triangle (Figure I). Equation below is a mathematical representation of power factor.

$$\cos\theta = \frac{P}{S}$$

where $\cos\theta$ = power factor (pf)
P = true power (watts)
S = apparent power (VA)

Power factor also determines what part of the apparent power is real power. It can vary from 1, (when the phase angle is 0°), to 0, (when the phase angle is 90°). In an inductive circuit, the current lags the voltage and is said to have a lagging power factor, as shown in Figure 2.

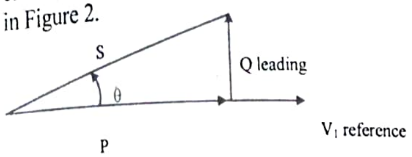


Fig: 1 Leading Power Factor

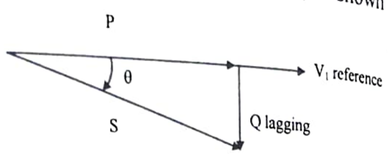


Fig: 2 Lagging Power Factor

In a capacitive circuit, the current leads the voltage and is said to have a leading power factor.

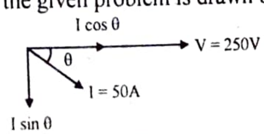
2nd Part:

1) If Power Factor is low the current rating of the electrical machinery increases which result in higher loss and overheating.
2) The Power Factor of a 3-phase system decreases, the current rises. The heat dissipation in the system rises proportionately by a factor equivalent to the square of the current rise.
3) Low power factor reduces an electrical system's distribution capacity by increasing current flow and causing voltage drops.
4) Low power factor shortens the lifespan of electrical appliances.

3.10. A circuit receives 50A current at a power factor of 0.8 lag from a 250V, 50 Hz, 1-ph A.C. supply. Calculate the capacitance of the capacitor which is required to be connected across the circuit to make the power factor unity. [WBUT 2011]

Answer:

The phasor diagram for the given problem is drawn below.



θ is power factor angle, $\theta = \cos^{-0.8} = 36.86°$

The component of current in phase with voltage $= I\cos\theta = 50\times 0.8 = 40A$

The component of current lagging the voltage by 90°

$I\sin\theta = 50\times 0.6 = 30\text{A}$ $(\because \sin 36.86° = 0.6)$

The role of the capacitor is to eliminate the lagging component of current.

Let the value of the attached capacitance $= C$ Farads

$$\frac{V}{X_c} = I$$

or, $V\omega C = I$

$$\omega C = \frac{I}{V} = \frac{30}{250} \text{ Siemens.}$$

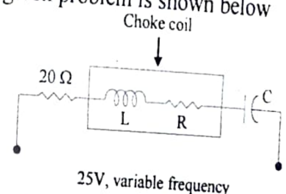
$$C = \frac{30}{250\times 2\times\pi\times 50}\text{ Farads} = \frac{30}{125\times 2\times\pi\times 100}\text{ Farads} = \frac{30}{25\times\pi\times 10^3}\text{ Farads} = 382\mu F$$

So the required capacitance to make the p.f = unity is $382\mu F$.

3.11. A 20Ω resistor, a choke coil having some inductance and some resistance and a capacitor are connected in series across a 25V variable frequency source. When frequency is 400 Hz, the current is maximum and its value is 0.5A and the potential difference across the capacitor is 150V. Calculate the resistance and the inductance of the choke coil and the capacitance of the capacitor. [WBUT 2011]

Answer:

The circuit diagram for the given problem is shown below



The source impedance $= 20 + R + j\omega L - j/\omega C$

Maximum current flows through the coil at resonance.

During resonance, $\omega L = \dfrac{1}{\omega C}$

Current during resonance, $I = \dfrac{V}{20+R}$

or, $\dfrac{25}{20+R} = 0.5$

Hence, $20 + R = 50$

or, $R = 30\Omega$

Voltage drop across the capacitor, $V_c = IX_c = I/\omega C$

or, $\omega C = \dfrac{0.5}{150}$ Siemens.

or, $C = \dfrac{0.5}{150\times 2\times\pi\times 400}\text{F} = \dfrac{5}{1500\times 2\times\pi\times 400}\text{F} = \dfrac{5}{120\times\pi\times 10^4}\text{F} = \dfrac{5\times 10^{-6}}{1.2\times\pi}\mu\text{F} = 1.33\mu\text{F}$

During resonance, $\omega L = \dfrac{1}{\omega C}$

$$\omega^2 = \frac{1}{LC}$$

$$L = \frac{1}{\omega^2 C} = \frac{1\times 1.33}{(2\times\pi\times 400)^2\times 10^{-6}}\text{H} = \frac{1.33}{64\times\pi^2\times 10^4\times 10^{-6}}\text{H} = \frac{133}{64\times\pi^2}\text{H} = 210\text{mH}$$

The resistance and inductance of choke coil are 30Ω and 210mH respectively. The capacitance of the capacitor is 1.33μF.

3.12. Prove that the current in a purely resistive circuit is in phase with applied A.C. voltage and current in a purely capacitive circuit leads applied voltage by 90° and also draw their waveforms. **[WBUT 2012]**

Answer:

In a purely resistive circuit:

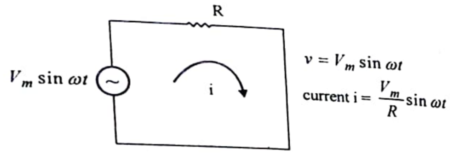
Consider a simple circuit consisting of a pure resistance R ohms connected across a voltage $v = V_m \sin \omega t$

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R}$$

i.e. $i = \left(\dfrac{V_m}{R}\right)\sin(\omega t)$

This is the equation giving instantaneous value of the current.

Circuit diagram of a purely resistive circuit is as drawn below.



Comparing this with standard equation

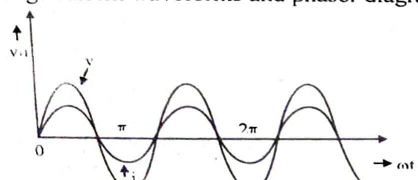
$$i = I_m \sin(\omega t + \phi)$$

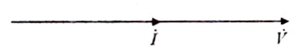
$$I_m = \frac{V_m}{R} \;\&\; \phi = 0$$

So, maximum value of alternating current i is $I_m = \dfrac{V_m}{R}$ while, as $\phi = 0$, it indicates that it is in phase with the voltage applied. There is no phase difference between the two.

The current is going to achieve its maximum (positive & negative) and zero whenever voltage is going to achieve its maximum (positive & negative) and zero values.

The voltage current waveforms and phasor diagram are drawn below.

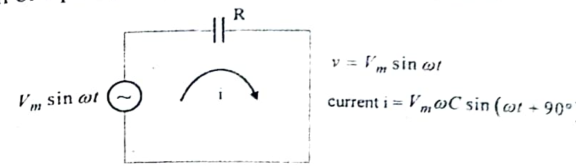




In a purely capacitive circuit:

Consider a simple circuit consisting of a pure capacitor of C farads, connected across a voltage given by the equation, $v = V_m \sin \omega t$

Circuit diagram of a purely capacitive circuit is as drawn below:



The current i charges the capacitor C. The instantaneous charge 'q' on the plates of the capacitor is given by

$q = c_v$

$q = CV_m \sin \omega t$

Now, current is rate of flow of charge.

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV_m \sin \omega t)$$

$$i = CV_m \frac{d}{dt}(\sin \omega t) = CV_m \omega \cos \omega t$$

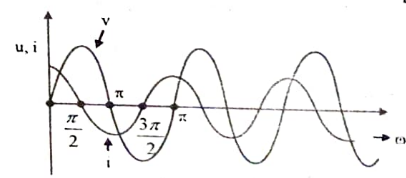
$$i = \frac{V_m}{\left(\frac{1}{\omega C}\right)}\sin\left(\omega t + \frac{\pi}{2}\right) = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

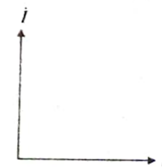
where, $I_m = \dfrac{V_m}{X_C}$ and $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC}$

The above equation clearly shows that the current is purely sinusoidal and having phase angle of $\dfrac{\pi}{2}$ radians i.e. 90°.

This means current leads voltage applied by 90°. The positive sign indicates leading nature of the current. If current is assumed reference, we can say that voltage across capacitor lags the current passing through the capacitor 90°.

The waveforms of voltage, current and the corresponding phasor diagrams are drawn below.





3.13. a) Derive the expression of quality factor of a series R-L-C circuit at resonance. **[WBUT 2013]**

Answer: ***Refer to Question No. 3.1.***

b) A coil of resistance 10Ω and inductance 0.02 H s connected in series with another coil of resistance 6Ω and inductance 15mH across a 230 V, 50 Hz supply. Calculate: **[WBUT 2013]**
(i) impedance of the circuit
(ii) voltage drop across each coil
(iii) the total power consumed by the circuit.

Answer:

i) Total resistance of the circuit $=(10+6)\Omega=16\Omega$

Total inductance of the circuit = 0.02 H + 15 mH= 20 mH + 15 mH = 35 mH

Total inductive reactance of the circuit $= 0.035\times 2\times\pi\times 50 = 10.99\ \Omega \cong 11\ \Omega$

Total impedance of the circuit $=\sqrt{R^2+\omega^2L^2}=\sqrt{16^2+11^2}=19.4\Omega$

Phase angle $=\tan^{-1}\dfrac{11}{16}=34.5°$

ii) The current flowing through the circuit $=\dfrac{230}{19.4}\text{A}=11.86\text{A}$

and it lags behind the supply voltage by 34.5°.

The impedance of the first coil

$$=(10+j2\pi\times50\times0.02)\Omega=\sqrt{10^2+(2\pi)^2}\,\Omega=\sqrt{100+(6.28)^2}\,\Omega=11.81\Omega$$

Phase angle $=\tan^{-1}\dfrac{2\pi}{10}=\tan^{-1}\dfrac{\pi}{5}=32.14°$

The voltage drop across first coil $=11.86\angle-34.5°\times11.81\angle32.14°V=140\angle-1.36°V$

Impedance of the second coil $=(6+j2\pi\times50\times0.015)\Omega$

$$=\sqrt{36+(1.5\pi)^2}\,\Omega=\sqrt{58.2}\Omega=7.629\Omega\simeq7.63\Omega$$

Phase angle $=\tan^{-1}\dfrac{1.5\pi}{6}=38°$

Voltage drop across the coil $=11.86\angle-34.5°\times7.63\angle38°V=90.5\angle3.5°V$

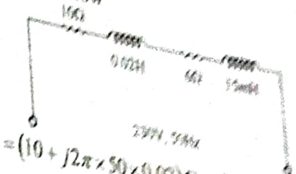
Ans. Voltage drop across first coil is 140V and it lags the supply voltage by 1.36° whereas voltage drop across second coil is 90.5V and it leads the supply voltage by 3.5°.

iii) Total power consumed in the two coils $=I^2(R_1+R_2)W=(11.86)^2\times16W=2.25kW$

3.14. a) A coil of resistance 10Ω and inductance 0.02H is connected in series with another coil of resistance 6Ω and inductance 15mH across a 230V, 50Hz supply. Calculate:
(i) impedance of the circuit
(ii) the voltage drop across each coil and
(iii) the total power consumed by the circuit. **[WBUT 2014, 2016]**

Answer:

a) The circuit diagram is drawn below



Impedance of branch 1 $=(10+j2\pi\times50\times0.02)\Omega=(10+j2\pi)\Omega=(10+j6.282)\Omega$

$$=\sqrt{100+(6.28)^2}\,\Big|\tan^{-1}\frac{6.282}{10}=11.81\underline{|32.14°}$$

Impedance of branch 2 $=(6+j2\pi\times50\times0.015)\Omega=(6+j1.5\pi)\Omega=(6+j4.71)\Omega$

$$=7.63\Omega\Big|\tan^{-1}\frac{4.71}{6}=7.63\underline{|38.13°}$$

(i) Total impedance of the circuit $=(10+j2\pi+6+j1.5\pi)\Omega$

$$=(16+j3.5\pi)\Omega=(16+j11)\Omega=19.4\underline{|34.5°}$$

Current through the circuit $\dfrac{230\underline{|0°}}{19.4\underline{|34.5°}}=11.86\underline{|-34.5°}$

Current through the circuit is 11.86A at 34.5° logging the supply voltage.

(ii) Voltage drop across coil 1 $=11.86\times11.81\underline{|-34.5°+32.14°}=140°\underline{|-2.36°}V$

Voltage drop across coil 1 is 140V lagging the supply voltage by 2.36.

Voltage drop across coil 2 $=11.86\times7.63\angle38.13°-34.15°=90.5\angle4°$

(iii) Current flowing through the circuit = 19.4 A

Total resistance =16Ω

Power consumption $=(19.4)^2\times16W=6021.76W=6kW$

b) Define power factor. Shown that the active power of a purely capacitive circuit over a complete cycle is zero. **[WBUT 2014, 2016]**

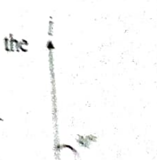
Answer:

1st Part: ***Refer to Question No. 3.9(b).***

2nd Part:

For a capacitance, $z=\dfrac{1}{\omega C}$ $\quad\phi=90°$

The phasor diagram is shown below with current leading the voltage by 90°.



Instantaneous power

$$=vi=V_m\sin\omega t\cdot\frac{V_m}{z}\sin(\omega t+\phi)$$

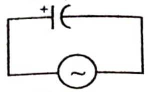
$$=\frac{V_m^2}{2}\cdot\omega C\quad[2\sin\omega t\sin(\omega t+\phi)]$$

$$=\frac{V_m^2\omega C}{2}\left[\cos\{\omega t-(\omega t+\phi)\}-\cos\{\omega t+\omega t+\phi\}\right]$$

$$=\frac{V_m^2\omega C}{2}\left[\cos(-\phi)-\cos(2\omega t+\phi)\right]$$

$\phi=90°$, hence $\cos(-\phi)=\cos\phi=0$ and average value of $\cos(2\omega t+\phi)=0$. Thus it can be shown that power dissipation in a pure capacitance in an a.c. circuit is equal to zero.

3.15. a) A capacitor of 100 µF is connected across a 200V, 50 Hz single phase supply. Calculate (i) the reactance of the capacitor, (ii) r.m.s. value of current, (iii) the maximum current. **[WBUT 2015]**



Answer:

a) (i) Reactance of the capacitor $=X_C=\frac{1}{j\omega C}=\frac{1}{2\pi\times f\times C}$

$$=\frac{1}{2\pi\times 50\times 100\times 10^{-6}}\Omega$$

$$=\frac{10^6}{\pi\times 10^4}\Omega=\frac{100}{\pi}\Omega=31.84\Omega$$

(ii) Taking the r.m.s voltage to be voltage to be 200V.

R.M.S. current $=\frac{200}{31.84}\text{A}=6.28\text{ A}$

(iii) The maximum current $=\left(6.28\times\sqrt{2}\right)\text{A}=8.88\text{ A}$.

b) What is meant by bandwidth? With a neat sketch of waveform find out the expression for the bandwidth of a resonant circuit. **[WBUT 2015]**

Answer:

Bandwidth is defined as the difference between the upper cut off frequency and the lower cut off frequency. These two frequencies are also termed as the half power frequencies of the network.

The current varies inversely with the variation in the impedance and therefore it is maximum at resonance frequency when impedance is minimum and decreases with the variation in frequency on both sides of the resonant frequency (as impedance is large).

The curve drawn below depicting the relation between circuit current and the frequency of the source voltage is called the resonance curve.

Dependence of the Resonance Curve

The shape of the Resonance Curve depends largely on the value of the resistance R. For small values of R it is sharply peaked and flat for larger values of R.

Depending on the value of R, the circuit is termed as ***sharply resonant*** or ***highly selective.*** On the other hand, high resistance circuits have flat resonance curves and poor selectivity.

However, it should be emphasized that the height of the response curve depends only upon the value of R for constant amplitude excitation, the width of the curve or the steepness of the sides depends upon the other two element values also. This width of the response curve which determines the selectively of a network is related to a quantity called bandwidth.

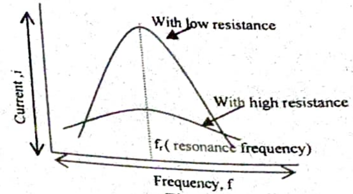


Fig: (a)

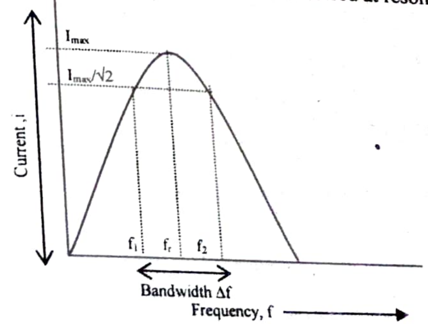
Selectivity of different resonant circuits is compared in terms of their bandwidths. Bandwidth is defined as the difference between the upper cut off frequency and the lower cut off frequency. These two frequencies are also termed as the half power frequencies of the network. When the source frequency assumes these two critical values, the power transferred to the circuit is equal to half the power transmitted during resonance. Hence the name half power frequencies. During this condition the current in the circuit is $1/\sqrt{2}$ times the current at resonant condition.

Bandwidth $\Delta f=f_2-f_1$

The actual power input at frequencies at f_1 and f_2, $P=I^2R=(I_{max}/\sqrt{2})^2R$

$=(I^2_{max}R)/2$

$=P_{max}/2$ ..... where P_{max} is the power transferred at resonance.



It can be established mathematically that the sharpness of the response curve of any resonant circuit is determined by the maximum amount of energy that can be stored in a circuit, compared to the energy that is lost during one complete period.

3.16. a) Prove that the current in purely capacitive circuit leads the applied voltage by an angle 90° and draw their waveforms. Also calculate the average power of capacitive circuit. **[WBUT 2015]**

Answer:

a) The capacitive reactance, $X_L = \dfrac{1}{j\omega C}$

In phasor form, $X_C = \dfrac{1}{\omega C\angle 90^\circ}$

Current through capacitance $= \dfrac{\text{Voltage across capacitance}}{\text{Capacitive reactance}}$

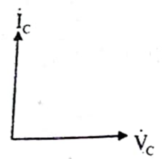
$$I_C = \frac{V_C}{X_C} = V_C \cdot \omega C\angle 90^\circ$$

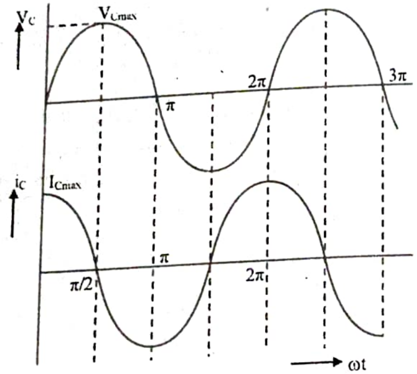
This clearly proves that the current phasor leads the voltage phasor by 90° for a capacitance.

Average power of a capacitive circuit $= V_C I_C \cos\phi_C$

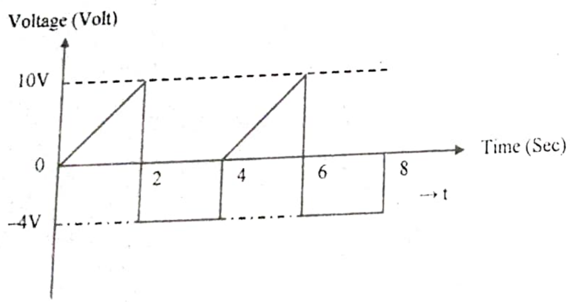
$$= V_C V_C \omega C \cos 90^\circ = V_C^2 \omega C \cos 90^\circ = 0$$

The phasor diagrams and voltage-current waveforms are drawn below:





b) Find the Form Factor of the given waveform. [WBUT 2015]



Answer:

Form factor $= \dfrac{\text{RMS value}}{\text{Average value}}$

Average value $= \dfrac{1}{T}\left[\int_0^T f(t)\,dt\right]$

$f(t) = 5t$ for $0 \le t \le 2$

$= -4$ for $2 \le t \le 4$

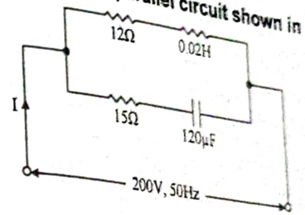
$\therefore$ Average value $= \dfrac{1}{4}\int_0^2 5t\,dt - \dfrac{1}{4}\int_2^4 4\,dt = \dfrac{1}{4}\times 5\times\left.\dfrac{t^2}{2}\right|_0^2 - \dfrac{1}{4}\left.4t\right|_2^4 = \dfrac{10-8}{4} = \dfrac{1}{2} = 0.5$

RMS value $= \sqrt{\dfrac{1}{4}\left[\int_0^2 (5t)^2\,dt + \int_2^4 (-4)^2\,dt\right]} = \sqrt{\dfrac{1}{4}\left[\left.\dfrac{25t^3}{3}\right|_0^2 + \left.16t\right|_2^4\right]}$

$= \sqrt{\dfrac{1}{4}\left[\dfrac{200}{3} + 32\right]} = \sqrt{\dfrac{296}{12}} = \sqrt{\dfrac{74}{3}} = 4.97$

Form factor $= \dfrac{4.97}{0.5} = 9.9$

3.17. Find the net current I of the ac parallel circuit shown in figure below. [WBUT 2016]



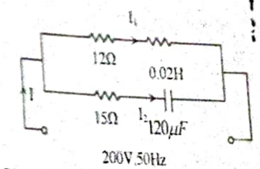
Answer:

Impedance Z_1

$= 12 + j0.02\times 2\times\pi\times 50 = 12 + j2\pi = (12 + j6.28)\Omega$

$= 13.54\angle 27.6^\circ\,\Omega$

current



$I_1 = \dfrac{200}{13.54\angle 27.6^\circ}A = 14.8\angle -27.6 A = 13.1 - j6.84 A$

$Z_2 = 15 - \dfrac{j}{2\times\pi\times 50\times 120\times 10^{-6}} = 15 - j\dfrac{1000}{12} = 15 - j\dfrac{250}{3} = 15 - j83.33$

$= 84.67\angle -80^\circ\,\Omega$

current $I_2 = \dfrac{200}{84.67} = 2.36\angle 80^\circ A = (0.41 + j2.32)A$

Hence, total current $= 13.1 - j6.84 + 0.41 + j2.32 = 13.51 - j4.52$

$= 14.25$ lagging the voltage by $\tan^{-1}\dfrac{4.52}{13.51}$ degrees $= 18.5^\circ$

3.18. Explain the method of measurement of balanced three phase power by two wattmeter method under different power factor conditions. [WBUT 2006, 2009]

OR,

Explain the 2-Wattmeter method of power measurement for a 3-ϕ balanced load. Draw the necessary phasor diagrams. Also show how the power factor can be measured from this method. [WBUT 2014]

OR,

Show that the power in a three phase circuit can be measured using 2 wattmeters. [WBUT 2015]

OR,

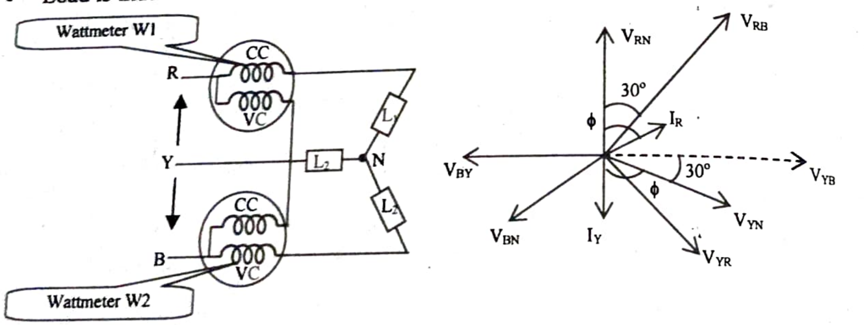
Draw the necessary phasor diagram of power measurement for 3 phase star connected balanced inductive load and show how power factor can be calculated from this method. [WBUT 2017]

Answer:

Power Measurement in three-phase Circuit: The two-wattmeter Method

Suppose

- The loads L_1, L_2 and L_3 are connected in star configuration, as shown in the figure below.
- The current coils of the two wattmeters are connected in any two lines, say the 'red' and 'blue' lines, and the voltage circuits are connected between these lines and the third line.
- V_{RN}, V_{YN} and V_{BN} to be the potential, differences (p.ds) across the loads.
- I_R, I_Y and I_B to be the corresponding values of the line (and phase) currents.
- Load is inductive in nature.



Three-phase power measurement for balanced load using two wattmeter method can be understood from the phasor diagram.

Presume, coil of wattmeter-I measures V_{RB} and current coil measures I_R.

Power measured by wattmeter-I = $V_{RB}\, I_R\, \cos(\phi - 30°)$

Similarly,

Presume coil of wattmeter-II measures V_{YB} and current coil measures I_Y.

Power measured by wattmeter-II = $V_{YB}\, I_Y\, \cos(30° + \phi)$

Total power measured

$$= V_{RB} I_R \cos(\phi - 30°) + V_{YB} I_Y \cos(30° + \phi)$$

$$= \sqrt{3} V_{ph} I_{ph} [\cos 30° \cos\phi + \sin 30° \sin\phi + \cos 30° \cos\phi - \sin 30° \sin\phi]$$

$$= \sqrt{3} V_{ph} I_{ph} [\sqrt{3}\cos\phi] = 3 V_{ph} I_{ph} \cos\phi = \sqrt{3} V_L I_L \cos\phi$$

Measurement of Power Factor using two-Watt-Meter Method

Let W_1 & W_2 be the readings of two Wattmeters.

$$W_1 = V_{LL} I_{LL} \cos(30° - \phi)$$

$$W_2 = V_{LL} I_{LL} \cos(30° + \phi)$$

$$\frac{W_1 + W_2}{W_1 - W_2} = \frac{V_{LL} I_{LL}\left[\cos(30° - \phi) + \cos(30° + \phi)\right]}{V_{LL} I_{LL}\left[\cos(30° - \phi) - \cos(30° + \phi)\right]} = \frac{2\cos 30° \cos\phi}{2\sin 30° \sin\phi} = \frac{2 \times \frac{\sqrt{3}}{2}\cos\phi}{2 \times \frac{1}{2}\sin\phi} = \frac{\sqrt{3}}{\tan\phi}$$

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} \Rightarrow \phi = \tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

$$\text{p.f.} = \cos\phi \qquad \Rightarrow \text{p.f.} = \cos\left\{\tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right\}$$

3.19. A three-phase 230 V load has a power factor 0.7. Two wattmeters are used to measure power which shows the input to be 10 kW. Find the reading of each wattmeter. [WBUT 2009(EVEN)]

Answer:

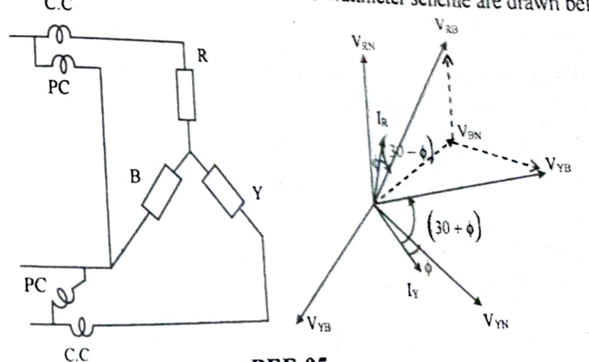
From the given data, $V_{LL} = 230\text{V}$ $\quad \cos\phi = 0.7 \quad \phi = \cos^{-1} 0.7 = 45.58°$

There phase power is given by $\sqrt{3} V_{LL} I_{LL} \cos\phi$

Let us assume that the load is connected in star.

For star connection, $I_{LL} = I_\phi = \dfrac{10\times10^3}{\sqrt{3}\times230\times0.7} A = 35.86 A$

The schematic diagram and phasor diagram for two-wattmeter scheme are drawn below.



The readings of two wattmeters are V_{BN}
$\sqrt{3}V_\phi I_\phi\cos(30°-\phi)$ & $\sqrt{3}V_\phi I_\phi\cos(30°+\phi)$

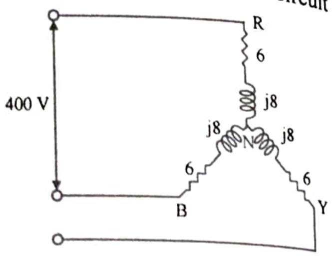
$$\left[\begin{aligned}&\because\sqrt{3}V_\phi I_\phi\cos(30°-\phi)+\sqrt{3}V_\phi I_\phi\cos(30°+\phi)\\&=\sqrt{3}V_\phi I_\phi\left[\frac{\sqrt{3}\cos\phi}{2}+\frac{\sin\phi}{2}+\frac{\sqrt{3}}{2}\cos\phi-\frac{\sin\phi}{2}\right]\\&=\sqrt{3}V_\phi I_\phi\cos\phi=\sqrt{3}V_{LL}I_{LL}\cos\phi\end{aligned}\right]$$

Hence the readings of two wattmeters are
$230\times35.86\cos15.58°W$ & $230\times35.86\cos75.58°W$
i.e., $7.945kW$ & $2.05kW$ respectively.

3.20. Three equal impedances $(6+j8)\ \Omega$ are connected a cross a 400 V, 3-phase, 50 Hz supply. Calculate-
i) the line current & the phase current
ii) power factor
iii) active & reactive drawn by load per phase. **[WBUT 2012]**

Answer:

(i) Assuming the three equal impedances $(6+j8)\Omega$ to be connected in star, the circuit diagram is drawn.



Let 400V be the line voltage.

The per phase voltage is $\frac{400}{\sqrt{3}}\text{V}=231\text{V}$

Per phase impedance $\sqrt{8^2+6^2}\ \Omega=10\Omega$

Hence, per phase current $=\frac{231}{10}\text{A}=23.1\text{ A}$

Line current = phase current for star connection = 23A.

(ii) Power factor angle, $\phi=\tan^{-1}\frac{8}{6}=53.1°$, Power factor $=\cos\phi=\cos53.1°=0.6$

(iii) Active power drawn per phase $=V_{ph}I_{ph}\cos\phi=231\times23.1\times0.6$ Watt $=3.202$ kW

Reactive power per phase

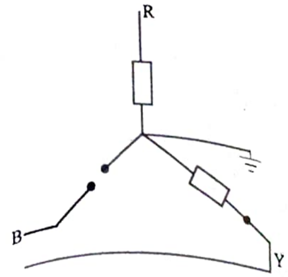
$=V_{ph}I_{ph}\sin\phi=231\times23.1\times0.8$ $\quad\left[\sin\phi=\sqrt{1-\cos^2\phi}=\sqrt{1-(0.6)^2}=0.8\right]$

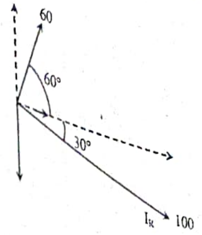
$=4.269\,\text{kVAR}$.

3.21. In a three phase four wire power distribution system, phase B is open while current through R & Y are $100\angle-30°$ & $60\angle60°$. Find the current through the neutral connection. **[WBUT 2015]**

Answer:

The current phasors along R & Y are drawn





The component of I_R along the reference x-direction
$=100\cos30°A=\frac{\sqrt{3}\times100}{2}A=86.6A$

Component of I_Y along the x-direction $=60\cos60°A=\frac{60}{2}A=30A$

Total current along x-direction $=(86.6+30A)=116.6A$

Component of I_Y along the y-direction $=60\sin60°A=\frac{60\times\sqrt{3}}{2}=51.96A$

Component of I_R along the y-direction $=-100\sin30°V=-\frac{100}{2}A=50A$

Resultant current along the y-direction $=(51.96-50)A=1.96V$

Resultant current $=\sqrt{(116.6)^2+(1.96)^2}=116.6A$

Leading the reference x-direction by $\tan^{-1}\frac{1.96}{116.6}=0.54$ degrees

The current through the neutral to balance this current $=116.6\angle180°A$

3.22. a) Explain the meaning of phase and phase difference of sinusoidal quantities. **[WBUT 2018]**

Answer:

For a sinusoidal quantity

$$E_1=E_m\sin(\omega t+\phi_1)$$
$$E_2=E_m\sin(\omega t+\phi_2)$$

where ϕ is the phase angle the angle between voltage and current.

$\therefore$ The phase difference of above two signal E_1 and E_2 is $(\phi_1-\phi_2)$.

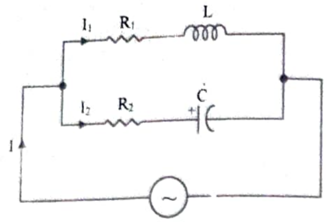
b) A coil of resistance of 30Ω and inductance 320mH is connected in parallel to a circuit consisting of a 75Ω resistor in series with 150μF capacitor. The circuit is connected to a 200V, 50Hz supply. Determine the supply current and circuit power factor. **[WBUT 2018]**

Answer:
The supply current

$$I = I_1 + I_2$$

$$I_1 = \frac{V}{R_1 + i\omega L}$$

$$I_2 = \frac{V}{R_2 - \dfrac{i}{\omega C}}$$



$$I_1 = \frac{200\angle 0^\circ}{30 + j314 \times 320 \times 10^{-3}}$$

$$I_2 = \frac{200\angle 0^\circ}{75 - \dfrac{j}{314 \times 150 \times 10^{-6}}}$$

$$\therefore \quad I = I_1 + I_2 = \frac{200\angle 0^\circ}{30 + 10.048j} + \frac{200\angle 0^\circ}{75 - 21.23j}$$

$$= \frac{200\angle 0^\circ(75 - 21.23j + 30 + 10.048j)}{(30 + 10.048j)(75 - 21.23j)} = 10.67\angle 34.33^\circ$$

Supply current is 10.67 amp and power factor is $\cos 34.33^\circ = 0.825$

c) At $t = 0$, the instantaneous value of 50Hz sinusoidal current is 5A and increase in magnitude further. Its rms value is 10A.
(i) Write the expression of its instantaneous value.
(ii) Find the current at $F = 0.01$ s and $t = 0.015$ s
(iii) Sketch the waveforms indicating these values. **[WBUT 2018]**

Answer:
Since the instantaneous value of current at $t = 0$ is greater than zero and it is increasing in nature, the phase lead can be computed in the following way.
The peak value of current

$$I_m = 10 \times \sqrt{2} = 14.14\text{ A}$$

$$i = I_m \sin(\omega t + \phi)$$

$$\omega = 2\pi f = 100\pi \text{ rad/sec}$$

At $t = 0$,

$$i = I_m \sin\phi$$

$$5 = 10 \times \sqrt{2}\sin\phi$$

Hence, $\sin\phi = \dfrac{1}{2\sqrt{2}} = 0.35$

$$\phi = 20.7^\circ$$

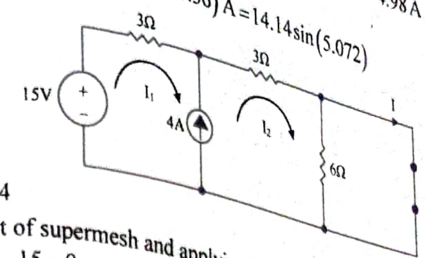
Hence, $i = 14.14\sin(\omega t + 20.7^\circ)$

(ii) The current at $t = 0.01$ sec

$= 14.14\sin(100\pi \times 0.01 + 0.36) = 14.14\sin(\pi + 0.36) = -4.98\text{ A}$

The current at $t = 0.015$ sec

$= 14.14\sin(100 \times \pi \times 0.015 + 0.36)\text{ A} = 14.14\sin(5.072)$



$$I_2 - I_1 = 4$$

Using the concept of supermesh and applying KVL in mesh I and mesh II, we get,

$$3I_1 + 3I_2 - 15 = 0 \quad \text{.... (1)}$$

[∵ No current flows through 6Ω resistor in mesh II since there is a parallel short circuit]

$$I_1 + I_2 = 5 \quad \text{.... (2)}$$

Adding Eqns. (1) and (2), we get,

$$2I_2 = 9$$

$$I_2 = 4.5\text{ A}$$

$$I_1 = 4.5 - 4 = 0.5\text{ A}$$

Now, $I = I_2$ since no current flows through 6Ω resistor
Hence, Norton equivalent current = 4.5 A