Association-Based Recommender System using Statistical Implicative Cohesion Measure

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***Abstract*—The strength of the association rule-based approach compared to other approaches in building recommender systems is that it can provide the deep explanations. Besides, evaluating the quality of generated rules to obtain the better recommendations is also necessary. This can be completed by using the statistical implicative cohesion measure - a measure used for finding the rules with strong implicative relationships. The higher the cohesion value of a rule is, the better the quality of that rule is. This paper proposes a new approach based on the association rules and the cohesion measure to discover the tendencies in a data set and recommend the top items to a user. The proposed recommender system is tested on the data sets Groceries and CourseRegistration. Depending on the purpose of users, they can change the thresholds on the measure to observe the tendencies as well as to get the top recommendations.**

Keywords—recommender system; association rule; statistical implicative cohesion measure,

I. INTRODUCTION

Recommender systems are techniques and software tools that provide suggestions for items to be of use to a user [6]. Recommender systems are widely used in a variety of services, such as eBay or Amazon. With a set of users, a set of items, and the explicit or implicit ratings representing the user evaluation for items, a recommender system: (1) - predicts missing ratings for items that have not yet viewed (or known or liked or listen, etc.) by users; (2) - provides a list of the best recommendations (N items with the highest predicted ratings) to a user. Four approaches for building recommender systems are: content-based filtering, collaborative filtering, association rule-based filtering, and hybrid filtering [1] [6] [8]. In the content-based filtering techniques, the system learns to recommend items similar to the ones that the user liked in the past. In the collaborative filtering techniques, the system recommends to the active user the items by identifying other users with similar tastes. In the association rule-based filtering techniques, the system relies on a set of association rules (constraints), interestingness measures, and a set of items to recommend items to a given user. In the last filtering techniques, the system is based on the combination of the above mentioned techniques.

The strength of the association rule-based filtering that is the weakness of other filtering techniques is transparency [1]. The transparency denotes the degree to which suggestions (recommendations) can be explained to users. The association rule-based filtering can provide deep explanations which take into account semantic item knowledge whereas other techniques only rely on the interpretation of the relationship to nearest neighbors or the similarity of the recommended item to rated items. In association rule based recommender systems, two measures that are widely applied to evaluate itemsets and generate a set of rules are support and confidence [20]. Moreover, to obtain the better recommendations, the quality of the generated association rules should also be evaluated. The objective interestingness measures using statistics derived from data set are devoted to this problem.

The statistical implicative cohesion measure [23] - an asymmetric objective interestingness measure - was developed to discover association rules with strong implicative relationship between the components of a rule. This paper proposes a new approach to develop a recommender system using association rules and the statistical implicative cohesion measure to discover the tendencies in a data set as well as recommend top N items to a user.

The paper is organized into 5 parts. The first part is the introduction. The second part briefly presents the statistical implicative cohesion measure and the algorithms proposed for calculating the cohesion values of the association rules. The third part describes the model of recommender system and the proposed algorithms used for building an association rule-based recommender system using the cohesion measure. The fourth part is the experiment. The last part concludes this paper.

II. Statistical Implicative Cohesion Measure

A. Association rule

TABLE I. A TRANSACTION DATABASE

	i_1	i_2	i_3		i_m
t_1	0	1	0	…	1
t_2	1	0	1	…	0
…	…	…	…	…	…
t_n	1	0	0	…	1

Let $I = \{i_1, i_2, \ldots, i_m\}$ be the set of different attributes (items); $D = \{t_1, t_2, \ldots, t_n\}$ be a transaction database in which each record t_i $(i{:}\,1..n)$ is a transaction, and t_i is a subset of items $(t_i \subseteq I)$, an association rule [20] is denoted by $X \rightarrow Y$ where X is called antecedence, Y is called consequence, X and Y are the subsets of items, and $X \cap Y = \emptyset$. An association rule represents the implicative trend between item sets.

B. Presentation of an association rule in the statistical implication

Let I be the set of items; D be a transaction database; $X \subset I$ $(Y \subset I)$ be the subset of items; $\bar{X}$ $(\bar{Y})$ be the complementary set of X (Y); n be the number of transactions; $n_X = card(X)$ be the number of transactions buy (or like or rate, or etc.) all items of X; $n_Y = card(Y)$ be the number of transactions buy (or like or rate, or etc.) all items of Y; and the counter-example number $n_{X\bar{Y}} = card(X \cap \bar{Y})$ be the number of transactions buy all items of X but do not buy Y.

TABLE II. THE CONTIGENCY TABLE FOR ITEMSETS X AND Y

	X	$\bar{X}$	
Y	n_{XY}	$n_{\bar{X}Y}$	n_Y
$\bar{Y}$	$n_{X\bar{Y}}$	$n_{\bar{X}\bar{Y}}$	$n_{\bar{Y}}$
	n_X	$n_{\bar{X}}$	n

In statistical implication, the presentation of an association rule $X \rightarrow Y$ can be expressed by four values: n, n_X, n_Y, and $n_{X\bar{Y}}$. $\{n, n_X, n_Y, n_{X\bar{Y}}\}$ is called the cardinality of an association rule.

The following algorithm is proposed for calculating the cardinalities of a rule set.

Algorithm 1. Algorithm for calculating the cardinalities of the rule set.

Input: a set of association rules ($ruleset$), a transaction database ($data$)

Output: Cardinalities of the rule set ($cardinalities$)

Steps:

1. Count the number of transactions n.
2. Calculate n_X
   - Transform the left hand side of $ruleset$ into a matrix $lhsRules$ in which $lhsRules[i,j] = TRUE$ if the item j is one element of the left hand side of rule i, and $lhsRules[i,j] = FALSE$ otherwise.
   - Calculate the matrix (cross) product $lhsProduct = lhsRules \ x \ t(data)$
   - Count n_X of each rule i $n_X[i] = rowSum(lhsProduct[i])$.
3. Calculate n_Y. The method for calculating n_Y is similar to the method for calculating n_X, except that it uses the right hand side of $ruleset$.
4. Calculate $n_{X\bar{Y}}$
   - Calculate n_{XY}. The method for calculating n_{XY} is similar to the method for calculating n_X, except that it uses both sides of $ruleset$.
   - Calculate $n_{X\bar{Y}}$ of each rule i: $n_{X\bar{Y}}[i] = n_X[i] - n_{XY}[i]$
5. Concatenate n, n_X, n_Y, and $n_{X\bar{Y}}$ with $ruleset$ to create the $cardinalities$.

C. Statistical implicative cohesion measure

The statistical implicative cohesion measure is first introduced in [23], and is an asymmetric objective interestingness measure. The objective of this measure is to discover the rules with a good implicative quality (i.e. rules with the strong implicative relationship between the antecedence and the consequence). Therefore, it is suitable for using the cohesion measure in analyzing the association rules to obtain useful ones.

The cohesion of an association rule $X \rightarrow Y$, denoted by $c(X,Y)$, is defined by (1).

$$c(X,Y) = \begin{cases} \left(1 - \left(-p\, log_2\, p - (1-p)\, log_2(1-p)\right)^2\right)^{\frac{1}{2}} \\ \quad \text{if } p > 0.5 \\ 0 \ \text{ otherwise} \end{cases} \tag{1}$$

Where $p = \varphi(X,Y)$ measures the implication intensity of the association rule $X \rightarrow Y$.

To accept or reject the relationship $X \rightarrow Y$, considering the number of counter-example $n_{X\bar{Y}}$ is the quite common method. The implication intensity measure [22] [23] was proposed to precisely express the unlikelihood of the counter-example number $n_{X\bar{Y}}$.

The implication intensity of the rule $X \rightarrow Y$, denoted by $\varphi(X,Y)$, is defined by (2).

$$\varphi(X,Y) = \begin{cases} 1 - Pr\left(Q(X,\bar{Y}) \leq q(X,\bar{Y})\right) = \dfrac{1}{2\pi} \displaystyle\int_{q(X,\bar{Y})}^{\infty} e^{-\frac{t^2}{2}}\, dt \\ \quad \text{if } n_Y \neq n \\ 0 \ \text{ otherwise} \end{cases} \tag{2}$$

In (2), $Q(X,\bar{Y})$ is a standardized random variable, and $q(X,\bar{Y})$ is the observed value of $Q(X,\bar{Y})$. They are defined by (3) and (4) respectively.

$$Q(X,\bar{Y}) = \frac{card(A \cap \bar{B}) - \frac{n_X(n-n_Y)}{n}}{\sqrt{\frac{n_X(n-n_Y)}{n}}} \quad (3)$$

$$q(X,\bar{Y}) = \frac{n_{X\bar{Y}} - \frac{n_X(n-n_Y)}{n}}{\sqrt{\frac{n_X(n-n_Y)}{n}}} \quad (4)$$

In (3), $card(A \cap \bar{B})$ is a random variable and follows a Poissonian distribution $P(\lambda)$ with λ defined by (5). The probability of $card(A \cap \bar{B}) \le n_{X\bar{Y}}$ is calculated by (6).

$$\lambda = \frac{n_X(n-n_Y)}{n} \quad (5)$$

$$\Pr(card(A \cap \bar{B}) \le n_{X\bar{Y}}) = \sum_{s=0}^{n_{X\bar{Y}}} \frac{\lambda^s}{s!} e^{-\lambda} \quad (6)$$

The proposed algorithm used for calculating the cohesion value of an association rule is the follow.

Algorithm 2. Algorithm for calculating the cohesion value of an association rule.

Input: an association rule ($rule$)

Output: the cohesion value of the rule (c)

Steps:

1. Calculate the implication intensity value $\varphi(X,Y)$of the $rule$
   - Set X consists of all items in the left hand side of the $rule\ X = lhs(rule)$
   - Set Y consists of all items in the right hand side of the $rule\ Y = rhs(rule)$
   - Calculate the cardinality of the $rule$ $\{n, n_X, n_{\bar{Y}}, n_{X\bar{Y}}\}$ by using the Algorithm 1
   - Calculate λ by following the Formula (5) $lambda = (n_X * (n - n_Y))/\,n$
   - Calculate sum by following the Formula (6)

     $sum = 0$

     for $s = 0$ to $n_{X\bar{Y}}$

     $sum = sum + ((lambda \wedge s)\,/\,factorial(s)) * exp(-lambda)$
   - Calculate the implicative intensity value by following the Formula (2) $\varphi(X,Y) = 1 - sum$
2. Calculate the cohesion value c of the $rule$ by following the Formula (1)

   $p = \varphi(X,Y)$

   if $(p < 0.5)$

   $c = 0$

   else

   $c = sqrt(1 - (-p * log(2,p) - (1-p) * log(2,1-p))\hat{\ }2)$

III. ASSOCIATION RULES BASED RECOMMENDER SYSTEM USING COHESION MEASURE

A. Recommender system

The model of a recommender system consists of a set of users $U = \{u_1, u_2, \dots, u_n\}$, a set of items $I = \{i_1, i_2, \dots, i_m\}$, and a rating matrix $R = (r_{jk})$ where $j = 1..n$ and $k = 1..m$ to record the rating of a user u_j for an item i_k. In the raring matrix R, r_{jk} may be empty if the user u_j has not yet rated the item i_k or does not know about it.

TABLE III. A RATING MATRIX OF A RECOMMENDER SYSTEM

	i_1	i_2	i_3		i_m
u_1	0	1	0	…	1
u_2	1	0	1	…	0
…	…	…	…	…	…
u_n	1	0	0	…	1

The rating matrix R of a recommender system is seen as a transaction database where each user u_j is treated as a transaction t_j containing the subset of items in I with a rating of 1. The whole transaction database is defined as $D = \{t_1, t_2, \dots, t_n\}$ in which the transaction j is $t_j = \{i_k \in I\,|r_{jk} = 1\}$.

B. Association rule-based recommender system using cohesion measure

An association rule-based recommender system is a recommender system that uses a dependency model of items described by a set of association rules to produce recommendations.

The dependency model is built by mining a set of association rules from the rating matrix. Many algorithms are developed for finding a set of association rules such as the algorithm of Agrawal and Srikant [21], the algorithm of Zaki [19]. To extract a set of useful rules, the quality of a rule has to be measured. The interestingness measures are devoted for this problem. The combination of the support and the confidence is the most common method used for quantifying the quality of a rule. However, the confidence is insensitive to the dilatation of the frequency of X, the frequency of Y, and the database size; other measures do not clearly specify the direction of the relationship [20]. Therefore, the cohesion measure is selected along with the support and confidence measures to analyze the association rules.

An association rule-based recommender system using the cohesion measure is proposed by the following algorithm.

Algorithm 3. Algorithm for creating an association rule-based recommender system using the cohesion measure.

Input: a transaction database ($data$), thresholds on two measures: support, confidence ($support\_threshold$, $confidence\_threshold$)

Output: an association rule-based recommender system using the cohesion measure

Steps:

1. Generate a set of rules ($ruleset$) by using the $Apriori$ algorithm and the thresholds on two measures support and confidence selected by users ($support\_threshold$, $confidence\_threshold$)
2. Calculate the cohesion $c(X, Y)$ for each rule of the ruleset by using the Algorithm 2
3. Sort the rules with their cohesion values in the descending order.

Then, the association rule-based recommender system using the cohesion measure is used for: discovering the tendencies in a transaction database, and suggesting the top items to a user. The algorithms are proposed as the follows.

Algorithm 4. Algorithm for discovering the tendencies in a transaction database.

Input: a transaction database ($data$) and a threshold on cohesion measure ($cohesion\_threshold$)

Output: the tendencies

Steps:

1. Build an association rule-based recommender system using the cohesion measure by the Algorithm 3
2. Display the tendencies (the groups of rules) in the transaction database where the right hand sides of rules of one group are the same and the cohesion values of each rule are equal or greater than the selected threshold

Algorithm 5. Algorithm for recommending the list of top N items to a user.

Input: a transaction database ($data$) and the items liked by a user u_a (T_a)

Output: the top N items that u_a can like

Steps:

1. Build an association rule-based recommender system using the cohesion measure by the Algorithm 3
2. Recommend to a user u_a items that u_a can like:
   - Find all matching rules $X \rightarrow Y$ for which $X \subseteq T_a$.
   - Recommend N right hand sides (Y) of the matching rules with the highest cohesion values.

IV. EXPREMENT

A. Experimental data

Two data sets used for the experiment are Groceries [16] and CourseRegistration [4].

The data set Groceries contains the real world transactions of a typical local grocery in 1 month. The data set consists of 9835 rows (transactions), 169 columns (categories of items), 43367 cells with value of true/1 (i.e. the category is purchased by users), and the remaining cells (empty cells) with value of false/0 (i.e. the category is not purchased or not known by users).

The data set CourseRegistration records the course registration information of 300 information technology students of the same cohort in one semester. In that semester, there are 34 courses (23 required courses and 11 elective courses). These elective courses are divided into two groups. The first group is composed of 7 courses related to the physical education whereas the second group consists of 4 courses about society knowledge. For each elective course group, a student just registers one course. The data set consists of 300 rows, 25 columns, 1519 cells with value of true/1 (i.e. the course is registered by students), and the remaining cells (empty cells) with value of false/0.

B. Scenarios

1) Discovering tendencies of the purchase

Steps of this scenario: (1) - Read the data set Groceries; (2) - Discover the tendencies of the purchase by using the Algorithm 4.

2) Recommending top N courses to a student

Steps of this scenario: (1) - Read the data set CourseRegistration; (2) - Recommend to a student u_a the top N courses that u_a should register by using the Algorithm 5.

C. Results

1) Discovering tendencies of the purchase

The support values of the items in the data set Groceries are shown in Figure 1. This distribution shows that 95% of items have the support value less than 10%, and about 50% of items have the support value less than 1%.

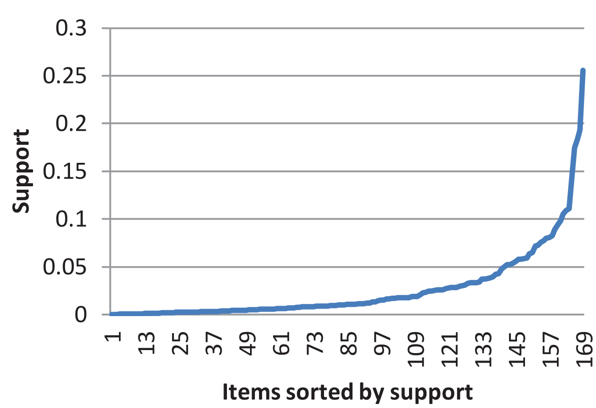

Fig. 1. The support distribution of items in the data set Groceries.

Many potential interesting rules that involve the items with the low support values might be eliminated by a support threshold [20]. To overcome this drawback, in our experiment, the selected min support is 1% (0.01). Because the confidence measure is used for calculating the reliability of the inference made by an association rule $X \rightarrow Y$ (i.e. the present of Y in the transactions that contain X), therefore, the threshold on confidence should be equal or greater than 50% (0.5).

If the selected min cohesion and the selected min confidence are 99% (0.99) and 50% respectively, two tendencies in purchasing categories are revealed in Table IV: buying other vegetables, and buying whole milk. The former (the rules from 1 to 4) shows that users will buy other vegetables if they bought root vegetables and fruit (citrus or tropical fruit), or bough root vegetables and yogurt/rolls (buns). The latter (the remaining rules) shows that users will buy whole milk if they bought categories such as vegetables (other or root), fruit (tropical or pip), yogurt, or whipped/sour cream.

With the proposed association rule-based recommender system, a user can change the thresholds on three measures (support, confidence, and cohesion) to observe the tendencies in a transaction database. If the threshold on the confidence is greater than 60% and the thresholds on the support and the cohesion are 1% and 99% respectively, two tendencies (buying other vegetables and buying whole milk) are not exist in the transaction database.

Discovering the tendencies helps the grocery managers identify the opportunities for cross-selling the products to the customers.

TABLE IV. TABLE SHOWS TENDENCIES OF THE PURCHASE

Tendencies	Presentation of Association Rule in the form X → Y	n	nX	nY	nXY_	Support	Confidence	Cohesion
Buying other vegetables	{citrus fruit,root vegetables} => {other vegetables}	9835	174	1903	72	0.010371	0.586207	1.000000000000000
	{tropical fruit,root vegetables} => {other vegetables}	9835	207	1903	86	0.012303	0.584541	1.000000000000000
	{root vegetables,yogurt} => {other vegetables}	9835	254	1903	127	0.012201	0.502092	0.999999999999995
	{root vegetables,rolls/buns} => {other vegetables}	9835	239	1903	119	0.011998	0.570048	0.999999999999978
Buying whole milk	{root vegetables,yogurt} => {whole milk}	9835	254	2513	111	0.014540	0.562992	1.000000000000000
	{tropical fruit,yogurt} => {whole milk}	9835	288	2513	139	0.015150	0.517361	1.000000000000000
	{other vegetables,yogurt} => {whole milk}	9835	427	2513	208	0.022267	0.512881	1.000000000000000
	{other vegetables,whipped/sour cream} => {whole milk}	9835	284	2513	140	0.014642	0.507042	1.000000000000000
	{tropical fruit,root vegetables} => {whole milk}	9835	207	2513	89	0.011998	0.570048	0.999999999999970
	{other vegetables,butter} => {whole milk}	9835	197	2513	84	0.011490	0.573604	0.999999999999935
	{other vegetables,domestic eggs} => {whole milk}	9835	219	2513	98	0.012303	0.552511	0.999999999999763
	{curd,yogurt} => {whole milk}	9835	170	2513	71	0.010066	0.582353	0.999999999999107
	{pip fruit,other vegetables} => {whole milk}	9835	257	2513	124	0.013523	0.517510	0.999999999995045
	{root vegetables,rolls/buns} => {whole milk}	9835	239	2513	114	0.012710	0.523013	0.999999999989650
	{yogurt,whipped/sour cream} => {whole milk}	9835	204	2513	97	0.010880	0.524510	0.999999999660849

2) Recommending top N courses to a student

For the data set CourseRegistration, if the thresholds on support, confidence and cohesion are equal to 1%, 50%, and 50% respectively, there are 463 extracted association rules. A student can change the thresholds on three measures (support, confidence, and cohesion) as well as the registered courses to obtain the top N courses she/he should register.

With the proposed thresholds, if a student has already registered the course IT System Administration (CT179), what are the top three courses that the student should register in the next step? In this case, only two courses meeting the defined constraints are suggested to the student as in Table V: Fundamentals of Information Technology (CT187) and Political Science (ML006). However, if the min cohesion is increased to 90%, there is only one suggestion – the course CT187.

If we change the number of courses already selected by the student, for example, the student registered: Computer Architecture (CT173) and Probability and Statistics (TN010), the top three recommended courses are: Data Structures (CT103), one course in the elective group related to the physical education (TCTC), and Calculus A2 (TN002) (Table V). If the min confidence is increased to 90%, only one recommendation for this case is the course CT103.

If a student has already registered the list of three courses: Introduction to Software Engineering (CT171), Probability and

Statistics (TN010), and one course in the elective group related to the society science (XHTC), she/he will be recommended the top three courses: TCTC, TN002, and CT103 (Table V). However, the cohesion value of the relationship between the registered courses and the recommended course CT103 is quite low, about 60%. Therefore, in this case, the best recommendations should be TCTC and TN002.

TABLE V. TABLE SHOWS RECOMMENDED COURSES TO A STUDENT

Registered Courses	Top 3 Recommended Courses	Support	Confidence	Cohesion
{CT179}	CT187	0.033333	0.714286	0.999383
	ML006	0.026667	0.571429	0.786578
{CT173, TN010}	CT103	0.086667	1.000000	0.995573
	TCTC	0.146667	0.550000	0.992906
	TN002	0.050000	0.576923	0.917153
{CT171, TN010, XHTC}	TCTC	0.146667	0.550000	0.992906
	TN002	0.036667	0.687500	0.863103
	CT103	0.050000	0.937500	0.595515

V. CONCLUSION

The paper proposes a new approach to build recommender systems by using the association rules and the statistical implicative cohesion measure. The support and confidence measures – symmetric measures – are used for evaluating itemsets to generate association rules while the cohesion measure – an asymmetric measure – is used for analyzing association rules. The cohesion measure discovers rules with a strong implicative relationship between the antecedence and the consequence. The proposed system not only discovers tendencies of transactions in a data set but also recommends the list of top N items to a user. The association rules based recommender system using the cohesion measure was experimented in two scenarios on data sets Groceries and CourseRegistration respectively. The results will be narrowed or extended by varying the threshold on measures, depending on the purpose of users. For the cohesion measure, the higher the values are set, the better the implicative quality rules are obtained.

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