

## ESC103F Engineering Mathematics and Computation: Tutorial #4

### Question 1:

- i) Find a 3x3 matrix with 3 independent columns and all nine entries equal to 1 or 2.
- ii) In part (i), what is the maximum possible number of 1's?

### Question 2:

Suppose matrix  $A$  is 5x2, with column vectors  $\vec{a}_1$  and  $\vec{a}_2$ . We are now going to add one more column to produce matrix  $B$ , now 5x3. Do  $A$  and  $B$  have the same column space if:

- i) the new column is the zero vector?
- ii) the new column is  $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ ?
- iii) the new column is  $\vec{a}_2 - \vec{a}_1$ ?

### Question 3:

Assume the vectors  $\vec{v}$  and  $\vec{w}$  are nonzero and non-parallel. Which of the following sets of vectors are linearly independent:

- i)  $\{\vec{0}, \vec{v}, \vec{w}\}$
- ii)  $\{\vec{v}, \vec{w}, 3\vec{v} - 4\vec{w}\}$
- iii)  $\{\vec{v}, \vec{w}, \vec{v} \times \vec{w}\}$

### Question 4:

If two 5x2 matrices  $A$  and  $B$  each have independent columns, so does the matrix  $A + B$ . Is this statement true or false?

### Question 5:

- i) Solve this system of equations  $S\vec{y} = \vec{c}$  for the unknowns in  $\vec{y}$  in terms of the constants in  $\vec{c}$  by taking advantage of the triangular structure of matrix  $S$ :

$$S\vec{y} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \vec{c} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

- ii) Express the solution in part (i) for  $\vec{y}$  as a matrix  $T$  times the vector  $\vec{c}$ .
- iii) The matrix  $T$  is called the inverse of matrix  $S$ . Are the columns of  $S$  independent or dependent?