ESC103F Engineering Mathematics and Computation: Tutorial #1

Question 1: Let $\overrightarrow{OP_1}$ and $\overrightarrow{OP_2}$ be the vectors in standard position of two points P_1 and P_2.

a) Using vector addition, derive an expression for the vector $\overrightarrow{P_1P_2}$ in terms of $\overrightarrow{OP_1}$ and $\overrightarrow{OP_2}$. (Hint: draw a diagram that locates the origin, P_1 and P_2.)

b) If the point M is $1/3^{rd}$ of the way from P_1 to P_2, derive a general expression for the vector $\overrightarrow{OM}$ in terms of $\overrightarrow{OP_1}$ and $\overrightarrow{OP_2}$. (Hint: draw a diagram that locates the origin, P_1, P_2 and M.) From this general expression, determine M if P_1=(1,2,3) and P_2=(4,5,6).

Question 2: Given points P(2,-1,4), Q(3,-1,2), A(0,2,1) and B(1,3,0), determine if $\overrightarrow{PQ}$ and $\overrightarrow{AB}$ are parallel.

Question 3: Let the points A, B, C, and D in the plane form a quadrilateral ABCD (a four-sided figure). Let E, F, G, and H be the midpoints of each side of the quadrilateral. Using a vector method approach, prove that the quadrilateral EFGH is a parallelogram. (Hint: draw a diagram showing all eight points.)

Question 4: Points A(-3, 2), B(1, -2) and C(7, 1) are given.

Find the coordinates of point D so that ABCD forms a parallelogram in order of points A, B, C, D. (Hint: plot the 3 points A, B, and C and show the approximate location of point D to form the parallelogram.)

Question 5: The linear combination of two vectors $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ defines a plane that goes through the origin. Sketch these two vectors in the xyz space.

i) Consider linear combinations $c\vec{v} + d\vec{w}$. Write an expression for a single vector in terms of *c* and *d* that defines the plane.

ii) Using your result from part (i), find a vector that is **not** in the plane.

Question 6: Find two different linear combinations of the three vectors $\vec{u} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$ that produce $\vec{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. If you take **any** three vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$, will there always be some linear combination that produces $\vec{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$?