

## ESC103F Engineering Mathematics and Computation: Tutorial #5

**Question 1:** Test the “truth” of the associative law  $(AB)C = A(BC)$ :

i)  $[1 \ 1] \begin{bmatrix} 1 \\ 1 \end{bmatrix} [1 \ 1 \ 1]$

ii)  $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

**Question 2:**

Let  $A = \begin{bmatrix} 2 & -2 & 1 & 6 & 0 \\ 1 & -1 & 0 & 2 & 0 \\ 3 & -3 & 0 & 6 & 1 \end{bmatrix}$ . We want to factor this matrix,  $A = CR$ .

- i) Construct matrix  $C$  from matrix  $A$  by going from left to right and putting each column of  $A$  into  $C$  if that column is not a combination of earlier columns.
- ii) Construct matrix  $R$ . Note: if  $C$  has  $r$  columns, then  $R$  must have  $r$  rows.

**Question 3:** If all columns of  $A = [\vec{a} \ \vec{a} \ \vec{a}]$  are vectors in  $R^n$  and where  $\vec{a} \neq \vec{0}$ , what are  $C$  and  $R$ , where  $A = CR$ ?

**Question 4:** Why is it not possible for a matrix with 4 rows and 7 columns to have 5 independent columns?

**Question 5:** Complete the 2x2 matrices to meet the requirements specified:

i)  $\begin{bmatrix} 3 & 6 \\ 5 & \square \end{bmatrix}$  (rank 1)

ii)  $\begin{bmatrix} 6 & \square \\ 7 & \square \end{bmatrix}$  (orthogonal columns)

iii)  $\begin{bmatrix} 2 & \square \\ 3 & 6 \end{bmatrix}$  (rank 2)

iv)  $\begin{bmatrix} 3 & 4 \\ \square & -3 \end{bmatrix}$  ( $A^2 = I$ )