

ESC103F Engineering Mathematics and Computation: Tutorial #5

Question 1: Test the “truth” of the associative law $(AB)C=A(BC)$:

i) $[1 \ 1] \begin{bmatrix} 1 \\ 1 \end{bmatrix} [1 \ 1 \ 1]$

ii) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

Question 2:

Let $A = \begin{bmatrix} 2 & -2 & 1 & 6 & 0 \\ 1 & -1 & 0 & 2 & 0 \\ 3 & -3 & 0 & 6 & 1 \end{bmatrix}$. We want to factor this matrix, $A = CR$.

- i) Construct matrix C from matrix A by going from left to right and putting each column of A into C if that column is not a combination of earlier columns.
- ii) Construct matrix R . Note: if C has r columns, then R must have r rows.

Question 3: If all columns of $A = [\vec{a} \ \vec{a} \ \vec{a}]$ are vectors in R^n and where $\vec{a} \neq \vec{0}$, what are C and R , where $A = CR$?

Question 4: Why is it not possible for a matrix with 4 rows and 7 columns to have 5 independent columns?

Question 5: Complete the 2x2 matrices to meet the requirements specified:

i) $\begin{bmatrix} 3 & 6 \\ 5 & \square \end{bmatrix}$ (rank 1)

ii) $\begin{bmatrix} 6 & \square \\ 7 & \square \end{bmatrix}$ (orthogonal columns)

iii) $\begin{bmatrix} 2 & \square \\ 3 & 6 \end{bmatrix}$ (rank 2)

iv) $\begin{bmatrix} 3 & 4 \\ \square & -3 \end{bmatrix}$ ($A^2 = I$)