

ESC103F Engineering Mathematics and Computation: Tutorial #4

Question 1:

- i) Find a 3×3 matrix with 3 independent columns and all nine entries equal to 1 or 2.
- ii) In part (i), what is the maximum possible number of 1's?

Question 2:

Suppose matrix A is 5×2 , with column vectors \vec{a}_1 and \vec{a}_2 . We are now going to add one more column to produce matrix B , now 5×3 . Do A and B have the same column space if:

- i) the new column is the zero vector?
- ii) the new column is $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$?
- iii) the new column is $\vec{a}_2 - \vec{a}_1$?

Question 3:

Assume the vectors \vec{v} and \vec{w} are nonzero and non-parallel. Which of the following sets of vectors are linearly independent:

- i) $\{\vec{0}, \vec{v}, \vec{w}\}$
- ii) $\{\vec{v}, \vec{w}, 3\vec{v} - 4\vec{w}\}$
- iii) $\{\vec{v}, \vec{w}, \vec{v} \times \vec{w}\}$

Question 4:

If two 5×2 matrices A and B each have independent columns, so does the matrix $A + B$. Is this statement true or false?

Question 5:

- i) Solve this system of equations $S\vec{y} = \vec{c}$ for the unknowns in \vec{y} in terms of the constants in \vec{c} by taking advantage of the triangular structure of matrix S :

$$S\vec{y} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \vec{c} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

- ii) Express the solution in part (i) for \vec{y} as a matrix T times the vector \vec{c} .
- iii) The matrix T is called the inverse of matrix S . Are the columns of S independent or dependent?