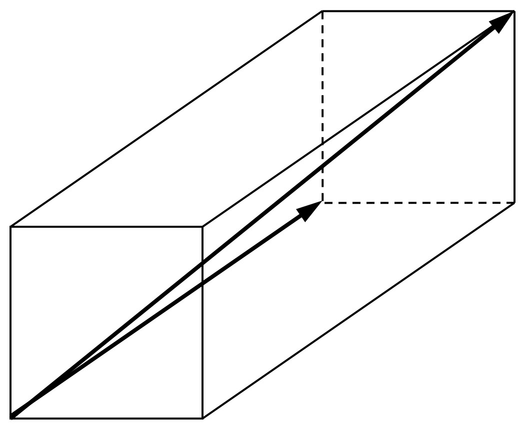
ESC103F Engineering Mathematics and Computation: Tutorial #2

Question 1: Consider the points located at A(1,1,1), B(2,2,3) and C(6,1,10). Find the angle ABC where B is the vertex.

Question 2: Let $\vec{u}$ and $\vec{v}$ be nonzero vectors in 2-D or 3-D and let $k = ||\vec{u}||$ and $l = ||\vec{v}||$. Prove that the vector $\vec{w} = l\vec{u} + k\vec{v}$ bisects the angle between $\vec{u}$ and $\vec{v}$.

Question 3: Given a rectangular solid with sides of lengths 1, 1, and $\sqrt{2}$, use a vector approach to find the angle between a body diagonal and one of the longest sides.



Question 4: Using cross product, find the area of the triangle having vertices A(1, 2), B(7, -2) and C(7, 20/3).

Question 5: Suppose you start with the standard Cartesian coordinate system for 2-D in terms of the unit vectors:

$$\vec{\imath} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } \vec{\jmath} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

We are now going switch to a new 2-D coordinate system where the new x-axis, denoted by x’, has been rotated 30 degrees counterclockwise from the original x-axis, and the new y-axis, denoted by y’, has been rotated 15 degrees clockwise from the original y-axis.

i) Make a sketch of the old and new coordinate systems.

ii) Show that the new unit vector $\vec{\imath}'$ along x’ can be expressed in terms of the original coordinate system as:

$$\vec{\imath}' = \begin{bmatrix} cos30^o \\ sin30^o \end{bmatrix}$$

iii) Derive a similar expression for the new unit vector $\vec{j}'$ along y' in terms of the original coordinate system.

iv) Express the vector $\begin{bmatrix}2\\1\end{bmatrix}$ in the original coordinate system as a linear combination of the new unit vectors associated with the new coordinate system.

Question 6: Using projections, show that the sum of the squares of the distances from a point $P = (x_1, y_1)$ to the perpendicular lines $ax + by = 0$ and $bx - ay = 0$ is equal to the square of the length of the vector $\overrightarrow{OP} = \begin{bmatrix}x_1\\y_1\end{bmatrix}$.

Question 7: Prove that if $\vec{u}$ and $\vec{v}$ are vectors in R^3, then $\vec{u} \cdot (\vec{u} \times \vec{v}) = 0$.

Question 8: Prove the Lagrange Identity, that is if $\vec{u}$ and $\vec{v}$ are vectors in R^3, then

$$||\vec{u} \times \vec{v}||^2 = ||\vec{u}||^2||\vec{v}||^2 - (\vec{u} \cdot \vec{v})^2.$$

Question 9: True or false? If $\vec{u}$ is orthogonal to $\vec{v} + \vec{w}$, then $\vec{u}$ is orthogonal to $\vec{v}$ and $\vec{w}$. Justify your answer with a proof (for true) or a counter example (for false).

Question 10: True or false? If $\vec{u}$ and $\vec{v}$ are nonzero vectors such that

$$||\vec{u} + \vec{v}||^2 = ||\vec{u}||^2 + ||\vec{v}||^2$$

then $\vec{u}$ and $\vec{v}$ are orthogonal. Justify your answer with a proof (for true) or a counter example (for false).

Question 11: True or false? If $\vec{u}$ and $\vec{v}$ are nonzero vectors, then $||\vec{u} + \vec{v}|| = ||\vec{u}|| + ||\vec{v}||$ if and only if $\vec{u}$ and $\vec{v}$ are parallel vectors. Justify your answer with a proof (for true) or a counter example (for false).