

ESC103F Engineering Mathematics and Computation: Tutorial #3

Question 1: Consider the plane that intersects the coordinate axes at $x = a$, $y = b$ and $z = c$, where a, b, c are nonzero scalars. Using the vector cross product, derive and show that the scalar equation of this plane is given by:

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Question 2: Find the vector equation for the line through $(-2, 5, 0)$ that is parallel to the planes $2x + y - 4z = 0$ and $-x + 2y + 3z + 1 = 0$.

Question 3: Let A, B and C(2, -1, 1) be the vertices of a triangle where \overrightarrow{AB} is parallel to $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, \overrightarrow{AC} is parallel to $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$, and angle $C = 90^\circ$. Find the vector equation of the line through B and C.

Question 4: Find all unit vectors parallel to the yz -plane that are orthogonal to the vector $\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$.

Question 5: The volume of a tetrahedron is given by:

$$\frac{1}{3}(\text{area of base})(\text{height})$$

Use this result to prove that the volume of a tetrahedron with sides defined by the vectors \vec{a} , \vec{b} and \vec{c} is given by $\frac{1}{6} |\vec{a} \cdot (\vec{b} \times \vec{c})|$, taking the base to be defined by vectors \vec{b} and \vec{c} . (Note: $\vec{a} \cdot (\vec{b} \times \vec{c})$ is referred to as the scalar triple product.)

Question 6: Find the distance between the point $P(-3,1,3)$ and the plane $5x + z = 3y - 4$.

Question 7: Show that the planes $3x - y + 6z = 7$ and $-6x + 2y - 12z = 1$ are parallel, and find the distance between the planes.

Question 8: Find the vector equation for the line in \mathbb{R}^3 that contains the point $P(-1,6,0)$ and is orthogonal to the plane $4x - z = 5$.

Question 9: Find the scalar equation for the plane that is represented by the vector equation:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 6 \end{bmatrix} + t \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} + s \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

Question 10: The equation $ax + by = 0$ represents a line through the origin in \mathbb{R}^2 if a and b are not both zero. What does this equation represent in \mathbb{R}^3 if you think of it as $ax + by + 0z = 0$? Explain.

Question 11: Consider the following planes in \mathbb{R}^3 :

$$1x + 0y + 0z = 2$$

$$0x + 1y + 0z = 3$$

$$0x + 0y + 1z = 4$$

- i) Construct a mental image of the row picture.
- ii) Express the 3 equations as a single vector equation.
- iii) Construct a mental image of the column picture.
- iv) What does the solution to these 3 equations represent in \mathbb{R}^3 ?

Question 12: Consider the following planes in \mathbb{R}^3 :

$$x + y + z = 2$$

$$x + 2y + z = 3$$

$$2x + 3y + 2z = 5$$

- i) Express these 3 equations as a single vector equation.
- ii) Find two linear combinations of the column vectors on the left-hand side that equal the right-hand side vector.
- iii) Show that it is only possible to find linear combinations of the column vectors on the left-hand side when the right-hand side vector is equal to $\begin{bmatrix} 4 \\ 6 \\ c \end{bmatrix}$ and c is equal to what value?
- iv) Express these 3 equations in the form $A\vec{v} = \vec{b}$.