ESC103F Engineering Mathematics and Computation: Tutorial #6

Question 1: Consider the system of linear equations given below:

$$\begin{aligned} 2x + 3y &= 1 \\ 10x + 9y &= 11 \end{aligned}$$

i) Using elimination, determine the equivalent upper triangular system.
ii) What are the two pivots associated with this upper triangular system?
iii) Use back substitution to find the unknowns (and check your solution).

Question 2: Consider the system of linear equations given below:

$$\begin{aligned} ax + by &= f \\ cx + dy &= g \end{aligned}$$

We will assume that the first pivot a is nonzero.

i) Elimination produces what formula for the second pivot?
ii) What condition must hold for the second pivot to be nonzero?
iii) Assuming the condition in part (ii) holds, what is the value for y?

Question 3:

A system of linear equations cannot have just two solutions. Let's examine why this is the case.

i) Consider a system of linear equations with 3 unknowns. Assume that two solutions are known, (x, y, z) and (X, Y, Z). What is another solution?
ii) If you know that 25 planes in R^3 meet at two points, where else do they meet?

Question 4:

Consider the system of linear equations below:

$$\begin{aligned} 2x + 5y + z &= 0 \\ 4x + dy + z &= 2 \\ y - z &= 3 \end{aligned}$$

The objective is to determine the equivalent upper triangular system.

i) What is the upper triangular system for the value of d in part (i)?

ii) What value of d produces a zero pivot in row 3?
iii) Is there a solution to this system for the value of d in part (iii)?

Question 5:

Consider matrix A given below:

$$A = \begin{bmatrix} a & 2 & 3 \\ a & a & 4 \\ a & a & a \end{bmatrix}$$

For which three values of a will elimination fail to produce 3 nonzero pivots?

Question 6:

i) Determine three elementary matrices E_1, E_2, E_3 that put matrix A into upper triangular form U,

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix}$$

where $E_3E_2E_1A = U$.

ii) Solve for one elementary matrix $E = E_3E_2E_1$.

iii) Include $\vec{b}$ to produce the augmented matrix,

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 4 & 6 & 1 & 0 \\ -2 & 2 & 0 & 0 \end{array}\right]$$

With our goal being to solve this system, we want to use elimination to convert $A\vec{x} = \vec{b}$ into $U\vec{x} = \vec{c}$. Determine $\vec{c}$ from E and $\vec{b}$.

iv) Working with $U\vec{x} = \vec{c}$, solve for $\vec{x}$ using back substitution.

Question 7: Consider plane #1: $x - 3y - z = 0$ and plane #2: $x - 3y - z = 12$. These planes are parallel because their normal vectors are parallel.

i) Give a vector equation for all points on each plane using the form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ? \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} ? \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} ? \\ 0 \\ 1 \end{bmatrix}$$

ii) From just looking at the two vector equations derived in part (i), how do you know the two planes are parallel?

Question 8: Consider matrix $A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$.

i) Find matrix R by first finding matrix C where $A = CR$.

ii) Use Gaussian elimination to find R by first finding R_0.

Question 9: Put as many 1's as possible in a 4x7 R_0 matrix that is in reduced row echelon form where the leading variables correspond to columns 2, 4 and 5.

Question 10: Consider the system $A\vec{x} = \vec{0}$ and matrix A is 3x5. Suppose column 4 of matrix A is all zeros. Then, x_4 is certainly what kind of variable? What is the special solution $\vec{x}$ associated with this variable x_4?

Question 11: Construct a matrix A whose column space contains $\begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$ and has $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ as a solution to $A\vec{x} = \vec{0}$. What other $A's$ would have these same properties?