A New Switching Superposition Strategy in Decode-Forward Relay System

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***Abstract*—In this correspondence paper, a decode-forward relay system with a source, a relay, and a destination is considered. A new switching superposition-coded (NSSC) relay scheme is proposed to improve error performance by increasing the equivalent squared minimum distance. On the receiver side, a low-complexity linear-combining successive-interference-cancellation decoder is proposed for the NSSC scheme. The theoretical and simulation results show that the NSSC scheme achieves better performance compared to the existing superposition-coded relay schemes.**

***Index Terms*—Maximum likelihood, pair-wise error probability, squared minimum distance.**

I. Introduction

Cooperative communication systems with the help of relay nodes have been studied to improve the achievable rate and communication reliability [1]–[3]. The two most important methods in cooperative relay systems are amplify-forward and decode-forward (DF) relaying [1]. On the other hand, superposition codes comprised by multiple superimposed Gaussian signals have been used in a relay system to improve the error performance and the spectral efficiency [4]–[8]. Superposition codes have also been designed with digital modulations [9]–[13], such as binary phase-shift keying (BPSK) or pulse amplitude modulation (PAM).

In [13], a suboptimal switched-power superposition-coded (SPSC) relay scheme was proposed for PAM by deriving an equivalent squared minimum distance (ESMD) [14] that determines the error probability. Although the SPSC scheme with the near maximum likelihood (near-ML) decoder achieves excellent performance, the near-ML decoder is not attractive for practical systems due to its high complexity. Furthermore, it is not easy to find a high-performance linear receiver for the SPSC scheme. In this paper, we propose a *new switching superposition-coded (NSSC)* DF relay scheme by changing the sign of one of the superimposed symbols in addition to exchanging their powers. Inspired by the cooperative maximum ratio combining (CMRC) [2], we also propose a low-complexity *linear-combining successive-interference-cancellation (LCSIC)* decoder which exhibits very good performance for the NSSC scheme. Comparing the error probabilities, the NSSC scheme exhibits better performance than the existing schemes for both near-ML and LCSIC decoders. Simulation results confirm the performance improvement of the proposed NSSC scheme under the perfect channel state information (CSI) and the channel estimation error (CEE).

Throughout the paper, the following notations are used. $\max_x f(x)$ and $\min_x f(x)$ mean the largest and smallest values of a function in its domain, respectively; $\arg\min_x f(x)$ denotes the points in the domain of a function at which the function values are maximized; and $x \sim \mathcal{CN}(0, \sigma^2)$ means that x is a circularly symmetric Gaussian random variable with zero mean and variance σ^2.

II. System Model

Consider a single-antenna DF relay system consisting of one source, one relay, and one destination. Half duplex transmission and frequency-flat quasi-static Rayleigh fading are assumed. It is also assumed that the relay knows the instantaneous CSI of the source-relay (SR) link and the destination knows the instantaneous CSIs of the SR, the SD, and the relay-destination (RD) links.

In the first phase, the source broadcasts a two-layer superposition codeword $x_S(\mathbf{s}) = \alpha_1 s_1 + \alpha_2 s_2$, $\mathbf{s} = (s_1, s_2)$, $s_i \in \{-1, +1\}$, $i \in \{1, 2\}$, $\alpha_1 > \alpha_2 > 0, \alpha_1^2 + \alpha_2^2 = 1$.[1] The received signals at the relay and destination are given by

$$\begin{aligned} y_{SR} &= h_{SR} x_S(\mathbf{s}) + z_{SR} \\ y_{SD} &= h_{SD} x_S(\mathbf{s}) + z_{SD}, \end{aligned} \tag{1}$$

where h_{SR} and h_{SD} are the channel coefficients of the SR and SD links, respectively, and $z_{SR} \sim \mathcal{CN}(0, \sigma^2)$ and $z_{SD} \sim \mathcal{CN}(0, \sigma^2)$ are the respective noise terms.[2]

In the second phase, the relay decodes the received signal by a maximum likelihood (ML) decoder and forwards a new codeword $x_R(\mathbf{s}_R) = \beta_1 s_1^R + \beta_2 s_2^R$, $s_i^R \in \{-1, +1\}$, $i \in \{1, 2\}$ $(\beta_1^2 + \beta_2^2 = 1)$ to the destination. Without cyclic redundancy check (CRC) codes at the relay, the decoded symbols $\mathbf{s}_R = (s_1^R, s_2^R)$ may be different from $\mathbf{s} = (s_1, s_2)$.

The received signal at the destination in the second phase is given by

$$y_{RD} = h_{RD} x_R(\mathbf{s}_R) + z_{RD}, \tag{2}$$

where h_{RD} is the channel coefficient of the RD link and $z_{RD} \sim \mathcal{CN}(0, \sigma^2)$ is the noise component in the second phase. Then the transmitted signal-to-noise ratio (SNR) is proportional to $\rho = 1/\sigma^2$.

III. New Switching Superposition-Coded (NSSC) Relay Scheme

Considering the error probability in the SR link, the ML decoder is too complicated to be analyzed [3]. Using the pair-wise error probability (PEP) [14] instead of the exact error probability, the full-diversity achievable near-ML decoder [3], [13, eq. (5)] at the

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[1]To simplify the analysis, only one-dimensional modulations such as BPSK are considered. It is not difficult to extend the result to two-dimensional modulation.

[2]The units of the signal power and noise power are in Watt and are ignored throughout the paper.

destination is written as

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \Big[|y_{\mathrm{SD}} - h_{\mathrm{SD}} x_{\mathrm{S}}(\mathbf{s})|^2 + \min_{\tilde{\mathbf{s}}_{\mathrm{R}}} \{ |y_{\mathrm{RD}} - h_{\mathrm{RD}} x_{\mathrm{R}}(\tilde{\mathbf{s}}_{\mathrm{R}})|^2 - \sigma^2 \ln P_{\mathrm{SR}}(\mathbf{s} \to \tilde{\mathbf{s}}_{\mathrm{R}}) \} \Big], \quad (3)$$

where $\mathbf{s} \in \{-1,+1\} \times \{-1,+1\}$, $\tilde{\mathbf{s}}_{\mathrm{R}} \in \{-1,+1\} \times \{-1,+1\}$, $P_{\mathrm{SR}}(\mathbf{s} \to \tilde{\mathbf{s}}_{\mathrm{R}}) = 1$ for $\tilde{\mathbf{s}}_{\mathrm{R}} = \mathbf{s}$, $P_{\mathrm{SR}}(\mathbf{s} \to \tilde{\mathbf{s}}_{\mathrm{R}}) = Q\left(\sqrt{\frac{|h_{\mathrm{SR}}|^2}{2\sigma^2} |x_{\mathrm{S}}(\mathbf{s} - \tilde{\mathbf{s}}_{\mathrm{R}})|^2}\right)$ for $\tilde{\mathbf{s}}_{\mathrm{R}} \neq \mathbf{s}$, and $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{y^2}{2}} dy$. Using the near-ML decoder, the maximum PEP is derived in [13, eq. (13)] and given by

$$\max_{\mathbf{s}, \tilde{\mathbf{s}}} P(\mathbf{s} \to \tilde{\mathbf{s}}) \leq 4 \exp\left(-\frac{D_{\min}^2}{4\sigma^2} \right) \quad (4)$$

where $D_{\min}^2$ is the ESMD of the system which is the minimum of the ESMDs for the broadcast channel (BC) $\mathrm{S} \to \mathrm{D}, \mathrm{R}$ and the multiple access channel (MAC) $\mathrm{S}, \mathrm{R} \to \mathrm{D}$ such as

$$\begin{aligned} D_{\min}^2 &= \min\{D_{\mathrm{BC}}^2, D_{\mathrm{MAC}}^2\}, \\ D_{\mathrm{BC}}^2 &= \min_{\mathbf{s}, \tilde{\mathbf{s}}} \left\{ |h_{\mathrm{SD}}|^2 |x_{\mathrm{S}}(\mathbf{s} - \tilde{\mathbf{s}})|^2 + \frac{1}{2} |h_{\mathrm{SR}}|^2 |x_{\mathrm{S}}(\mathbf{s} - \tilde{\mathbf{s}})|^2 \right\} \\ &= 4 \left(|h_{\mathrm{SD}}|^2 + \frac{1}{2} |h_{\mathrm{SR}}|^2 \right) \min\{(\alpha_1 - \alpha_2)^2, \alpha_2^2\}, \end{aligned} \quad (5)$$

and

$$D_{\mathrm{MAC}}^2 = \min_{\mathbf{s}, \tilde{\mathbf{s}}} \{ |h_{\mathrm{SD}}|^2 |x_{\mathrm{S}}(\mathbf{s} - \tilde{\mathbf{s}})|^2 + |h_{\mathrm{RD}}|^2 |x_{\mathrm{R}}(\mathbf{s} - \tilde{\mathbf{s}})|^2 \}.$$

D_{MAC}^2 is related to both $x_{\mathrm{S}}(\cdot)$ and $x_{\mathrm{R}}(\cdot)$ and can be rewritten as

$$D_{\mathrm{MAC}}^2 = \min\{D_1^2, D_2^2, D_3^2, D_4^2\}, \quad (6)$$

where

$$\begin{aligned} D_1^2 &= |h_{\mathrm{SD}}|^2 (\alpha_1 |s_1 - \tilde{s}_1| - \alpha_2 |s_2 - \tilde{s}_2|)^2 \\ &\quad + |h_{\mathrm{RD}}|^2 (\beta_1 |s_1 - \tilde{s}_1| - \beta_2 |s_2 - \tilde{s}_2|)^2 \\ D_2^2 &= |h_{\mathrm{SD}}|^2 (\alpha_1 |s_1 - \tilde{s}_1| + \alpha_2 |s_2 - \tilde{s}_2|)^2 \\ &\quad + |h_{\mathrm{RD}}|^2 (\beta_1 |s_1 - \tilde{s}_1| + \beta_2 |s_2 - \tilde{s}_2|)^2 \\ D_3^2 &= (|h_{\mathrm{SD}}|^2 \alpha_1^2 + |h_{\mathrm{RD}}|^2 \beta_1^2) |s_1 - \tilde{s}_1|^2 \\ D_4^2 &= (|h_{\mathrm{SD}}|^2 \alpha_2^2 + |h_{\mathrm{RD}}|^2 \beta_2^2) |s_2 - \tilde{s}_2|^2. \end{aligned}$$

Here D_1^2, D_2^2, and D_3^2 are the squared distances for s_1 whereas D_1^2, D_2^2, and D_4^2 are that for s_2. There are two existing superposition-coded relay schemes: the conventional one and the SPSC one.

- The conventional scheme, $\beta_1 = \alpha_1$ and $\beta_2 = \alpha_2$:
  In this case, the squared distances in (6) are:

$$\begin{aligned} D_1^2 &= 4(|h_{\mathrm{SD}}|^2 + |h_{\mathrm{RD}}|^2)(\alpha_1 - \alpha_2)^2 \quad (7) \\ D_2^2 &= 4(|h_{\mathrm{SD}}|^2 + |h_{\mathrm{RD}}|^2)(\alpha_1 + \alpha_2)^2 \\ D_3^2 &= 4(|h_{\mathrm{SD}}|^2 + |h_{\mathrm{RD}}|^2)\alpha_1^2 \\ D_4^2 &= 4(|h_{\mathrm{SD}}|^2 + |h_{\mathrm{RD}}|^2)\alpha_2^2. \quad (8) \end{aligned}$$

  Since D_3^2 is larger than D_4^2, s_1 always exhibits better error performance than s_2.

- The SPSC scheme, $\beta_1 = \alpha_2$ and $\beta_2 = \alpha_1$:
  To improve the performance, the SPSC scheme switches the power of s_1 and s_2 at the relay, i.e., $\beta_1 = \alpha_2$ and $\beta_2 = \alpha_1$. The squared distances D_1^2 and D_2^2 are the ones in (7) and (8), respectively, and

$$\begin{aligned} D_3^2 &= 4(|h_{\mathrm{SD}}|^2 \alpha_1^2 + |h_{\mathrm{RD}}|^2 \alpha_2^2) \quad (9) \\ D_4^2 &= 4(|h_{\mathrm{SD}}|^2 \alpha_2^2 + |h_{\mathrm{RD}}|^2 \alpha_1^2). \quad (10) \end{aligned}$$

  As a result, $D_{\min}^2$ becomes larger and the average symbol error probability (SEP) of s_1 and s_2 is improved.

Similarly, one can observe that D_2^2 is always larger than D_1^2. We propose a NSSC relay scheme that changes the sign of one of the superimposed symbols in addition to switching powers, i.e., $\beta_1 = -\alpha_2$ and $\beta_2 = \alpha_1$. Then we have

$$\begin{aligned} D_1^2 &= 4(|h_{\mathrm{SD}}|^2 (\alpha_1 - \alpha_2)^2 + |h_{\mathrm{RD}}|^2 (\alpha_1 + \alpha_2)^2) \quad (11) \\ D_2^2 &= 4(|h_{\mathrm{SD}}|^2 (\alpha_1 + \alpha_2)^2 + |h_{\mathrm{RD}}|^2 (\alpha_1 - \alpha_2)^2) \quad (12) \end{aligned}$$

and D_3^2 and D_4^2 as in (9) and (10). By lowering the higher value D_2^2 from (8) to (12) and raising the lower value D_1^2 from (7) to (11), the minimum of them becomes larger. Since there always exists the case of $D_{\mathrm{MAC}}^2 < D_{\mathrm{BC}}^2$ in random fading channels, increasing the value of D_{MAC}^2 improves performance according to (5) and (4).

IV. Linear-Combining Successive-Interference-Cancellation (LCSIC) Decoder

The near-ML decoder in (3) reduces the decoding complexity while achieving similar performance to the ML decoder [3]. However, it is still complicated to be applied in practical system. Since all transmittable signals $\tilde{\mathbf{s}}_{\mathrm{R}}$ at the relay are considered in (3), the decoding complexity order is $|\mathcal{A}|^4$ for $s_i \in \mathcal{A}, i \in \{1, 2\}$, where $|\mathcal{A}|$ means the the cardinality of the symbol set $\mathcal{A}$. For ease of use in practical systems, a low-complexity LCSIC scheme is proposed.

A. LCSIC Scheme Without Decoding Error at Relay

Consider the channels in (1) and (2) for $\mathbf{s}_{\mathrm{R}} = \mathbf{s}$. For the conventional scheme, we have $x_{\mathrm{R}}(\cdot) = x_{\mathrm{S}}(\cdot)$, i.e., the same symbols are transmitted through two independent channels. Thus the maximum ratio combining (MRC) can be applied as

$$\begin{aligned} \tilde{y} &= w_{\mathrm{SD}} y_{\mathrm{SD}} + w_{\mathrm{RD}} y_{\mathrm{RD}} \\ &= (w_{\mathrm{SD}} h_{\mathrm{SD}} + w_{\mathrm{RD}} h_{\mathrm{RD}})(\alpha_1 s_1 + \alpha_2 s_2) + w_{\mathrm{SD}} z_{\mathrm{SD}} + w_{\mathrm{RD}} z_{\mathrm{RD}}, \end{aligned} \quad (13)$$

where $w_{\mathrm{SD}} = h_{\mathrm{SD}}^*$ and $w_{\mathrm{RD}} = h_{\mathrm{RD}}^*$. When $x_{\mathrm{R}}(\cdot) \neq x_{\mathrm{S}}(\cdot)$, the weights applied in (13) could not achieve good performance. Maximizing the equivalent power of each symbol, we apply the weights $\alpha_1 w_{\mathrm{SD}}$ and $\beta_1 w_{\mathrm{RD}}$ for s_1 and $\alpha_2 w_{\mathrm{SD}}$ and $\beta_2 w_{\mathrm{RD}}$ for s_2.

Now the *LCSIC* is ready to be stated.

- Case-I: $w_{\mathrm{SD}} h_{\mathrm{SD}} \geq w_{\mathrm{RD}} h_{\mathrm{RD}}$
  We combine the received signals as

$$\begin{aligned} \tilde{y}_1 &= \alpha_1 w_{\mathrm{SD}} y_{\mathrm{SD}} + \beta_1 w_{\mathrm{RD}} y_{\mathrm{RD}} \\ &= (\alpha_1^2 w_{\mathrm{SD}} h_{\mathrm{SD}} + \beta_1^2 w_{\mathrm{RD}} h_{\mathrm{RD}}) s_1 \\ &\quad + (\alpha_1 \alpha_2 w_{\mathrm{SD}} h_{\mathrm{SD}} + \beta_1 \beta_2 w_{\mathrm{RD}} h_{\mathrm{RD}}) s_2 + \alpha_1 w_{\mathrm{SD}} z_{\mathrm{SD}} \\ &\quad + \beta_1 w_{\mathrm{RD}} z_{\mathrm{RD}}. \end{aligned} \quad (14)$$

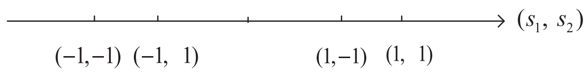

Fig. 1. An equivalent constellation diagram.

Since the equivalent power of s_1 is greater than that of s_2 for $\beta_1 = \pm\alpha_2$ and $\beta_2 = \alpha_1$, i.e., $(\alpha_1^2 w_{\mathrm{SD}} h_{\mathrm{SD}} + \beta_1^2 w_{\mathrm{RD}} h_{\mathrm{RD}})^2 \geq (\alpha_1\alpha_2 w_{\mathrm{SD}} h_{\mathrm{SD}} + \beta_1\beta_2 w_{\mathrm{RD}} h_{\mathrm{RD}})^2$, s_1 is determined first by using a decision rule $\tilde{y}_1 \underset{-1}{\overset{+1}{\gtrless}} 0$.

By canceling s_1 from the received signals, s_2 can be determined without interference. Since the combination in (14) enlarges the power of s_1 but not s_2, we reapply MRC for s_2 after canceling s_1 from both received signals and make decision

$$\tilde{y}_2 = w_{\mathrm{SD}}\alpha_2(y_{\mathrm{SD}} - h_{\mathrm{SD}}\alpha_1 s_1) + w_{\mathrm{RD}}\beta_2(y_{\mathrm{RD}} - h_{\mathrm{RD}}\beta_1 s_1) \underset{-1}{\overset{+1}{\gtrless}} 0. \tag{15}$$

- Case-II: $w_{\mathrm{SD}} h_{\mathrm{SD}} < w_{\mathrm{RD}} h_{\mathrm{RD}}$
  We combine the received signals corresponding to s_2:

$$\begin{aligned}\tilde{y}_2 &= \alpha_2 w_{\mathrm{SD}} y_{\mathrm{SD}} + \beta_2 w_{\mathrm{RD}} y_{\mathrm{RD}} \\ &= (\alpha_2^2 w_{\mathrm{SD}} h_{\mathrm{SD}} + \beta_2^2 w_{\mathrm{RD}} h_{\mathrm{RD}}) s_2 \\ &\quad + (\alpha_1\alpha_2 w_{\mathrm{SD}} h_{\mathrm{SD}} + \beta_1\beta_2 w_{\mathrm{RD}} h_{\mathrm{RD}}) s_1 + \alpha_2 w_{\mathrm{SD}} z_{\mathrm{SD}} \\ &\quad + \beta_2 w_{\mathrm{RD}} z_{\mathrm{RD}}.\end{aligned} \tag{16}$$

For $\beta_1 = \pm\alpha_2$ and $\beta_2 = \alpha_1$, $(\alpha_2^2 w_{\mathrm{SD}} h_{\mathrm{SD}} + \beta_2^2 w_{\mathrm{RD}} h_{\mathrm{RD}})^2 \geq (\alpha_1\alpha_2 w_{\mathrm{SD}} h_{\mathrm{SD}} + \beta_1\beta_2 w_{\mathrm{RD}} h_{\mathrm{RD}})^2$. Then s_2 is determined by checking $\tilde{y}_2 \underset{-1}{\overset{+1}{\gtrless}} 0$. s_1 is derived by canceling s_2 from the received signals and reapplying MRC.

B. Performance Analysis

The case of $w_{\mathrm{SD}} h_{\mathrm{SD}} \geq w_{\mathrm{RD}} h_{\mathrm{RD}}$ is analyzed only. For the other case, it is not difficult to follow.

1) Conventional Scheme with MRC: Observing the equivalent channel model in (13), the minimum distances for $s_i, i \in \{1, 2\}$ are

$$\begin{aligned}d_{1\min}^{\mathrm{Con}} &= 2(w_{\mathrm{SD}} h_{\mathrm{SD}} + w_{\mathrm{RD}} h_{\mathrm{RD}})(\alpha_1 - \alpha_2) \\ d_{2\min}^{\mathrm{Con}} &= 2(w_{\mathrm{SD}} h_{\mathrm{SD}} + w_{\mathrm{RD}} h_{\mathrm{RD}}) \min\{(\alpha_1 - \alpha_2), \alpha_2\}\end{aligned} \tag{17}$$

and the equivalent noise power is $P_{\mathbf{z}} = (|w_{\mathrm{SD}}|^2 + |w_{\mathrm{RD}}|^2)\sigma^2$. Then the maximum error probability for s_i is given as [14]

$$P_{ei}^{\mathrm{Con}} = Q\left(\sqrt{\frac{d_{i\min}^{2,\mathrm{Con}}}{2P_{\mathbf{z}}}}\right), \quad i \in \{1, 2\}.$$

2) SPSC and NSSC schemes with LCSIC: Since $\alpha_1^2 w_{\mathrm{SD}} h_{\mathrm{SD}} + \beta_1^2 w_{\mathrm{RD}} h_{\mathrm{RD}} \geq 0$ and $\alpha_1\alpha_2 w_{\mathrm{SD}} h_{\mathrm{SD}} + \beta_1\beta_2 w_{\mathrm{RD}} h_{\mathrm{RD}} \geq 0$, the minimum distance for s_1 in the constellation diagram in Fig. 1 is the distance between $(s_1, s_2) = (1, -1)$ and $(s_1, s_2) = (-1, 1)$ given by

$$\begin{aligned}d_{1\min}(\beta_1, \beta_2) = 2[&\alpha_1^2 w_{\mathrm{SD}} h_{\mathrm{SD}} + \beta_1^2 w_{\mathrm{RD}} h_{\mathrm{RD}} \\ &- (\alpha_1\alpha_2 w_{\mathrm{SD}} h_{\mathrm{SD}} + \beta_1\beta_2 w_{\mathrm{RD}} h_{\mathrm{RD}})]\end{aligned} \tag{18}$$

and the noise power is $P_{\mathbf{z}_1} = (\alpha_1^2|w_{\mathrm{SD}}|^2 + \alpha_2^2|w_{\mathrm{RD}}|^2)\sigma^2$. Then the maximum error probability for s_1 is

$$P_{e1} = Q\left(\sqrt{\frac{d_{1\min}^2(\beta_1, \beta_2)}{2P_{\mathbf{z}_1}}}\right).$$

Considering the MRC in (15), the minimum distance for s_2 is

$$d_{2\min} = 2\left[\alpha_2^2 w_{\mathrm{SD}} h_{\mathrm{SD}} + \alpha_1^2 w_{\mathrm{RD}} h_{\mathrm{RD}}\right]$$

and the noise power is $P_{\mathbf{z}_2} = (\alpha_2^2|w_{\mathrm{SD}}|^2 + \alpha_1^2|w_{\mathrm{RD}}|^2)\sigma^2$. Due to the error propagation of s_1, the error probability for s_2 is

$$P_{e2} = Q\left(\sqrt{\frac{1}{2}\min\left\{\frac{d_{1\min}^2(\beta_1, \beta_2)}{P_{\mathbf{z}_1}}, \frac{d_{2\min}^2}{P_{\mathbf{z}_2}}\right\}}\right).$$

3) Performance Comparison: Since $d_{\min}^{\mathrm{Con}} = \min\{d_{1\min}^{\mathrm{Con}}, d_{2\min}^{\mathrm{Con}}\}$ in (17) is maximized when $\alpha_1 - \alpha_2 = \alpha_2$, we let $\alpha_1 = \sqrt{\frac{4}{5}}$ and $\alpha_2 = \sqrt{\frac{1}{5}}$.

First, we compare the SPSC and NSSC schemes. Since $d_{2\min}$, $P_{\mathbf{z}_1}$, and $P_{\mathbf{z}_2}$ are the same for both schemes, only $d_{1\min}$ is compared as

$$\begin{aligned}d_{1\min}^{\mathrm{NS}} &= d_{1\min}(-\alpha_2, \alpha_1) \\ &= 2[\alpha_1(\alpha_1 - \alpha_2) w_{\mathrm{SD}} h_{\mathrm{SD}} + \alpha_2(\alpha_1 + \alpha_2) w_{\mathrm{RD}} h_{\mathrm{RD}}] \\ &\geq 2[\alpha_1(\alpha_1 - \alpha_2) w_{\mathrm{SD}} h_{\mathrm{SD}} - \alpha_2(\alpha_1 - \alpha_2) w_{\mathrm{RD}} h_{\mathrm{RD}}] \\ &= d_{1\min}(\alpha_2, \alpha_1) = d_{1\min}^{\mathrm{SP}}.\end{aligned} \tag{19}$$

We compare both NSSC and SPSC schemes with LCSIC to the conventional scheme to show their effectiveness. The ratios of the squared minimum distances and the noise power for the NSSC scheme are written as

$$\begin{aligned}\frac{d_{1\min}^{2,\mathrm{NS}}}{P_{\mathbf{z}_1}} &= \frac{4(\frac{2}{5} w_{\mathrm{SD}} h_{\mathrm{SD}} + \frac{3}{5} w_{\mathrm{RD}} h_{\mathrm{RD}})^2}{(\frac{4}{5}|w_{\mathrm{SD}}|^2 + \frac{1}{5}|w_{\mathrm{RD}}|^2)\sigma^2} \\ &= \frac{4(\frac{1}{5} w_{\mathrm{SD}} h_{\mathrm{SD}} + \frac{3}{10} w_{\mathrm{RD}} h_{\mathrm{RD}})(\frac{4}{5} w_{\mathrm{SD}} h_{\mathrm{SD}} + \frac{6}{5} w_{\mathrm{RD}} h_{\mathrm{RD}})}{(\frac{4}{5}|w_{\mathrm{SD}}|^2 + \frac{1}{5}|w_{\mathrm{RD}}|^2)\sigma^2} \\ &\geq \frac{4}{5\sigma^2}\Big(w_{\mathrm{SD}} h_{\mathrm{SD}} + w_{\mathrm{RD}} h_{\mathrm{RD}}\Big) = \frac{d_{1\min}^{2,\mathrm{Con}}}{P_{\mathbf{z}}} = \frac{d_{2\min}^{2,\mathrm{Con}}}{P_{\mathbf{z}}}\end{aligned} \tag{20}$$

and

$$\begin{aligned}\frac{d_{2\min}^{2,\mathrm{NS}}}{P_{\mathbf{z}_2}} &= \frac{4(\frac{1}{5} w_{\mathrm{SD}} h_{\mathrm{SD}} + \frac{4}{5} w_{\mathrm{RD}} h_{\mathrm{RD}})^2}{(\frac{1}{5}|w_{\mathrm{SD}}|^2 + \frac{4}{5}|w_{\mathrm{RD}}|^2)\sigma^2} \\ &\geq \frac{4}{5\sigma^2}(w_{\mathrm{SD}} h_{\mathrm{SD}} + w_{\mathrm{RD}} h_{\mathrm{RD}}) = \frac{d_{1\min}^{2,\mathrm{Con}}}{P_{\mathbf{z}}} = \frac{d_{2\min}^{2,\mathrm{Con}}}{P_{\mathbf{z}}}.\end{aligned} \tag{21}$$

Therefore, we have the maximum error probabilities for the NSSC scheme as

$$P_{e1}^{\mathrm{NS}} = Q\left(\sqrt{\frac{d_{1\min}^{2,\mathrm{NS}}}{2P_{\mathbf{z}_1}}}\right) \leq P_{e1}^{\mathrm{Con}}$$

and

$$P_{e2}^{\mathrm{NS}} = Q\left(\sqrt{\frac{1}{2}\min\left\{\frac{d_{1\min}^{2,\mathrm{NS}}}{P_{\mathbf{z}_1}}, \frac{d_{2\min}^{2,\mathrm{NS}}}{P_{\mathbf{z}_2}}\right\}}\right) \leq P_{e2}^{\mathrm{Con}}$$

at high SNR. It can be seen that the LCSIC decoder obtains good performance for the NSSC scheme. On the other hand, for the SPSC scheme, we have

$$\begin{aligned}\frac{d_{1\min}^{2,\mathrm{SP}}}{P_{\mathbf{z}_1}} &= \frac{4(\alpha_1-\alpha_2)^2(\alpha_1 w_{\mathrm{SD}}h_{\mathrm{SD}}-\alpha_2 w_{\mathrm{RD}}h_{\mathrm{RD}})^2}{(\alpha_1^2|w_{\mathrm{SD}}|^2+\alpha_2^2|w_{\mathrm{RD}}|^2)\sigma^2}\\ &= 4(\alpha_1-\alpha_2)^2\Big(w_{\mathrm{SD}}h_{\mathrm{SD}}-\frac{\alpha_2}{\alpha_1}w_{\mathrm{RD}}h_{\mathrm{RD}}\Big)\\ &\quad\cdot\frac{(\alpha_1^2 w_{\mathrm{SD}}h_{\mathrm{SD}}-\alpha_1\alpha_2 w_{\mathrm{RD}}h_{\mathrm{RD}})}{(\alpha_1^2|w_{\mathrm{SD}}|^2+\alpha_2^2|w_{\mathrm{RD}}|^2)\sigma^2}\\ &\le \frac{4}{5\sigma^2}\Big(w_{\mathrm{SD}}h_{\mathrm{SD}}+w_{\mathrm{RD}}h_{\mathrm{RD}}\Big)=\frac{d_{1\min}^{2,\mathrm{Con}}}{P_{\mathbf{z}}}=\frac{d_{2\min}^{2,\mathrm{Con}}}{P_{\mathbf{z}}}.\end{aligned}$$

The SPSC scheme could not achieve better performance than the conventional one using the LCSIC even though it achieves better performance than the conventional one using the near-ML decoder [13]. This means that the LCSIC decoder is not suitable for the SPSC scheme.

C. LCSIC Scheme With Decoding Error at the Relay

When the decoding error happens at the relay, the weights in (13) are not optimal. In [2], the CMRC is proposed by using the equivalent one-hop SNR of SR and RD links, γ_{eq}. The CMRC combines the received signals at the destination as in (13) by using the weights:

$$\begin{aligned} w_{\mathrm{SD}} &= \frac{\gamma_{\mathrm{SD}}}{\rho h_{\mathrm{SD}}} = h_{\mathrm{SD}}^*\\ w_{\mathrm{RD}} &= \frac{\gamma_{eq}}{\rho h_{\mathrm{RD}}} = \frac{\gamma_{eq}}{\rho|h_{\mathrm{RD}}|^2}h_{\mathrm{RD}}^* \approx \frac{\min\{|h_{\mathrm{SR}}|^2,|h_{\mathrm{RD}}|^2\}}{|h_{\mathrm{RD}}|^2}h_{\mathrm{RD}}^*, \end{aligned} \tag{22}$$

where γ_{eq} cannot be expressed as a closed-form, but can be well approximated by $\gamma_{eq} \approx \rho\min\{|h_{\mathrm{SR}}|^2, |h_{\mathrm{RD}}|^2\}$ at high SNR.

For $x_{\mathrm{R}}(\cdot) \neq x_{\mathrm{S}}(\cdot)$, instead of the equivalent SNR, the ESMD can be used to determine the combining weights. Similar to the equivalent SNR, the ESMD in (5) can also be expressed as the combination of the SD link and the equivalent one-hop link of SR and RD links. Moreover, when the decoding error happens at the relay, the decoded s_1 from the combination in (14) is likely to be wrong. Since small $|h_{\mathrm{SR}}|^2$ more possibly causes the transmission error on SR link, we revise the influence of $|h_{\mathrm{SR}}|^2$ by multiplying with a coefficient $\delta, 0 < \delta \le 1$. The weights for the LCSIC decoder in Sec. IV-A are

$$\begin{aligned} w_{\mathrm{SD}} &= h_{\mathrm{SD}}^*\\ w_{\mathrm{RD}} &= \min\{\delta * |h_{\mathrm{SR}}|^2, |h_{\mathrm{RD}}|^2\}\frac{h_{\mathrm{RD}}^*}{|h_{\mathrm{RD}}|^2}. \end{aligned} \tag{23}$$

Remark 1: Since $|w_{\mathrm{SD}}|^2 = w_{\mathrm{SD}}h_{\mathrm{SD}}$ and $|w_{\mathrm{RD}}|^2 \le w_{\mathrm{RD}}h_{\mathrm{RD}}$, the inequalities in (20) and (21) still hold. Therefore, the NSSC scheme using LCSIC with the weights in (23) obtains better performance than the conventional one.

Remark 2: Observing Sec. IV-A, the LCSIC decoder has complexity order $|\mathcal{A}|^2$, which is similar to the complexity of the CMRC and much less than the complexity order $|\mathcal{A}|^4$ of the near-ML decoder.

V. Simulation Results

In this section, the analytical results in the previous section will be confirmed by simulations. We assume that $h_{ij} \sim \mathcal{CN}(0, \sigma_{ij}^2)$ for

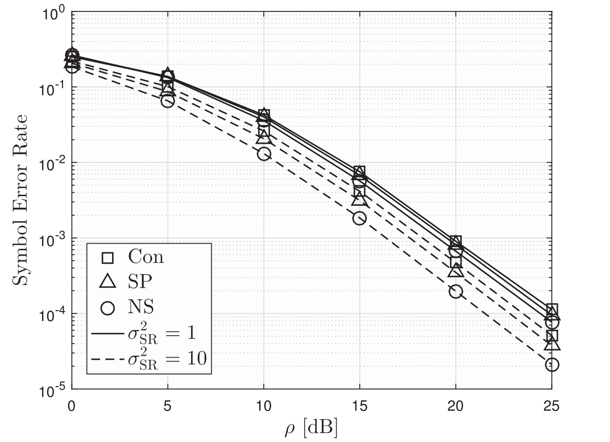

Fig. 2. SER comparison of superposition-coded relay schemes using the near-ML decoder on Rayleigh fading channels with $\sigma_{\mathrm{SD}}^2 = \sigma_{\mathrm{RD}}^2 = 1$.

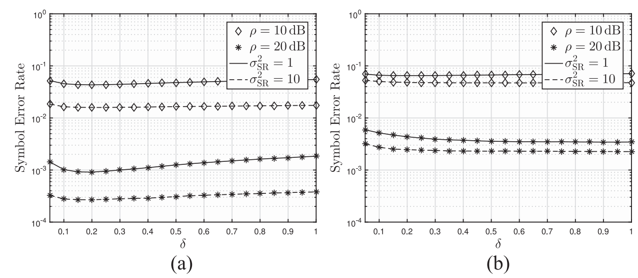

Fig. 3. SERs for the NSSC and SPSC relay schemes using LCSIC with various δ over Rayleigh fading channels of $\sigma_{\mathrm{SD}}^2 = \sigma_{\mathrm{RD}}^2 = 1$. (a) NSSC. (b) SPSC.

$(i,j) \in \{(\mathrm{S},\mathrm{R}), (\mathrm{S},\mathrm{D}), (\mathrm{R},\mathrm{D})\}$ and $\sigma_{\mathrm{SD}}^2 = \sigma_{\mathrm{RD}}^2 = 1$. It is assumed that decoding errors in the relay can occur depending on the channel state of SR link. Without feedback, a fixed power allocation, $\alpha_1 = \sqrt{4/5}$ and $\alpha_2 = \sqrt{1/5}$ which are the optimal solution for the conventional scheme, is used.

Fig. 2 compares the symbol error rates (SERs) of the three schemes with the near-ML decoder. Compared with the conventional and SPSC schemes, the NSSC scheme achieves 0.65 dB and 0.45 dB SNR gains for $\sigma_{\mathrm{SR}}^2 = 1$ and 2 dB and 1.2 dB for $\sigma_{\mathrm{SR}}^2 = 10$ at $\mathrm{SER} = 10^{-3}$, respectively.

Fig. 3(a) and (b) compare the SERs for the NSSC and SPSC schemes corresponding to various δ. One can observe that the NSSC scheme obtains the best performance at $\delta = 0.2$ for various ρ and σ_{SR}^2 while the SPSC scheme achieves the better performance on $0.3 \le \delta \le 1$.

In Fig. 4, we compare the NSSC and SPSC schemes to the conventional scheme by using the LCSIC decoder with the optimal δ.

We can observe that the proposed NSSC scheme with LCSIC exhibits better performance than the conventional one. The improvement is especially noticeable for a good SR link, which matches the practical relay system, where the transmission is assisted by a relay with a strong SR link. On the other hand, the SPSC scheme performs very bad with LCSIC. It shows again that the LCSIC is not suitable for the SPSC scheme as analyzed in Section IV-B.

Since the perfect CSI is not always available in practical system, we briefly discuss the error performance for imperfect CSI. We follow the estimated channel model in [15] as

$$h_{ij} = \hat{h}_{ij} + \epsilon_{ij}n_{ij} \tag{24}$$

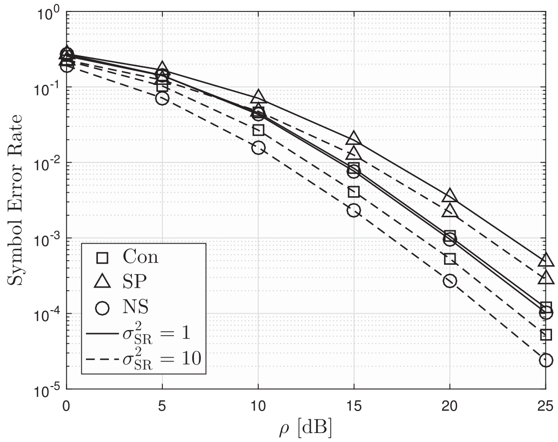

Fig. 4. SER comparison of superposition-coded relay schemes by linear decoders over Rayleigh fading channels of $\sigma^2_{\mathrm{SD}} = \sigma^2_{\mathrm{RD}} = 1$.

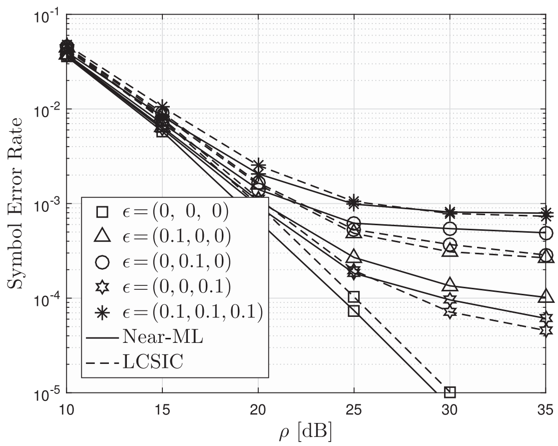

Fig. 5. SER comparison of decoding algorithms for the NSSC scheme over Rayleigh fading channel of $\sigma^2_{\mathrm{SR}} = \sigma^2_{\mathrm{SD}} = \sigma^2_{\mathrm{RD}} = 1$ with various CEE $\epsilon = (\epsilon_{\mathrm{SR}}, \epsilon_{\mathrm{SD}}, \epsilon_{\mathrm{RD}})$.

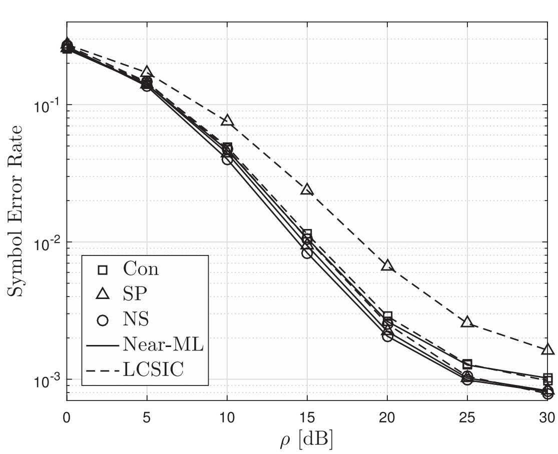

Fig. 6. SER comparison of decoding algorithms over Rayleigh fading channel of $\sigma^2_{\mathrm{SR}} = \sigma^2_{\mathrm{SD}} = \sigma^2_{\mathrm{RD}} = 1$ with CEE of $\epsilon = (0.1, 0.1, 0.1)$.

where $\epsilon_{ij} \in [0, 1]$ is a measure of the accuracy of the channel estimation, and $\hat{h}_{ij} \sim \mathcal{CN}(0, (1 - \epsilon^2_{ij})\sigma^2_{ij})$ is the estimation of the channel coefficient h_{ij} and independent of the estimation error, $n_{ij} \sim \mathcal{CN}(0, \sigma^2_{ij})$, for $(i, j) \in \{(\mathrm{S}, \mathrm{R}), (\mathrm{S}, \mathrm{D}), (\mathrm{R}, \mathrm{D})\}$. The value $\epsilon_{ij} = 0$ means that there is no CEE.

We compare the near-ML decoder and the LCSIC decoder using the NSSC scheme over Rayleigh fading channel of $\sigma^2_{\mathrm{SR}} = \sigma^2_{\mathrm{SD}} = \sigma^2_{\mathrm{RD}} = 1$ with $\epsilon = (\epsilon_{\mathrm{SR}}, \epsilon_{\mathrm{SD}}, \epsilon_{\mathrm{RD}})$ in Fig. 5. From the curves, one can observe the influence of the CEE: 1) there exist error floors at high SNR; 2) the influence of CEE on three links increases in order of RD, SR, SD; 3) the LCSIC decoder achieves better performance than the near-ML decoder in the cases of $\epsilon = (0, 0, 0.1), (0, 0.1, 0), (0.1, 0.1, 0.1)$ at high SNR. This again shows the superiority of the LCSIC decoder. Fig. 6 compares the three superposition-coded relay schemes in Rayleigh fading channel with CEE in all three links, i.e., $\epsilon = (0.1, 0.1, 0.1)$. Even though there are error floors at high SNR due to CEE, the NSSC scheme still achieves better performance than the existing schemes.

VI. Conclusion

The NSSC relay scheme is proposed to improve the error performance. To avoid the high complexity of the near-ML decoder, the LCSIC decoder is proposed for the easy use in practical systems. The NSSC relay scheme can also be applied in the multi-antenna relay systems by using orthogonal space-time codes such as the Alamouti code.

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