Switched-Power Two-Layer Superposition Coding in Cooperative Decode-Forward Relay Systems

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***Abstract*—In this paper, we consider a decode-forward relay system with a source, a relay, and a destination, where two-layer superposition codes are used at the source and the relay. An equivalent squared minimum distance (ESMD) that determines the error performance is derived by using an upper bound on the pair-wise error probability. Without deriving error probabilities, the error performance level for each of superimposed symbols can be shown in a straightforward manner by the ESMD. An optimal superposition-coded relay scheme and a suboptimal switched-power superposition coding scheme are proposed by improving the ESMD. Closed-form power allocation that maximizes the ESMD for the switched scheme is derived for 2-ary pulse amplitude modulation (PAM). An M-ary PAM generalization for the switched-power superposition-coded relay scheme is also presented. Simulation results show that significant signal-to-noise ratio gains are achieved in the optimal and switched-power superposition coding strategies for 2-ary and 4-ary PAM over the Rayleigh fading channel.**

***Index Terms*—Decode-forward (DF), diversity, maximum likelihood (ML), relay system, superposition code.**

I. Introduction

RECENTLY, cooperative communications with many relay strategies have been studied to improve the achievable rate and communication reliability. The classical three-terminal relay channel was first introduced by van der Meulen [1]. The cutset bound developed by Cover and El Gamal [2] set an upper bound on the capacity. Sendonaris, Erkip, and Aazhang presented an information theoretic model for a cooperative communication network and analyzed the achievable rate region and outage probability in the code division multiple access (CDMA) system [3], [4]. Laneman, Tse, and Wornell [5] developed various cooperative diversity algorithms for a source and destination pair based on relay amplifying or fully decoding and forwarding its received signals. These methods were referred to as amplify-forward (AF) and decode-forward (DF) relaying, respectively. Even though the relay operation for the AF relay is simple, the transceivers require expensive radio frequency amplifiers [6].

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Without cyclic redundancy check (CRC) codes at the relay, a maximum-likelihood (ML) decoder has been introduced for a single antenna and binary phase shift keying (BPSK) for the DF relay system [4]. A cooperative maximal ratio combining (C-MRC) of a three-terminal relay channel was proposed by deriving an equivalent signal-to-noise ratio (SNR) [6]. Ju and Kim [7] derived a closed-form formula of the approximated bit error probability (BEP) for the ML decoder in the DF relay system using M-ary pulse amplitude modulation (PAM) and M-ary quadrature amplitude modulation (QAM). Due to the complexity of the decision boundary [7], the ML decoder becomes very complicated when the modulation size or the number of antennas increases [8]. For this reason, a near-ML decoder for DF relay systems was proposed, and its near ML performance was shown [8].

Superposition codes were used to show the achievability of the lower bound on the relay channel capacity [2], [9], [10]. The superposition codes were constituted by multiple superimposed Gaussian signals. Several relay operation schemes with multi-layer superposition coding were also introduced [11], [12], where parts of Gaussian-signal layers are forwarded at the relay depending on the channel state information (CSI) or decoding results at the relay and destination. The error performance was analyzed in terms of the distortion exponent due to the difficulty in analyzing the received SNR in the point of view of information theory [13], [14]. In digital communication systems, superposition coding has been done with digital modulations, such as BPSK or PAM [15]–[18]. Symbol error probability, BEP, and pair-wise error probability (PEP) have been used to determine the error performance [19].

In this paper, we consider a superposition-coded DF relay system where a superposition of two modulated symbols is transmitted from the source in the first phase, and the relay forwards a reconstructed two-layer superposition code in the second phase. Unlike some investigations [20], [21] that use CRC codes at the relay to check its decoding error, we assume that CRC codes are not employed at the relay [4], [6]–[8], [22]. Thus, the relay may forward incorrect signals. As a performance benchmark, the PEP was derived in such a system [8]. Without superposition coding, the equivalent SNR was derived as a performance criterion [6]. However, when superposition coding is used at the source and the relay, it is very difficult to derive the equivalent individual SNR for each layer symbol similarly to a multiplexing multiple-input multiple-output (MIMO) system. Instead of the equivalent SNR, we define an equivalent squared minimum distance (ESMD) by deriving an upper bound on the PEP as in the point-to-point system [19].

With the ESMD, the error performance level for each layer can be compared without computing error probabilities.

Conventionally, the source and relay use the same two-layer superposition coding construction or the same modulation as in previous studies [6], [7]. In such conventional relay systems, the part of the ESMD related to one symbol is always larger than that corresponding to the other symbol. This results in a significant performance difference for two superimposed symbols. It is shown that a symbol with higher power assignment obtains much lower symbol error rate (SER) than the other one. One way to solve this problem is to do separate symbol-level power allocations in superposition coding at the source and the relay. We call it an *optimal superposition-coded* relay scheme. However, the separate symbol-level power allocation requires relatively high complexity. As an alternative, we propose a suboptimal scheme in which the relay uses the switched version of the symbol-level power allocation of the source. That is the power allocated to s_1^R (s_2^R) at the relay is the same as that of s_2 (s_1) at the source when the superimposed symbols at the source are s_1, s_2 and those at the relay are s_1^R, s_2^R. We call it a *switched-power superposition-coded* relay scheme.

In this paper, the optimal power allocations for above mentioned conventional, optimal, and switched-power superposition-coded relay schemes are handled for 2-ary PAM by maximizing the corresponding ESMD. The switched-power superposition-coded strategy is extended to M-ary PAM, and a closed-form power allocation is also derived. Note that it is not difficult to extend the superposition-coded schemes from PAM case to QAM case since QAM signals can be viewed as two parallel PAM signals. Numerical results show that the switched-power superposition-coded relay scheme achieves similar performance to the optimal superposition-coded scheme and much better error performance than the conventional one.

The main contributions of this paper are summarized as follows:

- The ESMD is defined as a new performance criterion in superposition-coded DF relay systems. Without derivation of error probabilities, the error performance level for each of superimposed symbols can be shown in a straightforward manner by the ESMD.
- With the ESMD, an optimal superposition-coded DF relay scheme is proposed, and its optimal power allocation is handled for 2-ary PAM.
- A suboptimal switched-power superposition-coded DF relay scheme is proposed by maximizing the ESMD. In addition, a closed-from power allocation that maximizes the ESMD for the switched scheme is derived for 2-ary PAM.
- The switched-power superposition coding scheme is generalized for M-ary PAM case. The significant SNR gain is achieved for 2-ary and 4-ary PAMs over Rayleigh fading channels.

The paper is organized as follows. In Section II, the system model and its ML and near-ML decoders are introduced. The ESMD is defined by deriving the PEP in Section III. In Section IV, the optimal and switched-power superposition-coded relay schemes are proposed by analyzing the ESMD for 2-ary PAM symbols. The closed-form power allocation for the switched scheme is also derived. The switched-power superposition coding scheme is extended to the general M-ary PAM case in Section V. In Section VI, the performance evaluation and discussion are given. Finally, we conclude the paper in Section VII.

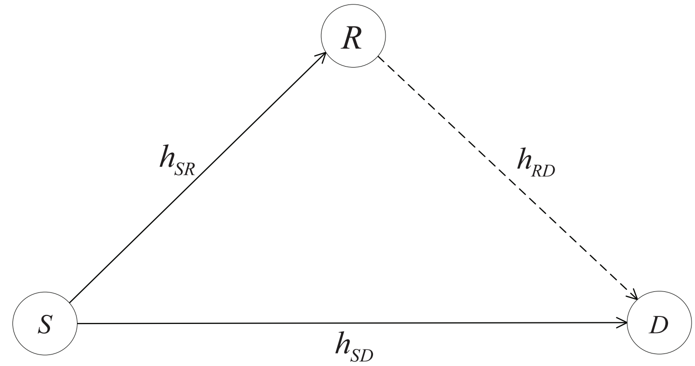


Fig. 1. The DF relay system. The solid line denotes the first phase transmission, and the dashed line denotes the second phase transmission.

II. System Description

A. System Model

We consider a DF relay system with one source, one relay, and one destination equipped with a single antenna as shown in Fig. 1. Half duplex transmission and frequency-flat quasi-static Rayleigh fading are assumed. It is also assumed that the relay knows the instantaneous CSI of the source-relay (SR) link, and the destination knows the instantaneous CSI of the SR, source-destination (SD), and relay-destination (RD) links. Let $\mathcal{A}$ be a set of message symbols from the M-ary signal constellation.

Unequal power-allocated superposition codes [15]–[18] are used at the source and the relay. In the first phase, the source broadcasts a two-layer superposition codeword $x_S(\mathbf{s}) = \sqrt{P_1}s_1 + \sqrt{P_2}s_2$ ($\mathbf{s} = (s_1, s_2) \in \mathcal{A}^2$, $E|s_i|^2 = 1$, $P_1 + P_2 = 1$) with average power P_S. The received signals at the relay and destination are given by

$$\begin{aligned} y_{SR} &= \sqrt{P_S} h_{SR} x_S(\mathbf{s}) + z_{SR} \\ y_{SD} &= \sqrt{P_S} h_{SD} x_S(\mathbf{s}) + z_{SD}, \end{aligned} \tag{1}$$

where the channel coefficients of the SR and SD links h_{SR} and h_{SD} are circularly symmetric Gaussian random variables with zero mean and variance σ_{SR}^2 and σ_{SD}^2. These distributions are denoted by $h_{SR} \sim \mathcal{CN}(0, \sigma_{SR}^2)$ and $h_{SD} \sim \mathcal{CN}(0, \sigma_{SD}^2)$, respectively. $z_{SR} \sim \mathcal{CN}(0, \sigma^2)$ and $z_{SD} \sim \mathcal{CN}(0, \sigma^2)$ represent the complex Gaussian random noise terms at the relay and the destination in the first phase.

In the second phase, the relay decodes $\mathbf{s}_R = (s_1^R, s_2^R) \in \mathcal{A}^2$ ($E|s_i^R|^2 = 1$) from the observation y_{SR}. It then reconstructs a codeword $x_R(\mathbf{s}_R) = \sqrt{P_1^R} s_1^R + \sqrt{P_2^R} s_2^R$ ($P_1^R + P_2^R = 1$) and forwards it to the destination with power P_R. The received signal at the destination in the second phase is given by

$$y_{RD} = \sqrt{P_R} h_{RD} x_R(\mathbf{s}_R) + z_{RD}, \tag{2}$$

where $h_{RD} \sim \mathcal{CN}(0, \sigma_{RD}^2)$ is the channel coefficient of the RD channel, and $z_{RD} \sim \mathcal{CN}(0, \sigma^2)$ is the random additive noise at the destination in the second phase.

Let $\Omega_{SR} = P_S|h_{SR}|^2$, $\Omega_{SD} = P_S|h_{SD}|^2$, and $\Omega_{RD} = P_R|h_{RD}|^2$. Then the instantaneous SNRs in the SR, SD, and RD links are Ω_{SR}/σ^2, Ω_{SD}/σ^2, and Ω_{RD}/σ^2, respectively.

B. ML and Near-ML Decoder

Let $P_{SR}(\mathbf{z}|\mathbf{s})$ be the probability that the relay decodes the received signal to $\mathbf{z}$ when the source transmits the codeword corresponding to the message vector $\mathbf{s}$. Considering all the possible signal vectors $\tilde{\mathbf{s}}_R \in \mathcal{A}^2$ at the relay, the ML decoder for DF relay system can be written as [8]

$$\begin{aligned}\hat{\mathbf{s}} &= \arg\max_{\mathbf{s}} p\left(y_{SD}, y_{RD}|\mathbf{s}\right)\\ &= \arg\max_{\mathbf{s}} \Bigg[-\frac{\left|y_{SD} - \sqrt{P_S}h_{SD}x_S(\mathbf{s})\right|^2}{\sigma^2}\\ &\quad + \ln\sum_{\tilde{\mathbf{s}}_R}\exp\left(\frac{-\left|y_{RD} - \sqrt{P_R}h_{RD}x_R(\tilde{\mathbf{s}}_R)\right|^2 + \sigma^2\ln P_{SR}(\tilde{\mathbf{s}}_R|\mathbf{s})}{\sigma^2}\right)\Bigg]\end{aligned} \tag{3}$$

where $P_{SR}(\mathbf{z}|\mathbf{s})$ is denoted by a summation of several $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{y^2}{2}}dy$ functions. This makes the decision boundaries confusing and the final error probability complicated. As an alternative, the PEP between $\mathbf{s}$ and $\mathbf{z}$,

$$P_{SR}(\mathbf{s} \to \mathbf{z}) = \begin{cases} 1 & \mathbf{z} = \mathbf{s} \\ Q\left(\sqrt{\frac{\Omega_{SR}}{2\sigma^2}|x_S(\mathbf{s}-\mathbf{z})|^2}\right) & \mathbf{z} \neq \mathbf{s} \end{cases} \tag{4}$$

can be used in (3). Although the PEP $P_{SR}(\mathbf{s} \to \mathbf{z})$ is not equal to $P_{SR}(\mathbf{z}|\mathbf{s})$, it can be a good substitution to simplify the expression of the decoder (3) [8]. The widely-used max-log approximation $\ln\sum_i e^{x_i} \approx \max_i x_i$ is also used [23], [24], [7]. Through these two steps, the ML decoder can be simplified to the so-called near-ML decoder [8] as

$$\begin{aligned}\hat{\mathbf{s}} =& \arg\min_{\mathbf{s}}\Bigg[\left|y_{SD} - \sqrt{P_S}h_{SD}x_S(\mathbf{s})\right|^2\\ &+\min_{\tilde{\mathbf{s}}_R}\left\{\left|y_{RD}-\sqrt{P_R}h_{RD}x_R(\tilde{\mathbf{s}}_R)\right|^2 - \sigma^2\ln P_{SR}(\mathbf{s}\to\tilde{\mathbf{s}}_R)\right\}\Bigg].\end{aligned} \tag{5}$$

The near-ML decoder obtains the same diversity and similar error performance compared to the ML decoder [8]. Due to the high complexity, the expression of the error probability of the DF relay system with the ML decoder is very complicated. To express the error probability in a closed-form, we use the near-ML decoder in the superposition-coded relay system (1) and (2).

III. Pairwise Error Probability and Equivalent Squared Minimum Distance

A. Pairwise Error Probability

In the point-to-point communication systems, the error probability is determined by the maximum PEP for possible symbol vectors $\mathbf{s}$ and $\tilde{\mathbf{s}}$. Also, the maximum PEP is determined by the squared minimum distance (SMD) [19] between any two transmitted signal points in the constellation. The structure of the near-ML decoder in (5) shows us the possibility of finding an equivalent SMD (ESMD) that maximizes the PEP.

In the near-ML decoder (5), the squared distances between received signals and the potentially transmitted signals at the source and the relay, $\left|y_{SD} - \sqrt{P_S}h_{SD}x_S(\mathbf{s})\right|^2$ and $|y_{RD} - \sqrt{P_R}h_{RD}x_R(\tilde{\mathbf{s}}_R)|^2$, are determined by the channel gains of SD and RD links, $|h_{SD}|$ and $|h_{RD}|$, and the distances of constellation points at the source and the relay, $|x_S(\tilde{\mathbf{s}}-\mathbf{s})|$ and $|x_R(\tilde{\mathbf{s}}-\tilde{\mathbf{s}}_R)|$. The PEP at the relay, $P_{SR}(\mathbf{s}\to\tilde{\mathbf{s}}_R)$ is also determined by the channel gains of the SR link, $|h_{SR}|$, and the distances of constellation points at the source $|x_S(\mathbf{s}-\tilde{\mathbf{s}}_R)|$ as shown in (4). Consequently, the performance of the near-ML decoder depends on the channel gains and the distances of constellation points at the source and the relay.

Therefore, we will derive the ESMD by computing the maximum PEP in this section. We assume that the source transmits signal $x_S(\mathbf{s})$, the relay transmits $x_R(\mathbf{s}_R)$, and the received signal at the destination y_{SD} and y_{RD} are given in (1) and (2). Due to the decoding error at the relay, the PEP between $\mathbf{s}$ and $\tilde{\mathbf{s}}$ at the destination is written as

$$P(\mathbf{s}\to\tilde{\mathbf{s}}) = \sum_{\mathbf{s}_R} P(\mathbf{s}\to\tilde{\mathbf{s}}|\mathbf{s},\mathbf{s}_R)P_{SR}(\mathbf{s}_R|\mathbf{s}). \tag{6}$$

Let $m([y_{SD}, y_{RD}], \mathbf{s}|\,\mathbf{s}, \mathbf{s}_R)$ be the metric of deciding $\mathbf{s}$ and $m([y_{SD}, y_{RD}], \tilde{\mathbf{s}}|\,\mathbf{s}, \mathbf{s}_R)$ be that for deciding $\tilde{\mathbf{s}}$ for the near-ML decoder in (5) when the source and the relay transmit $\mathbf{s}$ and $\mathbf{s}_R$, respectively. Then the conditional PEP between $\mathbf{s}$ and $\tilde{\mathbf{s}}$ in (6) can be written as:

$$\begin{aligned}&P(\mathbf{s}\to\tilde{\mathbf{s}}|\,\mathbf{s},\mathbf{s}_R)\\ &\quad = P\left(m([y_{SD}, y_{RD}], \mathbf{s}|\,\mathbf{s}, \mathbf{s}_R) > m([y_{SD}, y_{RD}], \tilde{\mathbf{s}}|\,\mathbf{s}, \mathbf{s}_R)\right)\end{aligned} \tag{7}$$

where

$$\begin{aligned}&m([y_{SD}, y_{RD}], \mathbf{s}|\,\mathbf{s}, \mathbf{s}_R)\\ &= \left|y_{SD} - \sqrt{P_S}h_{SD}x_S(\mathbf{s})\right|^2\\ &\quad + \min_{\tilde{\mathbf{s}}_R}\left\{\left|y_{RD} - \sqrt{P_R}h_{RD}x_R(\tilde{\mathbf{s}}_R)\right|^2 - \sigma^2\ln P_{SR}(\mathbf{s}\to\tilde{\mathbf{s}}_R)\right\}\\ &= |z_{SD}|^2 + \min_{\tilde{\mathbf{s}}_R}\Big\{\left|\sqrt{P_R}h_{RD}x_R(\mathbf{s}_R - \tilde{\mathbf{s}}_R) + z_{RD}\right|^2\\ &\qquad\qquad - \sigma^2\ln P_{SR}(\mathbf{s}\to\tilde{\mathbf{s}}_R)\Big\}\end{aligned} \tag{8}$$

and

$$\begin{aligned}&m([y_{SD}, y_{RD}], \tilde{\mathbf{s}}|\,\mathbf{s}, \mathbf{s}_R)\\ &= \left|y_{SD} - \sqrt{P_S}h_{SD}x_S(\tilde{\mathbf{s}})\right|^2\\ &+ \min_{\tilde{\mathbf{s}}_R}\left\{\left|y_{RD} - \sqrt{P_R}h_{RD}x_R(\tilde{\mathbf{s}}_R)\right|^2 - \sigma^2\ln P_{SR}(\tilde{\mathbf{s}}\to\tilde{\mathbf{s}}_R)\right\}\end{aligned}$$

$$
\begin{aligned}
&= \left|\sqrt{P_S} h_{SD} x_S(\mathbf{s}-\tilde{\mathbf{s}}) + z_{SD}\right|^2 \\
&+\min_{\tilde{\mathbf{s}}_R}\left\{\left|\sqrt{P_R} h_{RD} x_R(\mathbf{s}_R-\tilde{\mathbf{s}}_R)+z_{RD}\right|^2-\sigma^2 \ln P_{SR}(\tilde{\mathbf{s}}\to\tilde{\mathbf{s}}_R)\right\}.
\end{aligned} \tag{9}
$$

With high SNR, the last parts in (8) and (9) are written for $\mathbf{z} \neq \mathbf{s}$ as [8]:

$$
\begin{aligned}
\sigma^2 \ln P_{SR}(\mathbf{s}\to\mathbf{z}) &= \sigma^2 \ln Q\left(\sqrt{\frac{\Omega_{SR}}{2\sigma^2}|x_S(\mathbf{s}-\mathbf{z})|^2}\right) \\
&= -\frac{1}{4}\Omega_{SR}|x_S(\mathbf{s}-\mathbf{z})|^2.
\end{aligned} \tag{10}
$$

For $\mathbf{z}=\mathbf{s}$, $\sigma^2 \ln P_{SR}(\mathbf{s}\to\mathbf{z}) = 0$ from the definition in (4). Then, (10) is satisfied for both cases of $\mathbf{z}=\mathbf{s}$ and $\mathbf{z}\neq\mathbf{s}$. Applying (10) to the two metrics in (8) and (9), the summand in (6) for high SNR range can be computed as follows:

$$
P(\mathbf{s}\to\tilde{\mathbf{s}}|\,\mathbf{s},\mathbf{s}_R)P_{SR}(\mathbf{s}_R|\mathbf{s}) \le \exp\left(-\frac{1}{4\sigma^2}D^2(\mathbf{s},\tilde{\mathbf{s}}|\mathbf{s}_R)\right) \tag{11}
$$

where

$$
\begin{aligned}
&D^2(\mathbf{s},\tilde{\mathbf{s}}|\mathbf{s}_R) \\
&= \begin{cases}
\Omega_{SD}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2 \\
\quad +\min_{\tilde{\mathbf{s}}_R}\left\{\Omega_{RD}|x_R(\mathbf{s}-\tilde{\mathbf{s}}_R)|^2\right. \\
\quad\quad \left.+\frac{1}{2}\Omega_{SR}|x_S(\tilde{\mathbf{s}}-\tilde{\mathbf{s}}_R)|^2\right\} & \mathbf{s}_R=\mathbf{s} \\
\Omega_{SD}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2 \\
\quad\quad +\frac{1}{2}\Omega_{SR}|x_S(\mathbf{s}-\mathbf{s}_R)|^2 & \text{otherwise.}
\end{cases}
\end{aligned} \tag{12}
$$

The proof of (11) is given in Appendix A. The maximum PEP is expressed by:

$$
\begin{aligned}
\max_{\mathbf{s},\tilde{\mathbf{s}}} P(\mathbf{s}\to\tilde{\mathbf{s}}) &= \max_{\mathbf{s},\tilde{\mathbf{s}}}\sum_{\mathbf{s}_R} P(\mathbf{s}\to\tilde{\mathbf{s}}|\mathbf{s},\mathbf{s}_R)P_{SR}(\mathbf{s}_R|\mathbf{s}) \\
&\le M^2 \max_{\mathbf{s},\tilde{\mathbf{s}}}\max_{\mathbf{s}_R} P(\mathbf{s}\to\tilde{\mathbf{s}}|\mathbf{s},\mathbf{s}_R)P_{SR}(\mathbf{s}_R|\mathbf{s}) \\
&\le M^2 \exp\left(-\frac{1}{4\sigma^2}\min_{\mathbf{s},\tilde{\mathbf{s}},\mathbf{s}_R} D^2(\mathbf{s},\tilde{\mathbf{s}}|\mathbf{s}_R)\right).
\end{aligned} \tag{13}
$$

B. Definition of Equivalent Squared Minimum Distance (ESMD)

In this subsection, we define an equivalent squared minimum distance (ESMD) by simplifying the exponent of the maximum PEP in (13), i.e., $\min_{\mathbf{s},\tilde{\mathbf{s}},\mathbf{s}_R} D^2(\mathbf{s},\tilde{\mathbf{s}}|\mathbf{s}_R)$. We rewrite $\min_{\mathbf{s},\tilde{\mathbf{s}},\mathbf{s}_R} D^2(\mathbf{s},\tilde{\mathbf{s}}|\mathbf{s}_R)$ in the following steps.

1) For $\mathbf{s}_R=\mathbf{s}$, $\min_{\mathbf{s},\tilde{\mathbf{s}}} D^2(\mathbf{s},\tilde{\mathbf{s}}|\mathbf{s}_R)$ is handled in the following three cases related to $\tilde{\mathbf{s}}_R$:
   * The case of $\tilde{\mathbf{s}}_R=\mathbf{s}$

$$
\begin{aligned}
&\min_{\mathbf{s},\tilde{\mathbf{s}}} D^2(\mathbf{s},\tilde{\mathbf{s}}|\mathbf{s}_R=\mathbf{s}) = t_1 \\
&\triangleq \min_{\mathbf{s},\tilde{\mathbf{s}}}\left[\Omega_{SD}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2+\frac{1}{2}\Omega_{SR}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2\right]
\end{aligned}
$$

   * The case of $\tilde{\mathbf{s}}_R=\tilde{\mathbf{s}}$

$$
\begin{aligned}
&\min_{\mathbf{s},\tilde{\mathbf{s}}} D^2(\mathbf{s},\tilde{\mathbf{s}}|\mathbf{s}_R=\mathbf{s}) = t_2 \\
&\triangleq \min_{\mathbf{s},\tilde{\mathbf{s}}}\left[\Omega_{SD}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2+\Omega_{RD}|x_R(\mathbf{s}-\tilde{\mathbf{s}})|^2\right]
\end{aligned}
$$

   * The case of $\tilde{\mathbf{s}}_R\neq\mathbf{s}$ and $\tilde{\mathbf{s}}_R\neq\tilde{\mathbf{s}}$

$$
\begin{aligned}
&\min_{\mathbf{s},\tilde{\mathbf{s}}} D^2(\mathbf{s},\tilde{\mathbf{s}}|\mathbf{s}_R=\mathbf{s}) \\
&\ge \min_{\mathbf{s},\tilde{\mathbf{s}}}\left[\Omega_{SD}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2+\frac{1}{2}\Omega_{SR}\min_{\tilde{\mathbf{s}}_R}|x_S(\tilde{\mathbf{s}}-\tilde{\mathbf{s}}_R)|^2\right] \\
&\ge \left(\Omega_{SD}+\frac{1}{2}\Omega_{SR}\right)\min_{\mathbf{s},\tilde{\mathbf{s}}}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2 = t_1
\end{aligned}
$$

2) For $\mathbf{s}_R\neq\mathbf{s}$, we have

$$
\begin{aligned}
&\min_{\mathbf{s},\tilde{\mathbf{s}},\mathbf{s}_R\neq\mathbf{s}} D^2(\mathbf{s},\tilde{\mathbf{s}}|\mathbf{s}_R\neq\mathbf{s}) \\
&= \min_{\mathbf{s},\tilde{\mathbf{s}},\mathbf{s}_R\neq\mathbf{s}}\left[\Omega_{SD}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2+\frac{1}{2}\Omega_{SR}|x_S(\mathbf{s}-\mathbf{s}_R)|^2\right] \\
&= \left(\Omega_{SD}+\frac{1}{2}\Omega_{SR}\right)\min_{\mathbf{s},\tilde{\mathbf{s}}}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2 = t_1.
\end{aligned} \tag{14}
$$

In summary, $\min_{\mathbf{s},\tilde{\mathbf{s}},\mathbf{s}_R} D^2(\mathbf{s},\tilde{\mathbf{s}}|\mathbf{s}_R) = \min(t_1,t_2)$. Similarly to the point-to-point communication system [19], we define an ESMD between $\mathbf{s}$ and $\tilde{\mathbf{s}}$ as:

$$
\begin{aligned}
D^2_{\min} &= \min_{\mathbf{s},\tilde{\mathbf{s}},\mathbf{s}_R} D^2(\mathbf{s},\tilde{\mathbf{s}}|\mathbf{s}_R) \\
&= \min_{\mathbf{s},\tilde{\mathbf{s}}}\left[\Omega_{SD}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2\right. \\
&\left.+\min\left\{\frac{1}{2}\Omega_{SR}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2,\Omega_{RD}|x_R(\mathbf{s}-\tilde{\mathbf{s}})|^2\right\}\right].
\end{aligned} \tag{15}
$$

Lemma 1: The superposition-coded DF relay system (1) and (2) achieves the maximum diversity of two for nonzero P_i and P_i^R, $i=1,2$.

$$
\begin{aligned}
D^2_{\min} = \min\Bigg[&\min_{\tilde{s}_i\neq s_1,\tilde{s}_2\neq s_2}\Bigg\{\Omega_{SD}\left(\sqrt{P_1}|s_1-\tilde{s}_1|-\sqrt{P_2}|s_2-\tilde{s}_2|\right)^2 \\
&+\min\left(\Omega_{RD}\left(\sqrt{P_1^R}|s_1-\tilde{s}_1|-\sqrt{P_2^R}|s_2-\tilde{s}_2|\right)^2,\frac{1}{2}\Omega_{SR}\left(\sqrt{P_1}|s_1-\tilde{s}_1|-\sqrt{P_2}|s_2-\tilde{s}_2|\right)^2\right)\Bigg\}, \\
&\left(\Omega_{SD}P_1+\min(\Omega_{RD}P_1^R,\frac{1}{2}\Omega_{SR}P_1)\right)\min_{\tilde{s}_1\neq s_1}|s_1-\tilde{s}_1|^2, \\
&\left(\Omega_{SD}P_2+\min(\Omega_{RD}P_2^R,\frac{1}{2}\Omega_{SR}P_2)\right)\min_{\tilde{s}_2\neq s_2}|s_2-\tilde{s}_2|^2\Bigg]
\end{aligned} \tag{16}
$$

Proof: See Appendix B.

Lemma 1 shows that the superposition-coded relay system whose error performance is determined by the ESMD in (15) achieves the maximum diversity of two. Thus, the ESMD can be chosen as a simple and reasonable criterion for optimizing the error performance instead of the PEP.

C. Simplification of ESMD for Two-Layer Superposition Codes

Plugging $x_S(\mathbf{s}-\tilde{\mathbf{s}}) = \sqrt{P_1}(s_1-\tilde{s}_1) + \sqrt{P_2}(s_2-\tilde{s}_2)$ and $x_R(\mathbf{s}-\tilde{\mathbf{s}}) = \sqrt{P_1^R}(s_1-\tilde{s}_1) + \sqrt{P_2^R}(s_2-\tilde{s}_2)$ into (15), the respective ESMDs for the superimposed symbols s_1 and s_2 can also be achieved.

- When $\tilde{s}_1 \neq s_1, \tilde{s}_2 \neq s_2$:
  Each of $(\sqrt{P_1}(s_1-\tilde{s}_1) + \sqrt{P_2}(s_2-\tilde{s}_2))^2$ and $\left(\sqrt{P_1^R}(s_1-\tilde{s}_1) + \sqrt{P_2^R}(s_2-\tilde{s}_2)\right)^2$ in the case of $\operatorname{sign}(s_1-\tilde{s}_1) = -\operatorname{sign}(s_2-\tilde{s}_2)$ is smaller than that in the case of $\operatorname{sign}(s_1-\tilde{s}_1) = \operatorname{sign}(s_2-\tilde{s}_2)$. It turns out that

$$\begin{aligned} D^2_{\min} &= \min_{\tilde{s}_1\neq s_1, \tilde{s}_2\neq s_2}\Bigg[\Omega_{SD}\left(\sqrt{P_1}|s_1-\tilde{s}_1|-\sqrt{P_2}|s_2-\tilde{s}_2|\right)^2 \\ &\quad +\min\Bigg\{\Omega_{RD}\left(\sqrt{P_1^R}|s_1-\tilde{s}_1|-\sqrt{P_2^R}|s_2-\tilde{s}_2|\right)^2, \\ &\quad \frac{1}{2}\Omega_{SR}\left(\sqrt{P_1}|s_1-\tilde{s}_1|-\sqrt{P_2}|s_2-\tilde{s}_2|\right)^2\Bigg\}\Bigg]. \end{aligned}$$

- When $\tilde{s}_1 \neq s_1, \tilde{s}_2 = s_2$:

$$D^2_{\min} = \left(\Omega_{SD}P_1 + \min\left(\Omega_{RD}P_1^R, \frac{1}{2}\Omega_{SR}P_1\right)\right)\min_{\tilde{s}_1\neq s_1}|s_1-\tilde{s}_1|^2.$$

- When $\tilde{s}_1 = s_1, \tilde{s}_2 \neq s_2$:

$$D^2_{\min} = \left(\Omega_{SD}P_2 + \min\left(\Omega_{RD}P_2^R, \frac{1}{2}\Omega_{SR}P_2\right)\right)\min_{\tilde{s}_2\neq s_2}|s_2-\tilde{s}_2|^2.$$

Then, we have (16), shown at the bottom of the previous page. The first and second terms of $D^2_{\min}$ in (16) correspond to $\tilde{s}_1 \neq s_1$, and the first and third terms correspond to $\tilde{s}_2 \neq s_2$. Thus, we can simply say that the minimum of the first and second terms in (16) is the ESMD between s_1 and $\tilde{s}_1$ and determines the error probability of s_1. The minimum of the first and third terms in (16) is the ESMD between s_2 and $\tilde{s}_2$ and determines the error probability of s_2.

Based on the ESMDs, we propose new superposition-coded transmission schemes in the following section.

IV. Superposition-Coded Relay Schemes

In this section, we consider a superposition-coded DF relay system with 2-ary PAM, i.e., $s_1, s_2 \in \{-1, 1\}$. For the sake of simplicity, we assume $P_1 > P_2$, i.e., $P_1 > \frac{1}{2}$.

A. Conventional Superposition-Coded Scheme: $P_1^R = P_1$

Conventionally, the same superposition code construction $\sqrt{P_1}s_1^R + \sqrt{P_2}s_2^R$ is used at the relay. Plugging $P_1^R = P_1$, $P_2^R = P_2$ and $s_1, s_2 \in \{-1, 1\}$ into (16), we find that the third term is always smaller than the second term. The ESMD becomes:

$$D^{2,\mathrm{con}}_{\min,2\mathrm{PAM}} = 4\left(\Omega_{SD} + \min\left(\Omega_{RD}, \frac{1}{2}\Omega_{SR}\right)\right) \cdot \min\left[\left(\sqrt{P_1}-\sqrt{P_2}\right)^2, P_2\right]. \tag{17}$$

Since $(\sqrt{P_1}-\sqrt{P_2})^2$ is an increasing function and $P_2 = 1 - P_1$ is a decreasing function of P_1 in $P_1 \in (\frac{1}{2}, 1)$, the best power allocation happens when $\sqrt{P_1}-\sqrt{P_2} = \sqrt{P_2}$, i.e., $P_1^{\mathrm{con}} = \frac{4}{5}$. In this case, the superposition codeword with two 2-ary PAM symbols is the same as a single 4-ary PAM symbol. The ESMD is $D^{2,\mathrm{con}}_{\min,2\mathrm{PAM}} = \frac{4}{5}\left(\Omega_{SD} + \min(\Omega_{RD}, \frac{1}{2}\Omega_{SR})\right)$.

B. Optimal Superposition-Coded Scheme

Consider the ESMD in (16). As shown in Subsection IV-A, when $P_1^R = P_1$, the third term is always smaller than the second term, meaning that the ESMD for s_2 is smaller than that for s_1. For this reason, the SER of s_2 is worse than that of s_1, as shown in Fig. 2, where $P_S = P_R = 1$ and $SNR = 1/\sigma^2$. To achieve the SER of 10^{-2}, the required SNR for s_2 is larger than that of s_1 by approximately 3 dB on the Rayleigh fading channel in both the environments $(\sigma^2_{SR}, \sigma^2_{SD}, \sigma^2_{RD}) = (1, 1, 1)$ and $(\sigma^2_{SR}, \sigma^2_{SD}, \sigma^2_{RD}) = (10, 1, 1)$.

One way to improve the error performance of s_2 is to decrease P_1^R and increase P_2^R such that $P_1^R < P_2^R$. As a result, the third term is enlarged while the second term becomes smaller, and the first term is improved via the power allocation of P_1^R. Finally, the minimum value of the three terms (the overall ESMD) increases and the average SER improves. We derive the optimal power allocation which maximizes the ESMD in

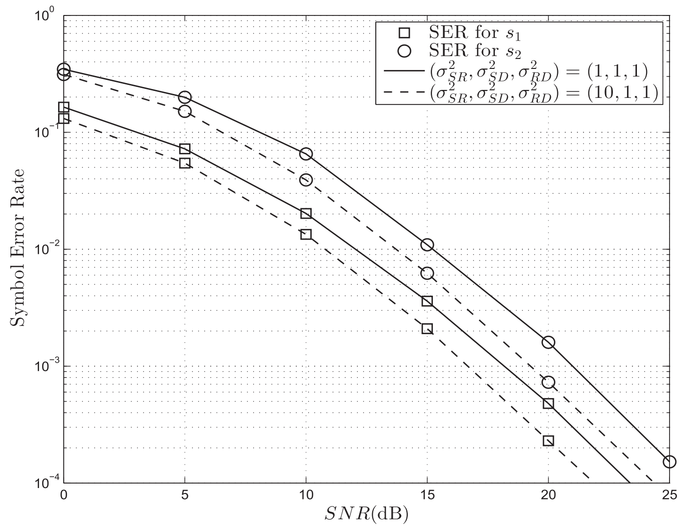


Fig. 2. Comparison of SERs of s_1 and s_2 for conventional superposition-coded relay scheme over Rayleigh fading channels.

(16) such as:

$$\left(P_1^{opt}, P_2^{R,opt}\right) = \arg \max_{P_1,\, P_2^R \in \left(\frac{1}{2},1\right)} D^2_{\min,2\mathrm{PAM}} \tag{18}$$

where

$$\begin{aligned} D^2_{\min,2\mathrm{PAM}} &= 4\min\Bigg[\Omega_{SD}\left(\sqrt{P_1}-\sqrt{P_2}\right)^2 \\ &\quad +\min\left\{\Omega_{RD}\left(\sqrt{P_1^R}-\sqrt{P_2^R}\right)^2, \frac{1}{2}\Omega_{SR}\left(\sqrt{P_1}-\sqrt{P_2}\right)^2\right\}, \\ &\quad \Omega_{SD}P_1 + \min\left(\Omega_{RD}P_1^R, \frac{1}{2}\Omega_{SR}P_1\right), \\ &\quad \Omega_{SD}P_2 + \min\left(\Omega_{RD}P_2^R, \frac{1}{2}\Omega_{SR}P_2\right)\Bigg]. \end{aligned}$$

It needs relatively high complexity to solve the above optimization problem. For this reason, we propose a simple scheme in the following subsection.

C. Switched-Power Superposition-Coded Scheme: ($P_1^R = P_2$ and $P_2^R = P_1$)

To avoid solving the high-complexity optimization problem in (18), we let $P_1^R = P_2$ and $P_2^R = P_1$ and find the optimal P_1 that maximizes the ESMD in (16), i.e.,

$$P_1^{\mathrm{swt}} = \arg \max_{P_1 \in (\frac{1}{2},1)} D^{2,\mathrm{swt}}_{\min,2\mathrm{PAM}} \tag{19}$$

where

$$\begin{aligned} D^{2,\mathrm{swt}}_{\min,2\mathrm{PAM}} &= 4\min\Bigg[\left(\Omega_{SD} + \min\left(\Omega_{RD}, \frac{1}{2}\Omega_{SR}\right)\right)\cdot(\sqrt{P_1}-\sqrt{P_2})^2, \\ &\quad \left(\Omega_{SD}P_1 + \min\left(\Omega_{RD}P_2, \frac{1}{2}\Omega_{SR}P_1\right)\right), \\ &\quad \left(\Omega_{SD}P_2 + \min\left(\Omega_{RD}P_1, \frac{1}{2}\Omega_{SR}P_2\right)\right)\Bigg]. \end{aligned} \tag{20}$$

We call this a *switched-power superposition-coded* relay scheme. With this switching, the minimum of the second and third terms is increased, and finally, the ESMD is enlarged.

Now, we solve the optimization problem in (19) as follows:

1) When $\frac{1}{2}\Omega_{SR} \leq \Omega_{RD}$:
The ESMD in (20) is the same as the right-hand side (RHS) in (17). The optimal power of P_1 is $P_1^{\mathrm{swt}} = \frac{4}{5}$.

2) When $\frac{1}{2}\Omega_{SR} > \Omega_{RD}$:
Let $a = \max(\Omega_{SD}, \Omega_{RD})$ and $b = \min(\Omega_{SD}, \Omega_{RD})$. Then we have $aP_2 + bP_1 \leq \Omega_{SD}P_1 + \Omega_{RD}P_2 < \Omega_{SD}P_1 + \frac{1}{2}\Omega_{SR}P_1$ and $aP_2 + bP_1 \leq \Omega_{SD}P_2 + \Omega_{RD}P_1$. The ESMD is simplified to

$$D^{2,\mathrm{swt}}_{\min,2\mathrm{PAM}} = 4\min(f_1, f_2, f_3)$$

where $f_1 = (a+b)\left(\sqrt{P_1}-\sqrt{P_2}\right)^2$, $f_2 = (aP_2 + bP_1)$, and $f_3 = (\Omega_{SD} + \frac{1}{2}\Omega_{SR})P_2$. For the functions f_1, f_2, and f_3, it is not difficult to find the following facts in $P_1 \in (\frac{1}{2}, 1)$ as illustrated in Fig. 3:

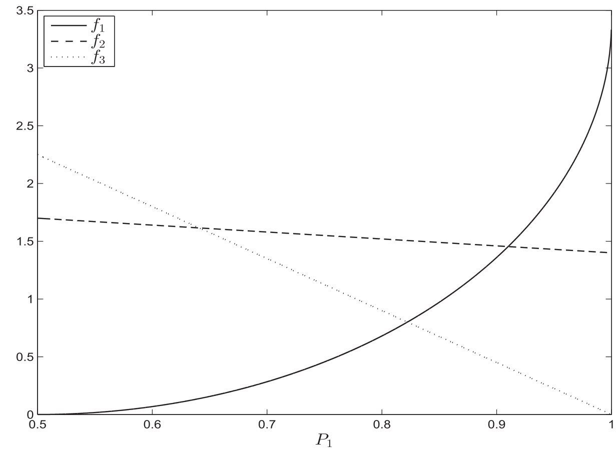


Fig. 3. An example for functions f_1, f_2, and f_3.

- f_1 is an increasing function of P_1;
- Both f_2 and f_3 are decreasing functions of P_1;
- f_2 and f_3 each have a unique cross point with f_1.

With these facts, the optimal power allocation of P_1 can be solved as

$$\begin{aligned} P_1^{\mathrm{swt}} &= \arg \max_{P_1 \in (\frac{1}{2},1)} \min(f_1, \min(f_2, f_3)) \\ &= \left\{P_1 | f_1 = \min(f_2, f_3),\, P_1 \in \left(\frac{1}{2}, 1\right)\right\} \\ &= \min_{j=2,3}\left\{P_1 | f_1 = f_j,\, P_1 \in \left(\frac{1}{2}, 1\right)\right\}. \end{aligned}$$

Let $\psi(\alpha,\beta,\gamma) = \frac{2\gamma^2-(\alpha-\beta)\beta+2\gamma\sqrt{\gamma^2-\alpha\beta}}{4\gamma^2+(\alpha-\beta)^2}$. The solutions of the equations $f_1 = f_2$ and $f_1 = f_3$ in $P_1 \in (\frac{1}{2}, 1)$ are $P_1^{f_1=f_2} = \psi(a,\, b,\, a+b)$ and $P_1^{f_1=f_3} = \psi\left(a+b,\, a+b-\Omega_{SD}-\frac{1}{2}\Omega_{SR},\, a+b\right)$, respectively. Thus the optimal P_1 for $\frac{1}{2}\Omega_{SR} > \Omega_{RD}$ is equal to $P_1^{\mathrm{swt}} = \min\left(P_1^{f_1=f_2}, P_1^{f_1=f_3}\right)$.

In summary, the optimal power allocation for the switched-power scheme is

$$P_1^{\mathrm{swt}} = \begin{cases} \min\left(P_1^{f_1=f_2}, P_1^{f_1=f_3}\right) & \text{if } \frac{1}{2}\Omega_{SR} > \Omega_{RD} \\ \frac{4}{5} & \text{otherwise.} \end{cases} \tag{21}$$

V. Generalization of Switched-Power Superposition-Coded Relay for M-ary PAM

In this section, we extend the result for 2-ary PAM to M-ary PAM. We assume that s_1 and s_2 use the same M-ary PAM, i.e., $s_1, s_2 \in \left\{\pm\sqrt{\frac{3}{M^2-1}}, \pm 3\sqrt{\frac{3}{M^2-1}}, \ldots, \pm(M-1)\sqrt{\frac{3}{M^2-1}}\right\}$. Then the distance between s_i and $\tilde{s}_i$ is in the set $\mathcal{D} = \{d_{\min}, 2d_{\min}, \ldots, (M-1)d_{\min}\}$, where $d_{\min} = \min_{\tilde{s}_i \neq s_i} |s_i - \tilde{s}_i|^2 = 2\sqrt{\frac{3}{M^2-1}}$. Then, $d_{\max} = \max_{\tilde{s}_i \neq s_i} |s_i - \tilde{s}_i|^2 = (M-1)$ $d_{\min}$. To simply distinguish two symbols s_1 and s_2 based on the value of the superposition code $x_S(\mathbf{s}) = \sqrt{P_1}s_1 +$

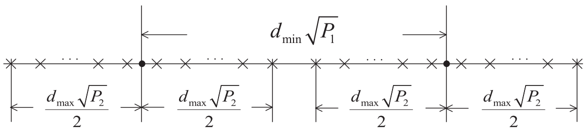


Fig. 4. Relationship between P_1 and P_2.

$\sqrt{P_2}s_2$ as shown in Fig. 4, we assume $\sqrt{P_1}d_{\min} > \sqrt{P_2}d_{\max} = \sqrt{P_2}(M-1)d_{\min}$, i.e., $P_1 > \frac{(M-1)^2}{(M-1)^2+1}$. We let $\mathcal{T}=(\frac{(M-1)^2}{(M-1)^2+1}, 1)$. Then $P_1 \in \mathcal{T}$.

For the conventional superposition-coded relay scheme, the optimal power allocation is $P_1^{\text{con}} = \frac{M^2}{1+M^2}$, and the ESMD is

$$\begin{aligned} D^{2,\text{con}}_{\min,\text{MPAM}} &= d^2_{\min} \min\left[(\sqrt{P_1}-(M-1)\sqrt{P_2})^2,\, P_2\right] \\ &\quad \cdot \left(\Omega_{SD} + \min\left(\Omega_{RD},\, \frac{1}{2}\Omega_{SR}\right)\right) \qquad (22) \\ &= \frac{d^2_{\min}}{M^2+1}\left(\Omega_{SD} + \min\left(\Omega_{RD},\, \frac{1}{2}\Omega_{SR}\right)\right). \end{aligned}$$

In contrast to the situation for 2-ary PAM in Section IV, the ESMD in (16) for the M-ary PAM case could not be simplified in a straightforward way. Let $d_1 = |s_1 - \tilde{s}_1|$ and $d_2 = |s_2 - \tilde{s}_2|$. Then the first term in (16) is the minimum of the following two terms:

$$\begin{aligned} &\left(\Omega_{SD}+\frac{1}{2}\Omega_{SR}\right) \min_{d_1,d_2\in\mathcal{D}} \left(\sqrt{P_1}d_1-\sqrt{P_2}d_2\right)^2 \\ &\quad = d^2_{\min}\left(\Omega_{SD}+\frac{1}{2}\Omega_{SR}\right)\left(\sqrt{P_1}-(M-1)\sqrt{P_2}\right)^2 \qquad (23) \end{aligned}$$

and

$$\begin{aligned} &\min_{d_1,d_2\in\mathcal{D}}\left[\Omega_{SD}\left(\sqrt{P_1}d_1-\sqrt{P_2}d_2\right)^2 + \Omega_{RD}\left(\sqrt{P_1}d_2-\sqrt{P_2}d_1\right)^2\right] \\ &\quad = d^2_{\min}\min_{1\le m\le M-1}\left[a\left(\sqrt{P_1}-m\sqrt{P_2}\right)^2 + b\left(m\sqrt{P_1}-\sqrt{P_2}\right)^2\right] \qquad (24) \end{aligned}$$

where $a = \max(\Omega_{SD}, \Omega_{RD})$ and $b = \min(\Omega_{SD}, \Omega_{RD})$. The proof of (24) is given in Appendix C.

Considering all possible values of $\tilde{s}_1$ and $\tilde{s}_2$, the ESMD for the switched-power superposition coding with M-ary PAM is rewritten as

$$\begin{aligned} &D^{2,\text{swt}}_{\min,\text{MPAM}} \\ &= d^2_{\min}\min\Bigg[\min_{1\le m\le M-1}\Big\{a\left(\sqrt{P_1}-m\sqrt{P_2}\right)^2 \\ &\qquad\qquad + b\left(m\sqrt{P_1}-\sqrt{P_2}\right)^2\Big\}, \\ &\qquad \left(\Omega_{SD}+\frac{1}{2}\Omega_{SR}\right)\left(\sqrt{P_1}-(M-1)\sqrt{P_2}\right)^2, \\ &\qquad \left(\Omega_{SD}P_1 + \min(\Omega_{RD}P_2,\, \frac{1}{2}\Omega_{SR}P_1)\right), \\ &\qquad \left(\Omega_{SD}P_2 + \min(\Omega_{RD}P_1,\, \frac{1}{2}\Omega_{SR}P_2)\right)\Bigg]. \qquad (25) \end{aligned}$$

It is not difficult to find that the ESMD for the switched-power superposition-coded relay scheme in (25) is greater than or equal to that for the conventional one in (22).

Now, we solve the following optimization problem:

$$P_1^{\text{swt}} = \arg\max_{P_1\in\mathcal{T}} D^{2,\text{swt}}_{\min,\text{MPAM}}. \qquad (26)$$

1) When $\frac{1}{2}\Omega_{SR} \le \Omega_{RD}$:
The ESMD is the same as the RHS in (22) and $P_1^{\text{swt}} = \frac{M^2}{1+M^2}$.
2) When $\frac{1}{2}\Omega_{SR} > \Omega_{RD}$:
Similarly to the 2-ary PAM case, the ESMD is simplified to

$$D^{2,\text{swt}}_{\min,\text{MPAM}} = d^2_{\min}\min\left(\min_{1\le m\le M} f_{1,m},\, f_2,\, f_3\right) \qquad (27)$$

where $f_{1,m} = a(\sqrt{P_1}-m\sqrt{P_2})^2 + b(m\sqrt{P_1}-\sqrt{P_2})^2$, $m = 1,\ldots,M-1$, $f_{1,M} = (\Omega_{SD}P_2 + \frac{1}{2}\Omega_{SR})(\sqrt{P_1} - (M-1)\sqrt{P_2})^2$, $f_2 = aP_2 + bP_1$, and $f_3 = (\Omega_{SD} + \frac{1}{2}\Omega_{SR})P_2$. By handling $D^{2,\text{swt}}_{\min,\text{MPAM}}$ in (27), we derive the optimal power allocation for $\frac{1}{2}\Omega_{SR} > \Omega_{RD}$ as

$$\begin{aligned} P_1^{\text{swt}} &= \arg\max_{P_1\in\mathcal{T}}\min\left(\min_{1\le m\le M} f_{1,m},\, f_2,\, f_3\right) \\ &= \min\left[\max_{1\le m\le M} P_1^{f_{1,m}=f_2},\, \max_{1\le m\le M} P_1^{f_{1,m}=f_3}\right] \qquad (28) \end{aligned}$$

where $P_1^{f_{1,m}=f_j} = \begin{cases} P_1^{1,m,j} & \text{if } P_1^{1,m,j} \in \mathcal{T} \\ \emptyset & \text{otherwise} \end{cases}$ for $j = 2, 3$, $P_1^{1,m,2} = \psi(a + (m-1)\,b,\, (m-1)\,a + b,\, m(a+b))$, $P_1^{1,m,3} = \psi(a + mb,\, ma + b - (\Omega_{SD} + \frac{1}{2}\Omega_{SR}),\, m(a+b))$ for $m = 1,\ldots,M-1$, $P_1^{f_{1,M}=f_2} = \psi((\Omega_{SD} + \frac{1}{2}\Omega_{SR}) - b,\, (M-1)^2(\Omega_{SD} + \frac{1}{2}\Omega_{SR}) - a,\, (M-1)(\Omega_{SD} + \frac{1}{2}\Omega_{SR}))$, and $P_1^{f_{1,M}=f_3} = \frac{M^2}{1+M^2}$.

The proof of Equation (28) is given in Appendix D.

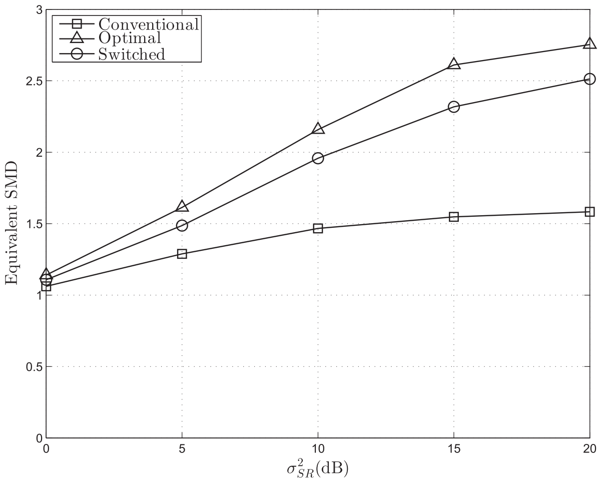
Summarizing the results for $\frac{1}{2}\Omega_{SR} \le \Omega_{RD}$ and $\frac{1}{2}\Omega_{SR} > \Omega_{RD}$, the optimal P_1 is represented by

$$P_1^{\text{swt}} = \begin{cases} \min\limits_{j\in\{2,3\}}\max\limits_{1\le m\le M} P_1^{f_{1,m}=f_j}, & \text{if } \frac{1}{2}\Omega_{SR} > \Omega_{RD} \\ \frac{M^2}{1+M^2} & \text{otherwise.} \end{cases} \qquad (29)$$

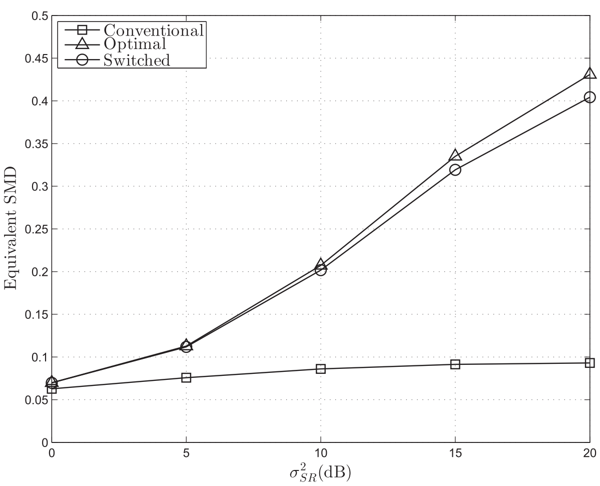
VI. Performance Evaluation and Discussion

In the previous sections, the optimal superposition-coded relay scheme and switched-power superposition-coded relay scheme are proposed. It is also shown that both have larger ESMD than the conventional scheme. Moreover, a closed-form for the optimal power, P_1^{swt}, was derived for the switched scheme.

In this section, we simulate the relay systems on Rayleigh fading channel with $\sigma^2_{SD} = \sigma^2_{RD} = 1$ and various values of σ^2_{SR}. "Conventional" means the conventional superposition-coded relay scheme in Subsection IV-A, and "Optimal" denotes

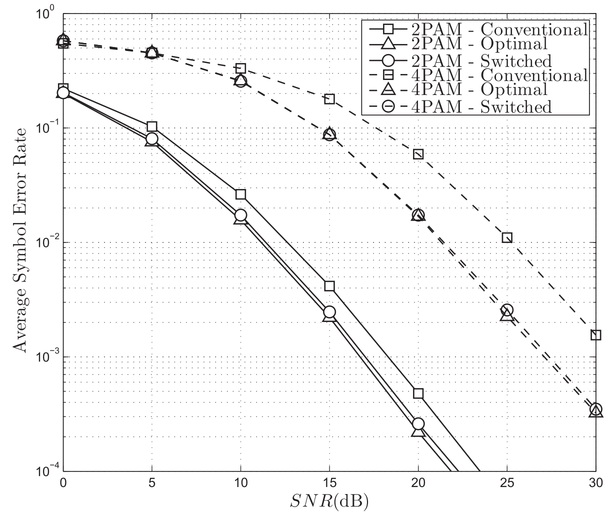


(a) 2-ary PAM

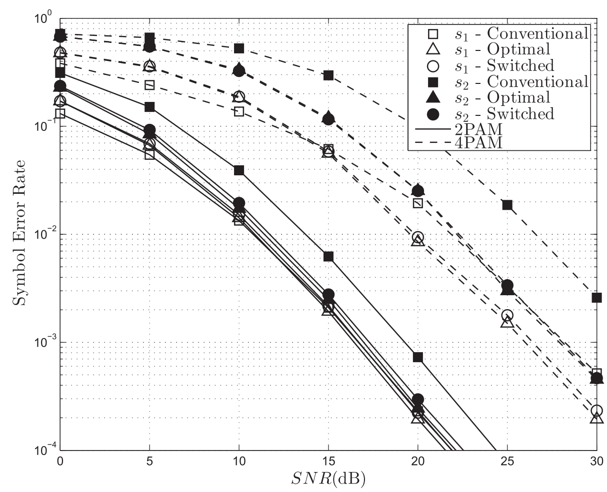


(b) 4-ary PAM

Fig. 5. Comparison of ESMD for the conventional, optimal, and switched-power superposition-coded relay schemes with $\sigma_{SD}^2 = \sigma_{RD}^2 = 1$ and various σ_{SR}^2.



(a) Average SER



(b) Respective SER

Fig. 6. Comparison of SERs for the conventional, optimal, and switched-power superposition-coded relay schemes with $(\sigma_{SR}^2, \sigma_{SD}^2, \sigma_{RD}^2) = (10, 1, 1)$.

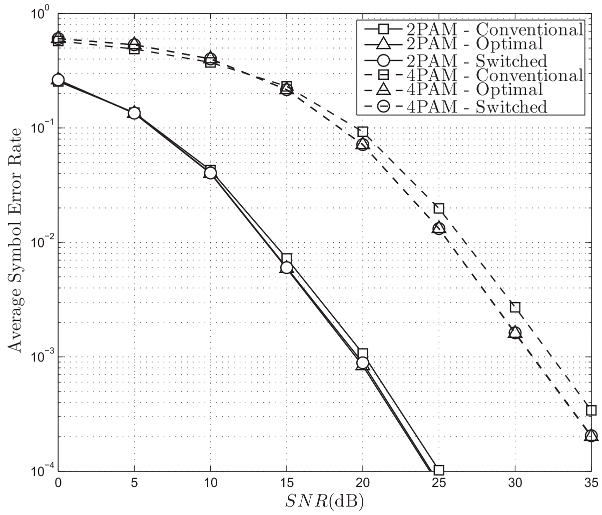
the optimal superposition-coded relay scheme as discussed in Subsection IV-B. "Switched" means the switched-power superposition-coded relay scheme proposed in Subsection IV-C. We evaluate and discuss the ESMD and the error performance of the systems when $P_S = P_R = 1$.

First, we compare the average ESMD for various values of σ_{SR}^2. Fig. 5 shows that the ESMD of the switched scheme is quite close to that of the optimal scheme and larger than that of the conventional scheme. With the increase of σ_{SR}^2, the ESMD of the switched scheme sees a big improvement, which matches the practical relay system where the transmission is assisted by a relay with a strong SR link.

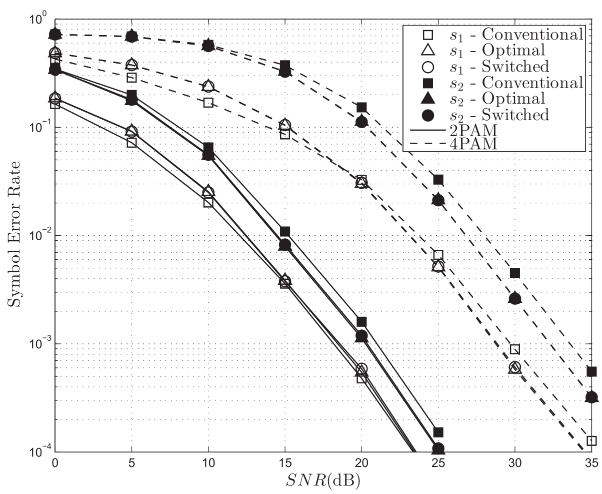
A comparison of SERs is also shown. Figs 6 and 7 show that the average SER of s_1 and s_2 in the switched-power superposition-coded relay scheme is very similar to that in the optimal scheme and better than that in the conventional scheme. Without channel coding, we compare the SNR improvements for the SER of 10^{-2}. We assume $SNR = 1/\sigma^2$.

Compared with the conventional superposition-coded relay scheme, the switched-power scheme has a 1.2 dB SNR gain for 2-ary PAM (solid line in Fig. 6(a)) and a 3.6 dB SNR gain for 4-ary PAM (dashed line in Fig. 6(a)) on the channel conditions $(\sigma_{SR}^2, \sigma_{SD}^2, \sigma_{RD}^2) = (10, 1, 1)$. Even for a fair channel, i.e., $(\sigma_{SR}^2, \sigma_{SD}^2, \sigma_{RD}^2) = (1, 1, 1)$, the SNR improvement of the proposed switched-power scheme is 0.4 dB (solid line in Fig. 7(a)) and 1 dB (dashed line in Fig. 7(a)) for 2-ary and 4-ary PAM symbols, respectively.

As shown in Figs 6(b) and 7(b), the SER of s_2 is worse than that of s_1 at over 3 dB and 4.3 dB for 2-ary PAM and 4-ary PAM, respectively, for both cases of $(\sigma_{SR}^2, \sigma_{SD}^2, \sigma_{RD}^2) = (10, 1, 1)$ and $(\sigma_{SR}^2, \sigma_{SD}^2, \sigma_{RD}^2) = (1, 1, 1)$ in the conventional superposition coding scheme. The reason is the small ESMD for s_2 as analyzed in Subsection IV-B. By applying the switched scheme, the SNR for s_2 is improved by 2 dB and 4.3 dB for 2-ary and 4-ary PAM with $(\sigma_{SR}^2, \sigma_{SD}^2, \sigma_{RD}^2) = (10, 1, 1)$, respectively. With $(\sigma_{SR}^2, \sigma_{SD}^2, \sigma_{RD}^2) = (1, 1, 1)$, the SNR gains for s_2 are 0.7 dB and 1.2 dB for 2-ary and 4-ary PAM, respectively. On the other hand, the SER for s_1 has a 0.2 dB loss and a 2 dB gain for 2-ary and 4-ary PAM on $(\sigma_{SR}^2, \sigma_{SD}^2, \sigma_{RD}^2) = (10, 1, 1)$ and a 0.5 dB loss and a 0.6 dB gain for 2-ary and



(a) Average SER



(b) Respective SER

Fig. 7. Comparison of SERs for the conventional, optimal, and switched-power superposition-coded relay schemes with $(\sigma_{SR}^2, \sigma_{SD}^2, \sigma_{RD}^2) = (1, 1, 1)$.

TABLE I
SNR IMPROVEMENT FOR THE SWITCHED-POWER SUPERPOSITION CODING SCHEME COMPARED WITH THE CONVENTIONAL SUPERPOSITION CODING SCHEME FOR 2-ARY AND 4-ARY PAMS AT $SER = 10^{-2}$

	$\sigma_{SR}^2 = 10$		$\sigma_{SR}^2 = 1$	
	2-ary	4-ary	2-ary	4-ary
Average SER	1.2 dB	3.6 dB	0.4 dB	1 dB
SER of s_1	-0.2 dB	2 dB	-0.5 dB	0.6 dB
SER of s_2	2 dB	4.3 dB	0.7 dB	1.2 dB

4-ary PAM for $(\sigma_{SR}^2, \sigma_{SD}^2, \sigma_{RD}^2) = (1, 1, 1)$, respectively. The average SERs in Figs 6(a) and 7(a) are obtained from the contributions of s_1 and s_2 mentioned above. The results are shown in Table I. These results show the effectiveness of the switched scheme.

VII. CONCLUSION

In this paper, we defined the ESMD as a new criterion of error performance in superposition-coded DF relay systems. Without derivation of the error probability, the level of the error performance can be shown in a straightforward manner by the ESMD. Unlike the equivalent SNR, the ESMDs for the individual superimposed symbols can also be derived. The optimal and switched-power superposition-coded DF relay schemes were proposed by analyzing the ESMD. In addition, the closed-form power allocation that maximizes the ESMD for the switched scheme was derived for 2-ary PAM and M-ary PAM. Significant SNR gains were achieved in the simulation for 2-ary and 4-ary PAM over Rayleigh fading channels.

The switched scheme can also be used in other scenarios. For example, if the SD link is blocked, two relays can assist the transmission, with one relay transmitting the same superposition codeword $\sqrt{P_1}s_1^{R_1} + \sqrt{P_2}s_2^{R_1}$, and the other transmitting the switched-power superposition codeword $\sqrt{P_1}s_2^{R_2} + \sqrt{P_2}s_1^{R_2}$. Furthermore, the switched-power superposition-coded DF relay scheme could also be extended to the multi-layer case, such as $\sqrt{P_1}s_1 + \sqrt{P_2}s_2 + \cdots + \sqrt{P_L}s_L$ is transmitted from the source, $\sqrt{P_1}s_2^{R_1} + \cdots + \sqrt{P_{L-1}}s_L^{R_1} + \sqrt{P_L}s_1^{R_1}$ is transmitted from the first relay R_1, $\sqrt{P_1}s_3^{R_2} + \cdots + \sqrt{P_{L-2}}s_L^{R_2} + \sqrt{P_{L-1}}s_1^{R_2} + \sqrt{P_L}s_2^{R_2}$ is transmitted from the second relay R_2 and so on. The total ESMD will be enlarged by changing the powers for $s_1, \ldots, s_L$. The switching strategies for multi-layer superposition-coded multi-relay systems can be one of the next research topics.

APPENDIX A
PROOF OF (11)

We first give the following theorem [8] that helps solve this problem.

Theorem 1 ([8, Theorem 1]): Let c_1 and c_2 be complex values satisfying $|c_2|^2 > |c_1|^2$ and x be a complex Gaussian random variable with distribution $\mathcal{CN}(0, \sigma^2)$. Then, for $\sigma^2 \to 0$, $|c_2 + x|^2 \geq |c_1 + x|^2$ in probability, i.e.,

$$\lim_{\sigma^2 \to 0} P\left(|c_2 + x|^2 \geq |c_1 + x|^2\right) = 1.$$

■

Then $P(\mathbf{s} \to \tilde{\mathbf{s}}|\mathbf{s}, \mathbf{s}_R) P_{SR}(\mathbf{s}_R|\mathbf{s})$ for different cases of $\mathbf{s}_R = \mathbf{s}$ and $\mathbf{s}_R \neq \mathbf{s}$ in high SNR are considered.

• For $\mathbf{s}_R = \mathbf{s}$:

Applying (10) to (8) and (9) and using Theorem 1 in (8) ($\tilde{\mathbf{s}}_R = \mathbf{s}$ is the solution of the min function in (8) in high SNR), we have

$$m([y_{SD}, y_{RD}], \mathbf{s}|\, \mathbf{s}, \mathbf{s}_R) = |z_{SD}|^2 + |z_{RD}|^2$$

and

$$\begin{aligned} m&\left([y_{SD}, y_{RD}], \tilde{\mathbf{s}}|\, \mathbf{s}, \mathbf{s}_R\right) \\ &= \left|\sqrt{P_S}h_{SD}x_S(\mathbf{s} - \tilde{\mathbf{s}}) + z_{SD}\right|^2 \\ &\quad + \min_{\tilde{\mathbf{s}}_R}\left\{\left|\sqrt{P_R}h_{RD}x_R(\mathbf{s} - \tilde{\mathbf{s}}_R) + z_{RD}\right|^2 + \frac{1}{4}\Omega_{SR}|x_S(\tilde{\mathbf{s}} - \tilde{\mathbf{s}}_R)|^2\right\}. \end{aligned}$$

Plugging these two metrics into (7), the summand in (6) is written as

$$
\begin{aligned}
&P(\mathbf{s}\to\tilde{\mathbf{s}}|\mathbf{s},\mathbf{s}_R=\mathbf{s})P_{SR}(\mathbf{s}_R=\mathbf{s}|\mathbf{s})\\
&\le P\Bigg(|z_{SD}|^2+|z_{RD}|^2>\left|\sqrt{P_S}h_{SD}x_S(\mathbf{s}-\tilde{\mathbf{s}})+z_{SD}\right|^2\\
&\quad+\min_{\tilde{\mathbf{s}}_R}\left\{\left|\sqrt{P_R}h_{RD}x_R(\mathbf{s}-\tilde{\mathbf{s}}_R)+z_{RD}\right|^2+\frac{1}{4}\Omega_{SR}|x_S(\tilde{\mathbf{s}}-\tilde{\mathbf{s}}_R)|^2\right\}\Bigg)\\
&=\max_{\tilde{\mathbf{s}}_R}P\Big(2\mathrm{Re}\left(\sqrt{P_S}h_{SD}x_S(\mathbf{s}-\tilde{\mathbf{s}})z_{SD}^*+\sqrt{P_R}h_{RD}x_R(\mathbf{s}-\tilde{\mathbf{s}}_R)z_{RD}^*\right)\\
&\qquad<-\Big(\Omega_{SD}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2+\Omega_{RD}|x_R(\mathbf{s}-\tilde{\mathbf{s}}_R)|^2\\
&\qquad\qquad+\frac{1}{4}\Omega_{SR}|x_S(\tilde{\mathbf{s}}-\tilde{\mathbf{s}}_R)|^2\Big)\Big)
\end{aligned}\tag{30}
$$

where $\mathrm{Re}(\cdot)$ and $(\cdot)^*$ denote the real part and the complex conjugate of a complex number, respectively, and $2\mathrm{Re}\left(\sqrt{P_S}h_{SD}x_S(\mathbf{s}-\tilde{\mathbf{s}})z_{SD}^*+\sqrt{P_R}h_{RD}x_R(\mathbf{s}-\tilde{\mathbf{s}}_R)z_{RD}^*\right)$ is a real Gaussian random variable with mean zero and variance $2\sigma^2\left(\Omega_{SD}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2+\Omega_{RD}|x_R(\mathbf{s}-\tilde{\mathbf{s}}_R)|^2\right)$. Then we have (31), shown at the bottom of the page.

- For $\mathbf{s}_R\neq\mathbf{s}$:

Applying (10) and setting $\tilde{\mathbf{s}}_R=\mathbf{s}_R$ in (8) gives the upper bound as

$$m([y_{SD},y_{RD}],\mathbf{s}|\,\mathbf{s},\mathbf{s}_R)\le|z_{SD}|^2+|z_{RD}|^2+\frac{1}{4}\Omega_{SR}|x_S(\mathbf{s}-\mathbf{s}_R)|^2.$$

From Theorem 1, the lower bound on the metric in (9) in high SNR is:

$$m([y_{SD},y_{RD}],\tilde{\mathbf{s}}|\,\mathbf{s},\mathbf{s}_R)\ge\left|\sqrt{P_S}h_{SD}x_S(\mathbf{s}-\tilde{\mathbf{s}})+z_{SD}\right|^2+|z_{RD}|^2.$$

Using these bounds gives (32), shown at the bottom of the page, where (a) is from the fact that $2\mathrm{Re}\left(\sqrt{P_S}h_{SD}x_S(\mathbf{s}-\tilde{\mathbf{s}})z_{SD}^*\right)$ is a real Gaussian random variable with mean zero and variance $2\sigma^2\Omega_{SD}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2$. This completes the proof of (11). □

Appendix B
Proof of Lemma 1

From the result of maximum PEP in (13) and the ESMD in (15), we have (33), shown at the bottom of the page, where (b) is due to the independent exponential random variables Ω_{SD}, Ω_{SR}, and Ω_{RD} with respective rate parameters $1/(P_S\sigma_{SD}^2)$, $1/(P_S\sigma_{SR})^2$, and

$$
\begin{aligned}
&P(\mathbf{s}\to\tilde{\mathbf{s}}|\mathbf{s},\mathbf{s}_R=\mathbf{s})P_{SR}(\mathbf{s}_R=\mathbf{s}|s)\\
&\quad\le\max_{\tilde{\mathbf{s}}_R}Q\left(\sqrt{\frac{\left(\Omega_{SD}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2+\Omega_{RD}|x_R(\mathbf{s}-\tilde{\mathbf{s}}_R)|^2+\frac{1}{4}\Omega_{SR}|x_S(\tilde{\mathbf{s}}-\tilde{\mathbf{s}}_R)|^2\right)^2}{2\sigma^2\left(\Omega_{SD}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2+\Omega_{RD}|x_R(\mathbf{s}-\tilde{\mathbf{s}}_R)|^2\right)}}\right)\\
&\quad\le\exp\left(-\frac{1}{4\sigma^2}\min_{\tilde{\mathbf{s}}_R}\left(\Omega_{SD}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2+\Omega_{RD}|x_R(\mathbf{s}-\tilde{\mathbf{s}}_R)|^2+\frac{1}{2}\Omega_{SR}|x_S(\tilde{\mathbf{s}}-\tilde{\mathbf{s}}_R)|^2\right)\right)
\end{aligned}\tag{31}
$$

$$
\begin{aligned}
&P(\mathbf{s}\to\tilde{\mathbf{s}}|\mathbf{s},\mathbf{s}_R\neq\mathbf{s})P_{SR}(\mathbf{s}_R\neq\mathbf{s}|s)\\
&\quad\le P\left(|z_{SD}|^2+|z_{RD}|^2+\frac{1}{4}\Omega_{SR}|x_S(\mathbf{s}-\mathbf{s}_R)|^2>\left|\sqrt{P_S}h_{SD}x_S(\mathbf{s}-\tilde{\mathbf{s}})+z_{SD}\right|^2+|z_{RD}|^2\right)P_{SR}(\mathbf{s}\to\mathbf{s}_R)\\
&\quad=P\left(2\mathrm{Re}\left(\sqrt{P_S}h_{SD}x_S(\mathbf{s}-\tilde{\mathbf{s}})z_{SD}^*\right)<-\Omega_{SD}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2+\frac{1}{4}\Omega_{SR}|x_S(\mathbf{s}-\mathbf{s}_R)|^2\right)P_{SR}(\mathbf{s}\to\mathbf{s}_R)\\
&\quad\overset{(a)}{\le}Q\left(\sqrt{\frac{\Omega_{SD}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2-\frac{1}{2}\Omega_{SR}|x_S(\mathbf{s}-\mathbf{s}_R)|^2}{2\sigma^2}}\right)Q\left(\sqrt{\frac{\Omega_{SR}|x_S(\mathbf{s}-\mathbf{s}_R)|^2}{2\sigma^2}}\right)\\
&\quad\le\exp\left(-\frac{1}{4\sigma^2}\left(\Omega_{SD}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2+\frac{1}{2}\Omega_{SR}|x_S(\mathbf{s}-\mathbf{s}_R)|^2\right)\right)
\end{aligned}\tag{32}
$$

$$
\begin{aligned}
&\max_{\mathbf{s},\tilde{\mathbf{s}}}E\left\{P(\mathbf{s}\to\tilde{\mathbf{s}})\right\}\\
&\quad\le M^2E\left\{\exp\left(-\frac{1}{4\sigma^2}D_{\min}^2\right)\right\}\\
&\quad\le M^2\max_{\mathbf{s},\tilde{\mathbf{s}}}E\left\{\exp\left(-\frac{1}{4\sigma^2}\Omega_{SD}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2\right)\right\}E\left\{\exp\left(-\frac{1}{8\sigma^2}\Omega_{SR}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2\right)+\exp\left(-\frac{1}{4\sigma^2}\Omega_{RD}|x_R(\mathbf{s}-\tilde{\mathbf{s}})|^2\right)\right\}\\
&\quad\overset{(b)}{=}M^2\max_{\mathbf{s},\tilde{\mathbf{s}}}\frac{1}{\frac{P_S\sigma_{SD}^2}{4\sigma^2}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2+1}\cdot\left[\frac{1}{\frac{P_S\sigma_{SR}^2}{8\sigma^2}|x_S(\mathbf{s}-\tilde{\mathbf{s}})|^2+1}+\frac{1}{\frac{P_R\sigma_{RD}^2}{4\sigma^2}|x_R(\mathbf{s}-\tilde{\mathbf{s}})|^2+1}\right]
\end{aligned}\tag{33}
$$

$1/(P_R\sigma_{RD}^2)$. Plugging $x_S(\mathbf{s}-\tilde{\mathbf{s}}) = \sqrt{P_1}(s_1-\tilde{s}_1)+\sqrt{P_2}(s_2-\tilde{s}_2)$ and $x_R(\mathbf{s}-\tilde{\mathbf{s}}) = \sqrt{P_1^R}(s_1-\tilde{s}_1)+\sqrt{P_2^R}(s_2-\tilde{s}_2)$ to (33), the maximum PEP is proportional to σ^4 for nonzero P_i and P_i^R, $i=1,2$ in high SNR, and then this system achieves a diversity of two. This result can also be obtained from Theorem 2 in the previous study [8]. □

Appendix C
Proof of (24)

Consider the left-hand side (LHS) in (24). For $\Omega_{SD} \geq \Omega_{RD}$, d_1 and d_2 that minimize $\Omega_{SD}(\sqrt{P_1}d_1-\sqrt{P_2}d_2)^2+\Omega_{RD}(\sqrt{P_1}d_2-\sqrt{P_2}d_1)^2$ should satisfy $(\sqrt{P_1}d_1-\sqrt{P_2}d_2)^2 < (\sqrt{P_1}d_2-\sqrt{P_2}d_1)^2$, i.e., $d_1 \leq d_2$ due to $\sqrt{P_1} > (M-1)\sqrt{P_2}$ and $d_{\min} \leq d_1, d_2 \leq (M-1)d_{\min}$. For $\Omega_{SD} < \Omega_{RD}$, $d_1 \geq d_2$ should be satisfied. Let $m_1 = \min(d_1,d_2)/d_{\min}$, and $m_2 = \max(d_1,d_2)/d_{\min}$. Since $a = \max(\Omega_{SD},\Omega_{RD})$, $b = \min(\Omega_{SD},\Omega_{RD})$, we have:

$$\begin{aligned}
&\min_{d_1,d_2\in\mathcal{D}}\left[\Omega_{SD}\left(\sqrt{P_1}d_1-\sqrt{P_2}d_2\right)^2+\Omega_{RD}\left(\sqrt{P_1}d_2-\sqrt{P_2}d_1\right)^2\right]\\
&=\min_{d_1,d_2\in\mathcal{D}}\left[a\left(\sqrt{P_1}\min(d_1,d_2)-\sqrt{P_2}\max(d_1,d_2)\right)^2\right.\\
&\qquad\left.+b\left(\sqrt{P_1}\max(d_1,d_2)-\sqrt{P_2}\min(d_1,d_2)\right)^2\right]\\
&=d_{\min}^2\min_{1\leq m_1\leq m_2\leq(M-1)}\left\{a\left(m_1\sqrt{P_1}-m_2\sqrt{P_2}\right)^2\right.\\
&\qquad\left.+b\left(m_2\sqrt{P_1}-m_1\sqrt{P_2}\right)^2\right\}\\
&\overset{(c)}{=}d_{\min}^2\min_{1\leq m\leq M-1}\left\{a\left(\sqrt{P_1}-m\sqrt{P_2}\right)^2+b\left(m\sqrt{P_1}-\sqrt{P_2}\right)^2\right\}
\end{aligned}$$

where (c) is due to the fact that $m_1\sqrt{P_1}-m_2\sqrt{P_2} \geq \sqrt{P_1}-m_2\sqrt{P_2}$ and $m_2\sqrt{P_1}-m_1\sqrt{P_2} \geq \sqrt{P_1}-m_1\sqrt{P_2}$. □

Appendix D
Proof of (28)

For functions $f_{1,m}, m=1,\ldots,M$ and f_j, $j=2,3$ in $P_1 \in \mathcal{T}$, the following facts hold:

1) $f_{1,m}, m=1,\ldots,M$ are increasing functions of P_1 and

$$a'+b' < f_{1,m} < a+mb, (1\leq m\leq M-1)$$
$$0 < f_{1,M} < \Omega_{SD}+\frac{1}{2}\Omega_{SR}$$

where $a' = \frac{a(M-1-m)^2}{(M-1)^2+1}$ and $b' = \frac{b(m(M-1)-1)^2}{(M-1)^2+1}$.

2) f_2 and f_3 are decreasing functions of P_1 and

$$\frac{a+b(M-1)^2}{(M-1)^2+1} > f_2 > b$$
$$\frac{\Omega_{SD}+\frac{1}{2}\Omega_{SR}}{(M-1)^2+1} > f_3 > 0$$

3) f_2 and f_3 each have a cross point with $f_{1,M}$

4) f_2 and f_3 are each smaller than $f_{1,m}$ (and do not cross with $f_{1,m}$) or have a cross point with $f_{1,m}$ for $m=1,\ldots,M-1$

5) f_2 and f_3 each have a cross point with $\min_{1\leq m\leq M} f_{1,m}$

6) $\min_{1\leq m\leq M} f_{1,m}$ and $\min(f_2,f_3)$ have a cross point.

Using these facts, the optimal power allocation of P_1 can be derived with the following steps:

- From facts 1), 2), and 6), the optimal P_1 that maximizes the ESMD in (27) is the solution of $\min_{1\leq m\leq M} f_{1,m} = \min(f_2,f_3)$
- From facts 1), 2), and 5), the solution of $\min_{1\leq m\leq M} f_{1,m} = \min(f_2,f_3)$ is the minimum value of the solutions of $\min_{1\leq m\leq M} f_{1,m} = f_2$ and $\min_{1\leq m\leq M} f_{1,m} = f_3$
- From facts 1), 2), and 3), the solution of $\min_{1\leq m\leq M} f_{1,m} = f_j$ is the maximum value of the solutions of $f_{1,m} = f_j$, $1\leq m\leq M$ in $P_1 \in \mathcal{T}$ for $j=2,3$

With these steps, the optimal power allocation is

$$\begin{aligned}
P_1^{\mathrm{swt}} &= \arg\max_{P_1\in\mathcal{T}}\min\left(\min_{1\leq m\leq M} f_{1,m}, \min(f_2,f_3)\right)\\
&=\left\{P_1 \mid \min_{1\leq m\leq M} f_{1,m} = \min(f_2,f_3), P_1\in\mathcal{T}\right\}\\
&=\min_{j=2,3}\left[\left\{P_1 \mid \min_{1\leq m\leq M} f_{1,m} = f_j, P_1\in\mathcal{T}\right\}\right]\\
&=\min_{j=2,3}\left[\max_{1\leq m\leq M}\left\{P_1 \mid f_{1,m} = f_j, P_1\in\mathcal{T}\right\}\right].
\end{aligned}$$

By computation, the solution of the equation $f_{1,m} = f_j$ in $\mathcal{T}$ is derived as $P_1^{f_{1,m}=f_j} = \begin{cases} P_1^{1,m,j} & \text{if } P_1^{1,m,j}\in\mathcal{T} \\ \emptyset & \text{otherwise}\end{cases}$, for $j=2,3$ where $P_1^{1,m,2} = \psi(a+(m-1)b, (m-1)a+b, m(a+b))$ and $P_1^{1,m,3} = \psi(a+mb, ma+b-(\Omega_{SD}+\frac{1}{2}\Omega_{SR}), m(a+b))$, $m=1,\ldots,M-1$. The respective solutions of $f_{1,M} = f_2$ and $f_{1,M} = f_3$ are $P_1^{f_{1,M}=f_2} = \psi((\Omega_{SD}+\frac{1}{2}\Omega_{SR})-b, (M-1)^2(\Omega_{SD}+\frac{1}{2}\Omega_{SR})-a, (M-1)(\Omega_{SD}+\frac{1}{2}\Omega_{SR}))$ and $P_1^{f_{1,M}=f_3} = \frac{M^2}{1+M^2}$. This completes the proof of Equation (28). □

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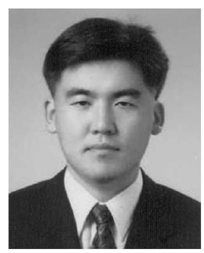

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