Statistics Assignment - 1

Q1)

a) Find the probability that no one has the same seat for both courses (exactly; you should leave your answer as a sum).

→ Define random variable X that makes the number of students that take the same seat in both classes. If we donate S_j that j^{th} student has the same seat, we have following.

$$P(X=0) = 1 - P(X \geq 1) = 1 - P(\cup_j S_j)$$

using inclusion-exclusion formula & the symmetry, we have

$$P\left(\bigcup_j S_j\right) = \sum_j (-1)^{j-1} \binom{100}{j} P\left(\bigcap_{k=1}^{j} S_k\right)$$

The probability of j^{th} student on their seats is simply $\frac{(100-j)!}{100!}$

Thus we have

$$P\left(\bigcup_j S_j\right) = \sum_j (-1)^{j-1} \binom{100}{j} \frac{(100-j)!}{100j} = \sum_{j=1}^{100} \frac{(-1)^{j-1}}{j!}$$

So,

$$P(X=0) = 1 - \sum_{j=1}^{100} \frac{(-1)^{j-1}}{j!} = \sum_{j=0}^{100} \frac{(-1)^j}{j!}$$

b) - - -

→ Defining indicator random variable I_j that indicates if S_j has occurred we have

$$X = \sum_{j=1}^{100} I_j$$

we know that $P(I_j) = \frac{1}{100}$ & that we can approximate.

$$P((I_j = 1) \cap (I_k = 1)) = \frac{1}{100} \cdot \frac{1}{99} \approx \left(\frac{1}{100}\right)^2 = P(I_j = 1) P(I_k = 1)$$

So, It is independent random variables. Next we can approximate X with poisson distribution with parameter $\lambda = E(X) = 100E(I_1) = 1$

So,

$$P(X=0) \approx \frac{1^0}{0!} e^{-1} \approx 0.37$$

c)

→ Use Poisson approximation to finally obtain that

$P(X \geq 2) = 1 - P(X=0) - P(X=1) \approx 1 - e^{-1} - e^{-1} = 1 - 2e^{-1}$

≈ 0.26