Assignment - 6

Q1)

a)

→ Use the multinominal idea to obtain that the joint PMF is simply

$$P(X=x, Y=y, Z=z) = \frac{n!}{x!y!z!} = \left(\frac{1}{3}\right)^x \left(\frac{1}{3}\right)^y \left(\frac{1}{3}\right)^z = \frac{n!}{x!y!z!}\left(\frac{1}{3}\right)^n$$

for $x+y+z=n$, otherwise it is equal to zero

b)

→ first observe that the game decisive if and only if there is one and only one random variable (out of X, Y, Z) that is equal to zero. So let's consider the case where $X=0$. Then we have to have that $Y=K$ for $K=1, \dots, n-1$. Hence $Z=n-K$. The probability in this case is

$$P(\text{decisive}, X=0) = \sum_{K=1}^{n-1} P(X=0, Y=K, Z=n-K)$$

$$= \sum_{K=1}^{n-1} \frac{n!}{K!(n-K)!}\left(\frac{1}{3}\right)^n = \left(\frac{1}{3}\right)^n \cdot (2^n - 2)$$

where we have used $\sum_{K=1}^{n-1} \frac{n!}{K!(n-K)!} = \sum_{K=1}^{n-1}\binom{n}{K} = \sum_{K=0}^{n}\binom{n}{K} - \binom{n}{0} - \binom{n}{n}$

$$= 2^n - 2.$$

Here $Z=0$ & $Y=0$, Hence required probability is

$$P(\text{decisive}) = 3P(\text{decisive}, X=0) = \frac{2^n-2}{3^n-1}$$

c)

→ from part(b), plugging $n=5$ we see that the probability that the game is decisive is equal to $\frac{2^5-2}{3^5-1} = \frac{10}{27}$. Now we are interested in what happing with probability when $n \to \infty$

$$\lim_{n\to\infty} P(\text{decisive}) = \lim_{n\to\infty} \frac{2^n - 2}{3^n - 1} = \lim_{n\to\infty} \frac{2^n}{3^n - 1} - \lim \frac{2}{3^n - 1}$$

$$= 3 \lim_{n\to\infty} \frac{2^n}{3^n} - \lim \frac{2}{3^n} = 0 - 0 = 0$$