Chapter 1
MAGNETIC CIRCUITS

1.1 Magnetic Field

Magnetic fields are fundamental mechanism by which energy is converted from one form to another in motors, generators, and transformers. There are four basic principles, which describe how magnetic fields are used in these devices.

1. A current-carrying wire produces a magnetic field in the area around it.
2. A time varying magnetic field induces a voltage in a coil of wire if it passes through that coil (BASIS OF TRANSFORMER ACTION).
3. A current-carrying wire in the presence of a magnetic field has a force induced on it (BASIS OF MOTOR ACTION).
4. A moving wire in the presence of a magnetic field has a voltage induced in it (BASIS OF GENERATOR ACTION).

1.2 PRODUCTION OF MAGNETIC FIELD & PRINCIPLES

1.2.1 **AMPERE'S CIRCUITAL LAW**.

Basic law governing the production of magnetic field by a current is **AMPERE'S CIRCUITAL LAW**.

$$\oint \vec{H}.d\vec{l} = \int_A \vec{J}.d\vec{A} = I_{net} \qquad (1\text{-}1)$$

Line integral of the magnetic field intensity *H* around a closed path in a magnetic field = Surface integral of current density *J* over any surface bounded by the closed path.

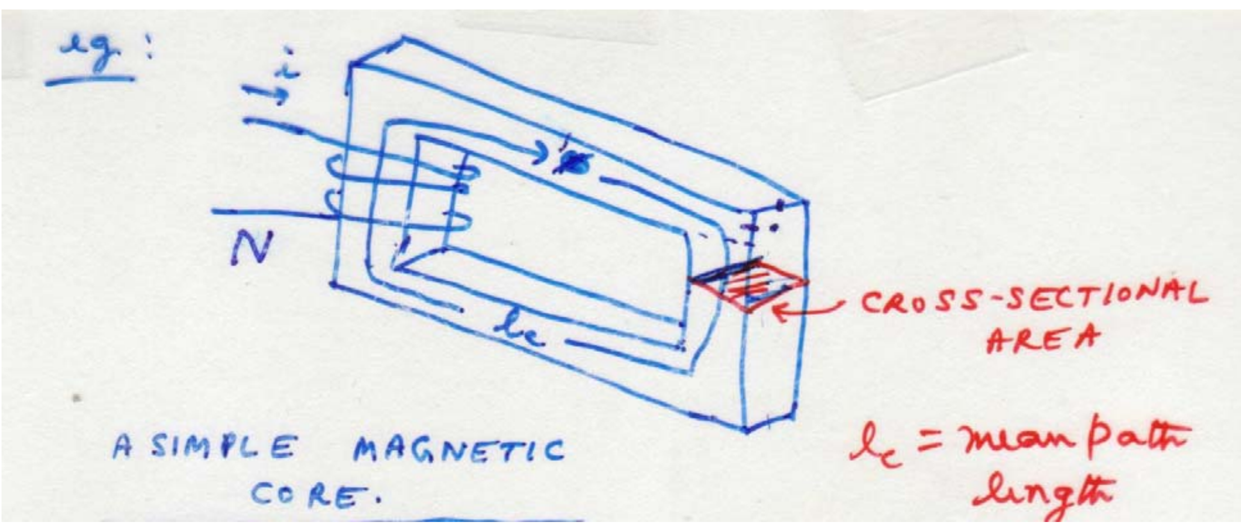


Fig. 1.1 A simple magnetic core wound with *N* turns of wire and carrying a current *i*.

Magnetic field produced remains inside the core.

$$Hl_c = (Ni) \qquad \text{(Amper's law)} \qquad \text{(I)}$$

Magnitude of $\vec{H}$ Ampere-turns

$$\boxed{H = \frac{(Ni)}{l_c} \qquad \text{(A.t/m)}} \qquad (1\text{-}2)$$

1.2.2 Relation between the flux density *B* (Webers/m^2) and *H* is given by

$$\boxed{B = \mu H \qquad (\text{Wb/m}^2)} \qquad \text{(II)}$$

μ is the permeability of the material used, $\mu = \mu_o \mu_r$

$\mu_o = 4\pi \times 10^{-7}$ H/m (Free Space)

μ_r = relative permeability of the material compared to free space.

Magnetic circuits contain

Materials with $\mu \cong \mu_0$.
eg: 1. air,copper, wood
2. Paramagnetic materials
(μ slightly < μ_0)
3. Diamagneitc materials
(μ slightly > μ_0)

Materials with $\mu >> \mu_0$
eg.: Ferromagnetic
$\mu_r \cong 2000$ to 6000 or more.
(eg: Iron core)

e.g.: For the earlier example (using (1-2) in (II)):

$$B = \mu H = \mu \left[\frac{(Ni)}{l_c} \right] \qquad (\text{Wb/m}^2 \text{ or Tesla}) \qquad (1\text{-}3)$$

1.2.3 TOTAL Flux:

$$\boxed{\text{TOTAL FLUX, } \phi = \int_A \vec{B} . d\vec{A}} \qquad \text{Webers} \qquad \text{(III)}$$

e.g.: If flux density is constant,

$$\phi = BA = \frac{\mu N i A}{l_c} \qquad \text{Wb} \qquad (1\text{-}4)$$

A = cross-sectional area of the core (m^2).

1.3 MAGNETIC CIRCUITS

Current in a coil of wire around a core produces a magnetic flux in the core. This is analogous to a voltage in an electric circuit producing current. Therefore, we can develop magnetic circuit models and solve these circuits.
Similar to voltage in an electric circuit, magneto motive force (mmf) $\mathcal{F}$ in a magnetic circuit.

Magneto motive force (mmf),

$$\mathcal{F} = Ni \qquad \text{At} \qquad (1\text{-}5)$$

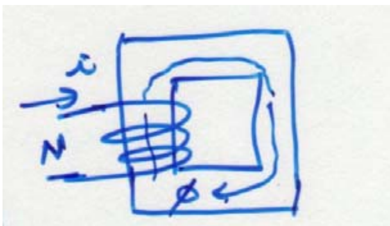
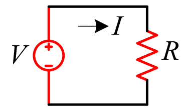


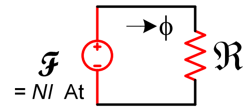
Fig. 1.2(a) Magnetic system.



$$I = \frac{V}{R}$$

ELECTRIC CIRCUIT

(b)



$$\phi = \frac{\mathcal{F}}{\Re}$$

EQUIVALENT MAGNETIC CIRCUIT

(c)

Fig. 1.2 Analogue between (b) ELECTRIC CIRCUIT and (c) EQUIVALENT MAGNETIC CIRCUIT (or Electric circuit analog). $\Re$ is reluctance, ϕ is the flux seen by the mmf $\mathcal{F}$.

Permeance $\quad \mathcal{P} = \dfrac{1}{\Re}$ (1-6)

$$\therefore \phi = \frac{\mathcal{F}}{\Re} = \mathcal{F}\mathcal{P} \qquad \text{Wb.} \qquad (1\text{-}7)$$

e.g. (continued): $\phi = \dfrac{\mu N i A}{l_c} = (Ni)\left[\dfrac{\mu A}{l_c}\right] = (\mathcal{F})(\dfrac{1}{\Re}) = \mathcal{F}\mathcal{P}$ Wb. (1-8)

$$\therefore \Re = \frac{l_c}{\mu A} \qquad \text{H}^{-1} \text{ or At/wb.} \qquad (1\text{-}9)$$

(Note: units of μ is H/m).

Table 1.1 Analogy between Magnetic circuit and DC Electric circuit.

Magnetic Circuit	**Electric Circuit**
Flux, ϕ	**Current, I**
MMF, $\mathcal{F}$	**Voltage, V**
Reluctance, $\Re = \dfrac{l_c}{\mu A}$	**Resistance, $R = \dfrac{l}{\sigma A}$**
Permeance, $\mathcal{P} = \dfrac{1}{\Re}$	**Conductance, $G = 1/R$**
Permeability, μ	**Conductivity, σ**
Ohm's Law, $\phi = \dfrac{\mathcal{F}}{\Re}$	**Ohm's Law, $I = \dfrac{V}{R}$**

MAGNETIZATION (*B - H*) CURVES:

Curves showing variation of *B* (flux density) versus *H* (field intensity) are available from manufacturers of magnetic materials. Approximate values of 3% to 5% errors are tolerable in reading such curves.

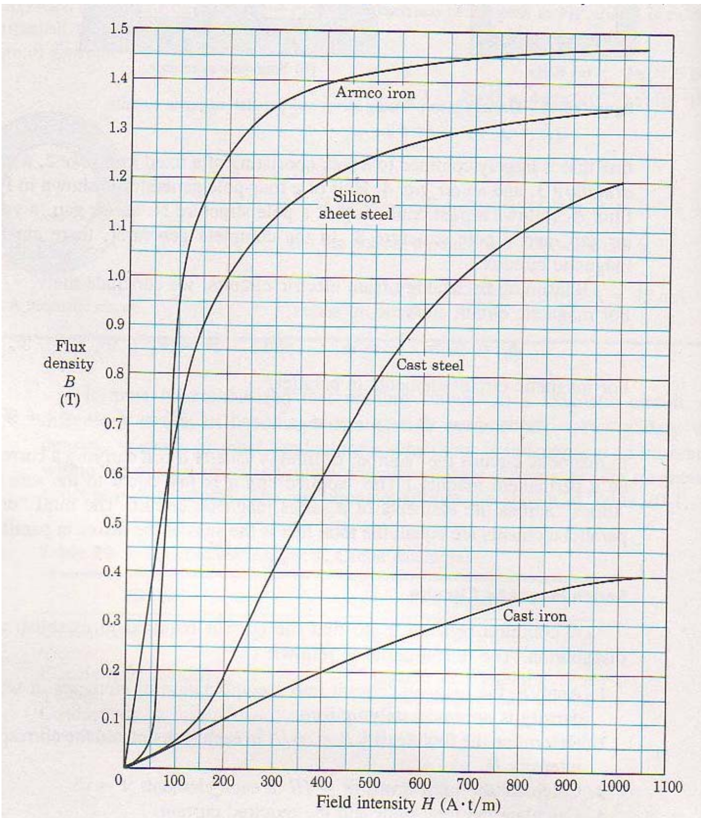


Fig. 1.3(a) *B – H* curves for common magnetic matewrials.

BH Curve: e.g. Iron.

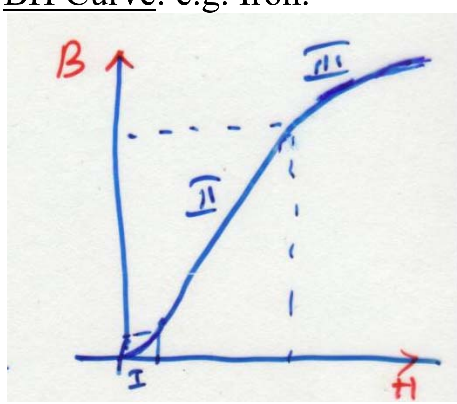


Fig. 1.3(b)

Region I: Initial permeability $\mu_i = \lim_{H \to 0} \frac{B}{H}$ (1-10a)

Region II: Linear $\mu = \frac{B}{H}$ (1-10b)

Region III: Differential permeability $\mu_d = \frac{dB}{dH}$ (1-10c)

1.4.1 *Reliances in series:* Equivalent reluctance $\Re_{ser}$ of series connected reluctances is

$$\Re_{ser} = \Re_1 + \Re_2 + \ldots\ldots \tag{1-11}$$

Example:

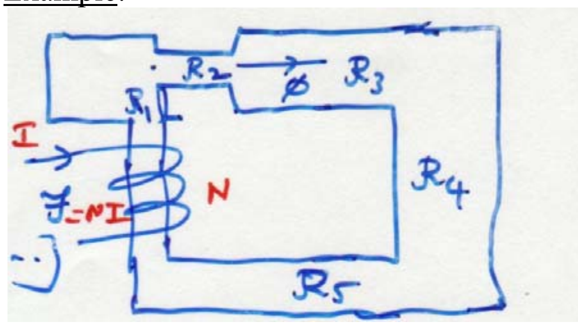


Fig. 1.4 Example of Reluctances in series.

Proof: MMF applied = $\mathcal{F}$ = NI = sum of MMF drops = $\Re_1\phi + \Re_2\phi + \ldots\ldots$

If $\Re_{ser}$ is the equivalent reluctance of $\Re_1, \Re_2, \ldots\ldots$ connected in series, then

$$\mathcal{F} = \Re_{ser}\phi \tag{1-12a}$$

$$\therefore \mathcal{F} = \Re_{ser}\phi = \Re_1\phi + \Re_2\phi + \ldots\ldots \tag{1-12b}$$

i.e. $$\Re_{ser} = \Re_1 + \Re_2 + \ldots\ldots \tag{1-12c}$$

1.4.2 *Reluctances in parallel:*

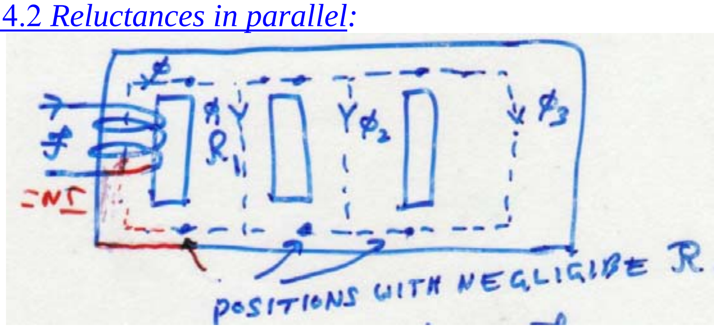


Fig. 1.5 Example of Reluctances in parallel.

$$\phi_1 = \frac{\mathcal{F}}{\Re_1}; \quad \phi_2 = \frac{\mathcal{F}}{\Re_2} \quad \ldots \tag{1-13a}$$

If $\Re_{par}$ is the equivalent of parallel combination of all reluctances,

$$\phi = \frac{\mathcal{F}}{\Re_{par}} = \phi_1 + \phi_2 + \ldots. = \frac{\mathcal{F}}{\Re_1} + \frac{\mathcal{F}}{\Re_2} + \ldots\ldots \tag{1-13b}$$

$$\therefore \frac{1}{\Re_{par}} = \frac{1}{\Re_1} + \frac{1}{\Re_2} + \ldots\ldots \tag{1-13c}$$

1.5 Fringing: Results from flux lines appearing along the sides and edges of magnetic members separated by air.

Effect: Effective air-gap increases. Fringing increases with the length of the air-gap.

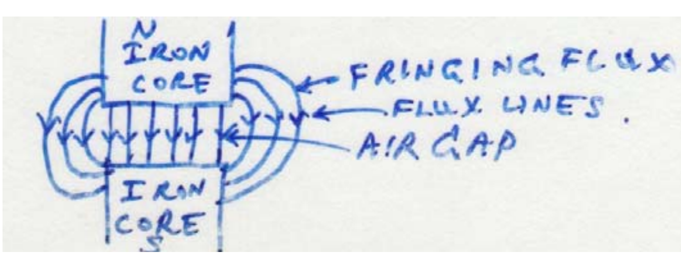


Fig. 1.6 Fringing across an air-gap.

1.6 KIRCHHOFF'S LAWS FOR THE MAGNETIC CIRCUIT

First Law [Kirchhoff's Flux Law (*KFL*)]: The total magnetic flux towards a junction is equal to the total magnetic flux away from that junction. (Algebraic sum of magnetic flux in a junction = 0).

Example:

$$\phi_L = \phi_M + \phi_N \quad \text{or} \quad \phi_L - \phi_M - \phi_N = 0 \qquad (1\text{-}14)$$

In general $\sum \phi_i = 0.$

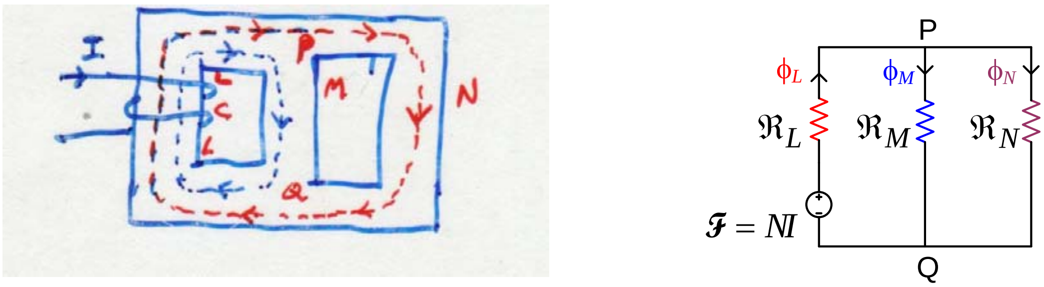


Fig. 1.7(a) Magnetic core with a coil. (b) Equivalent magnetic circuit

Second Law [Kirchhoff's MMF Law (*KMMFL*)]: The algebraic sum of all the magneto-motive-forces (mmfs) around any closed path in a magnetic circuit = 0.

Example (Fig. 1.7): Total mmf produced by coil C, $\mathcal{F} = NI = H_L l_L + H_M l_M$

$$= H_L l_L + H_N l_N$$

and $\quad \therefore \; H_M l_M - H_N l_N = 0 \qquad (1\text{-}15a)$

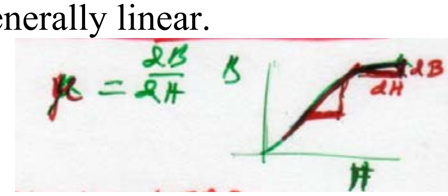
Or

$$\mathcal{F} = NI = \Re_L \phi_L + \Re_M \phi_M = \Re_L \phi_L + \Re_N \phi_N$$

$$\Re_M \phi_M - \Re_N \phi_N = 0. \qquad (1\text{-}15b)$$

1.7 DIFFERENCES BETWEEN MAGNETIC AND ELECTRIC CIRCUITS

(1) $\mathcal{F} - \phi$ characteristic is non-linear.
   $V - I$ characteristic of a DC resistive circuit is generally linear.
   [Saturation decreases μ of the material:
   since dB is small in saturation region
   for the same dH in the linear region].



(2) Fluxes leak-out via leakage paths.
   Leakage increases with magnetic saturation.
   In DC resistive circuit, current is confined to the conductor.

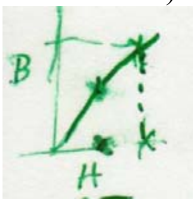
(3) Fringing: only in magnetic circuit.

(4) I^2R losses in a dc resistive circuit. But no $\phi^2\Re$ loss in a magnetic circuit.

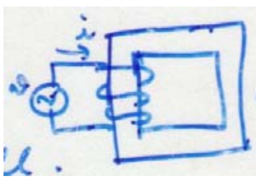
(5) Current density is limited by heating – in an electric circuit.
   Flux density is limited by saturation in a magnetic circuit.

1.8 AC OPERATION OF MAGNETIC CIRCUITS

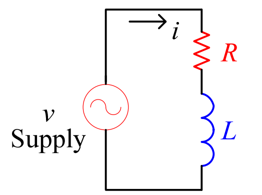
When a dc voltage is applied to a coil wound on a core, flux f established in a core is determined by the magnetic field intensity *H* (A-t/m) and properties of core defined by *B-H* curve and *A* (area of cross-section) of the core.



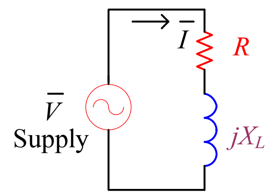
Current in the coil, $I = \frac{V}{R}$, where *V* is the applied DC voltage to the coil and *R* = resistance of the coil.
When a sinusoidal ac voltage is applied to the coil,



(a)



(b) Time domain circuit



(c) Phasor (Frequency domain)circuit

Fig. 1.8(a) Coil wound on a magnetic core and supplied by an ac source.
Fig. 1.8(b) and (c) Equivalent circuits for the coil when supplied by a sinusoidal source.

$v = \sqrt{2}\, V\cos(\omega t)$ Volts, where *V* is the root mean square (rms) voltage.

Phasor current: $\bar{I} = \dfrac{\bar{V}}{Z}$, (1-16a)

where phasor voltage is $\bar{V} = V\angle 0^{o}$ Volts, and impedance is $Z = R + j(\omega L) = |Z|\angle\,\phi^{o}$ Ω, (1-16b)

$$|Z| = \sqrt{(R^2 + X_L{}^2}\ \Omega, \quad \phi = \tan^{-1}\left(\frac{X_L}{R}\right), \tag{1-16c}$$

R is the resistance of the coil and is usually very small, Reactance of *L*, $X_L = \omega L$,
$\omega = 2\pi f$ rad/s, f = supply frequency (Hz).
Then current in the coil (in time domain) is $i = \sqrt{2}\,(V/|Z|)\cos(\omega t - \phi)$ Amps.

The magnetic flux in the core is that just required to produce an induced voltage ≅ applied voltage.

1.8.1 FARADY's LAW OF ELECTROMAGNETIC INDUCTION

: A time varying flux linking a coil induces an emf (or voltage) in the coil. This voltage is proportional to the number of turns and the time rate of change of flux ϕ;

i.e., $e = N\left(\dfrac{d\phi}{dt}\right)$ Volts. (1-17)

Flux linkage, $\lambda = N\phi$ Wb-turn (Wb-t) (1-18)

$\therefore\quad e = \dfrac{d\lambda}{dt}$ (1-19)

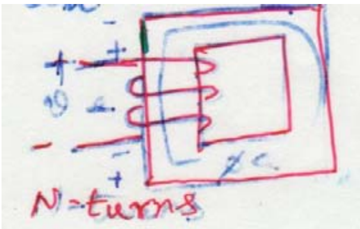


Fig. 1.9

The polarity of e is given by Lenz's law: direction of e is such as to force a current in the coil, opposing the change in ϕ.

If $R_{coil} = 0$ and no leakage flux, then

$$v = e = N\left(\frac{d\phi}{dt}\right) \qquad (1\text{-}20)$$

$$\therefore\ \phi = \frac{1}{N}\int_0^t v.dt \quad (\therefore \text{ voltage determines flux in ac circuit}). \qquad \text{Wb} \qquad (1\text{-}21)$$

1.8.2 EDDY CURRENTS: According to **Faraday's law**, time varying magnetic field (flux) linking an electric circuit induces a voltage in the circuit. Since these circuits are closed (solid core), induced voltages give rise to currents; called as eddy currents. This results in heating of the core (similar to I^2R losses).

Empirical formula: $P_e = (K_e)f^2B_m^2$ W/kg. (1-22)

B_m = maximum flux density in the core.

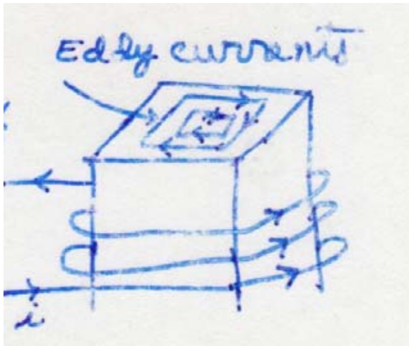


Fig. 1.10 Eddy currents in the core when the coil is supplied by an ac source.

1.8.3 HYSTERESIS: When exciting current is ac, hysteresis loop is traced out once each cycle. Area of complete hysteresis loop is the energy lost per cycle.

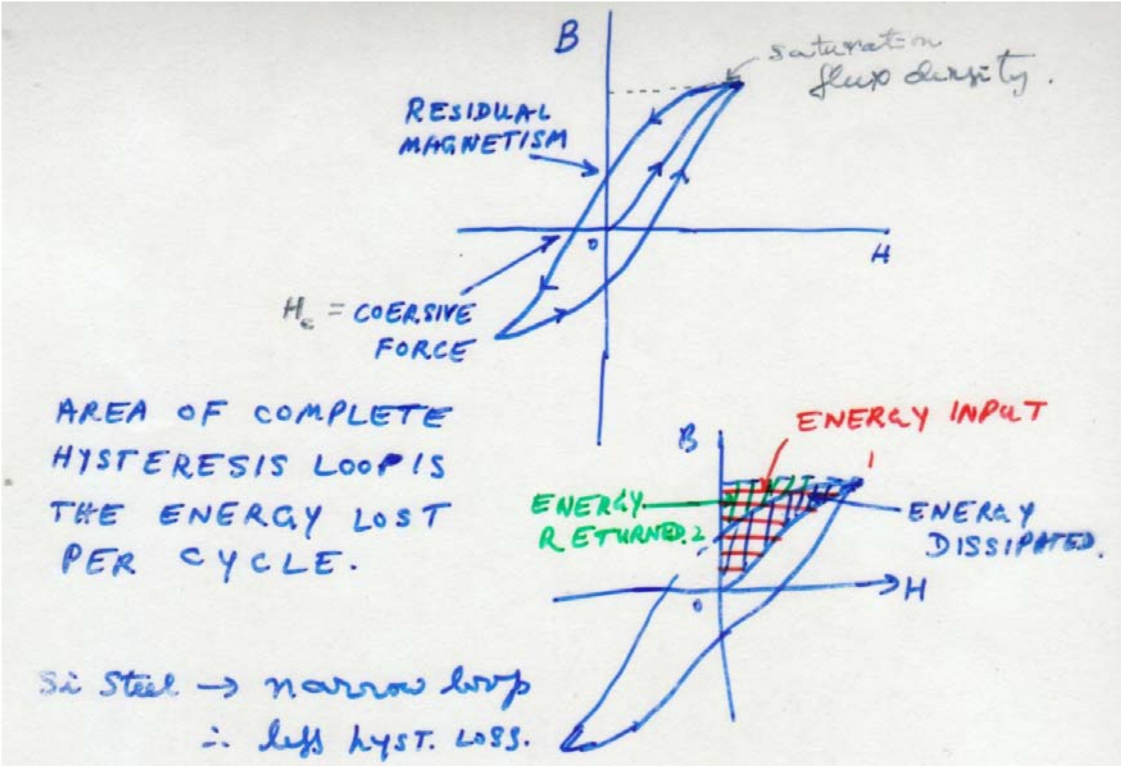


Fig. 1.11 Hysteresis loop

Silicon (Si) Steel has narrow hysteresis loop, ∴ less hysteresis loss.

Empirical formula (Steinmetz): $P_e = (K_h) f B_m^{\,n}$ W/kg., $n = 1.5$ to 2.5. (1-23)

IRON LOSS (also called as CORE LOSS) = EDDY CURRENT LOSS + HYSTERESIS LOSS.

1.8.4 REDUCTION of IRON LOSS:

(a) To reduce eddy current, use laminated core -- thin sheets with thin layers of insulation between the laminations (increases volume).
(lamination thickness = 0.5 mm to 5mm).
Eddy current loss is approximately $\propto$ (lamination thickness)^2.

Stacking factor = $\dfrac{\text{Volume actually occupied by magnetic material}}{\text{Total Volume of core}} < 1.$

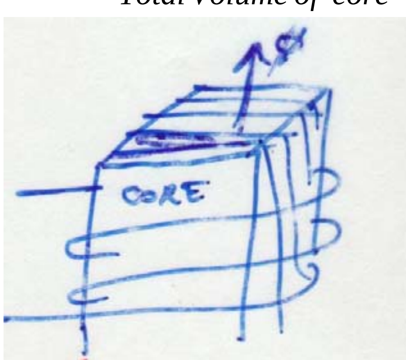


Fig. 1.12 Laminated core.

(b) Use good quality steel with narrow hysteresis loop to reduce the hysteresis loss.

1.9 INDUCTANCE:

Definition: $L = \frac{\lambda}{I} = \frac{N\phi}{I}$ Henry (H) (1-24)

λ is the flux linkage, $\phi = \phi_c + \phi_l$, where ϕ_c = core flux and ϕ_l = leakage flux.

$$L = f(N,\phi,I) \qquad (1\text{-}25)$$

However, L is not a function of I except at saturation.

Example:

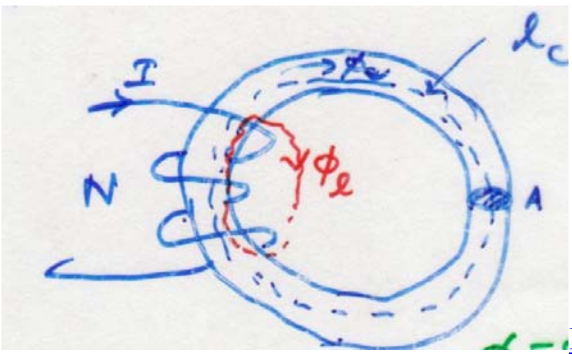


Fig. 1.13 Toroidal core wound with N turns of wire.

$$\mathcal{F} = NI \qquad \text{At}$$

$$\text{Total flux, } \phi = \frac{\mathcal{F}}{\Re} = \frac{NI}{\Re} \qquad \text{Wb.} \qquad (1\text{-}26)$$

$$\boxed{\therefore L = \frac{N\phi}{I} = \left(\frac{N}{I}\right)\left[\frac{NI}{\Re}\right] = \frac{N^2}{\Re} = N^2\mathcal{P}} \qquad \text{H.} \qquad (1\text{-}27)$$

If l_c is the mean length of the core and A is the area of cross-section, then

$\Re = \frac{l_c}{\mu A}$; and substituting for $\Re$ in (1-27),

$$\boxed{\therefore L = \frac{N^2(\mu A)}{l_c}} \qquad \text{H.} \qquad (1\text{-}28)$$

μ is not a function of I (if core is unsaturated).

The inductance determined dependent on $N\phi$ (ϕ is the total flux) and called SELF-INDUCTANCE of the coil (since $\mathcal{F}$ or I and λ are associated with same coil).

Leakage inductance, $L_l = \frac{N\phi_l}{I}$ H. (1-29)

1.10 Mutual inductance:

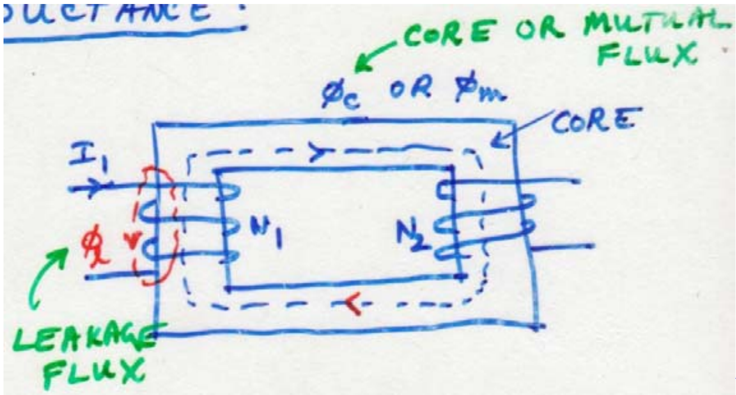


Fig. 1.14 Two coils wound on the same core.

Mutual flux ϕ_c links both N_1 and N_2. Leakage flux ϕ_l **links only N_1.**

Definition: Mutual inductance between two coils,

$$L_{12} = \frac{\text{Flux Linking Second Coil}}{\text{Current in First Coil}} = \frac{N_2\phi_m}{I_1} \qquad (1\text{-}30)$$

Since $\phi = \phi_m + \phi_l$, $\phi_m < \phi$: therefore, $\phi_m = k\phi$. Wb (1–31)

where k is the coupling coefficient and $k < 1$.

If $\Re$ is the reluctance seen by $\mathcal{F} = N_1 I_1$ (At),

$$\phi = \frac{\mathcal{F}}{\Re} = \frac{N_1 I_1}{\Re} \quad \text{Wb.} \qquad (1\text{-}32)$$

$$\therefore L_{12} = \frac{N_2(\phi_m)}{I_1} = \frac{N_2(k\phi)}{I_1} = \frac{N_2(k)}{I_1}\left[\frac{N_1 I_1}{\Re}\right] = \frac{kN_1N_2}{\Re} \quad \textbf{H.} \qquad (1\text{-}33)$$

Similarly, $L_{21} = \dfrac{\text{Flux Linking First Coil}}{\text{Current in Second Coil}} = \dfrac{N_1\phi_m}{I_2}$ (1-34)

$$L_{12} = L_{21} = M \qquad (1\text{-}35)$$

$k = \dfrac{L_{12}}{\sqrt{L_1L_2}} = \dfrac{M}{\sqrt{L_1L_2}}$ [Using (1-33) and $L_1 = \dfrac{N_1^2}{\Re}$, $L_2 = \dfrac{N_2^2}{\Re}$] (1-36)

$0 < k < 1$.

[Note: $\dfrac{L_{12}}{\sqrt{L_1L_2}} = \dfrac{\left[\dfrac{kN_1N_2}{\Re}\right]}{\sqrt{\left(\dfrac{N_1^2}{\Re}\right)\left[\dfrac{N_2^2}{\Re}\right]}} = k$]

Example 1:

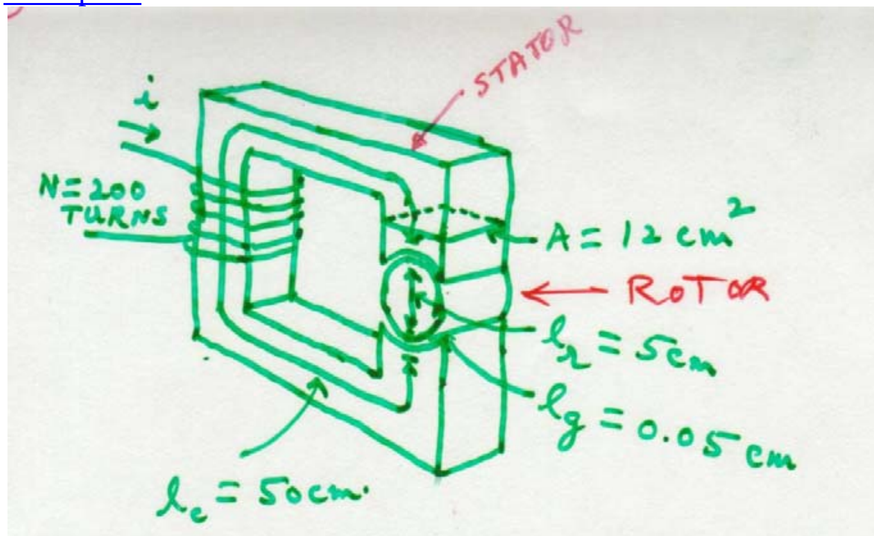
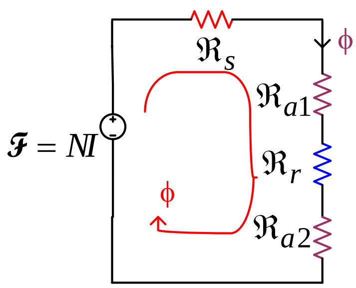


Fig. Ex #1: A SIMPLIFIED DIAGRAM OF A ROTOR and STATOR FOR A DC MOTOR.

Fig. Ex #1 shows a simplified rotor and stator of a DC motor.
The mean length of the stator, l_c = 50 cm, and it's cross-sectional area, A = 12 cm^2,
Mean length of rotor, l_r = 5 cm, and it's cross-sectional area = 12 cm^2 (assume).
Each air-gap between the rotor and the stator is 0.05 cm wide, and the cross-sectional area of each air-gap (including FRINGING) is 14 cm^2. The iron of core has μ_r = 2000, and N = 200 turns on the core. Current in the coil, i = 1 A, what will the resulting FLUX DENSITY in the air-gaps be?

Solution: First find M.M.F. applied to the core and the RELUCTANCE of the flux path.
$\therefore$ ϕ_{TOTAL} in the core can be found, and flux density can be calculated.



Equivalent magnetic circuit (or electric circuit analog) for the magnetic system.

Reluctance of STATOR, $\Re_s = \dfrac{l_s}{\mu_0 \mu_r A_s}$

$$= \frac{0.5(m)}{(4\pi \times 10^{-7}) \times (2000) \times (0.0012 m^2)}$$ = 166,000 A.turns/Wb (or H^{-1}).

Reluctance of ROTOR, $\Re r = \dfrac{l_r}{\mu_0 \mu_r A_r}$

$$= \frac{0.05(m)}{(4\pi \times 10^{-7}) \times (2000) \times (0.0012 m^2)}$$ = 16,600 A.turns/Wb.

Reluctance of AIR-GAP, $\Re_a = \dfrac{la}{\mu_0 \mu_r A_a}$

$$= \frac{0.0005(m)}{(4\pi \times 10^{-7}) \times (1) \times (0.0014 m^2)} = 284{,}000 \text{ A.turns/Wb.}$$

The total reluctances of the flux path is

$\Re_{TOT} = \Re_s + \Re_r + \Re_{a1} + \Re_{a2} = 166{,}000 + 16{,}600 + 2 \times (284{,}000) = 751{,}000$ A.turns/Wb.
The net mmf applied to the core is

$\mathcal{F} = Ni = (200 \text{ turns}) \times (1 \text{ A}) = 200$ At.

$\therefore$ Total flux in the core is $\phi = \dfrac{\mathcal{F}}{\Re_{TOTAL}} = \dfrac{200}{751{,}000} = 0.000266$ Wb.

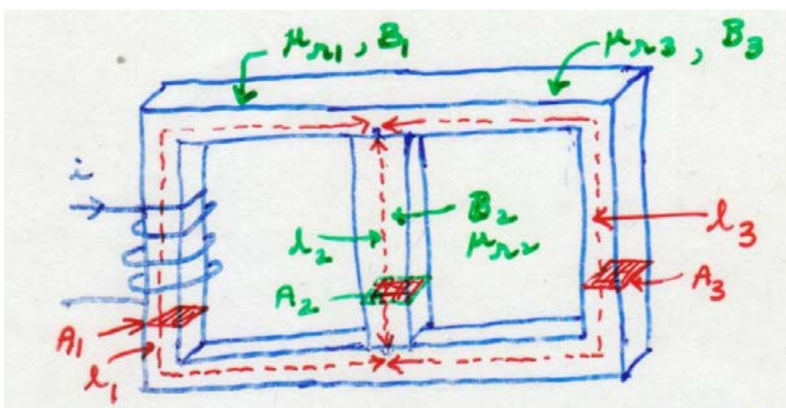
$\therefore$ Magnetic flux density in Motor's air-gap, $B = \dfrac{\phi}{A} = \dfrac{0.000266(Wb)}{0.0014(m^2)}$

$= \underline{0.19 \text{ Wb/m}^2}$

EXAMPLE 2: In the magnetic system shown,

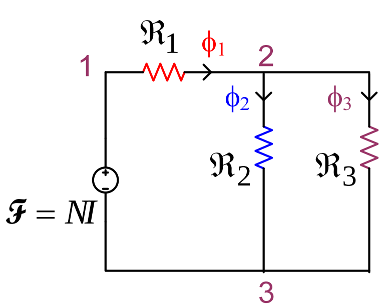
$l_1 = l_3 = 300$ mm, $l_2 = 100$ mm,
$A_1 = A_3 = 200$ mm^2, $A_2 = 400$ mm^2
$\mu_{r1} = \mu_{r3} = 2250$, $\mu_{r2} = 1350$
No. of turns in the coil, $N = 25$.

Determine the flux densities B_1, B_2 and B_3 in the three branches of the circuit when coil current is 0.5 A.



SOLUTION:

The Equivalent magnetic circuit (or electric circuit analog) for the magnetic system is as shown below:



$$\Re_1 = \frac{l_1}{\mu_0\mu_r A_1} = \Re_3 = \frac{l_3}{\mu_0\mu_r A_3}$$

$$= \frac{100\times10^{-3}(m)}{(4\pi\times10^{-7})\times(2250)\times(200\times10^{-6}m^2)} = 0.531 \times 10^6 \text{ A.turns/Wb (or H}^{-1}\text{).}$$

$$\Re_2 = \frac{l_2}{\mu_0\mu_r A_2} = \frac{100\times10^{-3}(m)}{(4\pi\times10^{-7})\times(1350)\times(400\times10^{-6}m^2)} = 0.148 \times 10^6 \text{ A.turns/Wb (or H}^{-1}\text{).}$$

Write mmf equations for two loops [Kirchhoff's MMF Law (*KMMFL*)]:

$$\mathcal{F} = \Re_1\phi_1 + \Re_2\phi_2 \qquad \text{At} \qquad (1)$$

$$0 = \Re_3\phi_3 - \Re_2\phi_2 \qquad \text{At} \qquad (2)$$

Also, Kirchhoff's Flux Law (*KFL*) to node 2: $\phi_1 = \phi_2 + \phi_3$ Wb. (3)

Substituting in (1) to (3):

$(N)(i) = 25 \times 0.5 = [0.531\ \phi_1 + 0.148\ \phi_2] \times 10^6$

$0 = -0.148\ \phi_2 + 0.531\ \phi_3$

$0 = -\phi_1 + \phi_2 + \phi_3$

Solve the above 3 equations for the fluxes:

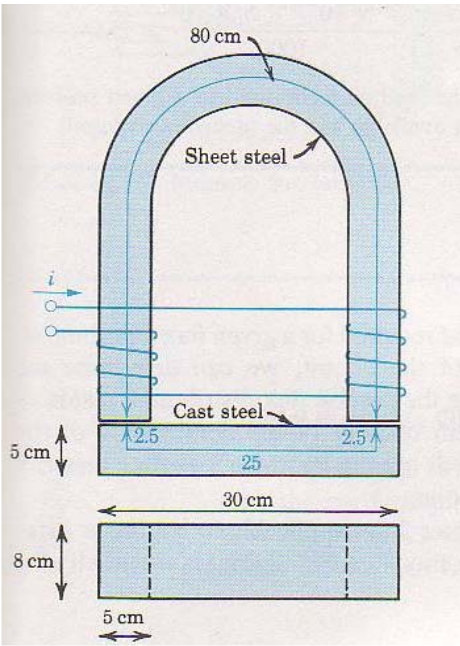
$\phi_1 = 19.3 \times 10^{-6}$ Wb., $\phi_2 = 15.1 \times 10^{-6}$ Wb., $\phi_3 = 4.21 \times 10^{-6}$ Wb.

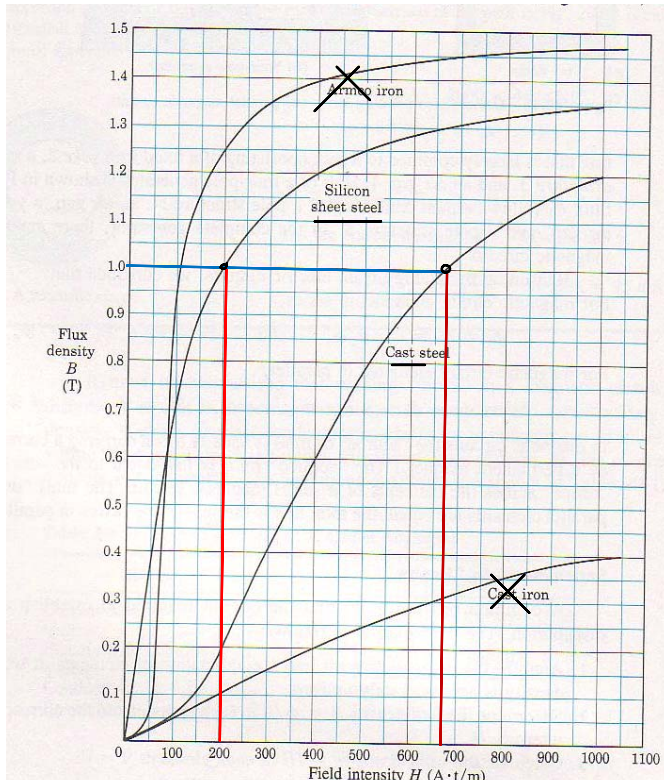
$$\therefore B_1 = \frac{\phi_1}{A_1} = \frac{19.3\times10^{-6}(Wb)}{200\times10^{-06}(m^2)} = \underline{0.0965}\ \underline{\text{T (or Wb/m}^2\text{)}}$$

$$B2 = \frac{\phi_2}{A_2} = \frac{15.1\times10^{-6}}{400\times10^{-06}} = \underline{0.0377}\ \underline{\text{T (or Wb/m}^2\text{)}}$$

$$B_3 = \frac{\phi_3}{A_3} = \frac{4.21\times10^{-6}}{200\times10^{-06}} = \underline{0.0210}\ \underline{\text{T (or Wb/m}^2\text{)}}$$

Example 3: Given the magnetic circuit shown, with 500 turns wound on each leg. Find the current required to establish a flux of 4 mWb across the 0.1 cm air-gaps (Fringing negligible). B-H curves are given.





Solution: We have two iron elements and two air-gaps in series.

SHEET STEEL: Length of sheet steel, l = 80 cms = 0.8 m.
Area of cross-section of sheet steel, A = 5 cm × 8 cm = 0.05 m × 0.08 m = 4 × 10^{-3} m^2

$$\text{Flux density, } B_1 = \frac{\phi}{A} = \frac{4\times10^{-3}(Wb)}{(0.05\times0.08)(m^2)} = 1 \text{ T.}$$

From B-H curve given, H = 200 At/m.
and MMF required, $\mathcal{F}_1 = Hl$ = 200 × 0.8 = <u>160 At.</u>

CAST STEEL:
Effective length of cast steel element is = 25 + 2.5 + 2.5 = 30 cms. = 0.3 m.

$$\text{Flux density, } B_2 = \frac{\phi}{A} = \frac{4\times10^{-3}(Wb)}{(0.05\times0.08)(m^2)} = 1 \text{ T.}$$

From B-H curve given, H = 670 At/m.
And MMF required, $\mathcal{F}_2 = Hl$ = 670 × 0.3 ≅ <u>200 At.</u>

AIR-GAPS:
Length of each air-gap, l = 1 mm = 1 × 10^{-3} m,
area of cross-section, A = 0.05 m × 0.08 m = 4 × 10^{-3} m^2

MMF required, $\mathcal{F}_3 = 2Hl = 2\dfrac{B}{\mu}l = 2\left(\dfrac{\phi}{A}\right)\left[\dfrac{l}{\mu_0\mu_r}\right] = \dfrac{2\times(4\times10^{-3})\times(1\times10^{-3})}{(4\times10^{-3})\times(4\pi\times10^{-7})\times1} \cong$ <u>1590 At.</u>

Total MMF required is = $\mathcal{F} = \mathcal{F}_1 + \mathcal{F}_2 + \mathcal{F}_3$

= 160 + 200 + 1590 = 1950 At.

MMFs of both coils are in the same direction.

$\therefore\ i = \dfrac{\mathcal{F}}{2N} = \dfrac{1950\,(At)}{(2\times500)\,(turns)}$ = <u>1.95 A.</u>

1.11 ENERGY STORED IN MAGNETIC CIRCUITS

An interpretation of inductance is as a measure of a circuit component to store energy in a magnetic field.

$E = \frac{1}{2}Li^2$ Joules.

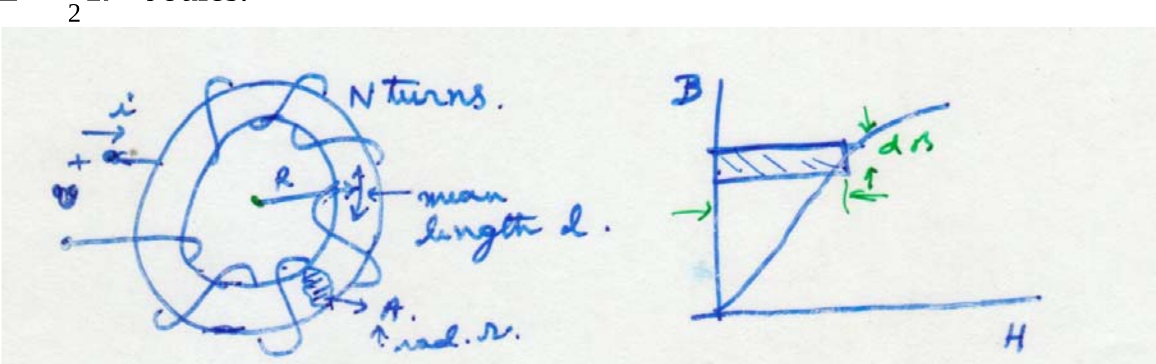


Fig. 1.15 Toroidal core wound with N turns of wire and it's strip of width dB on it's B-H curve.

MMF, $\mathcal{F} = Ni = Hl$ AT

Assume $r << R$; and therefore assume that B and H are uniform:

$$\phi = BA$$

Start with un-magnetized core; we can store energy by building up a current and creating a magnetic field. Magnetic energy comes from electric circuit and electrical input is:

$$w = \int_0^t vidt = \int_0^t N.\frac{d\phi}{dt}.(i)dt = \int_0^\phi Nid\phi \qquad \text{Joules} \qquad (*)$$

But $Ni = \mathcal{F} = Hl$ and $d\phi = (A)(dB)$

$\therefore\ Ni(d\phi) = Ni[A(dB)] = [Hl][A(dB)] = (lA)(H)dB$ [Note: lA = volume of the core.]

$\therefore$ Energy stored per unit volume is

$$w_v = \frac{w}{(lA)} = \int_0^B HdB = \text{area between magnetization curve and B-axis.}$$

If LINEAR magnetic characteristic; i.e., μ = constant $\quad B = \mu H$

$$w_v = \int_0^B \frac{B}{\mu} dB = \frac{1}{2}\frac{B^2}{\mu} \qquad \text{Joules/m}^3.$$

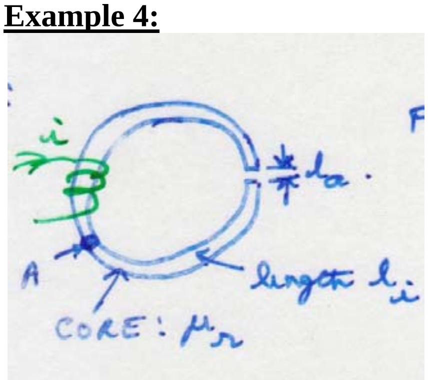
or
$$w_v = \int_0^H \mu H dH = \frac{1}{2}\mu H^2 \quad \text{Joules/m}^3$$

Using (*) and $L = \frac{\lambda}{i} = \frac{N^2}{\Re}$; $\quad \phi = \frac{\mathcal{F}}{\Re} = \frac{Ni}{\Re}$

[Note: $Li = \lambda = N\phi$, $\therefore$ $Ldi = Nd\phi$ *i.e.,* $\int_0^\phi Nid\phi = \int_0^i Lidi = \frac{1}{2}Li^2$]

$$\boxed{w = \frac{1}{2}Li^2 = \frac{1}{2}\Re\phi^2 \quad \text{Joules.}}$$

Example 4:



Flux density in core = B Tesla

Compare energy densities and total energies in iron and air. Neglect fringing. Assume uniform flux density. Also neglect leakage.

Solution: It is given that flux density in air is the same as in the iron core. Also, since fringing is neglected, area of cross sections of air and iron are the same ($A_a = A_i$).

Let energy density in air = w_{va} J/m^3 and energy in iron = w_{vi} J/m^3.

Then the ratio of energy densities, $\frac{w_{va}}{w_{vi}} = \frac{[B^2/(2\mu_0)]}{[B^2/(2\mu_0\mu_r)]} = \mu_r$.

Ratio of total energies, $\frac{w_a}{w_i} = \frac{[w_{va} \times l_a A_a]}{[w_{vi} \times l_i A_i]} = \mu_r \times \frac{l_a}{l_i}$

Take as an example, $\mu_r = 2000$, $l_a = 0.01 l_i$.

Then $w_a = [2000 \times 0.01] \times w_i = 20\, w_i$

That is 95.2% of total energy is stored in air!

Example 5: An iron ring of 20 cm mean diameter having a cross-section of 100 cm^2 is wound with 400 turns of wire. Calculate the exciting current required to establish a flux density of 1 WB/m^2 if $\mu_r = 1000$. What is the value of energy stored? Neglect leakage flux. $\mu_0 = 4\pi \times 10^{-7}$ H/m.

Solution: Flux density, $B = \mu_0\mu_r H = (\mu_0\mu_r)\frac{NI}{l}$ Wb/m^2 (or T).

Substituting the given values, $1 = (4\pi \times 10^{-7}) \times 1000 \times \frac{(400)I}{(\pi \times 0.2)}$

$\therefore$ Exciting current required to establish a flux density of 1 T is, $I = 1.25$ A

Inductance, $L = \frac{\lambda}{I} = \frac{N\phi}{I} = \frac{N}{I}.\left[\frac{\mathcal{F}}{\Re}\right] = \frac{N}{I}.\left[\frac{NI}{\left\{ l/(\mu_0\mu_r A)\right\}}\right]$

$= \frac{N^2(\mu_0\mu_r A)}{l}$

$= (400^2) \times (4\pi \times 10^{-7}) \times 1000 \times (100 \times 10^{-4})]/(\pi \times 0.2)$

$=$ 3.2 H.

Energy stored, $E = \frac{1}{2}LI^2 = \frac{1}{2} \times 3.2 \times (1.25)^2$ = 2.5 Joules.

[In this case, we can also use, $E = \frac{1}{2}\Re\phi^2 = \frac{1}{2} \times \left[\frac{l}{(\mu_0\mu_r A)}\right] \times (B \times A)^2$.]