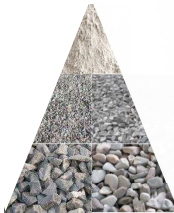


Reaction-diffusion model for oxidative aging of bitumen



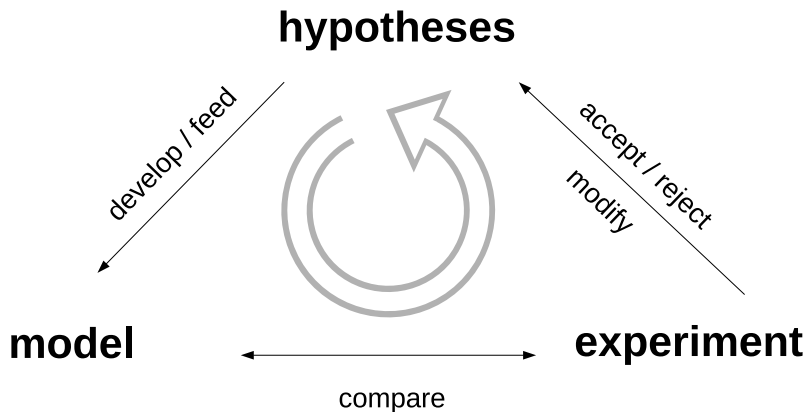
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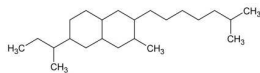
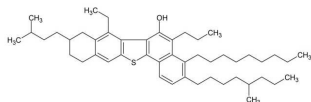
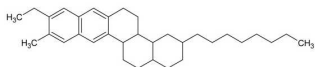
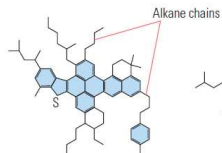


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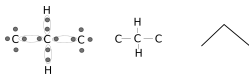


Uwe Mühlich



saturates²resins²aromatics / naphthenics²asphaltenes¹

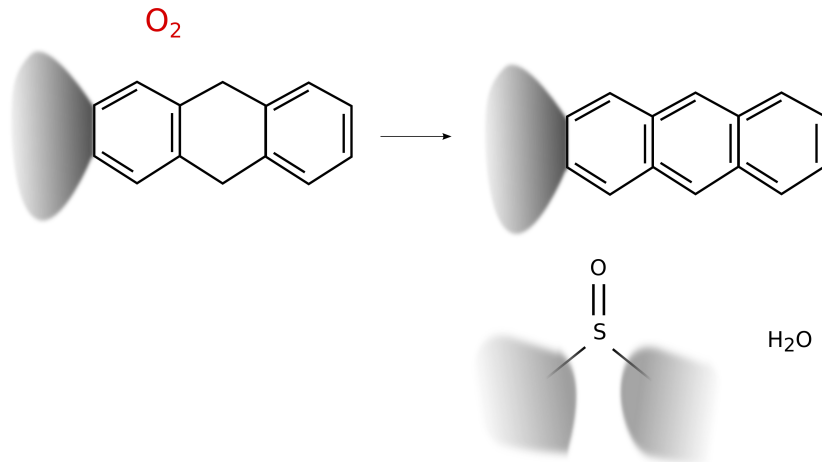
source: <http://www.interchem.at>



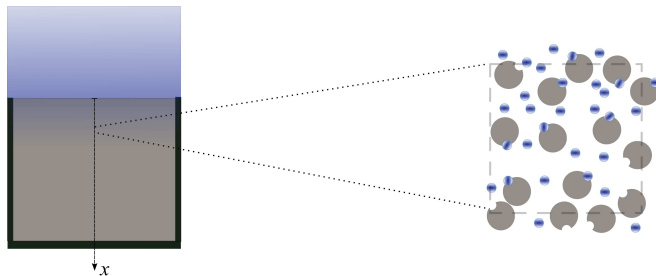
¹[Akbarzadeh et al., 2007]

²[Soenen, 2017]

example: “spurt” oxidation³ \approx 40 hours



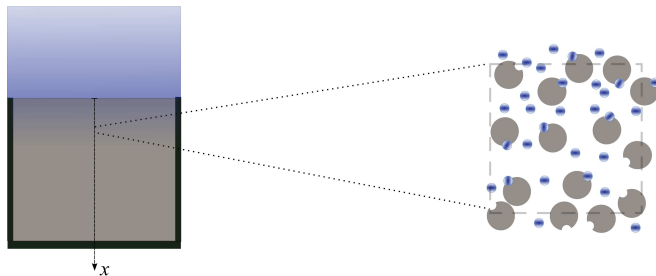
³[Petersen and Glaser, 2011]



$$\rho_1 \leftarrow \frac{n_1 m_1}{V}$$

$$\rho_2 \leftarrow \frac{n_2 m_2}{V}$$


$$\rho_3 \leftarrow \frac{n_3 m_3}{V}$$






$$\rho_1 \leftarrow \frac{n_1 m_1}{V}$$

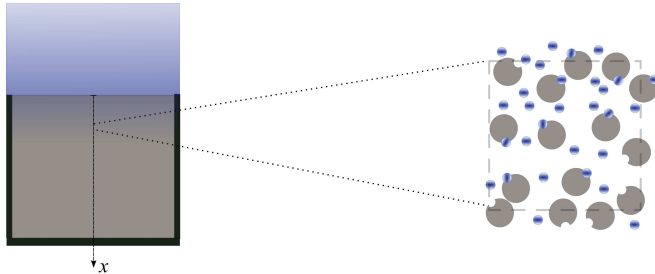

$$\rho_2 \leftarrow \frac{n_2 m_2}{V}$$


$$\rho_3 \leftarrow \frac{n_3 m_3}{V}$$


$$\partial_\tau \rho_1 + J'_1 = r_1$$


$$\partial_\tau \rho_2 + J'_2 = r_2$$


$$\partial_\tau \rho_3 + J'_3 = r_3$$



$$\rho_1 \leftarrow \frac{n_1 m_1}{V}$$

$$\rho_2 \leftarrow \frac{n_2 m_2}{V}$$

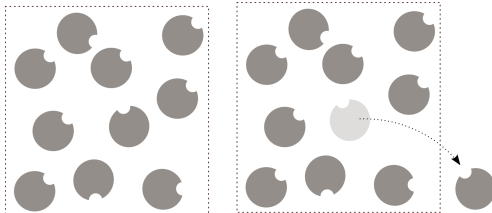
$$\rho_3 \leftarrow \frac{n_3 m_3}{V}$$

$$\partial_\tau \rho_1 + J'_1 = r_1$$

$$\partial_\tau \rho_2 + J'_2 = r_2$$

$$\partial_\tau \rho_3 + J'_3 = r_3$$

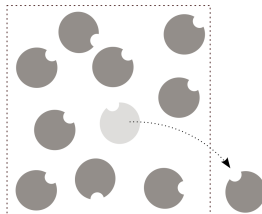
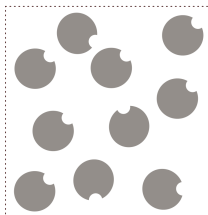
$$\partial_\tau \rho_\alpha + J'_\alpha = r_\alpha \quad , \quad \alpha = 1, \dots, N$$



chemical potential

change in energy by
removing / adding
one unit of particles

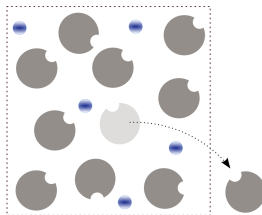
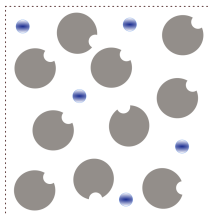
pure substance



chemical potential

change in energy by
removing / adding
one unit of particles

pure substance



mixture

TIP - one reaction and isothermal conditions

mass balances

$$\partial_{\tau} \rho_{\alpha} + J'_{\alpha} = r_{\alpha}$$

diffusion fluxes

$$J_{\alpha} = -\frac{1}{T} \sum_{\beta=1}^{\nu-1} L_{\alpha\beta} [\mu_{\beta} - \mu_{\nu}]'$$

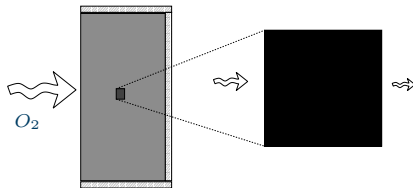
reaction kinetics

$$r_{\alpha} = \Lambda s_{\alpha} M_{\alpha}$$

chemical potentials

$$\begin{aligned} \mu_{\alpha} &= \mu_{\alpha}^0 + RT \ln a_{\alpha} \\ &= \mu_{\alpha}^0 + RT \ln \chi_{\alpha} \gamma_{\alpha} \end{aligned}$$

- 1 O_2
- 2 H_2O
- 3 aromatized (ARA*) with one S=O
- 4 aromatizable compounds with traces of sulfur (ARA)
 - resins
 - aromatics
 - asphaltenes
- 5 saturates

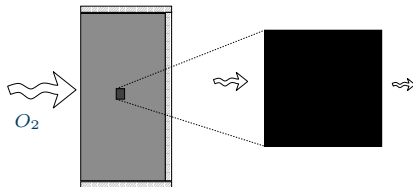


assumption:

every ARA member affected only once

⇒ average compounds

- 1 O_2
- 2 H_2O
- 3 aromatized (ARA*) with one S=O
- 4 aromatizable compounds with traces of sulfur (ARA)
 - resins
 - aromatics
 - asphaltenes
- 5 saturates



assumption:

every ARA member affected only once

⇒ average compounds

Typical elemental composition for heavy oil¹ (Gateau *et al.*, 2004).

Fraction	Weight percentage (%)	Elementary composition based on $C_{20} + (\%)$				
		C	H	N	O	S
Asphaltene	14.1	83.8	7.5	1.3	1.7	4.8
Resin	37.3	82.8	8.9	1.5	2.0	4.3
Aromatic	37.2	84.3	10.0	< 0.3	1.1	4.0
Saturate	11.4	86.6	13.0	< 0.3	< 0.2	< 0.1

¹ From Venezuela reserve.

stoichiometry



stoichiometric coefficients

$$s_1 = s_4 = -1, s_3 = s_2 = 1, s_5 = 0.$$

molar fraction relations

$$\chi_3 = \chi_2,$$

$$\chi_5 = \chi_5^0 [1 - \chi_1 - \chi_2],$$

$$\chi_4 = [1 - \chi_5^0][1 - \chi_1 - \chi_2] - \chi_2.$$

mass balances

$$\partial_\tau c_1 + \bar{J}'_1 = -\Lambda$$

$$\partial_\tau c_2 = \Lambda$$

- 1 O_2
- 2 H_2O
- 3 ARA^*
- 4 ARA
- 5 saturates

s_α	stoichiometric coeff's
χ_α	molar fraction $\frac{n_\alpha}{n_{\text{tot}}}$
c_α	molar concentrations $\frac{\rho_\alpha}{M_\alpha}$
M_α	molecular mass
Λ	reaction rate density $\frac{r_\alpha}{s_\alpha M_\alpha}$

Regular solution type activity model

[Vidal, 2003]

$$\ln \gamma_{\alpha} = \frac{v_{\alpha}^*}{RT} [\delta_{\alpha} - \bar{\delta}]^2$$

with

$$\bar{\delta} = \sum_{\alpha=1}^N \delta_{\alpha} \Phi_{\alpha}$$

$$\Phi_{\alpha} = \frac{v_{\alpha}^* \chi_{\alpha}}{\sum_{\beta=1}^N v_{\beta}^* \chi_{\beta}} = \frac{v_{\alpha}^* \chi_{\alpha}}{V^*}.$$

- 1 O_2
- 2 H_2O
- 3 ARA*
- 4 ARA
- 5 saturates

χ_{α}	molar fraction $\frac{n_{\alpha}}{n_{\text{tot}}}$
M_{α}	molecular mass
v_{α}^*	molar volume of pure substance in liquid state
δ_{α}	solubility parameter
Φ_{α}	volumetric fraction

diffusion part (constant T)

$$\begin{pmatrix} J_1 \\ \cancel{J_2} \\ \cancel{J_3} \\ \cancel{J_4} \\ \cancel{J_5} \end{pmatrix} = -\frac{1}{T} \begin{pmatrix} L_{11} & L_{12} & L_{13} & L_{14} \\ L_{21} & L_{22} & L_{23} & L_{24} \\ L_{31} & L_{32} & L_{33} & L_{34} \\ L_{41} & L_{42} & L_{43} & L_{44} \\ L_{51} & L_{52} & L_{53} & L_{54} \end{pmatrix} \cdot \begin{pmatrix} [\mu_1 - \mu_2]' \\ [\mu_3 - \mu_2]' \\ [\mu_4 - \mu_2]' \\ [\mu_5 - \mu_2]' \end{pmatrix}$$

$$[\mu_2 - \mu_5]_{,j} = \beta_1 [\mu_1 - \mu_5]'$$

$$[\mu_3 - \mu_5]_{,j} = \beta_2 [\mu_1 - \mu_5]'$$

$$[\mu_4 - \mu_5]_{,j} = \beta_3 [\mu_1 - \mu_5]'$$

$$J_1 = -\frac{L_{11}^*}{T} [\mu_1 - \mu_5]'$$

- | | |
|---|-----------|
| 1 | O_2 |
| 2 | H_2O |
| 3 | ARA* |
| 4 | ARA |
| 5 | saturates |

μ_α chemical potentials

\square' $\frac{\partial \square}{\partial x}$

L $\frac{RL_{11}^*}{\chi_1}$
(mobility)

$$A_{15} = \ln \frac{\gamma_1}{\gamma_5}$$

$$J_1 = -L \left\{ \left[1 + \frac{\chi_1}{1 - \chi_1 - \chi_2} + \chi_1 \frac{\partial A_{15}}{\partial \chi_1} \right] \chi_1' + \left[\frac{\chi_1}{1 - \chi_1 - \chi_2} + \chi_1 \frac{\partial A_{15}}{\partial \chi_2} \right] \chi_2' \right\}$$

diffusion part (constant T)

$$\begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \end{pmatrix} = -\frac{1}{T} \begin{pmatrix} L_{11} & L_{12} & L_{13} & L_{14} \\ L_{21} & L_{22} & L_{23} & L_{24} \\ L_{31} & L_{32} & L_{33} & L_{34} \\ L_{41} & L_{42} & L_{43} & L_{44} \\ L_{51} & L_{52} & L_{53} & L_{54} \end{pmatrix} \cdot \begin{pmatrix} [\mu_1 - \mu_2]' \\ [\mu_3 - \mu_2]' \\ [\mu_4 - \mu_2]' \\ [\mu_5 - \mu_2]' \end{pmatrix}$$

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$$[\mu_4 - \mu_5]_{,j} = \beta_3 [\mu_1 - \mu_5]'$$

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approximate Λ by a polynomial in activities [Pekar M. and Samohyl I., 2014]

$$\begin{aligned}\Lambda \approx P_{\Lambda} = B_0 &+ \frac{1}{2}B_{11}a_1^2 + B_{12}a_1a_2 + B_{13}a_1a_3 + B_{14}a_1a_4 + B_{15}a_1a_5 \\ &+ \frac{1}{2}B_{22}a_2^2 + B_{23}a_2a_3 + B_{24}a_2a_4 + B_{25}a_2a_5 \\ &+ \frac{1}{2}B_{33}a_3^2 + B_{34}a_3a_4 + B_{35}a_3a_5 \\ &+ \frac{1}{2}B_{44}a_4^2 + B_{45}a_4a_5 \\ &+ \frac{1}{2}B_{55}a_5^2\end{aligned}$$

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$$\begin{aligned}\Lambda \approx P_{\Lambda} = B_0 &+ \frac{1}{2}B_{11}a_1^2 + B_{12}a_1a_2 + B_{13}a_1a_3 + B_{14}a_1a_4 + B_{15}a_1a_5 \\ &+ \frac{1}{2}B_{22}a_2^2 + B_{23}a_2a_3 + B_{24}a_2a_4 + B_{25}a_2a_5 \\ &+ \frac{1}{2}B_{33}a_3^2 + B_{34}a_3a_4 + B_{35}a_3a_5 \\ &+ \frac{1}{2}B_{44}a_4^2 + B_{45}a_4a_5 \\ &+ \frac{1}{2}B_{55}a_5^2\end{aligned}$$

equilibrium

- affinity $A = - \sum_{\alpha=1}^N \mu_{\alpha} s_{\alpha}$ must vanish

$$\ln \left(\frac{a_1^{\ominus} a_4^{\ominus}}{a_2^{\ominus} a_3^{\ominus}} \right) = \frac{\mu_A^{\ominus}}{RT} \rightarrow K = \frac{a_1^{\ominus} a_4^{\ominus}}{a_2^{\ominus} a_3^{\ominus}}$$

- $\Lambda^{\ominus} = \Lambda(a_{\alpha} = a_{\alpha}^{\ominus})$ must vanish too

approximate Λ by a polynomial in activities [Pekar M. and Samohyl I., 2014]

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- $\Lambda^{\ominus} = \Lambda(a_{\alpha} = a_{\alpha}^{\ominus})$ must vanish too

$$\Lambda = B_{14}[a_1a_4 - Ka_2a_3]$$

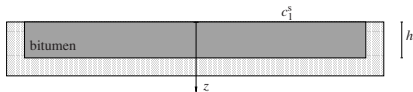
intermediate result:

$$\Lambda = B_{14} \left[\chi_1 \chi_4 \exp \left(\frac{v_1^*}{RT} [\delta_1 - \bar{\delta}]^2 + \frac{v_4^*}{RT} [\delta_4 - \bar{\delta}]^2 \right) - \chi_2 \chi_3 \exp \left(\frac{v_2^*}{RT} [\delta_2 - \bar{\delta}]^2 + \frac{v_3^*}{RT} [\delta_3 - \bar{\delta}]^2 \right) K \right]$$

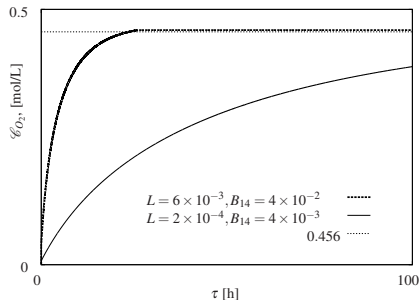
$$\Lambda = B_{14} \chi_1 \chi_4 \exp \left(\frac{v_1^*}{RT} [\delta_1 - \bar{\delta}]^2 + \frac{v_4^*}{RT} [\delta_4 - \bar{\delta}]^2 \right)$$

final result:

$$\Lambda = B_{14} \chi_1 \left[[1 - \chi_5^0][1 - \chi_1 - \chi_2] - \chi_2 \right] \exp \left(\frac{v_1^*}{RT} [\delta_1 - \bar{\delta}]^2 + \frac{v_4^*}{RT} [\delta_4 - \bar{\delta}]^2 \right)$$



$$\begin{aligned}
 h &= 0.1 \text{ cm} \\
 c_1^s &= 8.10 \times 10^{-7} \frac{\text{mol}}{\text{cm}^3} \\
 l_e &= 0.1/100 \\
 \Delta t_{\text{LB}} &= \frac{1}{6L} l_e^2 \\
 \chi_5^0 &= 0.35
 \end{aligned}$$



species		density $\frac{\text{g}}{\text{cm}^3}$	molecular weight $\frac{\text{g}}{\text{mol}}$	molar volume $\frac{\text{cm}^3}{\text{mol}}$	solubility	
					$\sqrt{\text{MPa}}$	$\sqrt{\frac{\text{g}}{\text{cm s}^2}}$
oxygen		1.114	32	28.0	14.0	44272.2
water		0.988	18	18.2	48.0	151790.4
ARA*				1709.0	20.0	63246.0
ARA	aromatics	1.006	440	437.0	21.0	
	resins	1.054	990	940.0	19.0	
	asphaltenes	1.200	4500	3750.0	20.0	
average				1709.0	20.0	63246.0
saturates		0.887	370	417.0	16.5	52177.9

$$\begin{aligned}\partial_\tau c_1 + \bar{J}'_1 &= -\Lambda \\ \partial_\tau c_2 &= \Lambda\end{aligned}$$

$$J_1 = -L \left\{ \left[1 + \frac{\chi_1}{1 - \chi_1 - \chi_2} + \chi_1 \frac{\partial A_{15}}{\partial \chi_1} \right] \chi'_1 + \left[\frac{\chi_1}{1 - \chi_1 - \chi_2} + \chi_1 \frac{\partial A_{15}}{\partial \chi_2} \right] \chi'_2 \right\}$$

$$\Lambda = B_{14} \chi_1 \left[[1 - \chi_5^0][1 - \chi_1 - \chi_2] - \chi_2 \right] \exp \left(\frac{v_1^*}{RT} [\delta_1 - \bar{\delta}]^2 + \frac{v_4^*}{RT} [\delta_4 - \bar{\delta}]^2 \right)$$

-
- only 2 purely phenomenological parameters (L, B_{14})
 - able to distinguish between different compositions
 - diffusion not significantly affected by the oxidation process
supports hypothesis of additional processes at molecular level



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A generalized regular solution model for asphaltene precipitation from n-alkane diluted heavy oils and bitumens.
Fluid Phase Equilibria, 232(1):159 – 170.



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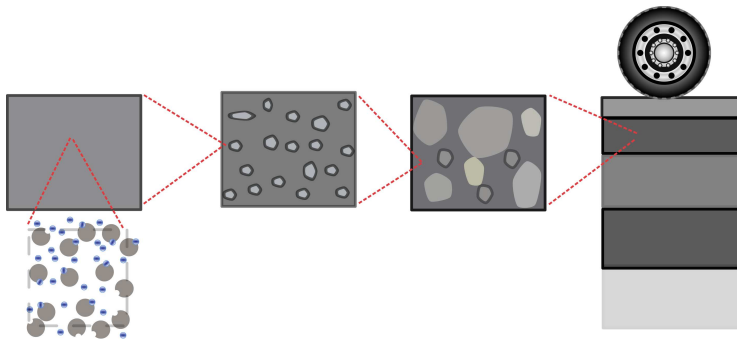
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Editions OPHRYS.



- second reaction $C=O$
- structure formation
- N_2 diffusion
- diffusion of rejuvenates
- mechanical part
- ...

- aging mortar
- diffusion rejuvenates
- mechanical properties

- aging asphalt
- diffusion rejuvenates
- mechanical properties