NASA Space Apps Challenge

Limassol, 2015

CHROMO SKY

Challenge: My Sky Color

ChromoSky

Introduction

This project aims to develop a tool that will allow anyone to make their contribution to the monitoring of the aerosol loading in the atmosphere, using the color of the sky as an indicator. The project consists of two parts: a desktop application for sky images processing and a website for data sharing. In the future it is planned to add the third part: a mobile app that will facilitate picture taking and will directly send the pictures to the processing application.

The aerosol loading of the atmosphere is measured from the difference between the actual color of the sky, obtained from the images, and the theoretically predicted color of the sky. Therefore the processing application works in two stages. On the first stage it uses the atmospheric data, current time and the user's location to predict the "normal" sky color. This is done through a mathematical model developed as a component of the project. On the second stage it processes the picture to obtain the actual color from its pixel data. This document summarizes both these stages and describes the mathematical model developed for the purpose of the project.

The position of the sun

The first factor affecting the color of the sky is the position of the sun. Hence, it is necessary to be able to predict its position on the sky in terms of its altitude (the angular height above the horizon) based on the observer's geographical coordinates (longitude and latitude) and the observer's time.

Since the solar position on the sky does not significantly change the sky color in short amounts of time (unless the sun is about to set or just rose) there is no need in high precision of its calculation. Therefore, we use the technique proposed by the **United States Naval Observatory**, guaranteeing the precision of about half a degree. We start by calculating the equatorial coordinates of the sun, using the following formulae (**equations 1 and 2**).

Equation 1	$RA = \tan^{-1}(\cos(\varepsilon) \times \tan[A + B \times n + C \times \sin(D + E \times n) + F \times \sin(2D + 2E \times n)])$
Equation 2	$\delta = \sin^{-1}(\sin(\varepsilon) \times \sin[A + B \times n + C \times \sin(D + E \times n) + F \times \sin(2D + 2E \times n)])$

RA and δ here refer to the right ascension and the declination of the sun (if the reader is not familiar with these concepts, she is advised to think of δ and RA simply as intermediate steps in the position calculation). n refers to the total number of days passed since 01/01/2000 12:00 UTC, expressed as a fraction. Other letters in the equations are empirically determined constants, given in the table below (note that all constants are given in degrees):

Constant	Value	Constant	Value	Constant	Value
A	280.460°	B	0.9856474°	C	1.915°
D	357.528°	E	0.9856003°	F	0.020°
ε	23.439281°	G	100.46°	H	0.985647°

The next step is to transform RA and δ into the altitude of the sun on the sky (θ), which is accomplished in the **equation 3** (λ and φ are the observer's latitude and longitude, while G and H are the constants from the table above; $\{x\}$ denotes the fractional part of x (e.g. $\{1.2\} = 0.2$)).

Equation 3	$\theta = \sin^{-1}(\sin(\delta)\sin(\lambda) + \cos(\delta)\cos(\lambda)\cos[G + Hn + \varphi + 360\{n + 0.5\} - RA])$

Atmospheric scattering

When light travels through a medium its photons can occasionally change their direction of motion – a phenomenon known as Rayleigh scattering. Due to the scattering of light in the atmosphere, an observer standing on the ground receives light not only from the Sun but from all directions in the sky. As will be shown later, blue light scatters more efficiently than other visible colors, which makes the sky blue. Nevertheless, other colors scatter as well, but to a lower extent determined by the composition of the atmosphere.

In order to detect an abnormality in the atmospheric color and, hence, an aerosol in the air, it is necessary to determine the "normal" color of the sky first. Therefore, we have to model the atmospheric scattering mathematically. For this project only the color of the sky in zenith (right above the observer) is important. If the sun is not in zenith itself, the scattering is necessary for this color to be observed. The **figure 1** demonstrates some possible paths of the photons "hitting" the observer:

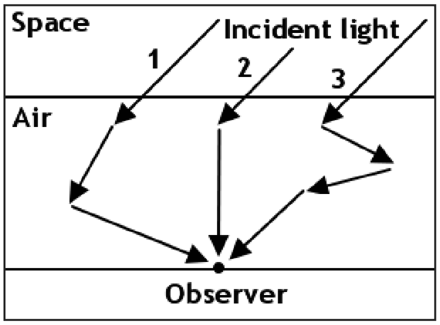

Figure 1. *Orders of scattering*

On the diagram the second ray underwent a single change in direction before it reached the observer. This type of scattering is called the first order scattering. The first ray had a second order scattering and the third ray a third order. On every scattering only a part of the original energy of the ray is inherited by the scattering ray, which implies that the second or third order scattered rays are expected to be much dimmer than the first order scattered rays. We use this fact for our advantage and ignore the second and the third order scatterings in the mathematical model to keep it simple.

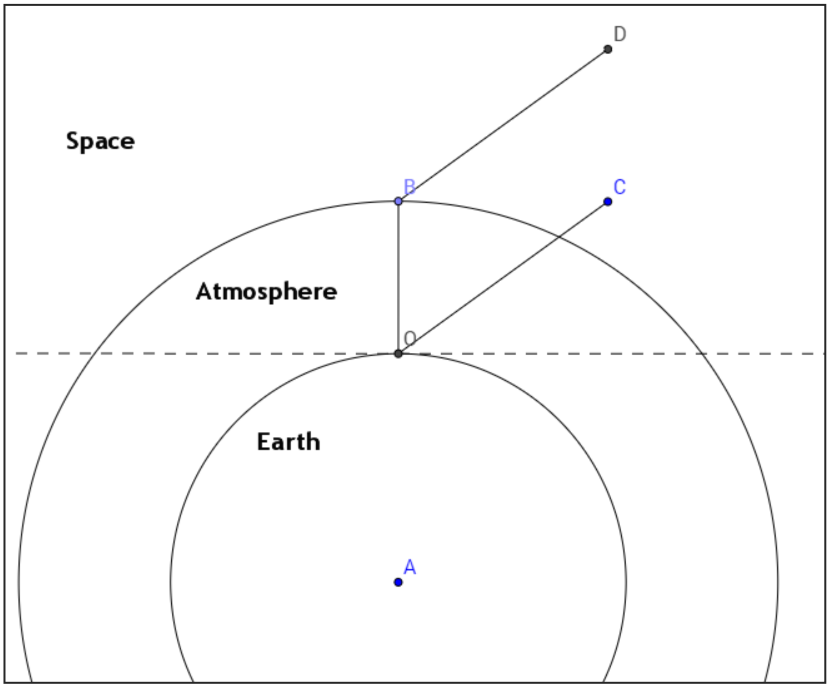

Figure 2. *Critical rays*

The **figure 2** demonstrates two rays **DB** and **CO** entering the atmosphere at the same angle to the ground. The paths of the rays inside the atmosphere on the diagram are only the extrapolations of their original directions and can be altered by scattering. The observer is located at the point **O**. Remember that both rays are allowed to be scattered only once. The ray **DB** must be scattered down immediately after it enters the atmosphere, since any scattering later, even if directed towards the observer, will not reach it from the zenith. The ray **CO** must remain unscattered until it reaches the point **O**, since any scattering before will not be detected by the observer as coming from the zenith, and after the point **O** it will hit the ground. Therefore, any ray responsible for the color of the zenith must have its atmospheric extrapolation going through a point on the line **BO**, which we name **M**. Note that this allows us to define any ray in terms of the distance **OM**.

To model the scattering mathematically it is necessary to know:

- The distance travelled by a ray before it got scattered (i.e. before it has reached the segment **BO**).
- The distance travelled by a ray while it is crossing the line **BO**. I.e. the distance on which it can be scattered.
- The distance travelled after scattering to the observer (**OM**).

Of course, the line **BO** is a mathematical construction and is infinitely thin, which makes the second distance zero. However, for our purposes it is necessary to define a range, within which the rays will still be assumed to be coming from the zenith. Please, refer to the **figure 3**.

In this case the incident ray is **CE**. We assume that this ray travels unscattered through the distance **FD**, which is the first distance required. Then it enters the region **DE** a scattering towards the observer in which would be considered arriving from the zenith. The region **DE** is formed by the angles **BOD** (where **O** is the observer) and **BOE**. Both angles are identical and equal to α, where α is an arbitrary **range** constant. The angle the ray makes with the ground is θ (**CMG**). The distance **DE** is the second distance required. The radius of the Earth with the atmosphere is R (**AB**), the radius of the Earth without the atmosphere is r (**AO**) and the distance **OM**, defining the ray, is equal to L.

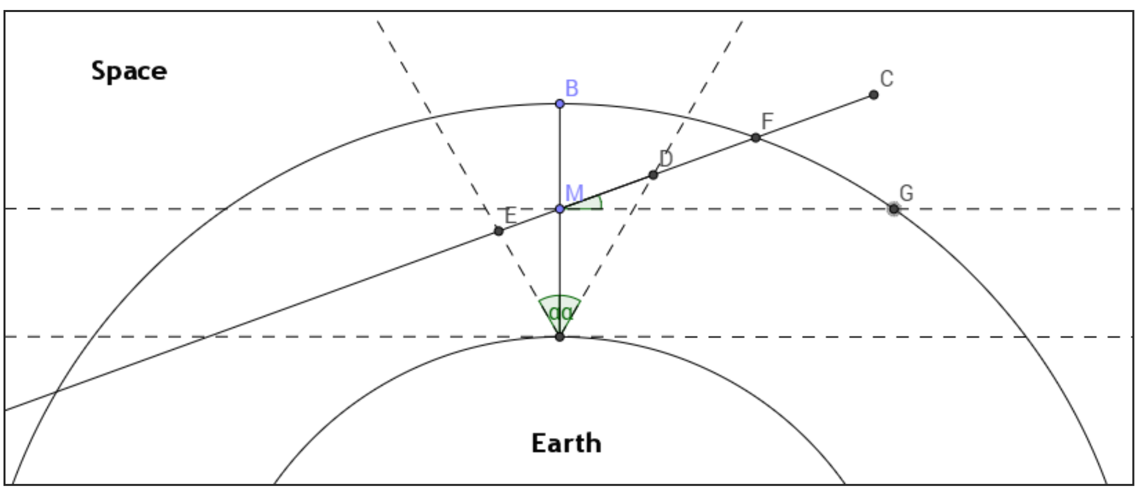

Figure 3. *The geometry of an incident ray (this figure is an addition to the **figure 2**).*

The distances required can be found using trigonometry as follows (the derivation is trivial and omitted).

Equation 4	$$\mathbf{FD} = R \times \cos\left(\theta + \sin^{-1}\left(\frac{(L+r)\cos\theta}{R}\right)\right) \times \sec(\theta) - \frac{L \times \sin(\alpha)}{\sin\left(\frac{\pi}{2} - \theta - \alpha\right)}$$
Equation 5	$$\mathbf{DE} = L \times \left[\csc\left(\frac{\pi}{2} - \alpha - \theta\right) + \sec(\alpha - \theta)\right] \times \sin(\alpha)$$

Finally, the third distance required is, on average, equal to **OM** if we assume the light to be scattered somewhere near **M**. This approximation is expected to be better as the range α is decreasing.

Atmospheric density

The next important feature in the mathematical model is the density of the atmosphere, since it affects the number of air molecules that can possibly scatter the photons of light. We are interested in three density values: the one between the points **D** and **F**, the one between the points **D** and **E** and the one between the points **O** and **M**. The density of the atmosphere depends on the altitude; hence, it is important to find the altitudes of the points **F**, **D**, **E** and **M** above the ground. The altitude of **M** is L by definition. Other altitudes can be computed using trigonometry to find **AF**, **AD** and **AE** and then subtracting r from the lengths of these segments. This is demonstrated below ($A(\mathbf{X})$ refers to the altitude of the point **X**). The derivation is trivial and will not be given.

Equation 6	$A(\mathbf{F}) = R - r$
Equation 7	$A(\mathbf{D}) = \sqrt{(L+r)^2 + \sec(\alpha+\theta)\sin(\alpha)\, L[L \times \sec(\alpha+\theta)\sin(\alpha) + 2(L+r)\sin(\theta)]} - r$
Equation 8	$A(\mathbf{E}) = \sqrt{(L+r)[r + L \times \cos(\alpha+\theta)\sec(\alpha-\theta)] + L^2 \times (\sec(\alpha-\theta))^2 \times \sin(\alpha)^2} - r$

When the altitudes are know the next step is to relate them to particular density values. Atmospheric pressure – the main determining factor – generally drops with altitude with the rate dependent on the gradient of temperature. The temperature of air nearly perfectly drops by $0.0065°$ per every meter of altitude up to the altitude of, approximately, 15 km, known as the troposphere. Then the temperature rises again for the next 20 km in a much less predictable fashion, defining the second layer of the atmosphere named stratosphere. Finally it starts to drop again in the mesosphere. It is apparent that an accurate model of the atmosphere would be extremely complicated. For the purposes of this project we used **NRLMSISE-00** model provided by NASA to output a table with atmospheric densities for each 10 meters of the altitude. Some values provided are given in the **Appendix A**. In order to find the average air densities in the regions **FD**, **DE** and **MO** we use the **equation 9**, where **X** and **Y** are arbitrary chosen points (e.g. **F** and **D**), $\langle\rho(\mathbf{XY})\rangle$ is the average air density in the region **XY** and $N_{00}(i)$ is the i^{th} value from the table in the **NRLMSISE-00**.

Equation 9	$\langle\rho(\mathbf{XY})\rangle = \dfrac{10}{\lvert A(\mathbf{Y}) - A(\mathbf{X})\rvert} \sum_{i=0.1A(\mathbf{X})}^{0.1A(\mathbf{Y})} N_{00}(i)$

Intensity reduction

While a light ray travels in the region **FD**, any scatterings will not affect the color of the sky in zenith. However, they will reduce the intensity of the ray, which is an effect necessary to be taken into account. The probability of a photon of light undergoing a scattering is given by the **equation 10**. Here λ is the wavelength of the photon, σ is the so called **scattering cross-section**, measured in m^2 and numerically equal to the probability of scattering per unit length travelled in a unit volume and ϕ is a constant for each gas, which depends on the diameter of its molecules and its refractive index.

Equation 10	$\sigma = \dfrac{\phi}{\lambda^4}$

The **equation 11**, hence, gives the fraction of light scattered per unit length travelled (s), where N is the number of scattering air molecules per unit volume in the medium that provides the scattering.

Equation 11	$$s = \sigma \times N = \frac{\phi}{\lambda^4} \times N$$

The number N will depend on three quantities: the density of air, which is given by the **equation 9**, the molar mass of the air (M_r) and the **Avogadro's constant** N_A, which is equal to 6.0221×10^{23}. Since air is not a single substance but rather a mixture of different gases, it is necessary to know the values of ϕ, M_r and the percentage abundance of each. The table below gives these details, as given by **VU University Amsterdam** and the **National Institute of Standards and Tech**.

Gas	**Scattering Constant** (ϕ)	**Molar mass** (M_r)	**Percentage abundance** (F)
Nitrogen (N_2)	4.09×10^{-56} m^6	28.02×10^{-3} kg mol^{-1}	75.47%
Oxygen (O_2)	3.26×10^{-53} m^6	32.00×10^{-3} kg mol^{-1}	23.20%
Carbon Dioxide (CO_2)	9.93×10^{-56} m^6	44.01×10^{-3} kg mol^{-1}	0.046%
Argon (Ar)	3.56×10^{-56} m^6	39.94×10^{-3} kg mol^{-1}	1.28%

Taking these values into account we rewrite the **equation 11** as the **equation 12**. The summation is used to add the values for each gas in the table above.

Equation 12	$$s = \langle \rho(\mathbf{FD}) \rangle \times N_A \times \sum_{\text{gases}} \left(\frac{\phi \times F}{\lambda^4 \times M_r} \right)$$

The total fractional remainder of intensity of light ($S_{\mathbf{FD}\ \text{Rem}}$) after the region **FD** due to scattering is, therefore (notice that we subtract the **equation 12** from one to get the reminder) :

Equation 13	$$S_{\mathbf{FD}\ \text{Rem}} = \left(1 - \langle \rho(\mathbf{FD}) \rangle \times N_A \times \sum_{\text{gases}} \left(\frac{\phi \times F}{\lambda^4 \times M_r} \right) \right)^{\mathbf{FD}}$$

Zenith scattering

The amount of light scattered in the region **DE** can be found using the **equation 13** as well; however, in this region we are interested only in the light scattered downwards towards the observer and not anywhere else. The amount of light scattered at a particular angle φ to its original direction of motion is proportional to $1 + \cos^2(\varphi)$. In order to reach the observer φ must be equal to the angle **DMO** ($\theta + \frac{\pi}{2}$) on the **figure 3**, accepting deviations within α. Let the total intensity of light that was scattered in **DE** at the angle φ be equal to I_{total}, which is a function of φ and λ. From the proportionality stated above I_{total} can be rewritten as $I_{\text{undir}}(\lambda)(1 + \cos^2(\varphi))$, where $I_{\text{undir}}(\lambda)$ is a function of λ only. The total amount of light scattered in all directions (I_{scatt}) is then given by the **equation 14**.

Equation 14	$$I_{\text{scatt}} = \int_0^{2\pi} I_{\text{undir}}(\lambda)(1 + \cos^2(\varphi))d\varphi = I_{\text{undir}}(\lambda) \times 3\pi$$

The amount of light scattered at a correct angle ($I_{\downarrow}$) is then given by the **equation 15**.

Equation 15	$I_{\downarrow} = \int_{\theta+\frac{\pi}{2}-\alpha}^{\theta+\frac{\pi}{2}+\alpha} (1+\cos^2(\varphi)) I(\lambda) d\varphi = I_{\text{undir}}(\lambda)(3\alpha - \cos(2^c)\cos(\alpha)\sin(\alpha))$ $\therefore I_{\downarrow} = \frac{I_{\text{scatt}}}{3\pi}(3\alpha - \cos(2)\cos(\alpha)\sin(\alpha))$

Note that if light is scattered at a correct angle, it doesn't immediately imply that it is directed downwards. It can be equally well directed upwards, into the page (**figure 3**), out of the page or anywhere in between. The probability of each direction in this case is identical, since they are equal from the perspective of the light ray. Therefore, $I_{\downarrow}$ should be further multiplied by $\frac{\alpha}{\pi}$ to obtain the total amount of light directed downwards within the deviation of α ($\frac{2\alpha}{2\pi} = \frac{\alpha}{\pi}$). The total fractional amount of light that remains after the scattering in the region **DE** in the main ray (the amount that reaches the point **E**) is given by the **equation 13** with **DE** instead of **FD**. The total fractional amount of scattered light is, therefore, one minus that value. Substitution to the **equation 15** and conversion of the absolute intensity $I_{\downarrow}$ into fractional intensity $S_{\mathbf{DE}\ \text{Scattered}}$ yields the **equation 16**.

Equation 16	$S_{\mathbf{DE}\ \text{Scattered}} = \left(1 - \left(1 - \langle\rho(\mathbf{DE})\rangle N_A \sum_{\text{gases}} \left(\frac{\phi \times F}{\lambda^4 \times M_r}\right)\right)^{\mathbf{DE}}\right) \frac{\alpha(3\alpha - \cos(2)\cos(\alpha)\sin(\alpha))}{3\pi^2}$

Finally, the ray has to make its path down to the observer (**OM**), where it will lose intensity equally to the region **FD**. In other words, the **equation 17** holds.

Equation 17	$S_{\mathbf{OM}\ \text{Rem}} = \left(1 - \langle\rho(\mathbf{OM})\rangle \times N_A \times \sum_{\text{gases}} \left(\frac{\phi \times F}{\lambda^4 \times M_r}\right)\right)^{\mathbf{OM}}$

To find the total fraction of light intensity in the ray that reaches the observer from the zenith (Ω) is obtained by combining the **equations 17, 16 and 13** as in the **equation 18**.

Equation 18	$\Omega(\lambda, L) = S_{\mathbf{FD}\ \text{Rem}} \times S_{\mathbf{DE}\ \text{Scattered}} \times S_{\mathbf{OM}\ \text{Rem}}$

Ω here is a function of two variables: the wavelength of light λ and the L distance.

Integration of rays

The **equation 18** is applicable to a single ray of light extrapolated through the point **M**. However, the total light intensity observed from the zenith is composed of all the possible rays. Hence, it is necessary to integrate the function $\Omega(\lambda, L)$ with respect to L in the region from $L = 0$ to $L = R - r$. For obvious reasons this integration is difficult to compute exactly, so we use the **Riemann Sum** approximation (**equation 19**, where Λ is the total observed intensity).

Equation 19	$\Lambda(\lambda) = \int_0^{\mathrm{R-r}} \Omega(\lambda, L)\, dL \approx d \times \sum_{i=0}^{\frac{R-r}{d}} \Omega(\lambda, i \times d)$

Here d is an arbitrary constant, which is preferred to be as small as possible for a higher precision. In our project we use the d value of 10 m.

Conversion to colors

The **equation 19** can be used to obtain the spectrum of light coming from the zenith. In order to do this $\Lambda(\lambda)$ must be multiplied with $\Phi(\lambda)$, where $\Phi(\lambda)$ is the intensity function of the solar spectrum. For the function $\Phi(\lambda)$ we use the table provided by the **American Society for Testing and Materials**, which is given in the **Appendix B**. Finally, when the spectrum $\Lambda(\lambda) \times \Phi(\lambda)$ is known we have to convert it to an RGB-color. This can be done as follows.

Firstly we convert $\Lambda(\lambda) \times \Phi(\lambda)$ into three numerical values X, Y and Z according to the **equations 20-22**.

Equation 20	$\mathrm{X} = 10^{-9} \sum_{i=35}^{70} \left(\Lambda(i \times 10^{-8}) \times \Phi(i \times 10^{-8}) \times \mathrm{CIE}_x(i \times 10^{-8})\right)$
Equation 21	$\mathrm{Y} = 10^{-9} \sum_{i=35}^{70} \left(\Lambda(i \times 10^{-8}) \times \Phi(i \times 10^{-8}) \times \mathrm{CIE}_y(i \times 10^{-8})\right)$
Equation 22	$\mathrm{Z} = 10^{-9} \sum_{i=35}^{70} \left(\Lambda(i \times 10^{-8}) \times \Phi(i \times 10^{-8}) \times \mathrm{CIE}_z(i \times 10^{-8})\right)$

Here we sum the zenith spectrum with the CIE function for every 10 nm of wavelength in the visible spectrum (from 350 nm to 700 nm). The CIE function refers to the sensitivity of various parts of a human eye (or a camera simulating a human eye) to various wavelength and its tabulated values are given in the **Appendix C**.

With these values we then calculate the relative proportions of the RGB values of the color, using the empirically derived **equation 23**.

Equation 23	$\begin{bmatrix} \mathrm{R} \\ \mathrm{G} \\ \mathrm{B} \end{bmatrix} = \begin{bmatrix} 3.240479 & -1.537150 & -0.498535 \\ -0.969256 & 1.875992 & 0.041556 \\ 0.055648 & -0.204043 & 1.057311 \end{bmatrix} \cdot \begin{bmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \end{bmatrix}$

The RGB values of the zenith sky color must be in the same ratio as R, G and B. The coefficient of proportion depends on each particular camera and its settings and can barely be predicted in general. However, the relative luminosity of the color for comparison purposes is numerically equal to Y.

Appendix A: NRLMSISE-00 atmospheric density model

Altitude [km]	Density [g cm^{-3}]	Altitude	Density	Altitude	Density
0	1.23×10^{-03}	34	9.63×10^{-06}	68	1.02×10^{-07}
1	1.11×10^{-03}	35	8.26×10^{-06}	69	8.80×10^{-08}
2	9.99×10^{-04}	36	7.10×10^{-06}	70	7.54×10^{-08}
3	9.05×10^{-04}	37	6.12×10^{-06}	71	6.45×10^{-08}
4	8.19×10^{-04}	38	5.28×10^{-06}	72	5.50×10^{-08}
5	7.41×10^{-04}	39	4.56×10^{-06}	73	4.69×10^{-08}
6	6.68×10^{-04}	40	3.95×10^{-06}	74	3.99×10^{-08}
7	6.00×10^{-04}	41	3.43×10^{-06}	75	3.40×10^{-08}
8	5.37×10^{-04}	42	2.99×10^{-06}	76	2.89×10^{-08}
9	4.77×10^{-04}	43	2.61×10^{-06}	77	2.46×10^{-08}
10	4.22×10^{-04}	44	2.28×10^{-06}	78	2.10×10^{-08}
11	3.71×10^{-04}	45	2.00×10^{-06}	79	1.78×10^{-08}
12	3.24×10^{-04}	46	1.76×10^{-06}	80	1.52×10^{-08}
13	2.81×10^{-04}	47	1.55×10^{-06}	81	1.29×10^{-08}
14	2.42×10^{-04}	48	1.37×10^{-06}	82	1.10×10^{-08}
15	2.08×10^{-04}	49	1.21×10^{-06}	83	9.31×10^{-09}
16	1.77×10^{-04}	50	1.08×10^{-06}	84	7.91×10^{-09}
17	1.50×10^{-04}	51	9.52×10^{-07}	85	6.72×10^{-09}
18	1.27×10^{-04}	52	8.44×10^{-07}	86	5.70×10^{-09}
19	1.07×10^{-04}	53	7.48×10^{-07}	87	4.83×10^{-09}
20	9.08×10^{-05}	54	6.63×10^{-07}	88	4.09×10^{-09}
21	7.68×10^{-05}	55	5.87×10^{-07}	89	3.47×10^{-09}
22	6.51×10^{-05}	56	5.20×10^{-07}	90	2.93×10^{-09}
23	5.52×10^{-05}	57	4.59×10^{-07}	91	2.48×10^{-09}
24	4.68×10^{-05}	58	4.05×10^{-07}	92	2.09×10^{-09}
25	3.98×10^{-05}	59	3.57×10^{-07}	93	1.76×10^{-09}
26	3.39×10^{-05}	60	3.14×10^{-07}	94	1.48×10^{-09}
27	2.88×10^{-05}	61	2.75×10^{-07}	95	1.24×10^{-09}
28	2.46×10^{-05}	62	2.41×10^{-07}	96	1.04×10^{-09}
29	2.10×10^{-05}	63	2.10×10^{-07}	97	8.66×10^{-10}
30	1.79×10^{-05}	64	1.83×10^{-07}	98	7.20×10^{-10}
31	1.53×10^{-05}	65	1.59×10^{-07}	99	5.97×10^{-10}
32	1.31×10^{-05}	66	1.38×10^{-07}	100	4.94×10^{-10}

Source: NASA / Community Coordinated Modeling Center

URL: http://ccmc.gsfc.nasa.gov/modelweb/models/nrlmsise00.php

Appendix B: Solar spectral radiation

Wavelength interval [nm]	Radiation intensity [W m^{-2}]
350-360	0.3370545
360-370	0.4253465
370-370	0.4752565
380-370	0.454778
390-370	0.553228
400-370	0.882604
410-370	0.932786
420-370	0.953269
430-370	0.950387
440-370	1.16698
450-370	1.28424
460-370	1.31081
470-370	1.33704
480-370	1.31148
490-370	1.35766
500-370	1.3401
510-370	1.30398
520-370	1.34282
530-370	1.36797
540-370	1.34862
550-370	1.35898
560-370	1.34258
570-370	1.32771
580-370	1.34258
590-370	1.30222
600-370	1.32862
610-370	1.31166
620-370	1.28514
630-370	1.29299
640-370	1.28701
650-370	1.22976
670-370	1.2733
680-370	1.27295
690-370	1.167955
700-310	1.15111

Source: American Society for Testing and Materials

URL: rredc.nrel.gov/solar/spectra/am1.5/

Appendix C: CIE Color Matching Table

Wavelength range [nm]	X-function value	Y-function value	Z-function value
350-360	3.77×10^{-03}	≈0	≈0
360-370	1.12×10^{-04}	≈0	7.40×10^{-04}
370-380	6.50×10^{-04}	≈0	3.70×10^{-03}
380-390	3.77×10^{-03}	4.15×10^{-04}	1.85×10^{-02}
390-400	2.21×10^{-02}	2.45×10^{-03}	1.10×10^{-01}
400-410	8.95×10^{-02}	9.08×10^{-03}	4.51×10^{-01}
410-420	2.04×10^{-01}	2.03×10^{-02}	$1.05\times10^{+00}$
420-430	2.92×10^{-01}	3.32×10^{-02}	$1.55\times10^{+00}$
430-440	3.48×10^{-01}	5.03×10^{-02}	$1.92\times10^{+00}$
440-450	3.22×10^{-01}	6.47×10^{-02}	$1.85\times10^{+00}$
450-460	2.49×10^{-01}	8.51×10^{-02}	$1.52\times10^{+00}$
460-470	1.81×10^{-01}	1.30×10^{-01}	$1.25\times10^{+00}$
470-480	8.18×10^{-02}	1.79×10^{-01}	7.55×10^{-01}
480-490	2.08×10^{-02}	2.38×10^{-01}	4.10×10^{-01}
490-500	2.46×10^{-03}	3.48×10^{-01}	2.38×10^{-01}
500-510	1.56×10^{-02}	5.20×10^{-01}	1.18×10^{-01}
510-520	7.96×10^{-02}	7.18×10^{-01}	5.65×10^{-02}
520-530	1.82×10^{-01}	8.58×10^{-01}	2.44×10^{-02}
530-540	3.10×10^{-01}	9.54×10^{-01}	9.85×10^{-03}
540-550	4.49×10^{-01}	9.89×10^{-01}	3.79×10^{-03}
550-560	6.13×10^{-01}	9.97×10^{-01}	1.43×10^{-03}
560-570	7.97×10^{-01}	9.73×10^{-01}	5.45×10^{-04}
570-580	9.64×10^{-01}	8.96×10^{-01}	2.12×10^{-04}
580-590	1.11	8.12×10^{-01}	8.49×10^{-05}
590-600	1.15	6.92×10^{-01}	3.55×10^{-05}
600-610	1.08	5.58×10^{-01}	1.55×10^{-05}
610-620	9.14×10^{-01}	4.23×10^{-01}	≈0
620-630	6.92×10^{-01}	2.98×10^{-01}	≈0
630-640	4.73×10^{-01}	1.94×10^{-01}	≈0
640-650	3.00×10^{-01}	1.19×10^{-01}	≈0
650-660	1.71×10^{-01}	6.67×10^{-02}	≈0
660-670	9.22×10^{-02}	3.56×10^{-02}	≈0
670-680	4.71×10^{-02}	1.81×10^{-02}	≈0
680-690	2.26×10^{-02}	8.66×10^{-03}	≈0
690-700	1.10×10^{-02}	4.20×10^{-03}	≈0

Source: University College London / Color and Vision Research Laboratory

URL: http://cvrl.ioo.ucl.ac.uk/cmfs.htm