Physics 170:
Statistical Mechanics and Thermodynamics

Lecture 10B

Administrative Notes

- **Final exam:**
  Wednesday 12/13, 3:30-6:30 pm in **Hewlett 102**

- You will be allowed 1 sheet of hand-written notes

- Review problems & 2016 exam on *Canvas*

- **My office hours:** Monday, 12:15 - 1:30 pm

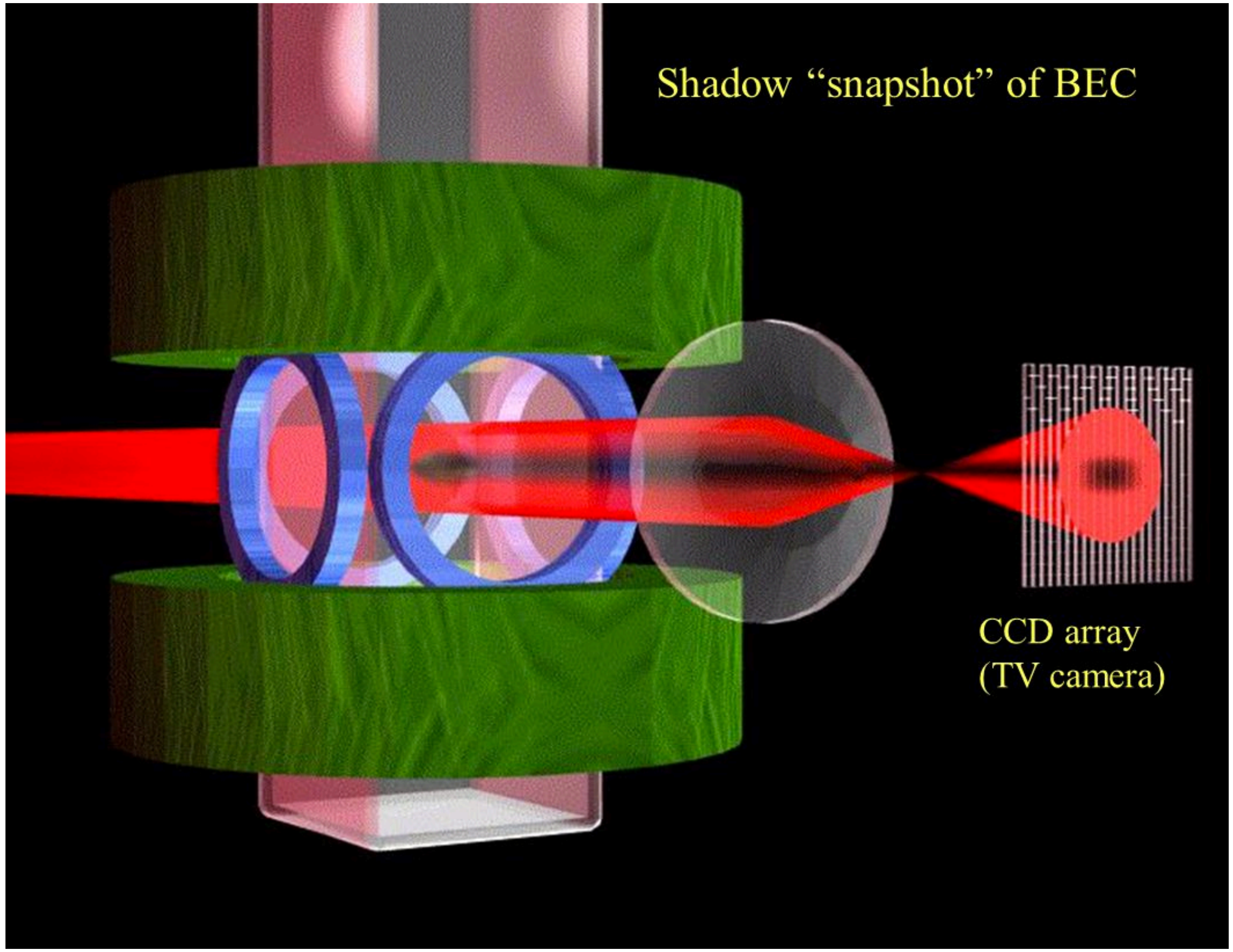
Today

- **Wrap up:** how can we prepare and detect ultra-cold quantum gases in the lab?

  Practice applying what you've learned in 170 outside the context of a textbook chapter.

- Consolidation of what we've learned in 170 (+ brief preview of 171)

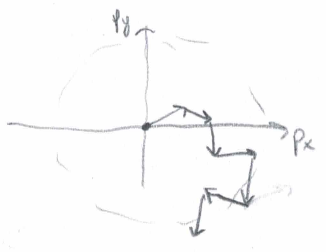
Detecting a BEC



Does the imaging light perturb the atoms?

c. *Recoil heating.* Does the light used for imaging appreciably perturb the atoms? Suppose that each atom absorbs on average $\langle N_p \rangle$ photons during the imaging process and reemits each one in a random direction. Correspondingly, each photon gives the atom a momentum kick of magnitude $\hbar k$ in a random direction, where k is the wavenumber of the light.

i. Estimate the average kinetic energy imparted to each atom by the random photon recoils, by modeling the process as a random walk in momentum space. Express your result in terms of $\langle N_p \rangle$, k, and the atomic mass m.



Radius $\sim \sqrt{N}\hbar k$

$$\langle p_x^2 + p_y^2 + p_z^2 \rangle = \hbar^2 k^2 \Longrightarrow \vec{p}_f = \vec{p}_0 + \sum_{i=1}^{N} \vec{p}_i$$

Total added momentum

$$\vec{P} = \sum_{i=1}^{N} \vec{p}_i \quad \text{(uncorrelated w/ } \vec{p}_0\text{)}$$

Total added kinetic energy

$$\frac{\langle P^2 \rangle}{2m} = \left\langle \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\vec{p}_i \cdot \vec{p}_j}{2m} \right\rangle = \left\langle \sum_{i=1}^{N} \frac{|\vec{p}_i|^2}{2m} \right\rangle = \frac{N\hbar^2 k^2}{2m}$$

p_i, p_j uncorrelated

ii. How does kinetic energy E_{rec} imparted by a *single* photon compare with the critical temperature for Bose-Einstein condensation? Give an estimate for the ratio τ_E / E_{rec} in terms of the atomic density n. Don't worry about numerical factors.

$$\frac{\tau_E}{E_{rec}} \sim \frac{n^{2/3}\hbar^2/m}{\hbar^2 k^2/m} \sim n^{2/3}/k^2$$

[or $n^{2/3}\lambda^2$, where λ = imaging wavelength]

iii. Consider a condensate of ^{23}Na atoms formed at a density of 10^{14} cm^{-3}. The atoms are imaged using an absorption line of wavelength $\lambda = 2\pi/k = 589$ nm. What is the ratio $\tau_E/E_{\rm rec}$ for these parameters?

$$\frac{2\pi}{k} \sim 600\,\text{nm} \Rightarrow k \sim \frac{2\pi}{6\times10^{-7}\,\text{m}} \approx 10^7/\text{m}$$

$$n = 10^{14}\,\text{cm}^{-3} = 10^{20}\,\text{m}^{-3} \Rightarrow n^{2/3} \approx n^{13.3} \sim 2\times10^{13}$$

$$\Rightarrow \frac{\tau_E}{E_{rec}} \sim \frac{n^{2/3}}{k^2} \sim \frac{2\times10^{13}}{10^{14}} \sim 0.2$$

iv. *Optional.* Find T_E and $T_{\rm rec} = E_{\rm rec}/k_B$ in units of Kelvin.

$$\lambda_E = \left(\frac{2.6}{n}\right)^{1/3} = 300\,\text{nm} = \frac{h}{\sqrt{2\pi m \tau_E}} \quad , \quad m_{Na} = 23\, m_p$$

$$\Rightarrow T_E = \tau_E / k_B = 1.5\,\mu K$$

$$T_{rec} = \frac{1}{k_B} \frac{\hbar^2 \left[\frac{2\pi}{589\,\text{nm}}\right]^2}{2 m_{Na}} = 1.2\,\mu K$$

$T_{rec} \sim T_E$ when careful about numerical factors

v. Based on your analysis, does the imaging light appreciably perturb the atoms? Will the recoil heating prevent you from obtaining an accurate image of the ultracold quantum gas? Why or why not?

* Yes: the imaging light heats the atoms by much more than τ_E
* We can still obtain an accurate image as long as we do it quickly: let atoms expand for long time t, then send <u>short</u> pulse of light to measure instantaneous position.
* Then need to make a new BEC for the next experiment.

3. *Ultracold atoms in a room-temperature apparatus.* Remarkably, by tricks of laser cooling and evaporative cooling, we can bring a small cloud of atoms to a temperature in the nanokelvin regime using a room-temperature apparatus.

   a. What precautions must we take to prevent the atoms from thermalizing with their surroundings?

   * Ultra-high vacuum to avoid collisions with any background gas molecules

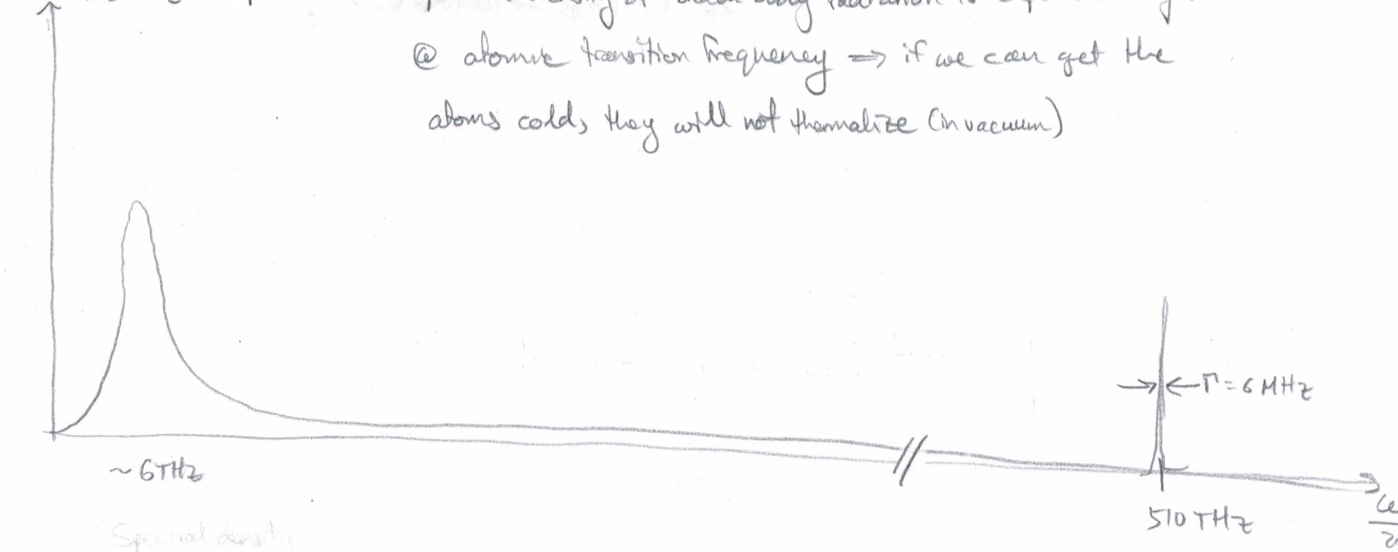
   * Avoid thermalization by absorption/emission of radiation . . . ??

   * Levitate magnetically or optically s.t. atoms are not touching room-temperature walls of vacuum chamber

   b. *Why aren't the atoms heated by blackbody radiation?* Consider the ^{23}Na spectral line at $\lambda = 589$ nm, corresponding to a frequency $\omega = 2\pi c/\lambda = 2\pi \times 510$ THz. The frequency width of this line is $\Gamma = 2\pi \times 10$ MHz. Sketch it on a single plot together with the blackbody radiation spectrum at room temperature. (You may use a broken axis.)

Room temperature: $\frac{\tau}{\hbar} = \frac{k_B(300K)}{\hbar} = \frac{(1.38\times10^{-23} J/K)(300K)}{1.05\times10^{-34} J\cdot s} \sim 4\times10^{13} s^{-1} \sim 2\pi \times 6 THz$

$u_\omega = \frac{\hbar\omega^3}{\pi^2 c^3} \times \frac{1}{e^{\hbar\omega/\tau}-1}$ $\Rightarrow$ spectral density of black-body radiation is exponentially small @ atomic transition frequency $\Rightarrow$ if we can get the atoms cold, they will not thermalize (in vacuum)



(order-of-magnitude)

c. Estimate the rate at which a sodium atom absorbs blackbody radiation on the $\lambda = 589$ nm spectral line, in units of photons/s. Assume that the atom has an effective cross-sectional area λ^2.

   i. What is the spectral density of blackbody radiation at the frequency $\omega = hc/\lambda$ of the atomic transition?
   ii. How is the radiation flux through a surface of area λ^2 related to the spectral density?
   iii. Within the spectral bandwidth Γ of the atomic transition, how many photons per second are incident on the surface of area λ^2?

(i) Recall DoS $\frac{\mathcal{D}(\omega)}{V} = \frac{\omega^2}{\pi^2 c^3}$

$$\Rightarrow \frac{u_\omega}{\hbar\omega} = \frac{\omega^2}{\pi^2 c^3} \frac{1}{e^{\hbar\omega/\tau} - 1}$$ [photons / bandwidth / volume]

(ii) Flux density $j_\omega \sim c \frac{u_\omega}{\hbar\omega} = \frac{\omega^2}{\pi^2 c^2} e^{-\hbar\omega/\tau}$

$\omega = \frac{2\pi c}{\lambda}$

(iii) Photon rate $\approx \Gamma \frac{j_\omega}{\hbar\omega} \times \lambda^2 \sim \frac{\Gamma \omega^2 \lambda^2 e^{-\hbar\omega/\tau}}{\pi^2 c^2} \sim e^{-\hbar\omega/\tau} \cdot \Gamma$

$$= e^{-(510\,\text{THz}/6\,\text{THz})} \times 2\pi \times 10\ \text{MHz}$$

$$= 1.2 \times 10^{-37} \times 6 \times 10^{7} \approx \boxed{10^{-29}/\text{s}}$$

$\ll$ 1/(Age of universe) !

. . . So if we can get atoms cold they will stay cold

— but how to get them cold in the first place?

① Laser cooling

∟ Low entropy: well defined frequency & phase

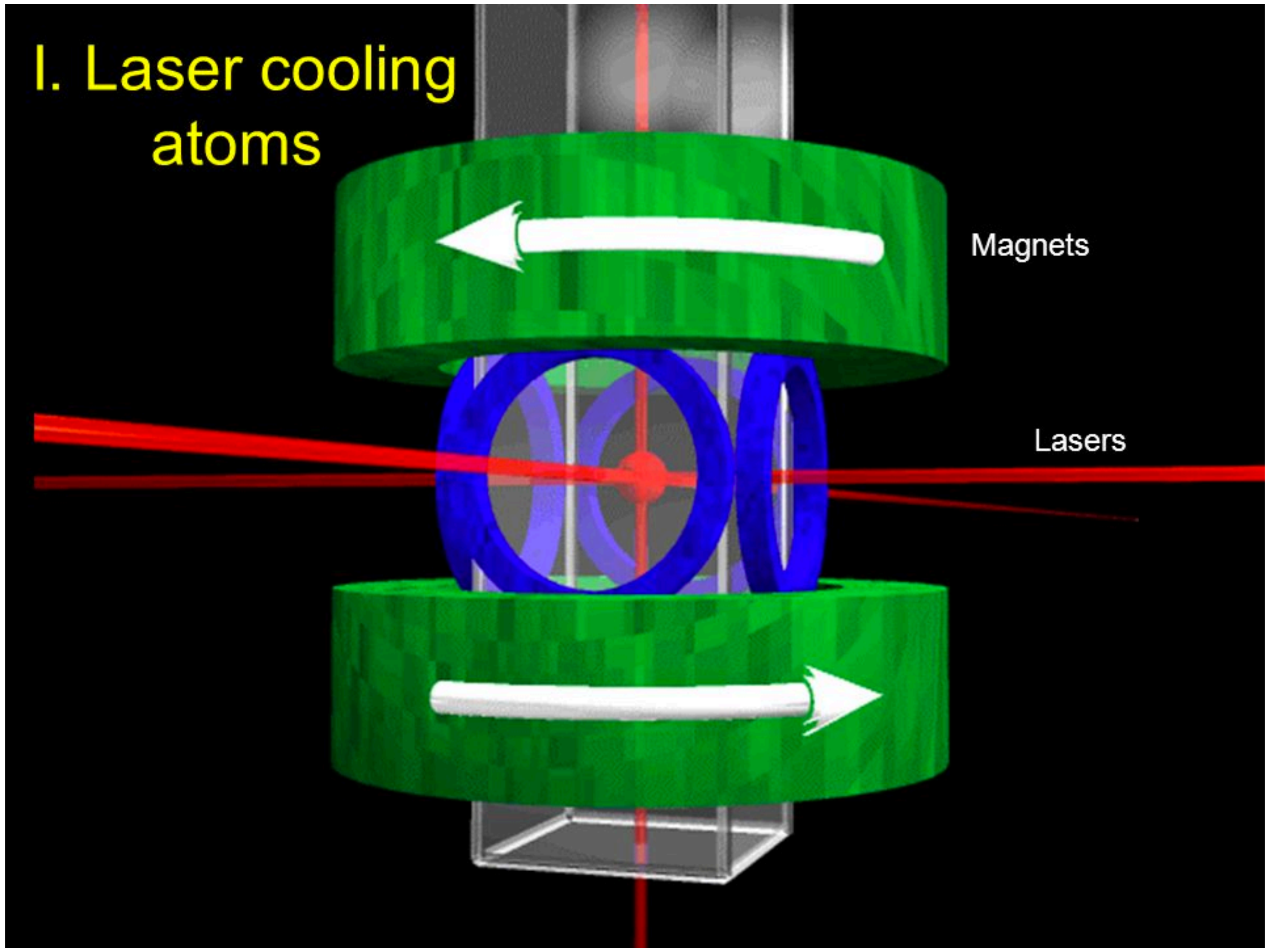
Dump entropy into spontaneously emitted photons

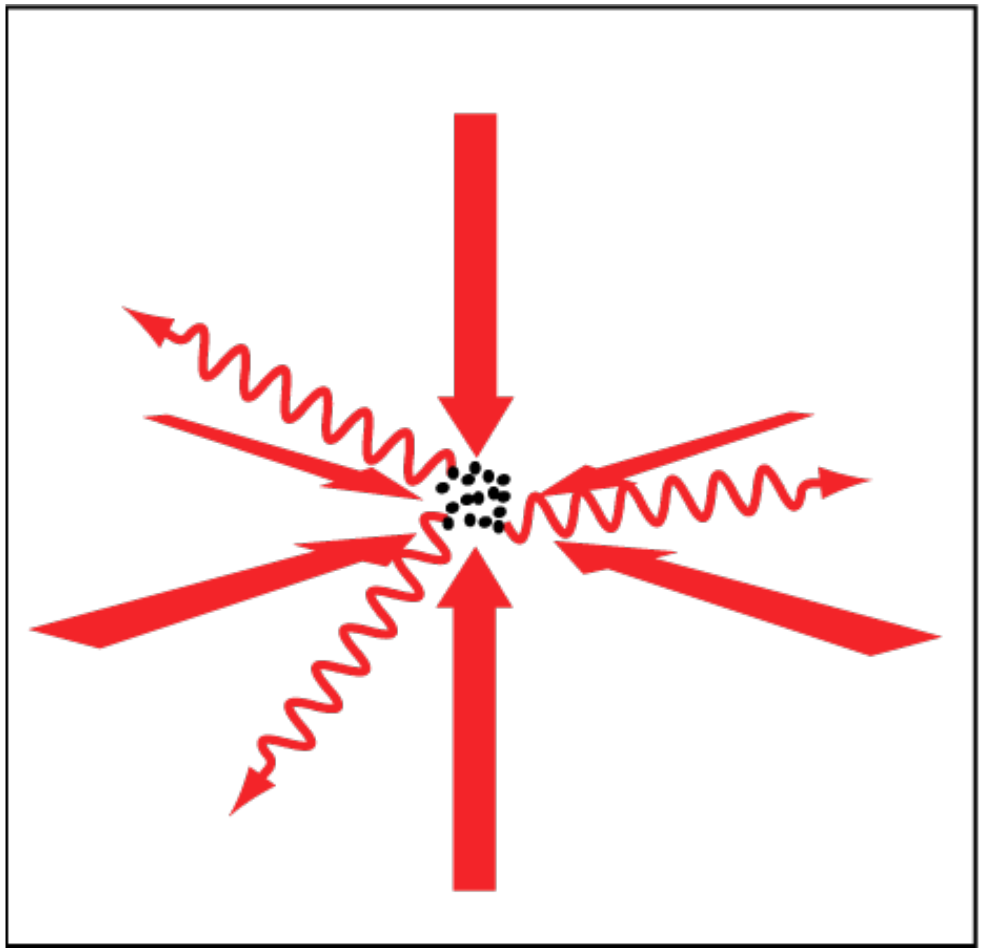
Limitation: Recoil energy (which we just calculated...)

② Evaporative cooling — c.f. coffee cooling by hot molecules leaving

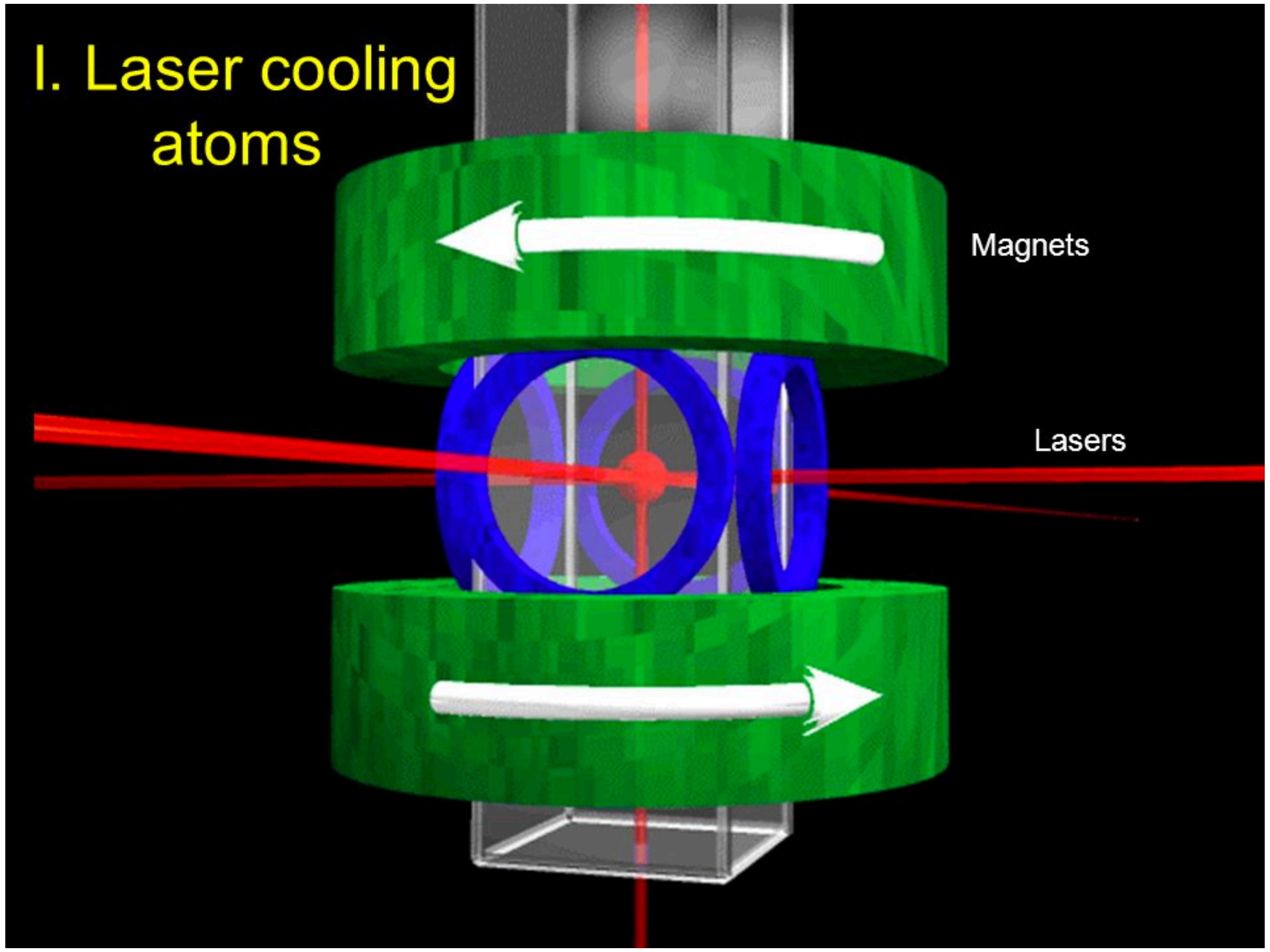
⟹ Slides

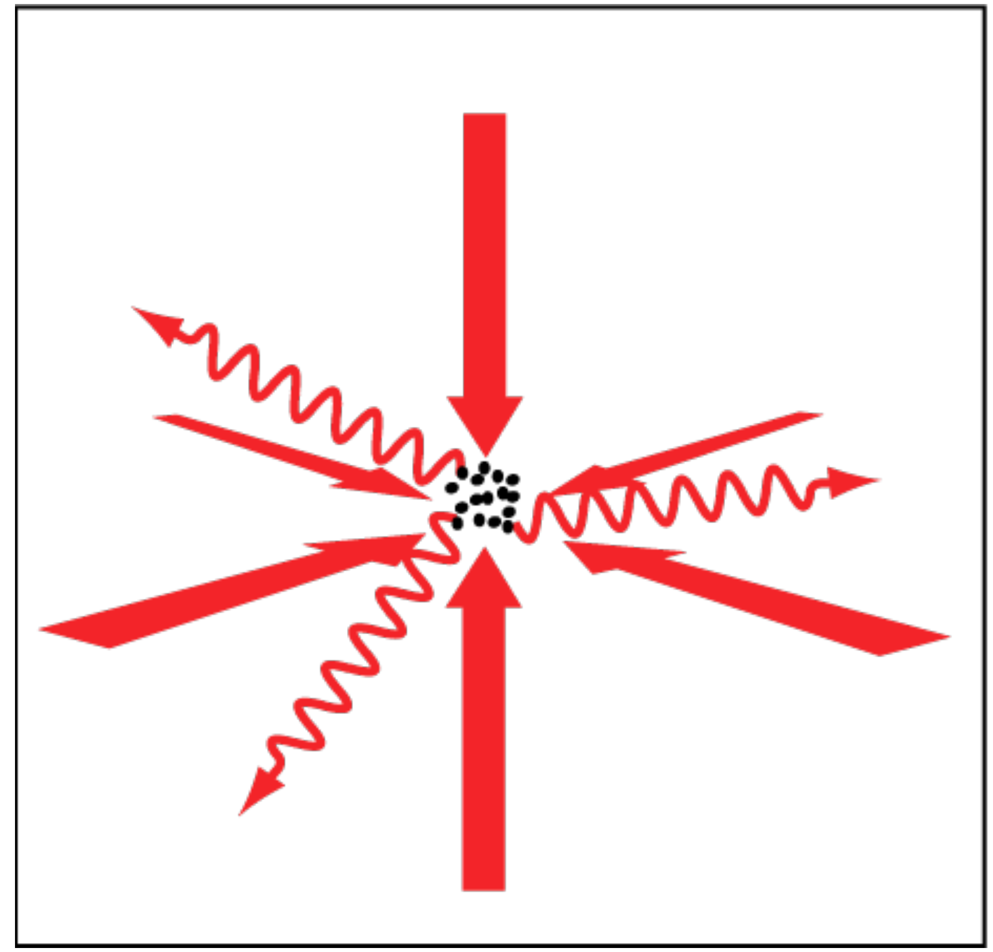
Laser Cooling





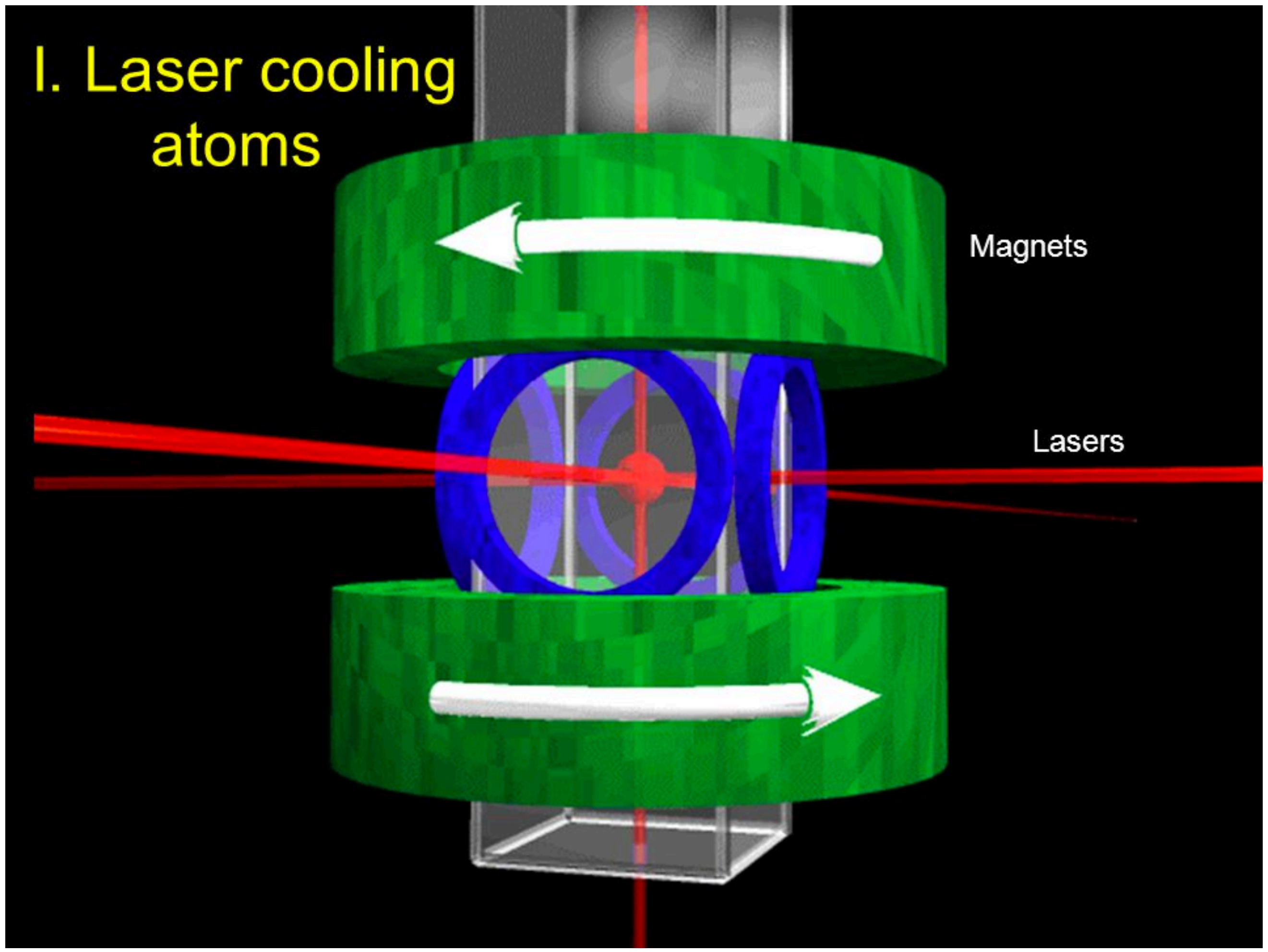
Laser Cooling

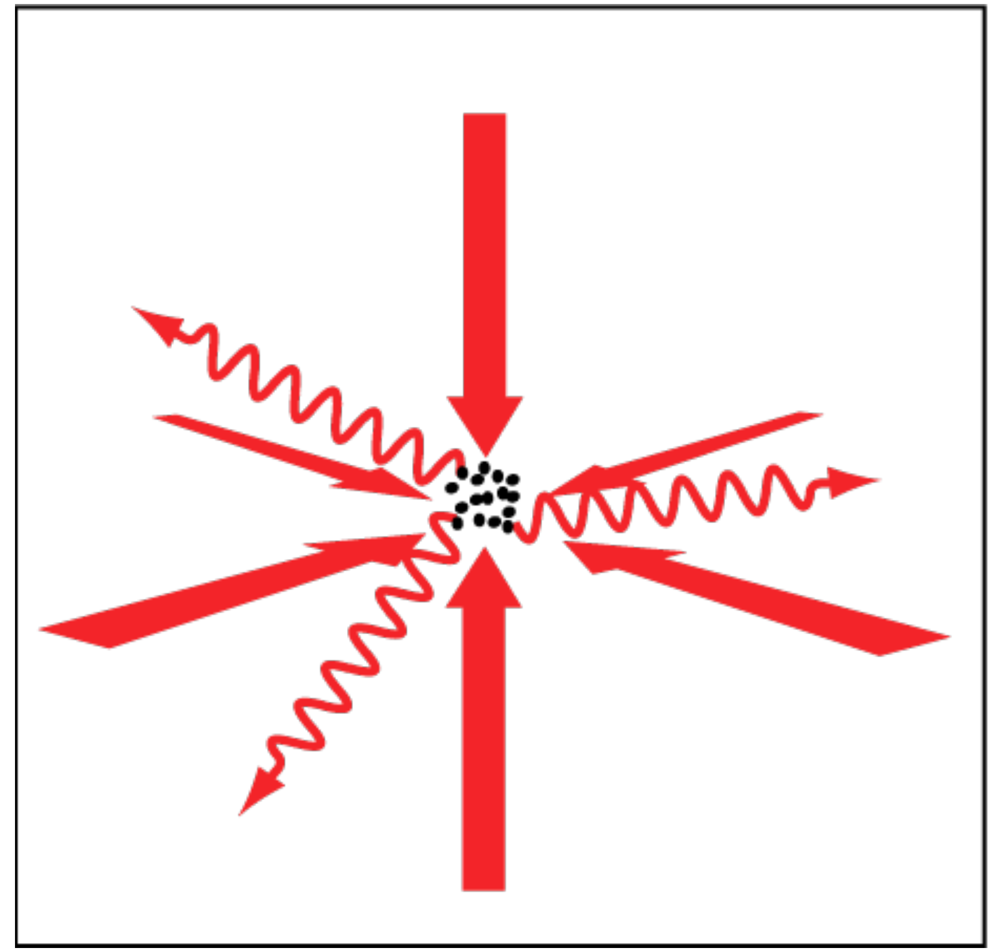




Spontaneous emission removes entropy…

Laser Cooling

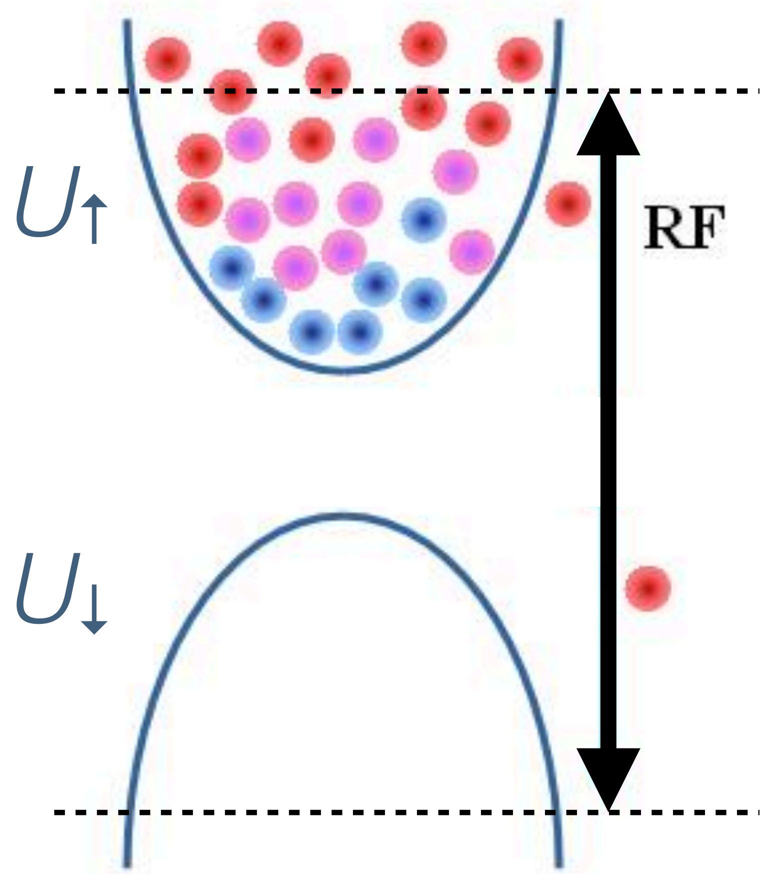




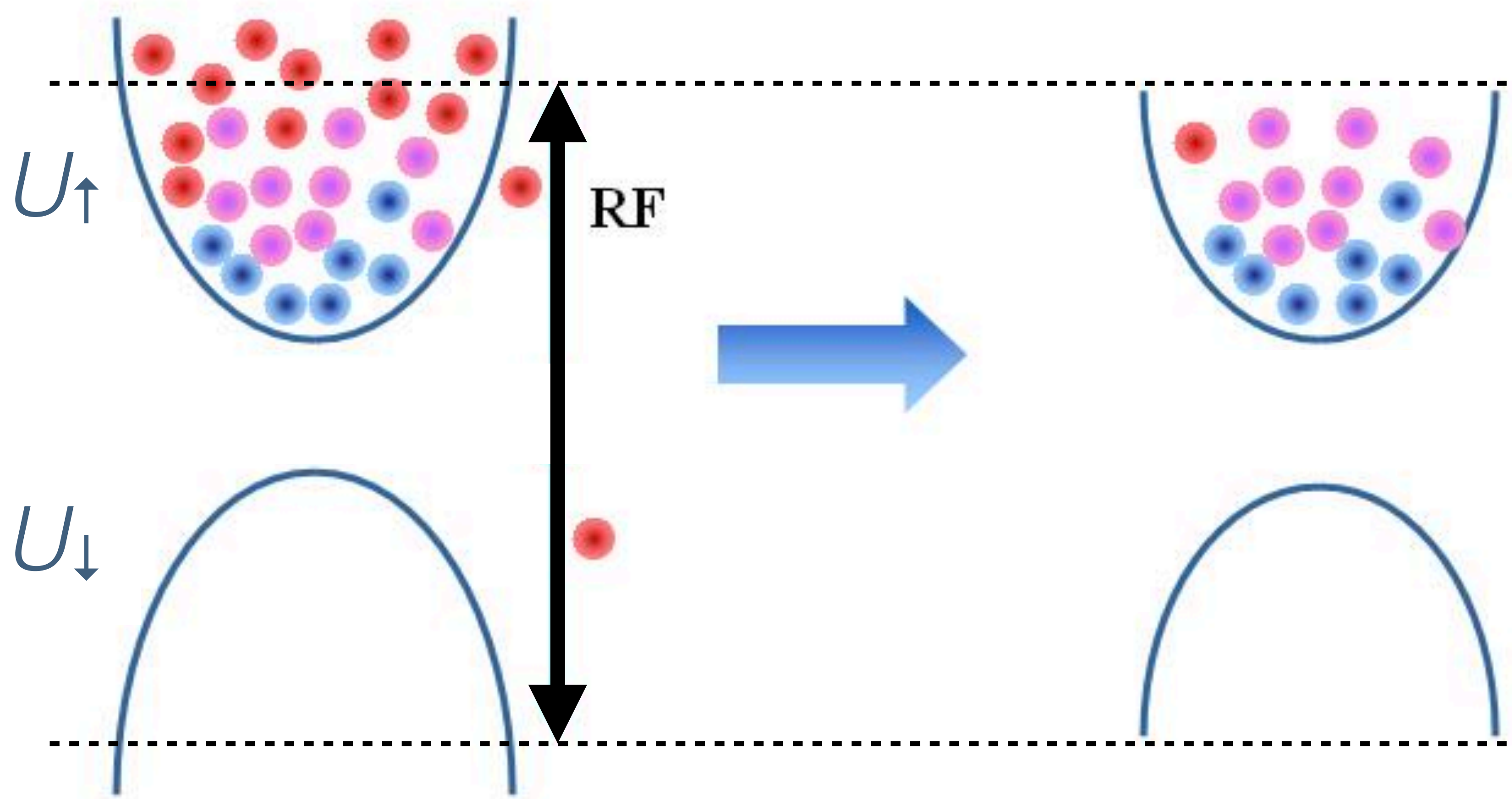
Spontaneous emission removes entropy…

…but ultimately recoil heating limits temperature

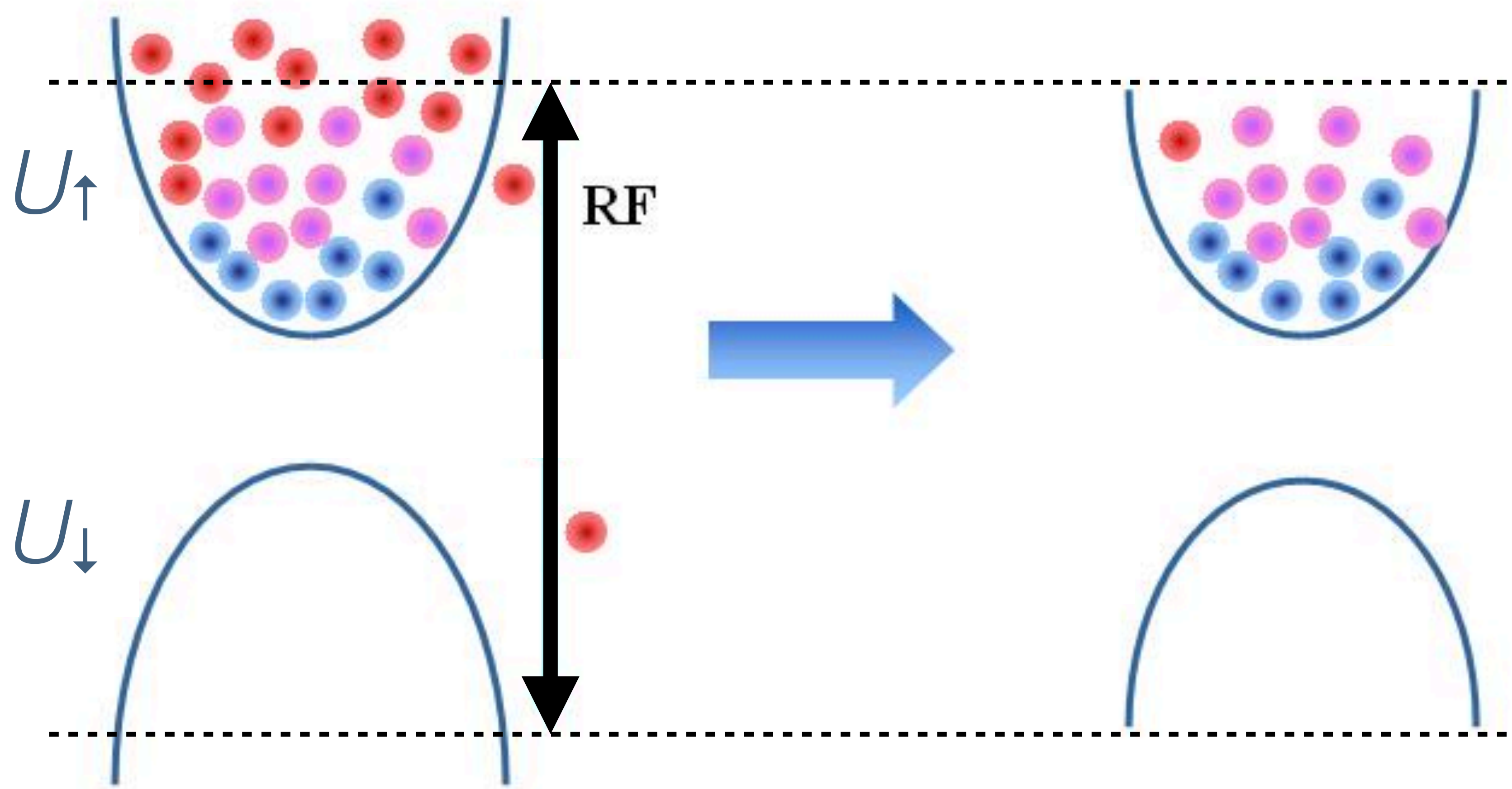
Evaporative Cooling



Evaporative Cooling



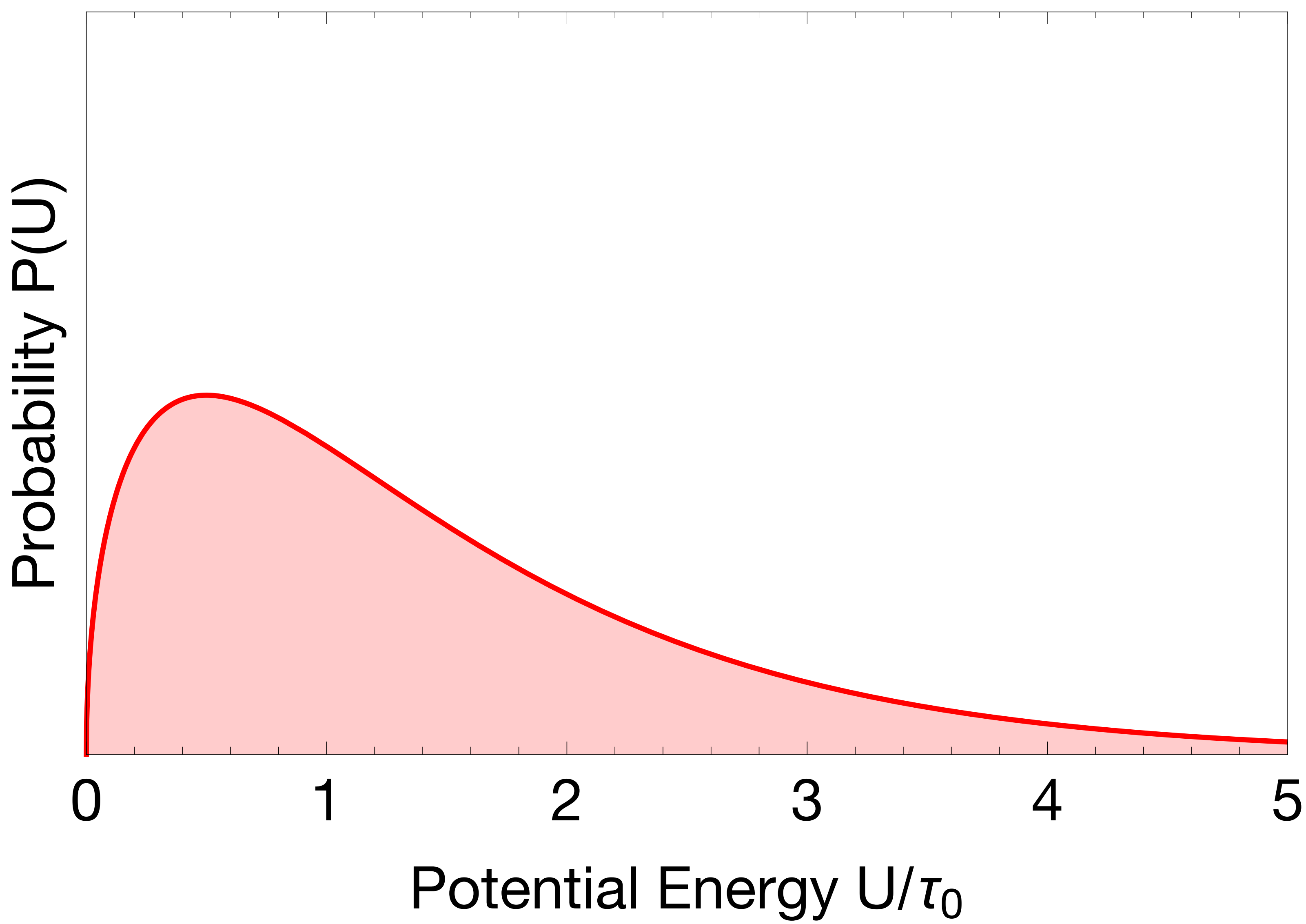
Evaporative Cooling



Reduces **temperature** but also reduces **atom number**…

…where should we place the "RF knife" to maximize increase in *phase space density*?

Evaporative Cooling



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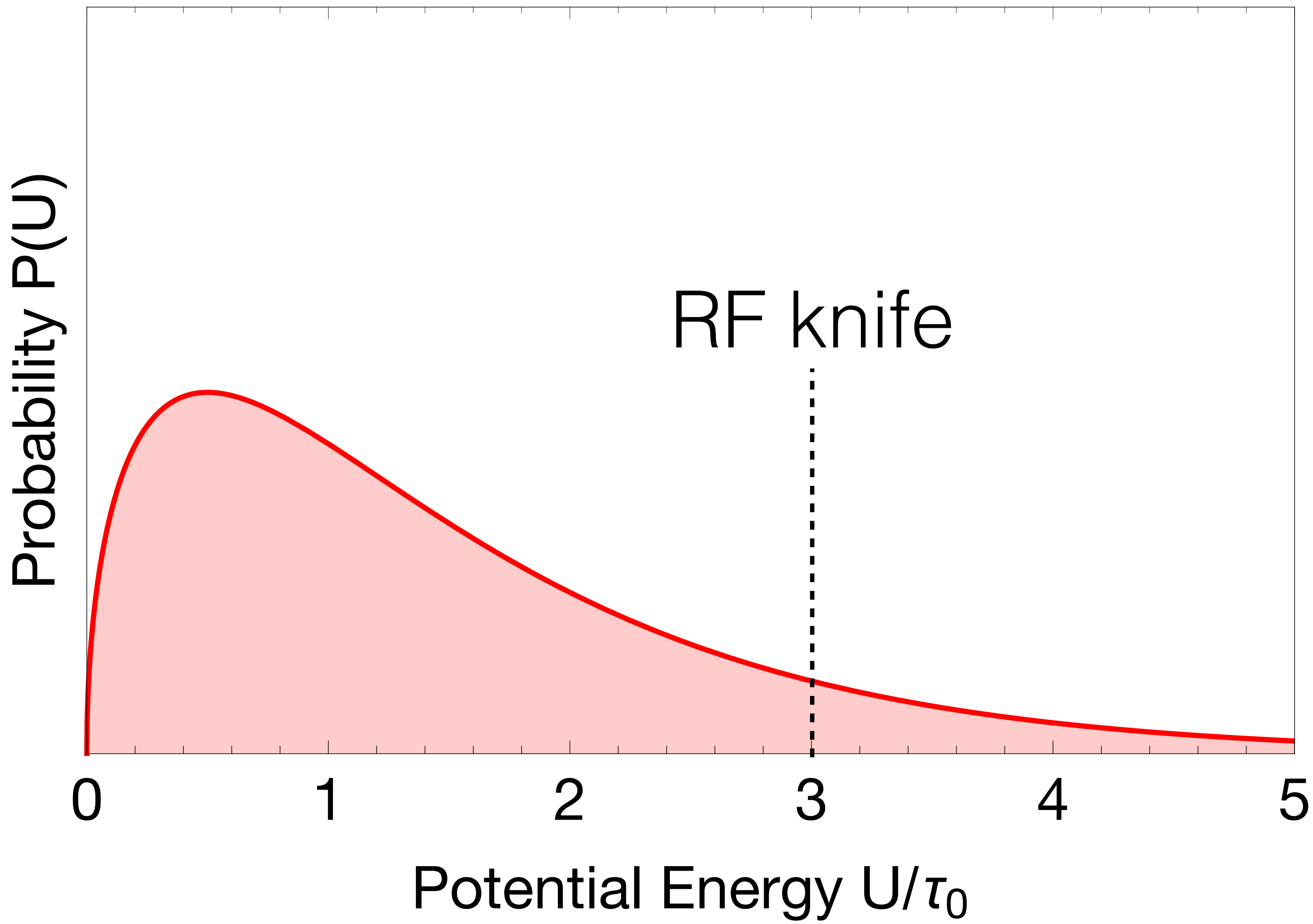

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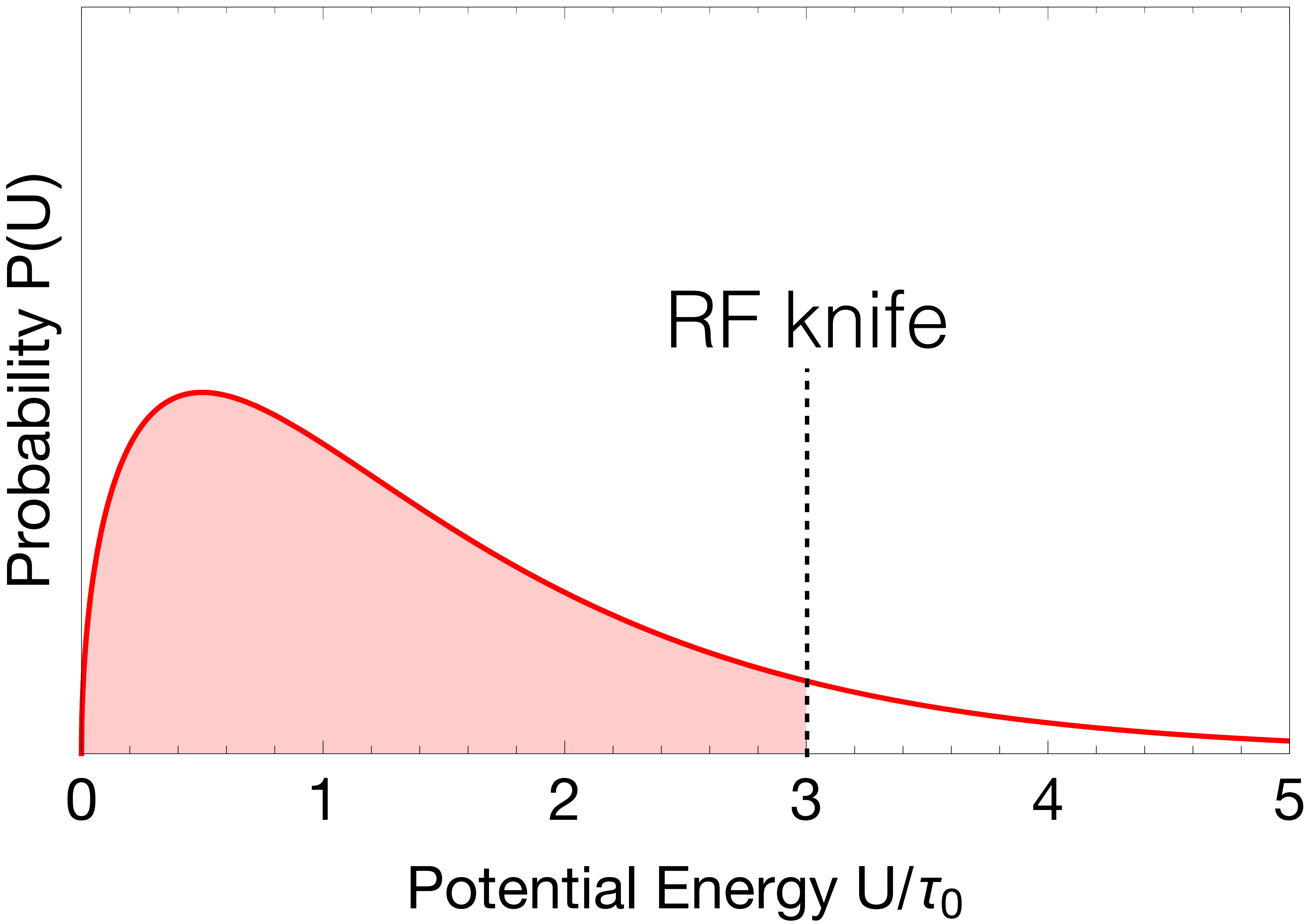

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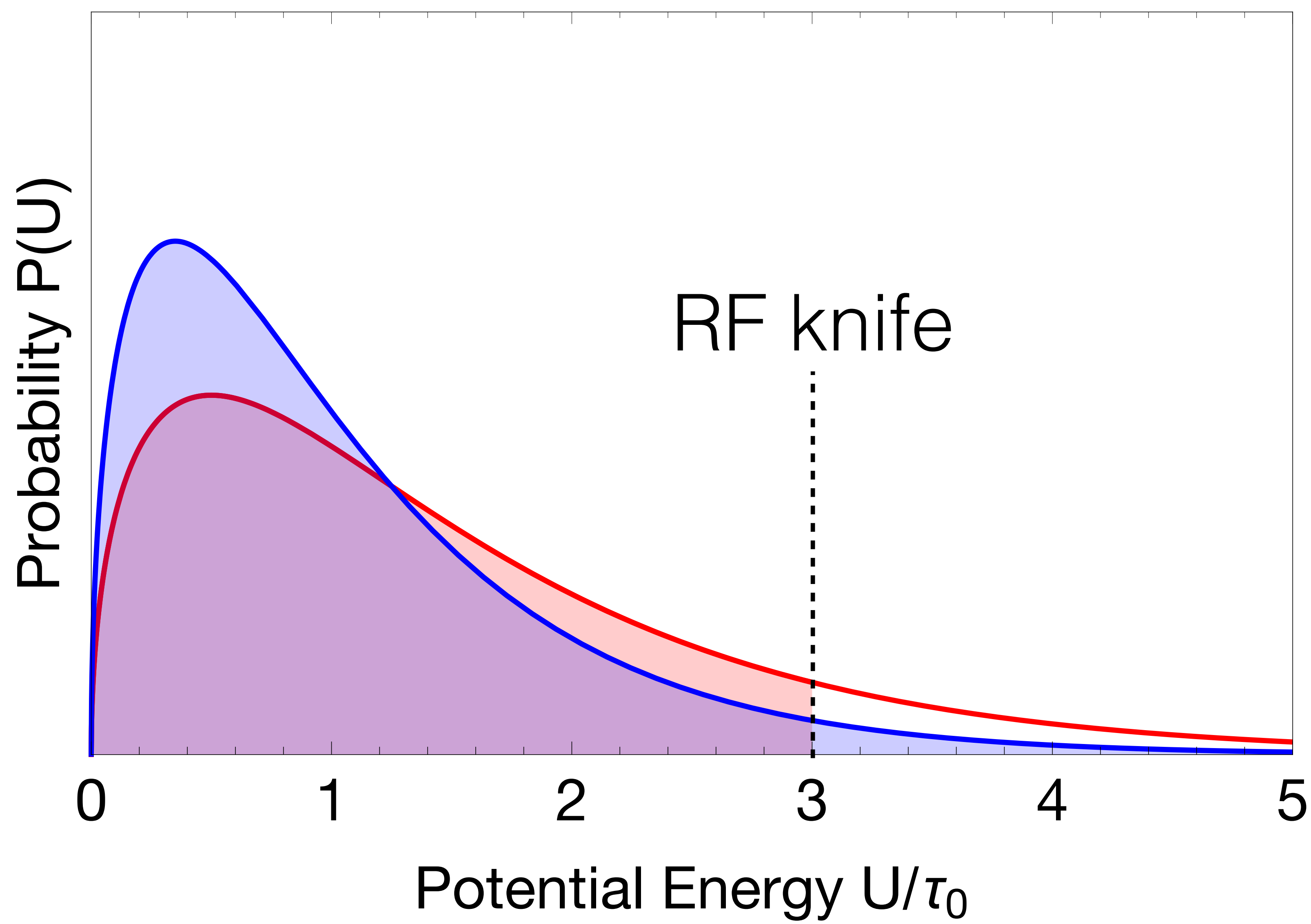

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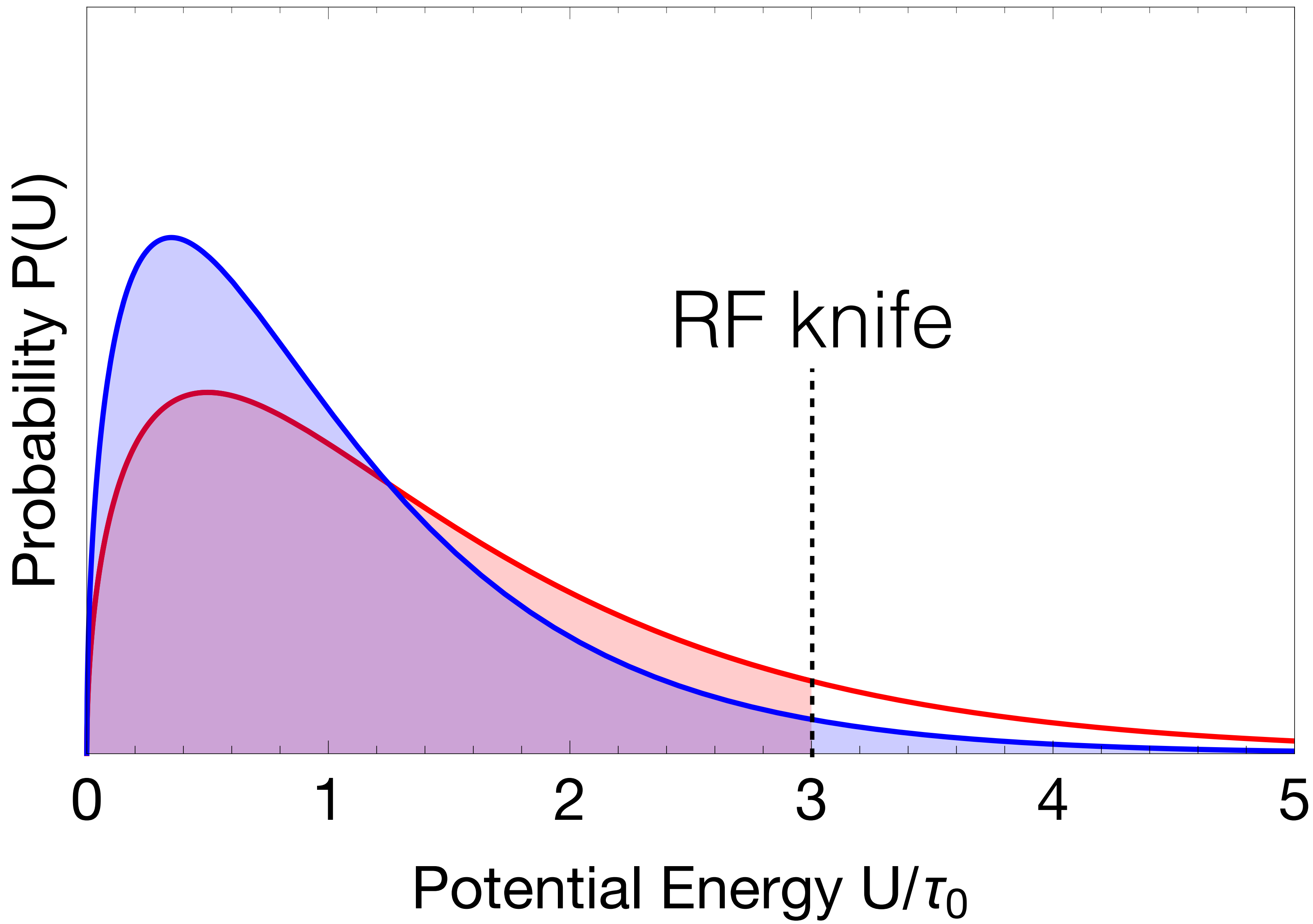

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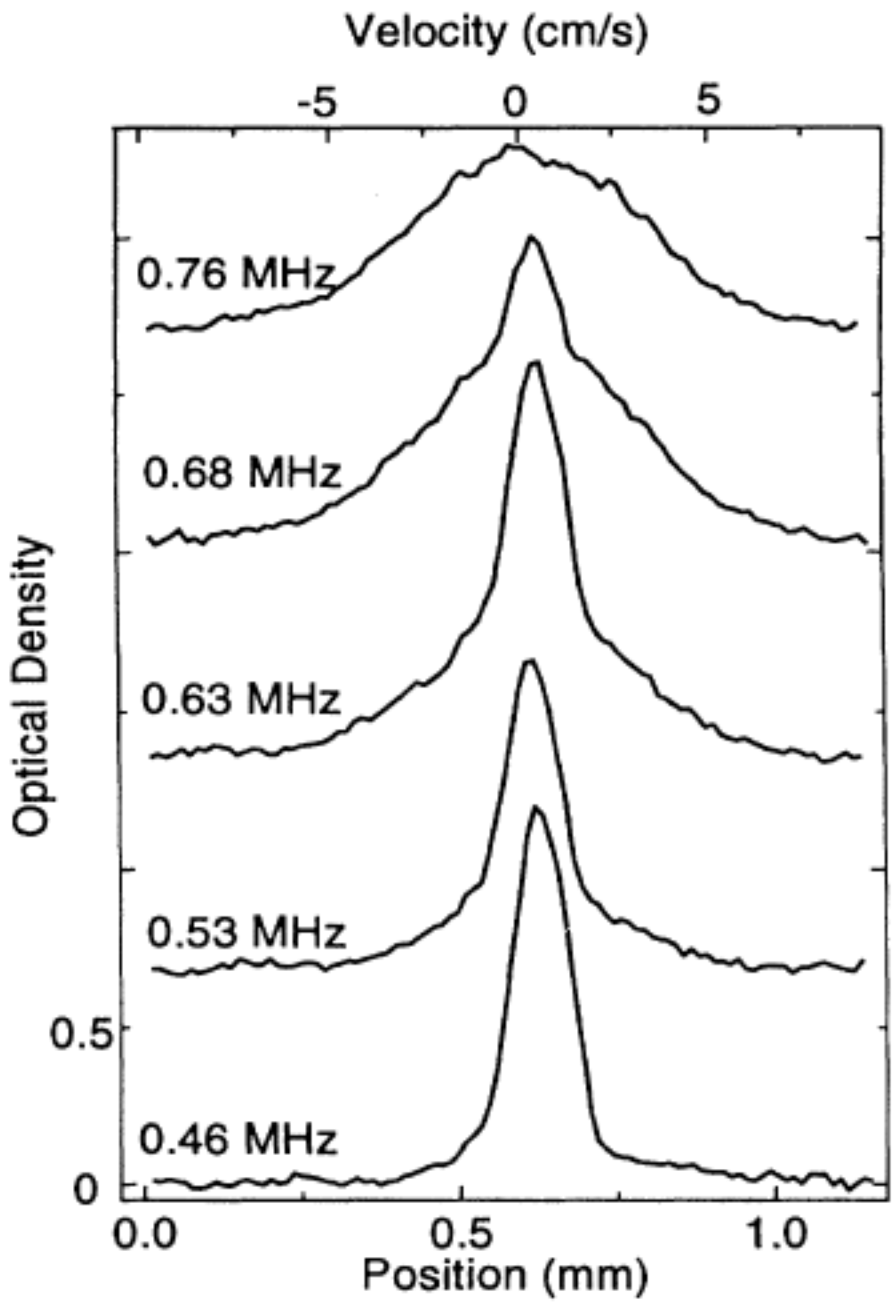
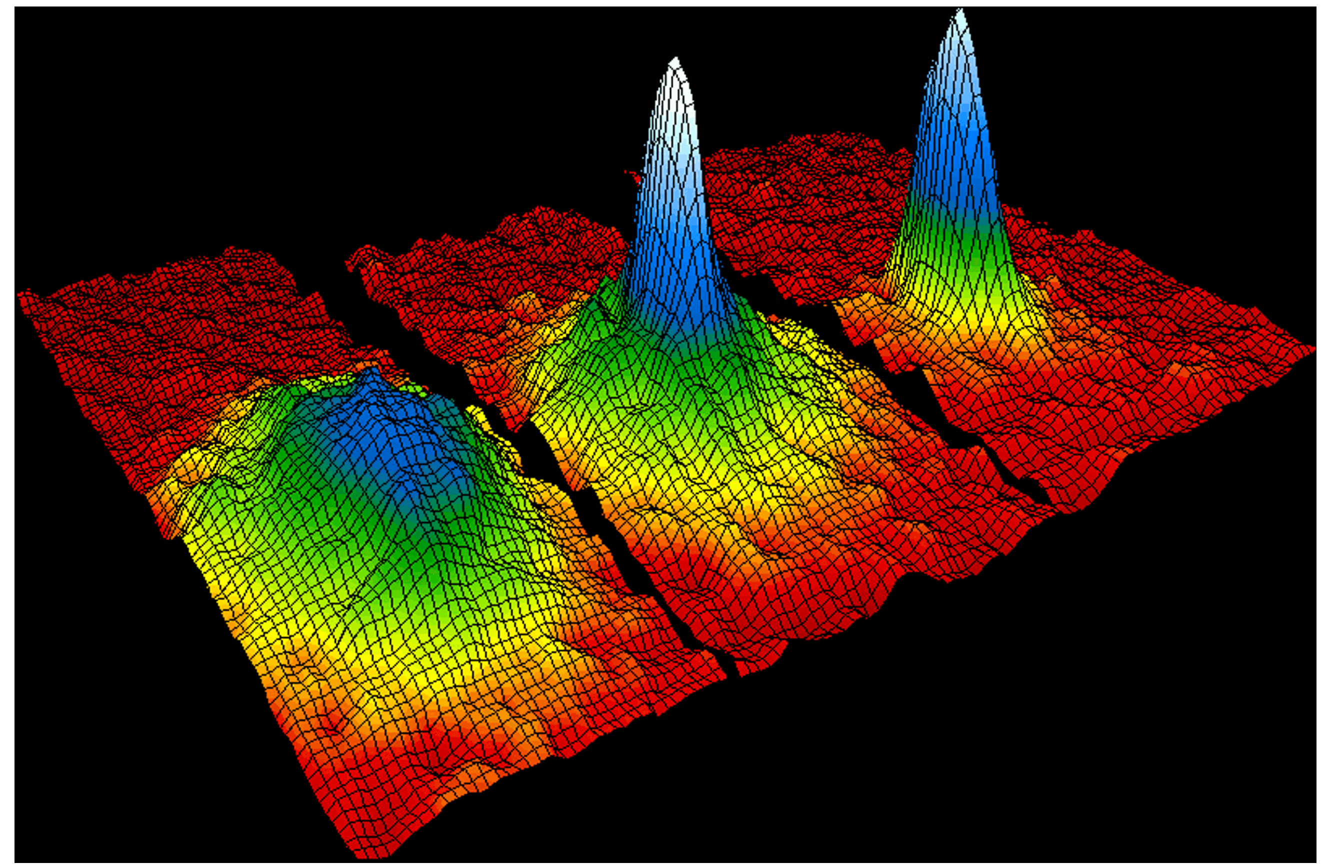
BEC



K.B. Davis, … & W. Ketterle, PRL (1995).

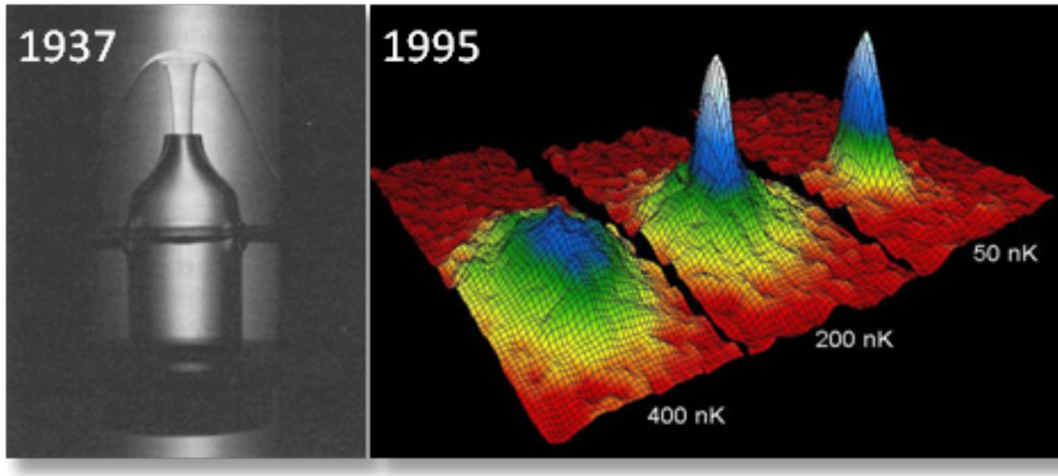
T = 2 μK, n~10^{14}/cm^3



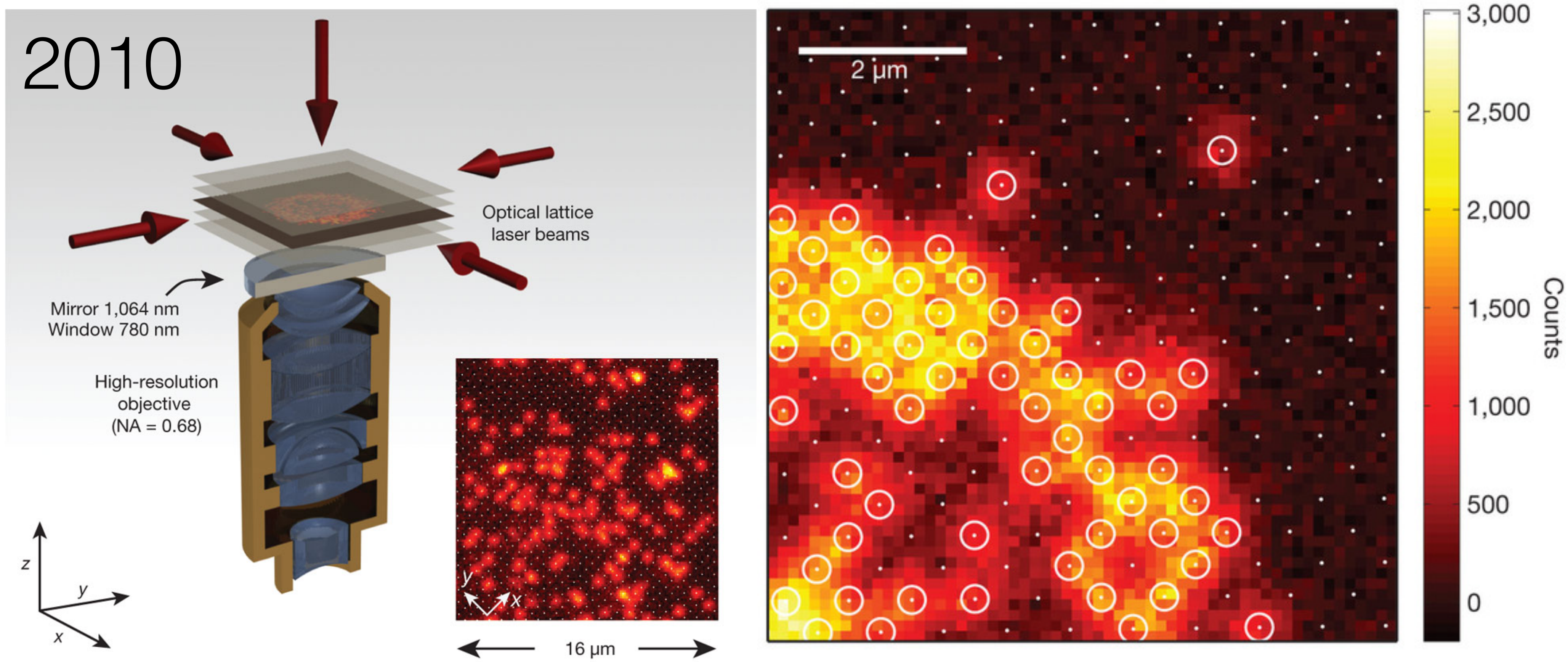
M.H. Anderson, … C.E. Wieman, & E.A. Cornell, *Science* (1995).

T = 170 nK, n~10^{12}/cm^3

Quantum Fluids & Gases

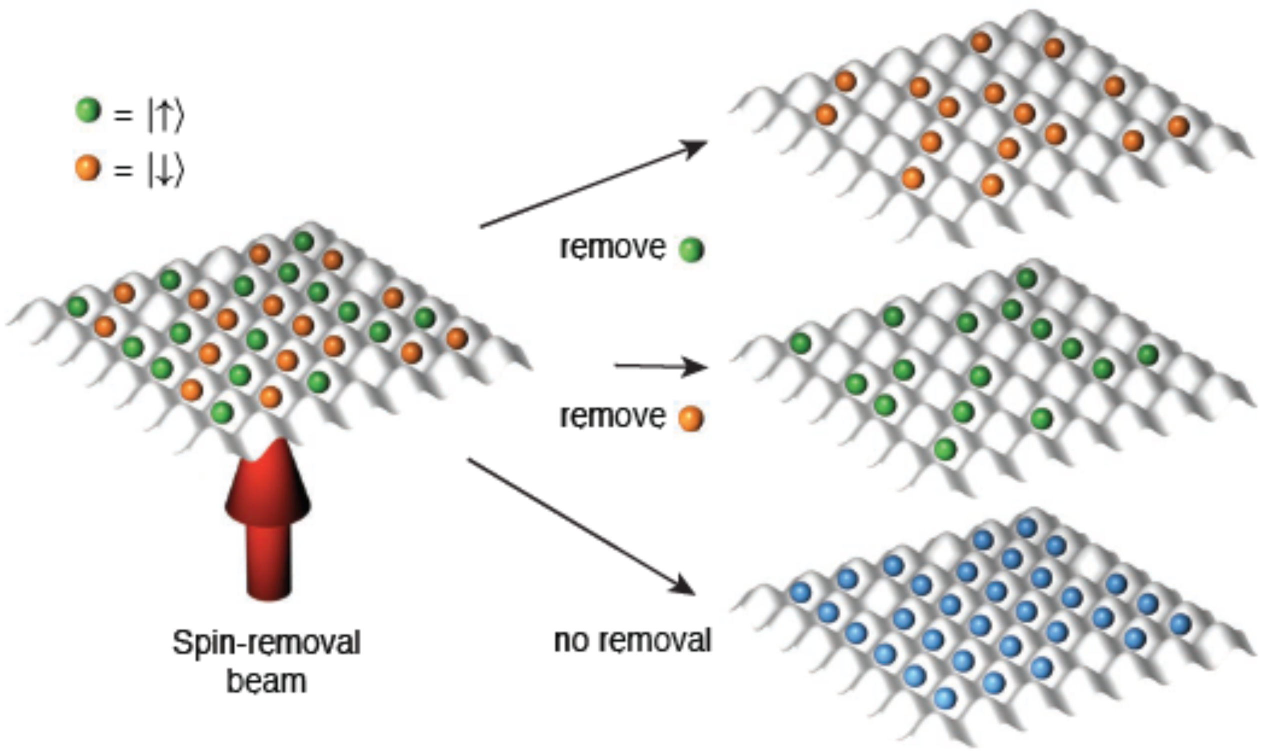


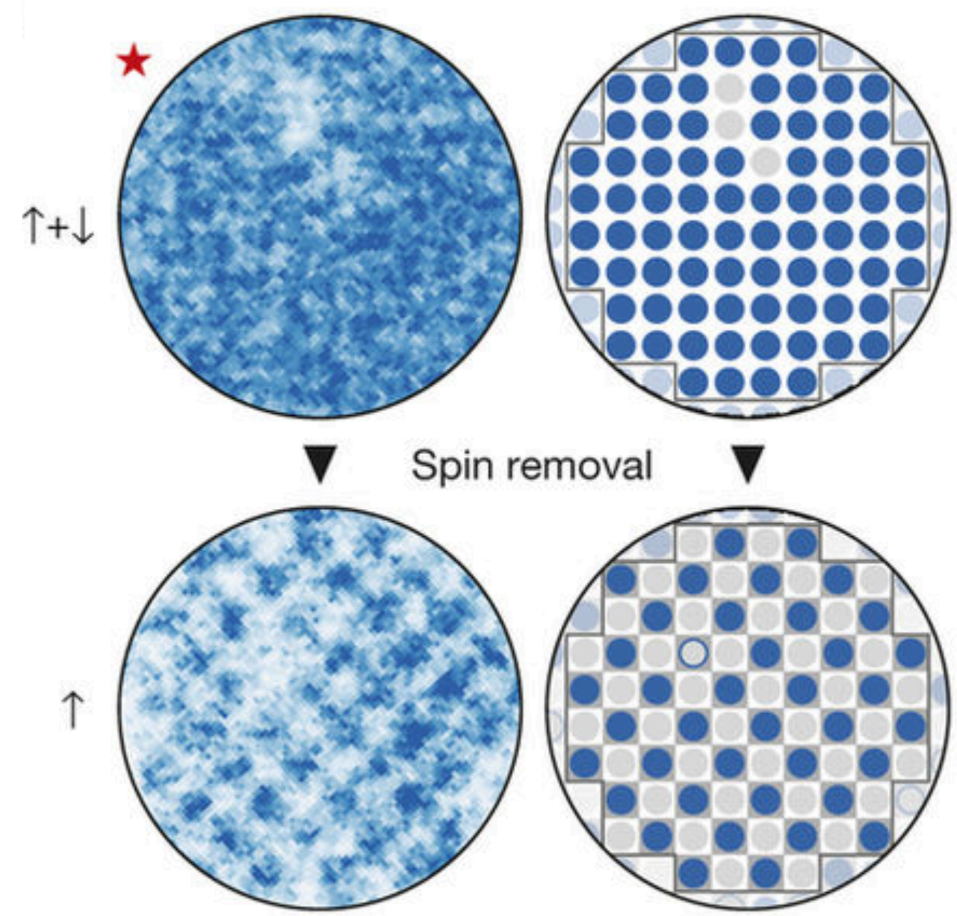
Source:
http://www-amop.phy.cam.ac.uk/amop-zh/Research2.html



JF Sherson *et al. Nature* **467**, 1-5 (2010).

Fermions Under the Microscope





Antiferromagnetism

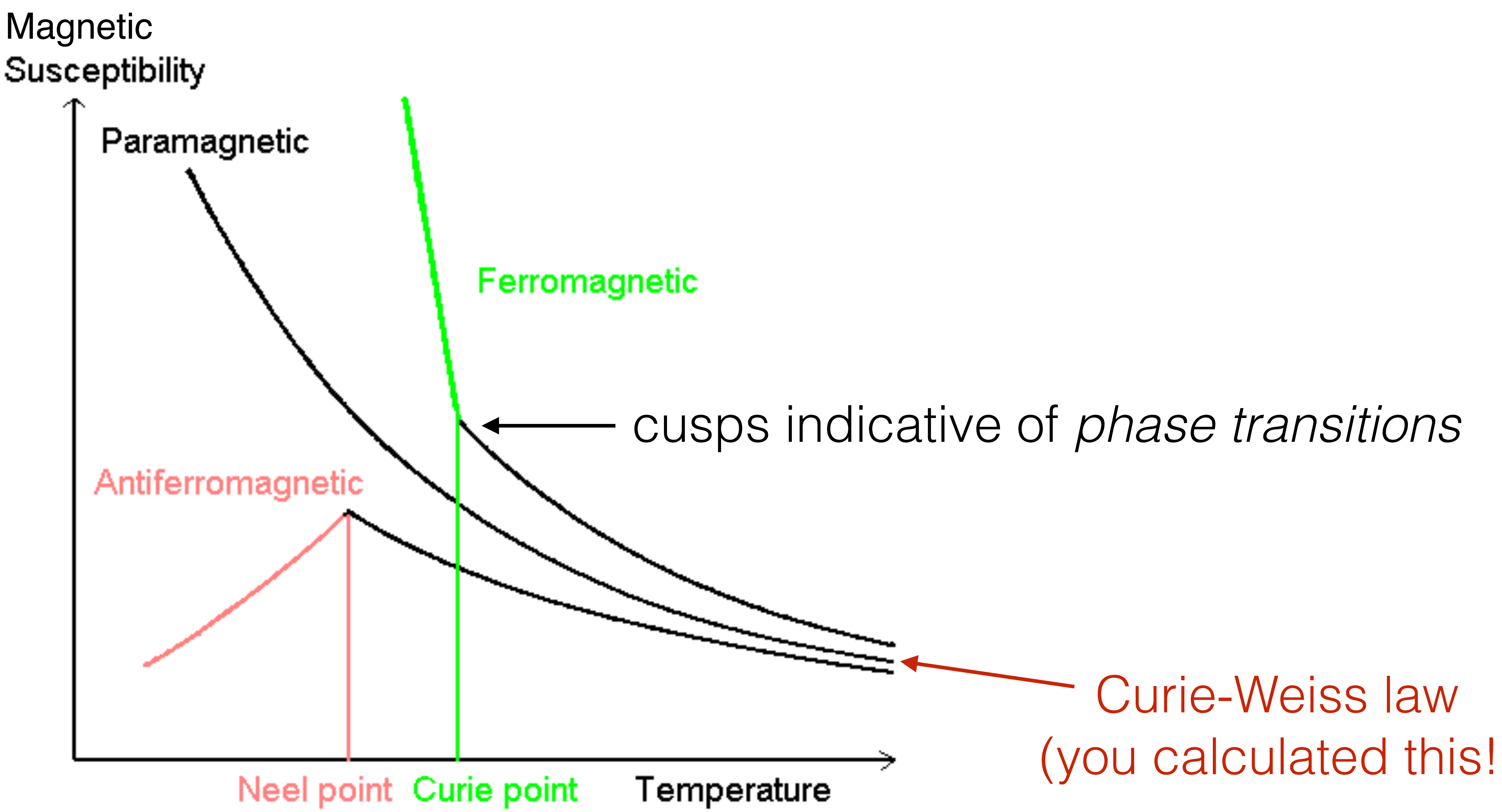
Physics 171 (+ 172)

- **Phase transitions,** e.g., due to magnetic interactions
- Chemical reactivity + chemical work
- Transport phenomena
  $\Rightarrow$ electrical and thermal conductivity, fluid flow

Physics 170

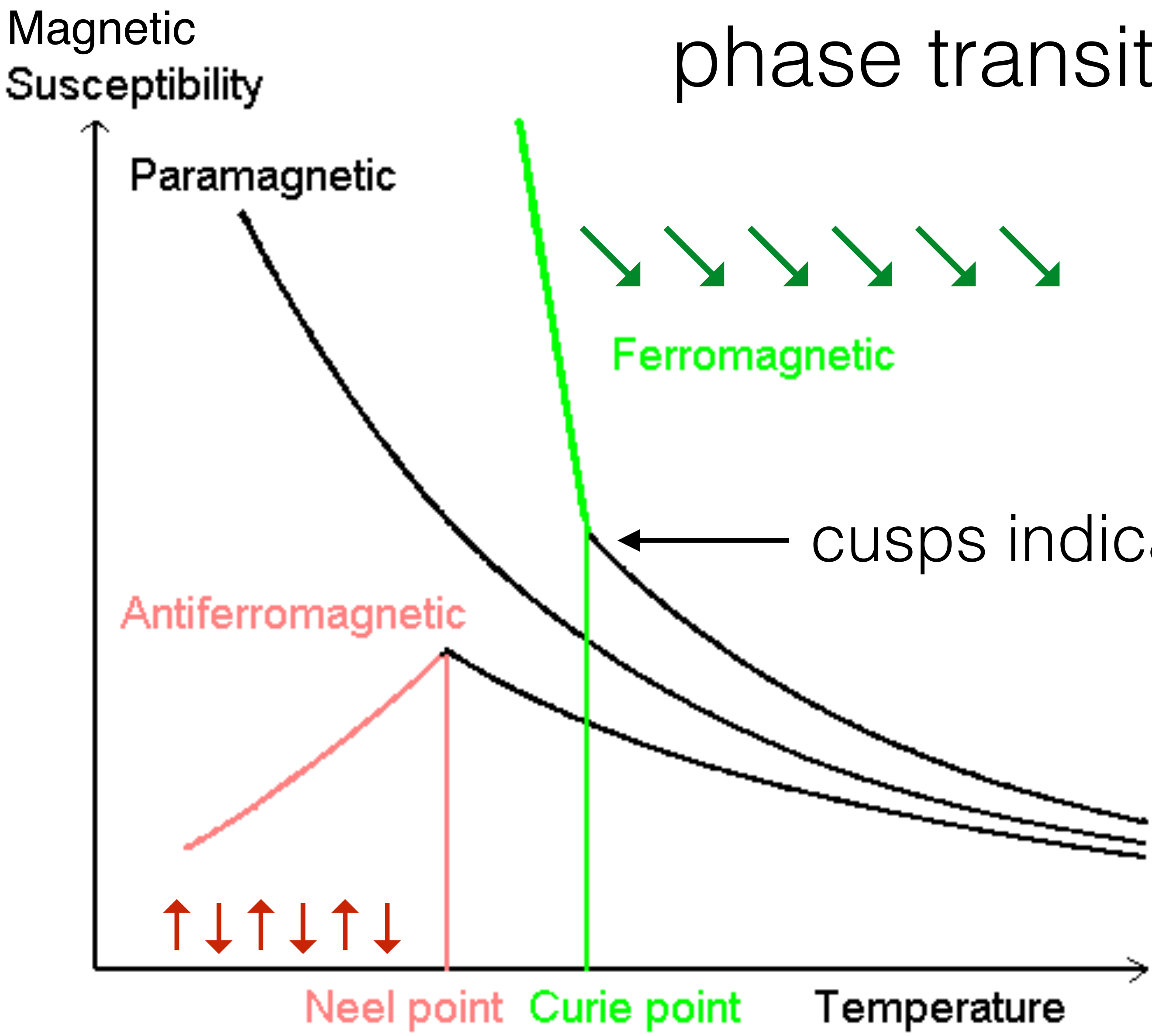
- Statistical mechanics of simple, non-interacting model systems
  $\Rightarrow$ gases (classical & quantum), paramagnet, harmonic oscillator
- Applications:
  photons, phonons, electrons in metals, stars
  generating work from heat

Detecting Phase Transitions



Detecting Phase Transitions

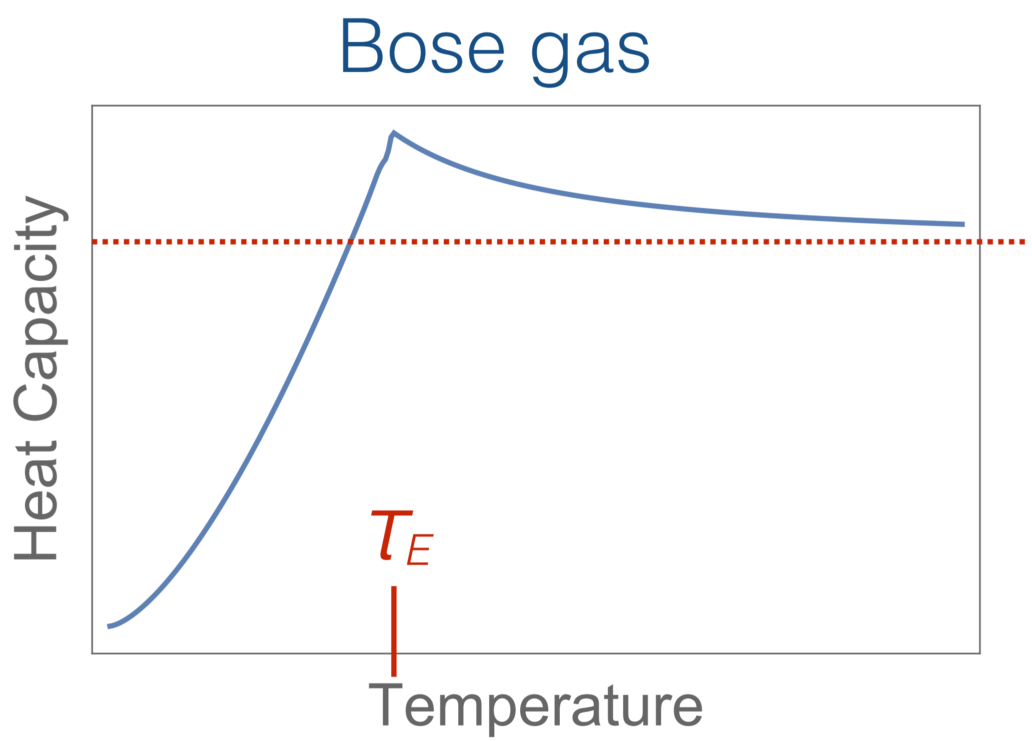
phase transition ⇔ broken symmetry



cusps indicative of *phase transitions*

Curie-Weiss law
(you calculated this!)

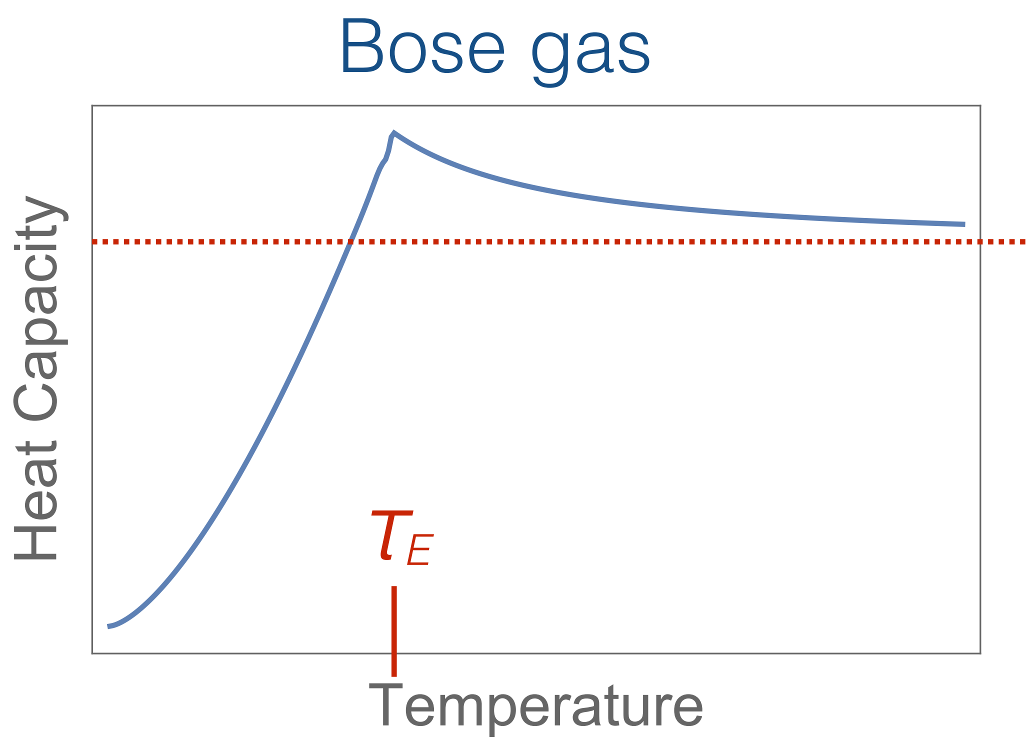
Heat Capacity of Bose Gas

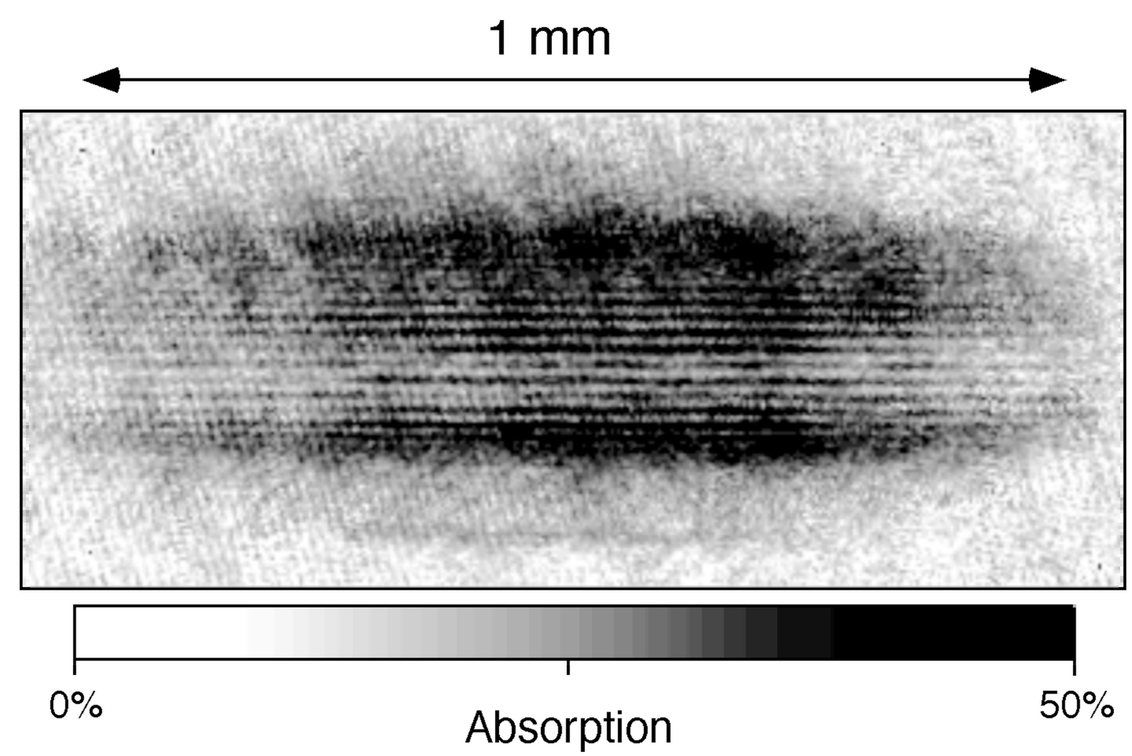


Cusp is indicative of a *phase transition*

- Bose-Einstein condensation in this case
- What is the broken symmetry?

Heat Capacity of Bose Gas





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Learning Goals

- Given a microscopic description of a physical system, you should be able to make predictions about its bulk properties. This includes deriving equations of state (e.g., ideal gas law, Curie-Weiss law) and response functions (such as heat capacity, compressibility, or magnetic susceptibility).

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- Better yet, be able to design the experiment—e.g., what could we measure to determine the vibrational frequency of a gas-phase diatomic molecule?

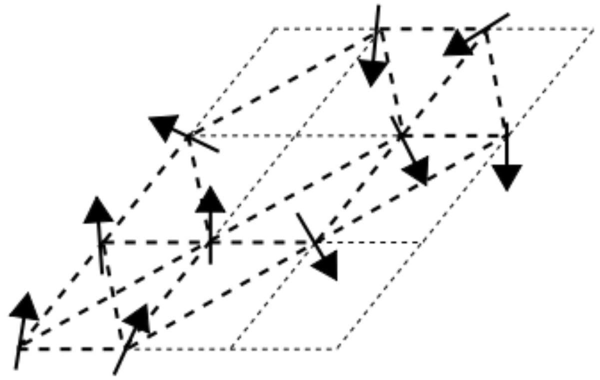
Response Functions

…are often the first place where new and interesting physics is found

- Phonons in solids

- Magnetic order

- *Absence* of magnetic order: e.g., spin glass
  $\Rightarrow$ violation of the 3rd law of thermodynamics?



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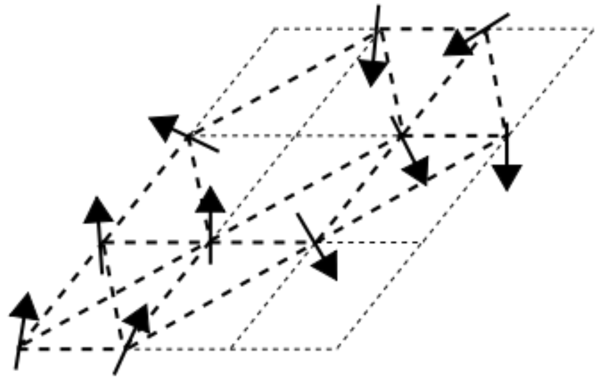

The entropy of a system approaches a constant value as the temperature approaches zero.

Learning Goals

- You should be able to articulate, explain, and apply the laws of thermodynamics.

  E.g., what is entropy, why does it tend to increase, and what fundamental limit does this tendency impose on our ability to generate work from heat?

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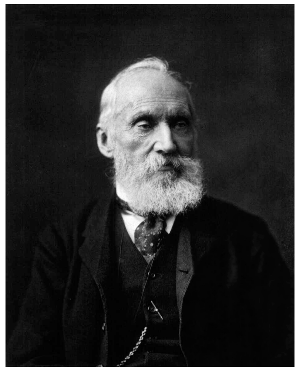
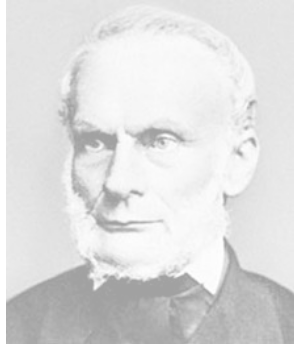


Kelvin: “It is impossible to construct a perfect heat engine.”

$\Updownarrow$



Clausius: “It is impossible to construct a perfect refrigerator.”

Learning Goals

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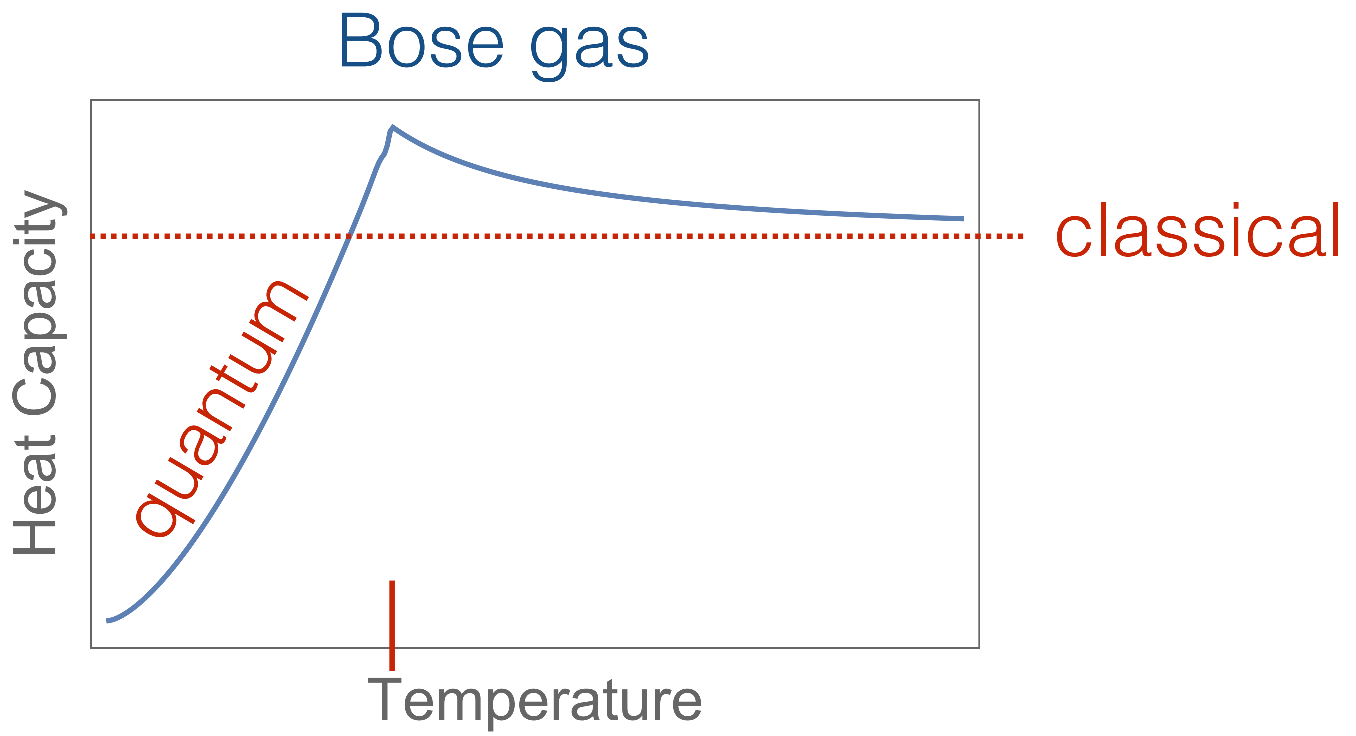

Learning Goals

Many of the model systems (random walk; harmonic oscillator) and statistical principles (central limit theorem; maximization of entropy) we study in statistical mechanics appear in diverse contexts both within physics and in other fields ranging from biology to machine learning to finance. Each time you encounter them, you should recognize them and be equipped to answer new questions by remembering or rederiving familiar results.