MIDTERM REVIEW

1. For a paramagnet, the fundamental thermodynamic relation is

$$dE = \tau d\sigma - MdB, \tag{1}$$

where M denotes the magnetization and $đW \equiv -MdB$ represents the work done on the system by a quasi-static variation of the magnetic field B. One can also see simply from the Hamiltonian that $dE = -MdB - BdM$. Verify explicitly that the heat absorbed by the magnet in a quasistatic process is $\tau d\sigma = -BdM$, as follows:

a. Calculate dS/dM directly from the Gaussian approximation to the multiplicity function. (What assumption must you make to apply the Gaussian approximation?)

b. Derive the equation of state of the paramagnet from the partition function and apply it to determine B/τ in the regime where the Gaussian approximation in part a. is valid.

c. Your results should confirm $d\sigma/dM = -B/\tau$ in the regime under consideration.

2. In a molecular crystal, each lattice site is occupied by a molecule. Consider such a lattice with N diatomic molecules. The potential between the two atoms, each of mass m, in a molecule is:

$$U(r) = \epsilon_0 \left[\left(\frac{a}{r}\right)^{12} - 2\left(\frac{a}{r}\right)^{6} \right] \tag{2}$$

where r is the separation between the two atoms in a molecule. The solid is in contact with a heat reservoir at a temperature T low enough so that $k_B T \ll \epsilon_0$.

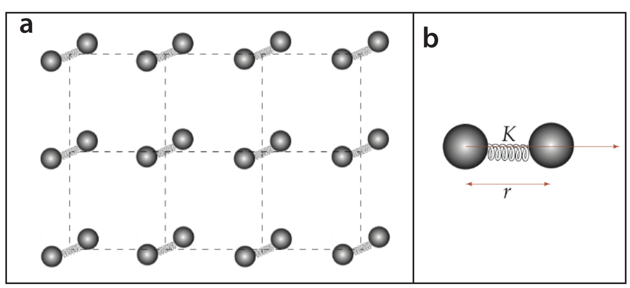

FIG. 1. **(a)** The crystal (with only 2 of 3 dimensions shown). **(b)** The molecule.

a. What is the equilibrium separation of the two atoms in the molecule?

b. What is the frequency for small vibrations of an individual molecule (you may want to first calculate the spring constant K)?

c. What is the average energy and the specifc heat of the solid due to the internal vibrations of the molecules of that solid?
Note: the specific heat c_V' is defined as the heat capacity per unit mass.

d. What is the condition for a "classical limit" for the vibrations of these molecules?

3. a. Show that the work performed by an ideal gas at temperature T as it expands in a piston from volume V_1 to volume V_2 is equal to the decrease ΔF in its free energy.
   b. Is ΔF larger or smaller than the decrease in the internal energy of the gas? What is the origin of the difference?

4. Consider an isolated system of N electric dipoles with total energy E. Each dipole has two possible quantum states with energies $\epsilon_{\pm} = \pm\mu\mathcal{E}$, where $\mathcal{E}$ is the magnitude of the applied electric field. The total energy of the system is E.
   a. Find the electric dipole moment of the sample in an electric field $\mathcal{E}$.
   b. Calculate the probability that electric dipole moments 2,4, and 8 will point up.
   c. Calculate the entropy of the system.
   d. Without using the formula above, explain physically what the entropy should be in the limits $\mathcal{E} \to \infty$ and $\mathcal{E} \to 0$.