PROBLEM SET 3

Reading: Kittel & Kroemer, Ch. 3, focusing on the following sections:

- Boltzmann Factor, pp. 58-63
- Helmholtz Free Energy, pp. 68-72

When should you start on which problems?

- Problem 1 immediately.
- Problems 2-7 after Lecture 3A (Wed).
- Problem 8 after Lecture 3B (Fri).

1. A one-dimensional simple harmonic oscillator has energy levels given by

$$\varepsilon_n = \left(n + \frac{1}{2}\right)\hbar\omega, \tag{1}$$

where ω is the characteristic (angular) frequency of the oscillator and the quantum number n can be any non-negative integer $n = 0, 1, 2, \ldots$. Suppose that such an oscillator is in thermal contact with a heat reservoir at a temperature T low enough so that $\tau \ll \hbar\omega$.

   a. Find the ratio of the probability of the oscillator being in the first excited state to the probability of its being in the ground state.

   b. Assuming that only the ground state and first excited state are appreciably occupied, find the mean energy of the oscillator as a function of the temperature τ.

2. *Magnetic susceptibility of a paramagnet* (adapted from K&K 3.3):

   a. Use the partition function to find an exact expression for the magnetization M and susceptibility $\chi = dM/dB$ as a function of temperature and magnetic field for the model system of spin-1/2 particles with magnetic moment μ. (Recall that the magnetization sets the energy $E = -MB$ in a magnetic field B.) Your result for the magnetization should agree with that which you derived on Problem Set 2 by another method:

$$M = N\mu \tanh\left(\frac{\mu B}{\tau}\right). \tag{2}$$

   b. Determine the limiting behavior of the magnetic susceptibility χ for $\tau \ll \mu B$ and for $\tau \gg \mu B$.

   c. Sketch $\chi(T)$.

3. K&K 3.4 (*Energy fluctuations*)

4. K&K 3.5 (*Overhauser effect*)

5. K&K 3.7 (*Zipper problem*)

6. K&K 3.9 (*Partition function for two systems*).

7. K&K 3.3 (*Free energy of a harmonic oscillator*)

8. *Work-load check:* how many hours did you spend on this problem set?