PROBLEM SET 5

Reading:

- Kapitulnik notes: pp. 38-44 of the PDF file on *Canvas*
- Kittel & Kroemer: Ch. 4, pp. 87-98

1. *Quadrupole trap.* A dilute, classical gas of N non-interacting atoms is trapped in a magnetic quadrupole field, which forms a potential

$$U(x, y, z) = \alpha\sqrt{x^2 + y^2 + 4z^2}. \tag{1}$$

   a. Calculate the peak density n_0 (i.e., the number of atoms per unit volume at the center of the trap) as a function of temperature τ. *Note:* you may wish to define a radial coordinate $\rho = \sqrt{x^2 + y^2 + \tilde{z}^2}$, where $\tilde{z} = 2z$.

   b. Calculate the Helmholtz free energy and entropy. Express your results in terms of the peak density n_0 and thermal de Broglie wavelength.

   c. State a condition for the classical treatment to be valid in terms of the peak density n_0 and temperature τ.

   d. By adiabatically reducing the trap gradient α, it is possible to lower the temperature τ.

      i. How does the temperature τ scale with trap gradient α at constant entropy?

      ii. Can this method of reducing the temperature bring an initially classical gas into the quantum regime (where the condition in c. is violated)? Why or why not?

2. Go through the midterm once again, now without the time pressure:

   - If you received fewer than 60% of the points on a given problem, submit the entire problem again.
   - If you received at least 60% of the points on a given problem, you may correct just the parts that you missed and explain your errors.

3. K&K problem 4.5 (*Surface temperature of the earth*).

4. K&K problem 4.6 (*Pressure of thermal radiation*).

5. K&K problem 4.9 (*Photon gas in one dimension*).