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EXERCISE 10A: QUANTUM GAS EXPERIMENTS

Objectives: Apply your broad knowledge of statistical mechanics to figure out…

- …how to detect a Bose-Einstein condensate
- …how we can study an ultracold quantum gas in a room-temperature apparatus

Useful past results:

- Thermal de Broglie wavelength: $\lambda_\tau = h/\sqrt{2\pi m\tau}$
- Condition for BEC in a 3D box: $n\lambda_\tau^3 > \zeta(3/2)$

1. *Bose-Einstein condensation (BEC).* Last time, we derived the condition for obtaining a macroscopic occupation of the ground state in a gas of bosons with number density $n = N/V$ in a box of volume $V = L^3$.

   a. How does the critical temperature τ_E scale with the number of bosons, their mass m, and the dimension L of the box? (Don't worry about numerical factors.)

   b. How does the critical temperature for Bose-Einstein condensation compare with the energy of the first excited state? Is the result surprising? Why or why not?

c. Re-express the condition for Bose-Einstein condensation as a condition on the *phase space density* $n/(\Delta p)^3$, where Δp represents the spread of the momentum distribution at temperature τ. How do you interpret your result?

d. For a given density n and temperature τ, what fraction of the atoms are in the ground state? First, calculate the *thermal fraction* N_e/N for $\tau < \tau_E$:

   i. …as a function of density $n = N/V$ and thermal de Broglie wavelength λ_τ.

   ii. …as a function of temperature τ and the critical temperature τ_E for Bose-Einstein condensation.

e. What is the thermal fraction N_e/N for $\tau > \tau_E$?

f. Sketch the condensate fraction $N_0/N = 1 - N_e/N$:

   i. …vs. density n at fixed temperature τ.

   ii. …vs. temperature τ at fixed density.

2. To detect a Bose-Einstein condensate, atoms are suddenly released from an optical trap, allowed to expand for a fixed time t, and then imaged on a CCD camera. In the limit of a long time of flight t, an atom's position in the image is directly proportional to its velocity at the time that the trap was switched off.

   a. Sketch a 1D cross-section of the image you would expect to observe for a Bose gas with a condensate fraction $N_0/N \sim 0.5$.
   b. Annotate your plot. What are the characteristic widths of the key features in terms of the time of flight t, the temperature τ, the mass m, and the size L of the box?

c. *Recoil heating.* Does the light used for imaging appreciably perturb the atoms? Suppose that each atom absorbs on average $\langle N_p \rangle$ photons during the imaging process and reemits each one in a random direction. Correspondingly, each photon gives the atom a momentum kick of magnitude $\hbar k$ in a random direction, where k is the wavenumber of the light.

   i. Estimate the average kinetic energy imparted to each atom by the random photon recoils, by modeling the process as a random walk in momentum space. Express your result in terms of $\langle N_p \rangle$, k, and the atomic mass m.

   ii. How does the kinetic energy E_{rec} imparted by a *single* photon compare with the critical temperature for Bose-Einstein condensation? Give an estimate for the ratio τ_E/E_{rec} in terms of the atomic density n. Don't worry about numerical factors.

iii. Consider a condensate of ^{23}Na atoms formed at a density of 10^{14} cm^{-3}. The atoms are imaged using an absorption line of wavelength $\lambda = 2\pi/k = 589$ nm. What is the ratio τ_E/E_{rec} for these parameters?

iv. *Optional.* Find T_E and $T_{\mathrm{rec}} = E_{\mathrm{rec}}/k_B$ in units of Kelvin.

v. Based on your analysis, does the imaging light appreciably perturb the atoms? Will the recoil heating prevent you from obtaining an accurate image of the ultracold quantum gas? Why or why not?

3. *Ultracold atoms in a room-temperature apparatus.* Remarkably, by tricks of laser cooling and evaporative cooling, we can bring a small cloud of atoms to a temperature in the nanokelvin regime using a room-temperature apparatus.

   a. What precautions must we take to prevent the atoms from thermalizing with their surroundings?

   b. *Why aren't the atoms heated by blackbody radiation?* Consider the ^{23}Na spectral line at $\lambda = 589$ nm, corresponding to a frequency $\omega = 2\pi c/\lambda = 2\pi \times 510$ THz. The frequency width of this line is $\Gamma = 2\pi \times 10$ MHz. Sketch it on a single plot together with the blackbody radiation spectrum at room temperature. (You may use a broken axis.)

c. Estimate the rate at which a sodium atom absorbs blackbody radiation on the $\lambda = 589$ nm spectral line, in units of photons/s. Assume that the atom has an effective cross-sectional area λ^2.

   i. What is the spectral density of blackbody radiation at the frequency $\omega = hc/\lambda$ of the atomic transition?

   ii. How is the radiation flux through a surface of area λ^2 related to the spectral density?

   iii. Within the spectral bandwidth Γ of the atomic transition, how many photons per second are incident on the surface of area λ^2?