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PROGRESS REPORT

For Period 14 February to 14 May

RESEARCH AND STUDY IN
SYSTEM OPTIMIZATION TECHNIQUES

Office of Astrodynamics and Guidance Theory
Aero and Astrodynamics Division
George C. Marshall Space Flight Center
National Aeronautics and Space Administration
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RESEARCH AND STUDY IN
SYSTEM OPTIMIZATION TECHNIQUES

SECTION I

GENERAL

This is the second quarterly progress report submitted in accordance with the provisions of Modification 4 to Contract NAS8-5222, "Research and Study in System Optimization Techniques." It covers the period from 14 February 1965 to 14 May 1965.

SECTION II

SUMMARY OF PRIOR PROGRESS

Stability of Motion Study

The previous results that have been obtained in the area of the stability of motion are presented in Honeywell MPG Report 1546-QR-1, 22 February 1965. It contains a mathematical analysis of the connection between differential equations and their Lyapunov functions.

Guidance Study

The literature on P-matrix predictive guidance was studied and the computer simulation to be programmed was decided upon. A three stage Thor vehicle was chosen as the model with a cross product steering law. Data on system parameters were gathered.

SECTION III

PROGRESS DURING REPORTING PERIOD

Stability of Motion Study

All of the books, reports and papers collected for the literature survey on methods and techniques for determining regions of stability have been studied. Over thirty articles were collected. Spot translation with an examination of the equations was used to decide the relevence of the Russian work to the fundamental problem. Some of the articles were found to be on unrelated topics. Other papers dealt with investigations of very special problems, and the techniques used could not be applied to our problems. Some of the results on estimates of domains of stability were so general that they provided little information about specific systems.

Several interesting results concerning the estimate of domains of stability have been found and some techniques for estimating domains of stability exist for certain types of nonlinear systems. In one of the translated papers, by Dobrotin, estimates of the domain of asymptotic stability are obtained for systems of the form

$$x^{(n)} + a_1 x^{(n-1)} + \ldots + a_n x = f(x,t),$$

where the a_1 are constant and f satisfies certain growth conditions. Also estimates of the solutions are obtained. The technique used is based upon successive approximations and might be generalized to other systems.

Although the literature survey has not been exhaustive, it appears that the area of research in stability under consideration is largely undeveloped and is reveiving some attention by mathematicians and engineers in both the U. S. and the U.S.S.R.

New results have been obtained for finding estimates of the domain of attraction for systems of the form

$$\dot{x} = Ax + R(x),$$

where A is a stability matrix and $R(x)$ contains higher ordered terms. The technique is based upon obtaining a Laypunov function for the system.

Guidance Study

The goal of this work is to evaluate P-matrix predictive guidance by computer simulation. A three-stage Thor vehicle was chosen and thrust and aerodynamic data were obtained from Douglas Aircraft Company. A rotating oblate earth model and the ARDC-59 atmosphere were used. A Douglas Thor trajectory simulation was duplicated, and the tip-off maneuver and cutoff time was changed to get a circular orbit injection. The adjoint matrix about this nominal circular orbit was generated and calculations using this adjoint matrix were stored on a magnetic tape. Using these guidance sensitivities a linear combination of the terminal errors was driven to zero by steering and cutoff commands.

A standard wind profile was used as a crosswind perturbation.
Thrust perturbations were made by changing the mass flow rate.
One percent changes of the mass flow rate plus the crosswind produced
differences between apogee and perigee of less than one mile. Both
apogee and perigee were either above or below a few miles of the
circular orbit radius. The apogee-perigee difference resulted
from using a first order scheme in predicting the terminal errors.
Similar errors will occur from gyro and accelerometer errors or cutoff
errors. Thus, the guidance scheme is satisfactory since the pertur-
bations chosen were extreme.

Both velocity and crossproduct steering laws were tried with the
gain increased or decreased a factor of ten from their best values.
Guidance was used during the whole flight or just the last 20 seconds
and no difference in apogee or perigee greater than one tenth of a
mile resulted. This is because it is not necessary to steer out
all the onboard errors to less than a hundredth of a foot per
second when the onboard state information is off a couple of feet
per second. Since Douglas reported that the Thor autopilot was
so good that for practical purposes programmed steering rates are
actual rates, body dynamics were not included in this simulation.

The decrease in both the apogee and perigee distance occurred
because the guidance mission chosen was to produce a circular
orbit in the same plane as the nominal orbit. It was found that

one could guide into a circular orbit using the correct terminal state even when extreme changes like a 15 per cent decrease in the mass flow rate occurred, but the orbit was 35 miles low. To prevent these low orbits the first stage should perhaps guide to the right altitude and then later stages guide to the correct circular velocity and zero terminal inertial flight path angle.

The results obtained will be documented soon after a few more runs are made.

SECTION IV

PLANS FOR NEXT QUARTER

The stability study will be directed toward basic questions concerned with domains of attraction of dynamical systems. A nonlinear system in Euclidean space can be studied as a linear dynamical system in a function space. The goal is to obtain results about how the nonlinearities affect the domain of attraction. Also a study of the stability of smoothly non-stationery systems will be carried on.

The guidance study is essentially completed and the results will be presented in a separate report.

SECTION V

EXPENDITURES

Total expenditures from 7 February through 16 May were $16,262. The contract is 96 per cent complete.