CACULUS ASSIGNMENT # 1

Name: Muhammad Uzair Atiq

Enroll: 01-136242-029

CALCULUS ASSIGNMENT #1

QUESTION . 1

SOLUTION

Area of circle: $A = \frac{\pi}{4} D^2$

Differentiate both sides w.r.t D

$$\frac{dA}{dD} = \frac{d}{dD}\left(\frac{\pi}{4} D^2\right)$$

$$= \frac{\pi}{4} \cdot 2D$$

$$= \frac{\pi}{2} \cdot D$$

As $D = 10m$

$$\frac{dA}{dD} = \frac{\pi}{2} \cdot 10$$

$$= \pi \cdot 5$$

$$= (3.14)(5)$$

$$\boxed{= 15.7 \ m^2/m \text{ or } 5\pi}$$ ANSWER

QUESTION #2

Given height function

$s = 24t - 0.8t^2$

(a) Velocity and Acceleration

→ velocity is derivative of height

$$v = \frac{ds}{dt} = \frac{d}{dt}(24t - 0.8t^2)$$

$$\boxed{v = 24 - 1.6t \text{ ms}}$$

→ Acceleration is derivative of velocity

$$a = \frac{dv}{dt} = \frac{d}{dt}(24 - 1.6t)$$

$$\boxed{a = -1.6 \text{ ms}^2}$$

$$\boxed{\begin{aligned} v &= 24 - 1.6t \text{ ms} \\ a &= -1.6 \text{ ms}^2 \end{aligned}}$$

(b) Time to reach highest point

At highest point velocity is zero
so using equation

$v = 24 - 1.6t$

we put $v = 0$

$0 = 24 - 1.6t$

$24 = 1.6t$

$t = \frac{24}{1.6}$

$t = 15s$

$\boxed{\text{Time} = t = 15\,s}$ ANSWER

(c) MAX HEIGHT

As we know max time to reach
highest point = 15s
so using height equation

$S = 24t - 0.8t^2$

put $t = 15$

$S = 24(15) - 0.8(15)^2$

$= 360 - 180$

$= 180\ m$

$\boxed{\text{Max. height} = 180\,m}$ Answer

d) Time to reach half max height

As max height = 180m

Half max height = $\frac{180}{2} = 90m$

put in equation of height

$90 = 24t - 0.8t^2$

$0.8t^2 - 24t + 90 = 0$

using quadratic formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{+24 \pm \sqrt{(-24)^2 - 4(0.8)(90)}}{2(0.8)}$

$t = \frac{24 \pm 16.97}{1.6}$

$t_2 = \frac{24 + 16.97}{1.6}$

$\boxed{t_2 = 25.6}$ on way down

$t = \frac{24 - 16.97}{1.6}$

$\boxed{t_1 = 4.3}$ on way up

e) HOW LONG IS Afloat

Solve $24t - 0.8t^2 = 0$

$t(24 - 0.8t) = 0$

~~either~~ $\boxed{t = 0s \quad \text{or} \quad t = 30s}$

QUESTION #3

equation $= x^2 + y^2 = 25$

points $= (3, -4)$

- substituting into equation

$(3)^2 + (-4)^2 = 25$

$9 + 16 = 25$

$25 = 25$

As L.H.S = R.H.S **SO POINT IS ON CURVE**

- **a) tangent line**

$\therefore y - y_1 = m(x - x_1)$

tangent line equation

$y - (-4) = m(x - 3)$

$y + 4 = m(x - 3)$

$y + 4 = \frac{3}{4}(x - 3)$

$y = \frac{3}{4}x - \frac{9}{4} + 4$

And to find tangent differentiate on both sides

$\frac{dy}{dx}(x^2 + y^2) = \frac{dy}{dx}(25)$

$2x + \frac{dy}{dx} \cdot 2y = 0$

$\frac{dy}{dx} = \frac{-2x}{2y} = \boxed{\frac{-x}{y}}$

points $(3, -4)$

$\frac{dy}{dx} = \frac{3}{4}$

$$\boxed{y = \frac{3}{4}x - \frac{25}{4}}$$

$\boxed{\text{~~Normal~~ Tangent line equation} = \frac{3}{4}x - \frac{25}{4}}$ Ans

b) NORMAL LINE

negative reciprocal slope $= -\frac{4}{3}$

put in tangent line

$y + \cancel{4} = -\frac{4}{3}(x - 3)$

$y = -\frac{4}{3}x + 4 - 4$

$$\boxed{y = -\frac{4}{3}x}$$

$\boxed{\text{Normal line ~~equation~~} = -\frac{4}{3}x}$ Answer

QUESTION · 4

function

$g(t) = -t^2 - 3t + 3$

a) Derivate $= -2t - 3$

set $-2t-3=0$ to find critical point

and $t = \frac{3}{2}$

So critical point $= -\frac{3}{2}$

- To check where $g(t)$ is increasing and decreasing we check values on left and right side of critical point

- For $t < -\frac{3}{2}$ we take -2

$\cancel{g(-2) = -(-2)^2 - 3(-2) + 3}$ x
$\cancel{= -4 + 6 + 3}$
$\cancel{= 5}$

$g'(-2) = -2(-2) - 3$
$= +4 - 3$
$= 1 > 0$ increasing

- for $t > -\frac{3}{2}$ we take 0

$g'(0) = -2(0) - 3$
$= -3 < 0$ decreasing

→ Local maximum at $-\frac{3}{2}$

$$g\left(-\frac{3}{2}\right) = -\left(-\frac{3}{2}\right)^2 - 3\left(-\frac{3}{2}\right) + 3$$

$$= -\frac{9}{4} + \frac{9}{2} + 3$$

$$= -2.25 + 4.5 + 3$$

$$- 5.25$$

• since function increases before $-\frac{3}{2}$ and decreases after $-\frac{3}{2}$ so this point is a local max.

• Function increases at $t < -\frac{3}{2}$, $(-\infty, -\frac{3}{2})$
• Function decreases at $t > -\frac{3}{2}$, $(\frac{-3}{2}, +\infty)$

• Local max $(-\frac{3}{2}, 5.25)$

QUESTION . 5

$$s(t) = 2t^3 - 14t^2 + 22t - 5 \quad , \ t \geq 0$$

• Velocity = $s'(t) = v = \frac{d}{dt}(2t^3 - 14t^2 + 22t - 5)$

$$= 6t^2 - 28t + 22$$

• Acceleration = $a = \frac{dv}{dt} = \frac{d}{dt}(6t^2 - 28t + 22)$

$$= 12t - 28$$

$$\boxed{\begin{aligned} v &= 6t^2 - 28t + 22 \\ a &= 12t - 28 \end{aligned}}$$

Answer

• MOTION OF PARTICLE

At Rest when $v=0$

$6t^2 - 28t + 22 = 0$

using quadratic formula

$$= \frac{-(-28) \pm \sqrt{(-28)^2 - 4(6)(22)}}{2(6)} = \frac{28 \pm 16}{12}$$

- $t = \frac{28+16}{12}$
  $= 3.667$

- $t = \frac{28-16}{12}$
  $= 1$

$\boxed{\text{so particle stops at } t=1 \text{ and } t = 3.67}$ Ans

QUESTION #6

- $r(x) = 9x$
- $c(x) = x^3 - 6x^2 + 15x$

PROFIT = Revenue - Cost

$p(x) = r(x) - c(x)$

$= 9x - x^3 + 6x^2 - 15x$

$= -x^3 + 6x^2 - 6x$

Derivative (FIRST)

$p'(x) = -3x^2 + 12x - 6$

CRITICAL POINT

$-3x^2 + 12x - 6 = 0$

$-3(x^2 - 4x + 2) = 0$

$x^2 - 4x + 2 = 0$

Using quadratic formula

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{4 \pm 2.83}{2}$$

$$x = \frac{4 + 2.83}{2}, \quad x = \frac{4 - 2.83}{2}$$

$x \approx 3.41, \quad x \approx 0.59$

Check at after Second derivative

$p'(x) = -3x^2 + 12x - 6$

$p''(x) = -6x + 12$

Put $P(x) = 3.41$ and 0.59

At $P(3.41)$
$= -6(3.41)+12$
$= -8.46$

, At $P(0.59)$
$= -6(0.59)+12$
$= 8.46$

So $P'(0.59) > 0$ is a local minimum

and $P'(3.41) < 0$ is a local maximum

$\boxed{\text{Max Profit occurs at } x = 3.41 \text{ thousand units}}$