

Date \_\_\_\_\_

# Linear Algebra Assignment

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Q. No. 01

Sol:-

OUTPUT		PURCHASED BY
GOODS	SERVICES	
0.8	0.3	Services
0.2	0.7	Goods

$P_g$  = annual output of goods sector

$P_s$  = annual output of services sector

For  $R_1$ :

$$P_s = 0.8 P_g + 0.3 P_s \quad \text{--- (i)}$$

For  $R_2$ :

$$P_g = 0.2 P_g + 0.7 P_s \quad \text{--- (ii)}$$

Converting (i) & (ii) homogeneous eqs

$$0.7 P_s - 0.8 P_g = 0$$

$$-0.7 P_s + 0.8 P_g = 0$$

Augmented Matrix:

$$\left[ \begin{array}{cc|c} -0.8 & 0.7 & 0 \\ 0.8 & -0.7 & 0 \end{array} \right]$$

$$R_2 = R_2 + R_1$$

$$\left[ \begin{array}{cc|c} -0.8 & 0.7 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

$$\begin{bmatrix} & -0.875 & : & 0 \\ & 0 & 0 & : & 0 \end{bmatrix}$$

## GENERAL SOLUTION

$$(P_g - 0.875)_{P_S} = 0$$

$$P_g = 0.875 P_S$$

$P_S$  is free

- Equilibrium remains unaffected until  $P_g = 0.875 P_S$  is satisfied

if  $P_S = 1000$ ,  $P_g = 0.875 \times 1000$

$$P_g = 875$$

## Q. No. 02

Sol:-

$$P_S = 300 \text{ million dollar}$$

$$P_C = 0.94 P_S \Rightarrow P_e = 0.85 P_S$$

$$P_C = 282, P_e = 255$$

No, there will be no change in equilibrium until ratios are not satisfied.

### Q. No. 03

Sol:-

CHEMICALS	OUTPUT FUEL	MACHINERY	PURCHASED BY
0.2	0.8	0.4	Chemicals
0.3	0.1	0.4	Fuel
0.5	0.1	0.2	Machinery

$p_c$  = output for chemical sector

$p_f$  = output for fuel sector

$p_m$  = output for machinery sector

For  $R_1$ ,

$$p_c = 0.2p_c + 0.8p_c + 0.4p_m$$

For  $R_2$

$$p_f = 0.3p_c + 0.1p_f + 0.4p_m$$

For  $R_3$

$$p_m = 0.5p_c + 0.1p_f + 0.2p_m$$

Conversion into homogeneous eq/s:

$$0.8p_c - 0.8p_f - 0.4p_m = 0$$

$$-0.3p_c + 0.9p_f - 0.4p_m = 0$$

$$-0.5p_c - 0.1p_f + 0.8p_m = 0$$

## Augmented Matrix

$$\left[ \begin{array}{ccc|c} 0.8 & -0.8 & -0.4 & : 0 \\ -0.3 & 0.9 & -0.4 & : 0 \\ -0.5 & 0.1 & 0.8 & : 0 \end{array} \right]$$

$10 \times (R_1, R_2, R_3)$

$$\left[ \begin{array}{ccc|c} 8 & -8 & -4 & : 0 \\ -3 & 9 & -4 & : 0 \\ -5 & -1 & 8 & : 0 \end{array} \right]$$

$\frac{1}{8} R_1$

$$\left[ \begin{array}{ccc|c} 1 & -1 & -\frac{1}{2} & : 0 \\ -3 & 9 & -4 & : 0 \\ -5 & -1 & 8 & : 0 \end{array} \right]$$

$R_2 + (3R_1)$ ,  $R_3 + (5R_1)$

$$\left[ \begin{array}{ccc|c} 1 & -1 & -0.5 & : 0 \\ 0 & 6 & -5.5 & : 0 \\ 0 & -6 & 5.5 & : 0 \end{array} \right]$$

$R_2 + R_1$ ,  $R_3 + 6R_2$

$$\left[ \begin{array}{ccc|c} 1 & 0 & -1.417 & : 0 \\ 0 & 1 & -0.917 & : 0 \\ 0 & 0 & 0 & : 0 \end{array} \right]$$

$$P_c = 1.147 \text{ pm}$$

$$P_f = 0.9417 \text{ pm}$$

pm is free

if  $pm = 100$

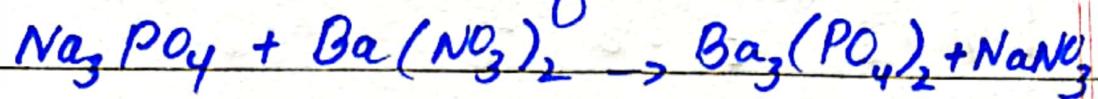
$$P_c = 1.147 (100) = 114.7$$

$$P_f = 0.9417 (100) = 94.17$$

$$pm = 100$$

## Question # 06

When solutions of sodium - - -



$\text{Na}_3\text{PO}_4 :$	3	$\text{Ba}(\text{NO}_3)_2 :$	0	sodium
	1		0	phosphorous
	4		6	oxygen
	0		1	barium
	0		2	nitrogen

$\text{Ba}_3(\text{PO}_4)_2 :$	0	$\text{NaNO}_3 :$	1	
	2		0	
	8		3	
	3		0	
	0		1	

The coefficients in the equation



	3	0	0	1
	1	0	2	0
$x_1$	4	+ $x_2$	6 = $x_3$	8 + $x_4$
	0	1	3	0
	0	2	0	1

Date \_\_\_\_\_  
Move the right terms to the left

$$\left[ \begin{array}{ccccc} 3 & 0 & 0 & -1 & 0 \\ 1 & 0 & -2 & 0 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array} \right]$$

Replacing  $R_1$  and  $R_3$

$$\left[ \begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array} \right] \quad \left[ \begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ -0 & 6 & 0 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array} \right]$$

$-3R_1 + R_2$   
 $-4R_1 + R_3$

Replacing  $R_2$  and  $R_4$

$$\left[ \begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 6 & 0 & -3 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array} \right] \quad \left[ \begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 18 & -3 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 0 & 6 & -1 & 0 \end{array} \right]$$

$-6R_2 + R_3$

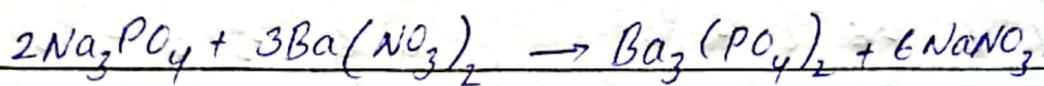
$$\left[ \begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad \left[ \begin{array}{ccccc} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$2R_3 + R_1$   
 $3R_3 + R_2$   
 $R_4 - R_5$   
 $R_4 - R_5$

The general solution is  $x_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix} x_4$

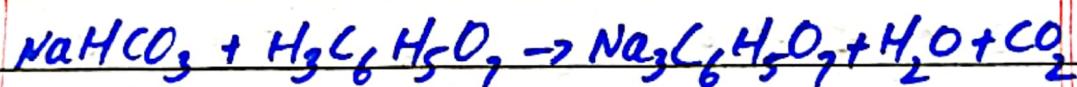
$$x_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} x_4 + x_3 = \begin{pmatrix} 1 \\ 6 \end{pmatrix} x_4 \text{ with } x_4 \neq 0$$

Take  $x_4 = 6$ . Then  $x_1 = 2, x_2 = 3, x_3 = 1$   
so, balanced equation is -



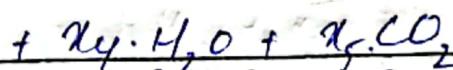
### Question # 07

Alka-Seltzer contains - - - -



$\text{NaHCO}_3$	$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}$	$\text{H}_3\text{C}_6\text{H}_5\text{O}_7$	$\begin{bmatrix} 0 \\ 8 \\ 6 \\ 7 \end{bmatrix}$	sodium
				hydrogen
				carbon
				oxygen

Now,



So,	$x_1$	$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}$	$x_2$	$\begin{bmatrix} 0 \\ 8 \\ 6 \\ 7 \end{bmatrix}$	$x_3$	$\begin{bmatrix} 3 \\ 5 \\ 6 \\ 7 \end{bmatrix}$	$x_4$	$\begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$	$x_5$	$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$

## Augmented matrix

$$\left[ \begin{array}{cccccc|cc} 1 & 0 & -3 & 0 & 0 & 0 & 0 & 0 \\ 1 & 8 & -5 & -2 & 0 & 0 & 0 & 0 \\ 1 & 6 & -6 & 0 & -1 & 0 & 0 & 0 \\ 3 & 7 & -7 & -1 & -2 & 0 & 0 & 0 \end{array} \right]$$

$$\left| \begin{array}{cccccc} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 8 & -2 & -2 & 0 & 0 \\ 1 & 6 & -6 & 0 & -1 & 0 \\ 3 & 7 & -7 & -1 & -2 & 0 \end{array} \right| \xrightarrow{R_2-R_1, R_3-R_1} \left| \begin{array}{cccccc} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 8 & -2 & -2 & 0 & 0 \\ 0 & 6 & -3 & 0 & -1 & 0 \\ 3 & 7 & -7 & -1 & -2 & 0 \end{array} \right| \xrightarrow{R_3-R_1} \left| \begin{array}{cccccc} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 8 & -2 & -2 & 0 & 0 \\ 0 & 6 & -3 & 0 & -1 & 0 \\ 0 & 0 & -4 & -1 & -1 & 0 \end{array} \right|$$

$$\left[ \begin{array}{cccccc} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 8 & -2 & -2 & 0 & 0 \\ 0 & 6 & -3 & 0 & 1 & 0 \\ 0 & 7 & 2 & -1 & -2 & 0 \end{array} \right] \xrightarrow{R_2 - 8R_1} \left[ \begin{array}{cccccc} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 6 & -3 & 0 & -1 & 0 \\ 0 & 7 & 2 & -1 & -2 & 0 \end{array} \right] \xrightarrow{R_3 - 6R_1} \left[ \begin{array}{cccccc} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 0 & 3 & 0 & -1 & 0 \\ 0 & 7 & 2 & -1 & -2 & 0 \end{array} \right]$$

$$\left[ \begin{array}{cccccc} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 0 & -3/2 & 3/2 & 1 & 0 \\ 0 & 1 & 2 & -1 & -2 & 0 \end{array} \right] \xrightarrow{R_3 - 6R_2} \left[ \begin{array}{cccccc} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 0 & -3/2 & 3/2 & -1 & 0 \\ 0 & 0 & 15/4 & 3/4 & -2 & 0 \end{array} \right] \xrightarrow{R_4 - 7R_2}$$

$$\left[ \begin{array}{cccccc} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 15/4 & 3/4 & -2 & 0 \end{array} \right] \xrightarrow{-2R_3/R_3} \left[ \begin{array}{cccccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 15/4 & 3/4 & -2 & 0 \end{array} \right] \xrightarrow{R_1 + 3R_3}$$

$$\left[ \begin{array}{cccccc|c} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{6} & 0 \\ 0 & 0 & 1 & -1 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{15}{4} & \frac{3}{4} & -2 & 0 \end{array} \right] \xrightarrow{R_3 + R_1} \left[ \begin{array}{cccccc|c} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{6} & 0 \\ 0 & 0 & 1 & -1 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{15}{4} & \frac{3}{4} & -2 & 0 \end{array} \right] \xrightarrow{\frac{4}{15}R_4} \left[ \begin{array}{cccccc|c} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{6} & 0 \\ 0 & 0 & 1 & -1 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & \frac{9}{2} & -\frac{7}{2} & 0 \end{array} \right] \xrightarrow{R_4 - \frac{15}{4}R_3}$$

$$\left[ \begin{array}{cccccc|c} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{6} & 0 \\ 0 & 0 & 1 & -1 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array} \right] \xrightarrow{\frac{2R_4}{9}} \left[ \begin{array}{cccccc|c} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{6} & 0 \\ 0 & 0 & 1 & -1 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array} \right] \xrightarrow{R_1 + 3R_4} \left[ \begin{array}{cccccc|c} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{6} & 0 \\ 0 & 0 & 1 & -1 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

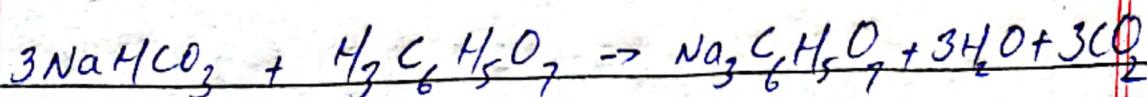
$$\left[ \begin{array}{cccccc|c} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & -1 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array} \right] \xrightarrow{R_2 + \frac{R_4}{2}} \left[ \begin{array}{cccccc|c} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array} \right] \xrightarrow{R_3 + R_4} \left[ \begin{array}{cccccc|c} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

General solution:

$$x_1 = x_5, \quad x_2 = \left(\frac{1}{3}\right)x_5, \quad x_3 = \left(\frac{1}{3}\right)x_5, \quad x_4 = x_5$$

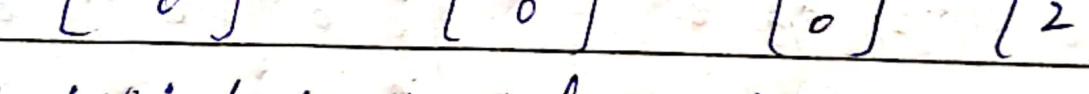
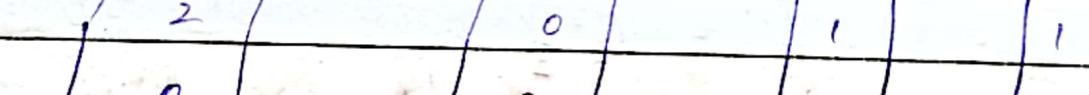
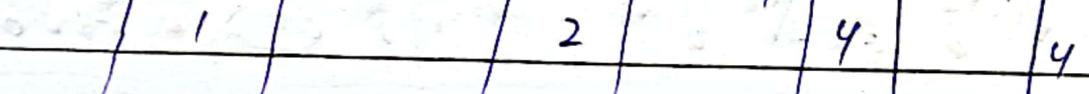
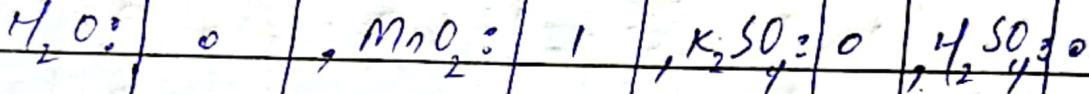
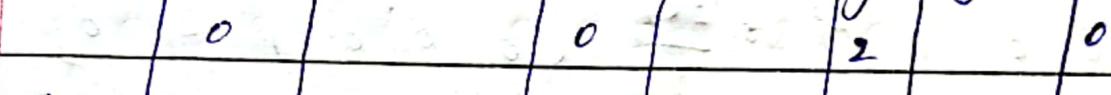
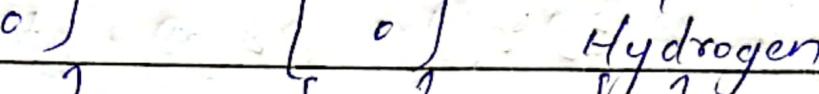
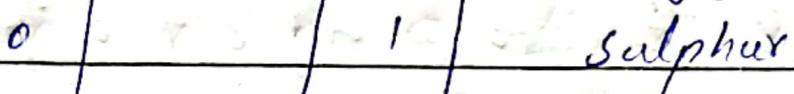
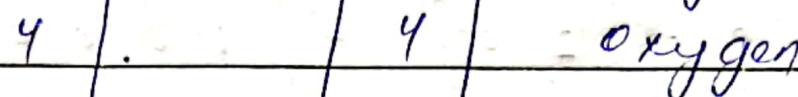
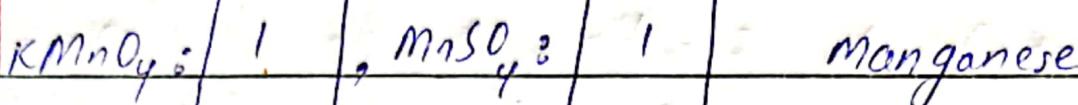
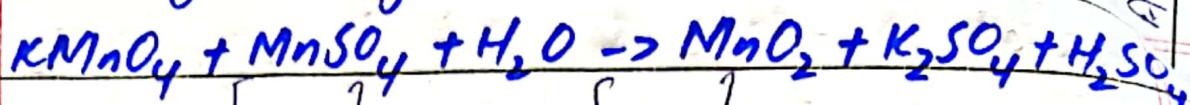
and  $x_5$  is free. Take  $x_5 = 3$ . Then

$$x_1 = x_4 = 3, \quad x_2 = x_3 = 1. \text{ Balanced eq.}$$

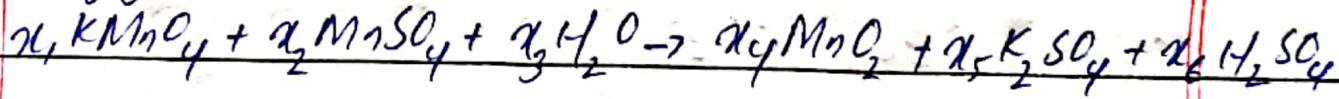


### Question # 08

The following reaction - - - .



Coefficients in chemical equation:



Vector Equation:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ x_1 & 1 & x_2 & 1 & x_3 & 0 \\ 4 & 4 & 1 & 2 & 4 & 4 \\ 0 & 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

Move the terms to the left side:

Augmented matrix:

$$\left[ \begin{array}{ccccccc} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 & 0 & 0 \\ 4 & 4 & 1 & -2 & -4 & -4 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 \end{array} \right]$$

$$\left[ \begin{array}{ccccccc} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 4 & 1 & -2 & 4 & -4 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 \end{array} \right] \quad R_2 - R_1$$

$$\left[ \begin{array}{ccccccc} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & -4 & -4 & 0 \\ 0 & 0 & 0 & -1 & -3 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 \end{array} \right] \quad R_3 - 4R_1$$

$$\left[ \begin{array}{ccccccc} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -4 & -4 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & -4 & 8 & 6 & 0 \end{array} \right] \quad R_4 - R_2$$

$$\left[ \begin{array}{ccccccc} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -4 & -4 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & -4 & 8 & 6 & 0 \end{array} \right] \quad R_5 - 2R_3$$

Data

General

$x_1 = x_6$

$x_5 = 0$

Taking

$$\left| \begin{array}{ccccccc} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & 0 & -4 & 2 & 0 \end{array} \right| \quad R_3 - 2R_4$$

$$\left| \begin{array}{ccccccc} 1 & 0 & 0 & 0 & 0 & -6 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \end{array} \right| \quad R_1 + 2R_5$$

$$\left| \begin{array}{ccccccc} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \end{array} \right| \quad R_2 + R_5$$

$$\left| \begin{array}{ccccccc} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -5/2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \end{array} \right| \quad R_4 + 3R_5$$

General solution:

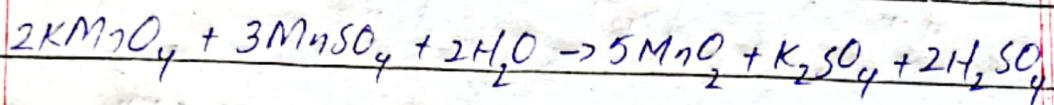
$$x_1 = x_6, x_2 = (1.5)x_6, x_3 = x_6, x_4 = (2.5)x_6$$

$x_5 = 0.5x_6$  and  $x_6$  is free

Taking  $x_6 = 2$ . Then

$$x_1 = x_3 = 2, x_2 = 3, x_4 = 5 \text{ and } x_5 = 1$$

Balanced equation:



Question # 09



Setting up vectors that list the atoms

per molecule. The vector equation:

	1	0	3	0	0	0	lead
	6	0	0	0	0	1	nitrogen
$x_1$	0	$+x_2$	$= x_3$	$0 + x_4$	$2 + x_5$	$0 + x_6$	chromium
	0	2	0	0	1	0	manganese
	0	8	4	3	2	1	oxygen

Augmented matrix

$$\left[ \begin{array}{ccccccc|c} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 8 & -4 & -3 & -2 & -1 & 0 \end{array} \right]$$

$$\left[ \begin{array}{ccccccc|c} 1 & 0 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 18 & 0 & 0 & -1 & 0 & R_2 - 6R_1 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 & \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 & \\ 0 & 8 & -4 & -3 & -2 & -1 & 0 & \end{array} \right]$$

swap  $R_2$  and  $R_3$

$$\left[ \begin{array}{ccccccc|c} 1 & 0 & -3 & 0 & 0 & 0 & 0 & \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 & \\ 0 & 0 & 18 & 0 & 0 & -1 & 0 & \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 & \\ 0 & 8 & -4 & -3 & -2 & -1 & 0 & \end{array} \right]$$

$$\left[ \begin{array}{cccccc|c} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 18 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 4 & -1 & 0 & R_4 - 2R_2 \\ 0 & 0 & -4 & 13 & -2 & -1 & 0 \end{array} \right] \quad \left[ \begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & -1/6 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 4 & -1 & 0 & 0 \\ 0 & 0 & -4 & 13 & -2 & -1 & 0 \end{array} \right] \quad R_1 + 3R_3$$

$$\left[ \begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & -1/6 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & -1/4 & 0 & 0 \\ 0 & 0 & 0 & 13 & -2 & -11/9 & 0 \end{array} \right] \quad \left[ \begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & -1/6 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & -1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5/4 & 11/9 & 0 \end{array} \right] \quad R_2 + 2R_4 \quad R_5 - 13R_1$$

$$\left[ \begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & -\frac{1}{6} & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{22}{45} & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{18} & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 1 & -\frac{44}{45} & 0 \end{array} \right] \xrightarrow{\text{R}_1 + \frac{P_1}{6}, \text{R}_2 + \frac{P_2}{45}, \dots} \left[ \begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & -\frac{1}{6} & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{22}{45} & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{18} & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 1 & -\frac{44}{45} & 0 \end{array} \right] \xrightarrow{4R_5/5} \left[ \begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & -\frac{1}{6} & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{22}{45} & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{18} & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 1 & -\frac{44}{45} & 0 \end{array} \right] \xrightarrow{R_4 + \frac{P_5}{4}} \left[ \begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & -\frac{1}{6} & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{22}{45} & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{18} & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 1 & -\frac{44}{45} & 0 \end{array} \right]$$

General solution is:

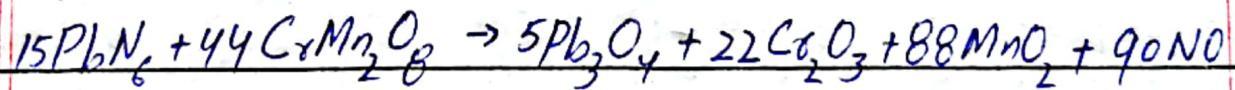
$$x_1 = \left( \frac{1}{6} \right) x_6, x_2 = \left( \frac{22}{45} \right) x_6, x_3 = \left( \frac{1}{18} \right) x_6, x_4 = \left( \frac{1}{4} \right) x_6$$

$$x_5 = \left( \frac{44}{45} \right) x_6 \quad \text{and } x_6 \text{ is free. Taking } x_6 = 90$$

Then,

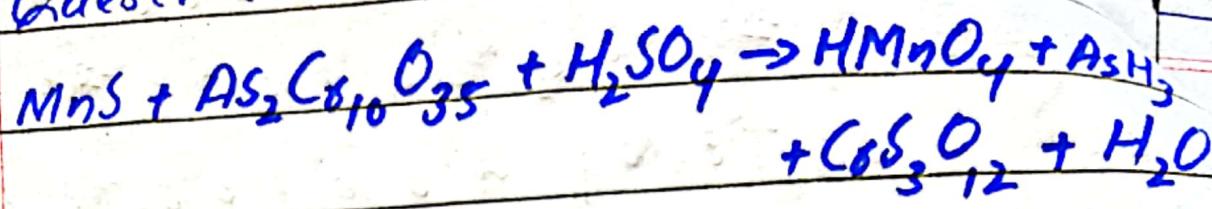
$$x_1 = 15, x_2 = 44, x_3 = 5, x_4 = 22, x_5 = 88$$

Balanced equation is:



Date \_\_\_\_\_

Question # 10



vector equation:

1	0	0	1	0	0	0	0	manganese
1	0	1	0	0	3	0	0	sulphur
$x_1$	$0 + x_2$	$2 + x_3$	$0 = x_4$	$0 + x_5$	$1 + x_6$	$0 + x_7$	$0$	arsenic
0	10	0	0	0	1	0	0	chromium
0	35	4	4	0	12	1	0	oxygen
0	0	2	1	3	0	2	0	hydrogen

Augmented matrix:

1	0	0	-1	0	0	0	0	
1	0	1	0	0	-3	0	0	
0	2	0	0	-1	0	0	0	
0	10	0	0	0	-1	0	0	
0	35	4	-4	0	-12	-1	0	
0	0	2	-1	-3	0	-2	0	

$$\left[ \begin{array}{ccccccc|c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 35 & 4 & -4 & 0 & -12 & -1 & 0 \\ 0 & 0 & 2 & -1 & -3 & 0 & -2 & 0 \end{array} \right] R_2 - R_1$$

swap  $R_2$  and  $R_3$

$$\left[ \begin{array}{ccccccc|c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 35 & 4 & -4 & 0 & -12 & -1 & 0 \\ 0 & 0 & 2 & -1 & -3 & 0 & -2 & 0 \end{array} \right]$$

$$\left[ \begin{array}{ccccccc|c} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & -1 & 0 & 0 \\ 0 & 35 & 4 & -4 & 0 & -12 & -1 & 0 \\ 0 & 0 & 2 & -1 & -3 & 0 & -2 & 0 \end{array} \right] R_2/2 \quad R_4 - 10R_2$$

Date \_\_\_\_\_

$$\left[ \begin{array}{ccccccc} 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 5 & -1 & 0 \\ 0 & 0 & 4 & -4 & \frac{35}{2} & -12 & -1 \\ 0 & 0 & 2 & -1 & -3 & 0 & -2 \end{array} \right] R_5 - 3R_3$$

$$\left[ \begin{array}{ccccccc} 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 5 & -1 & 0 \\ 0 & 0 & 0 & -8 & \frac{35}{2} & 0 & -1 \\ 0 & 0 & 2 & -1 & -3 & 0 & -2 \end{array} \right] R_5 - 4R_3$$

swap  $R_4$  and  $R_5$

$$\left[ \begin{array}{ccccccc} 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 \\ 0 & 0 & 0 & -8 & \frac{35}{2} & 0 & -1 \\ 0 & 0 & 0 & 0 & 5 & -1 & 0 \\ 0 & 0 & 0 & -3 & -3 & 6 & -2 \end{array} \right] R_7 - 2R_3$$

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1	0	0	0	-35/16	0	1/8	0	$R_1 + R_4$
0	1	0	0	-1/2	0	0	0	
0	0	1	1	0	3	0	0	
0	0	0	1	-35/16	0	1/8	0	$-R_1/8$
0	0	0	0	5	-1	0	0	
0	0	0	-3	-3	6	-2	0	

1	0	0	0	0	-35/16	0	1/8	0
0	1	0	0	0	-1/2	0	0	0
0	0	1	0	0	35/16	-3	-1/8	0
0	0	0	1	0	-35/16	0	1/8	0
0	0	0	0	1	1	-15	0	0
0	0	0	0	0	-153/16	6	-13/8	0
0	0	0	0	0	-153/16	6	-13/8	$R_6 + 3R_4$

1	0	0	0	0	-7/16	1/8	0	$R_1 + \frac{35R_5}{16}$
0	1	0	0	0	-1/10	0	0	$R_2 + \frac{R_5}{2}$
0	0	1	0	0	-41/16	-1/8	0	$R_3 - \frac{35R_5}{16}$
0	0	0	1	0	0	1/8	0	
0	0	0	0	1	1	-15	0	0
0	0	0	0	0	-153/16	6	-13/8	0

Date \_\_\_\_\_

1	0	3	0	0	0	-7/16	1/8	0
0	1	0	0	0	0	-1/10	0	0
0	0	1	0	0	0	-41/16	1/8	0
0	0	0	1	0	0	-7/16	1/8	0
0	0	0	0	1	0	-1/5	0	0
0	0	0	0	0	0	327/80	-13/8	0

$$\begin{aligned}x_1 &= \frac{-16}{327} x_5 \\x_2 &= \frac{1}{16} x_5 \\x_3 &= \frac{1}{16} x_5 \\x_4 &= \frac{1}{16} x_5\end{aligned}$$

$$R_4 + \frac{35R_5}{16}$$

$$R_6 + \frac{153R_5}{16}$$

1	0	0	0	0	0	-16/327	0	$R_1 + \frac{7R_6}{16}$
0	1	0	0	0	0	-13/327	0	$R_2 + \frac{R_6}{10}$
0	0	1	0	0	0	-41/16	1/8	0
0	0	0	1	0	0	-7/16	1/8	0
0	0	0	0	1	0	-1/5	0	0
0	0	0	0	0	1	-130/327	0	$80R_6$

327

1	0	0	0	0	0	-16/327	0	
0	1	0	0	0	0	-13/327	0	
0	0	1	0	0	0	-374/327	0	$R_3 + \frac{41R_6}{16}$
0	0	0	1	0	0	-16/327	0	$R_4 + \frac{7R_6}{16}$
0	0	0	0	1	0	-26/327	0	$R_5 + \frac{R_6}{5}$
0	0	0	0	0	1	-130/327	0	

General solution:

$$x_1 = \left( \frac{16}{327} \right) x_7, x_2 = \left( \frac{13}{327} \right) x_7, x_3 = \left( \frac{374}{327} \right) x_7$$

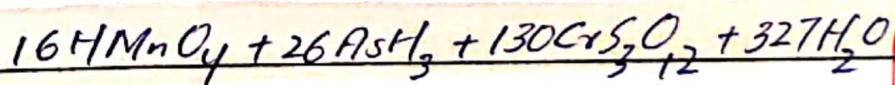
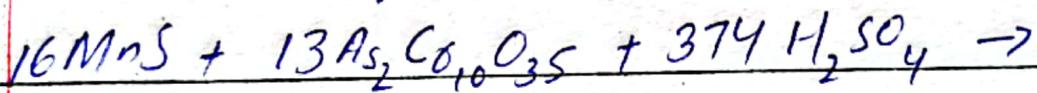
$$x_4 = \left( \frac{16}{327} \right) x_7, x_5 = \left( \frac{26}{327} \right) x_7, x_6 = \left( \frac{130}{327} \right) x_7$$

and  $x_7$  is free. Take  $x_7 = 327$ . Then,

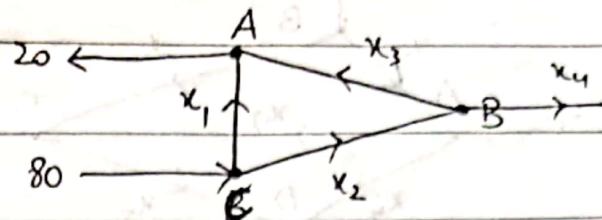
$$x_1 = 16, x_2 = 13, x_3 = 374, x_4 = 16$$

$$x_5 = 26, x_6 = 130$$

Balanced equation



### Question no (11)



Write eq. for each node :

Node	Flow in	Flow out
------	---------	----------

$$A \quad x_1 + x_3 = 20$$

$$B \quad x_1 + x_2 = x_3 + x_4$$

$$C \quad 80 = x_1 + x_2$$

$$\text{Total flow : } 80 = x_4 + 20$$

Rearrange the equations :

$$x_1 + x_3 = 20$$

$$x_2 - x_3 - x_4 = 0$$

$$x_1 + x_2 = 80$$

$$- x_4 = 60$$

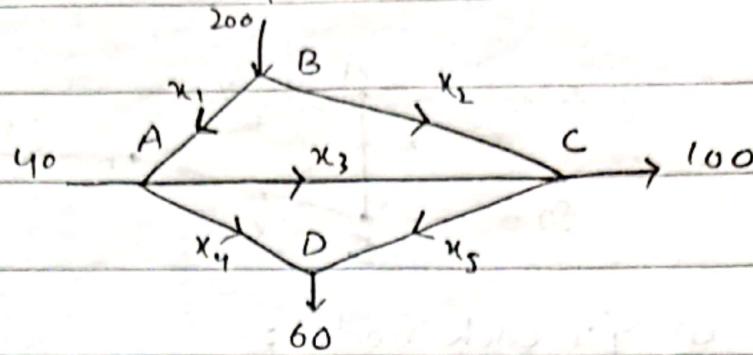
Reduce the augmented matrix :

$$\left[ \begin{array}{cccc|c} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 1 & 1 & 0 & 0 & 80 \\ 0 & 0 & 0 & 1 & 60 \end{array} \right] \rightarrow \left[ \begin{array}{cccc|c} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & 0 & 60 \\ 0 & 0 & 0 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\left\{ \begin{array}{l} x_1 = 20 - x_3 \\ x_2 = 60 + x_3 \\ x_3 \text{ is free} \cdot \text{ Since } x_1 \neq \text{negative, the largest value for } x_3 \text{ is 20.} \\ x_4 = 60 \end{array} \right.$$

### Question NO (12)



Node A :  $x_1 = x_3 + x_4 + 40$

Node B :  $200 = x_1 + x_2$

Node C :  $x_2 + x_3 = x_5 + 100$

Node D :  $x_4 + x_5 = 60$

Total flow :  $200 = 200$

Rearrange the equations :

$$x_1 - x_3 - x_4 = 40$$

$$x_1 + x_2 = 200$$

$$x_2 + x_3 - x_5 = 100$$

$$x_4 + x_5 = 60$$

Reduce the augmented matrix :

$$\left[ \begin{array}{ccccc} 1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array} : \begin{array}{c} 40 \\ 200 \\ 100 \\ 60 \end{array} \right] \sim \left[ \begin{array}{ccccc} 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} : \begin{array}{c} 100 \\ 100 \\ 60 \\ 0 \end{array} \right]$$

$$\left\{ \begin{array}{l} x_1 = 100 + x_3 - x_5 \\ x_2 = 100 - x_3 + x_5 \end{array} \right. \quad \text{when } x_4 = 0, x_5 = 60$$

$$x_3 \text{ is free}$$

$$x_4 = 60 - x_5$$

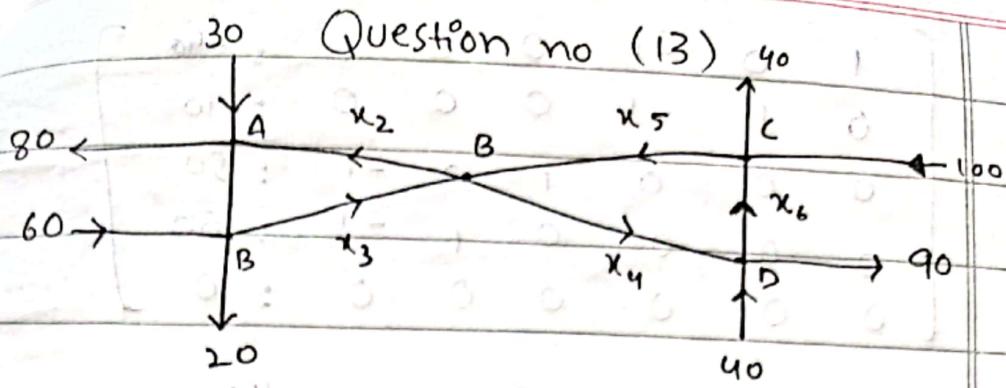
$$x_5 \text{ is free}$$

$$\left\{ \begin{array}{l} x_1 = 40 + x_3 \\ x_2 = 160 - x_3 \end{array} \right.$$

$$x_3 \text{ is free}$$

$$x_4 = 0$$

$$x_5 = 60$$



A :  $x_2 + 30 = x_1 + 80$

B :  $x_3 + x_5 = x_2 + x_4$

C :  $x_6 + 100 = x_5 + 40$

D :  $x_4 + 40 = x_6 + 90$

E :  $x_1 + 60 = x_3 + 20$

Rearrange the equations :

$$x_1 - x_2 = -50$$

$$x_2 - x_3 + x_4 - x_5 = 0$$

$$x_5 - x_6 = 60$$

$$x_4 - x_6 = 50$$

$$x_1 - x_3 = -40$$

Reduce the augmented matrix :

$$\left[ \begin{array}{ccccccc} 1 & -1 & 0 & 0 & 0 & 0 & : -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & : 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & : 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & : 50 \\ 1 & 0 & -1 & 0 & 0 & 0 & : -40 \end{array} \right]$$

$$\left[ \begin{array}{ccccccc} 1 & -1 & 0 & 0 & 0 & 0 & : -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & : 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & : 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & : 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & : 0 \end{array} \right]$$

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$$\left[ \begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & 0 & : -40 \\ 0 & 1 & -1 & 0 & 0 & 0 & : 10 \\ 0 & 0 & 0 & 1 & 0 & -1 & : 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & : 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & : 0 \end{array} \right]$$

general solution is

$$\begin{cases} x_1 = x_3 - 40 \\ x_2 = x_3 + 10 \\ x_3 \text{ is free} \\ x_4 = x_6 + 50 \\ x_5 = x_6 + 60 \\ x_6 \text{ is free} \end{cases}$$

### Question NO (14)

A :  $x_1 = x_2 + 100$

B :  $x_2 + 50 = x_3$

C :  $x_3 = x_4 + 120$

D :  $x_4 + 150 = x_5$

E :  $x_5 = x_6 + 80$

F :  $x_6 + 100 = x_1$

Rearrange the equations :

$$x_1 - x_2 = 100$$

$$x_2 - x_3 = -50$$

$$x_3 - x_4 = 120$$

$$x_4 - x_5 = -150$$

$$x_5 - x_6 = 80$$

$$-x_1 + x_6 = -100$$

Reduce the augmented matrix:

$$\left[ \begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ -1 & 0 & 0 & 0 & 0 & 1 & 100 \end{array} \right]$$

$$\left[ \begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\left[ \begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & -1 & 100 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 50 \\ 0 & 0 & 0 & 1 & 0 & -1 & -70 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

General solution is

$$\left\{ \begin{array}{l} x_1 = 100 + x_6 \\ x_2 = x_6 \\ x_3 = 50 + x_6 \\ x_4 = -70 + x_6 \\ x_5 = 80 + x_6 \\ x_6 \text{ is free} \end{array} \right.$$

Since  $x_4$  cannot be negative, the minimum value of  $x_6$  is 70.