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LINEAR ALGEBRA

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ASSIGNMENT 02

MATRIX DETERMINANT

The determinant of matrix is a special scalar value that can be computed from square matrix. It is denoted by : $|A|$ or $\det(A)$.

Properties :

Following are some major properties of determinant.

(i)

Determinant of identity matrix is always one. i.e. $|A| = 1$

Example :

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$|I| = (1 \times 1) - (0 \times 0) = 1$$

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(ii)

Consider A be the matrix and its determinant as det(A) then:

$$\det(kA) = k^n \det(A)$$

Example:

Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$ $\det(A) = 2-2 = 1$

Now if we have to find det(2A). Then multiply A by 2

$\det(2A) = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix} = 12-8 = 4$

According to property:

$\det(2A) = 2^2 \det(A)$

$= 4(1) = 4$

(iii)

Consider A as a matrix. If A has rows/columns equal to zero, then:

$$\det(A) = 0$$

Example:

Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$

$|A| = 0-0 = 0$

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(iv)

Let A be the matrix then,

$$\det(A) = \det(A^t)$$

Example:

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $|A| = 4 - 6 = -2$ — (i)

$A^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, $|A^t| = 4 - 6 = -2$ — (ii)

Since eq (i) and (ii) are equal thus property of $|A| = |A^t|$ exists.

(v)

Consider a matrix A. If matrix A has two identical rows and columns than determinant of A shall be zero.

Example:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\det(A) = |A| = (1 \times 1) - (1 \times 1) = 0$$

(vi)

Let A be the square matrix, if we interchange it any of two rows / columns then its sign for determinant changes. i.e,

$$|A| = -|A|$$

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Example :

Let $A = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$, $|A| = 16 - 24 = -8$ — (i)

Now interchange R1 and R2

$A = \begin{bmatrix} 6 & 8 \\ 2 & 4 \end{bmatrix}$, $|A| = 24 - 16 = 8$ — (ii)

Observing eq (i) and (ii)

$$-|A| = |A|$$

(vii)

Consider two square matrix A and B then :

$$\det(AB) = \det(A)\det(B)$$

Example :

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$$AB = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$\det(AB) = 950 - 946 = 4$ — (i)

$\det(A) = (4 \times 1) - (2 \times 3) = -2$

$\det(B) = (8 \times 5) - (6 \times 7) = -2$

$\det(A) \times \det(B) = 4$ — (ii)

eq (i) and (ii) satisfies the property

(viii)

If elements of row or column are multiplied by non-zero constant, determinant is multiplied by same constant.

Example:

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\det(A) = 4-6 = -2$ — (i)

Multiply 2 with R1 of A, A becomes

$A' = \begin{bmatrix} 2 & 4 \\ 3 & 4 \end{bmatrix}$

$|A'| = 8-12 = -4$ — (ii)

From eq (i) and eq (ii), It is clear that

$|A| = 2|A'|$

(ix)

If the elements of determinant above or below the diagonal becomes zero then determinant equal product of diagonal elements. This is known as **Triangular Property**.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ 0 & b_2 & b_3 \\ 0 & 0 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & 0 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 b_2 c_3$$

Example :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{bmatrix}$$ then $\det(A) = 45$ — (i)

$|A| = 1 \times 5 \times 9 = 45$

Finding $|A|$ by normal method to check if property exist.

$$|A| = 1\begin{vmatrix} 5 & 6 \\ 0 & 9 \end{vmatrix} - 2\begin{vmatrix} 0 & 6 \\ 0 & 9 \end{vmatrix} + 3\begin{vmatrix} 0 & 5 \\ 0 & 0 \end{vmatrix}$$

$$= 1(45-0) - 2(0-0) + 3(0-0)$$

$$= 45 \text{ — (ii)}$$

Since eq (i) and (ii) are same thus property satisfies.

(X)

Consider a matrix A ;

$$\begin{bmatrix} a_1 + b_1 & c_1 & d_1 \\ a_2 + b_2 & c_2 & d_2 \end{bmatrix}$$

then $|A| = \begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \end{vmatrix} + \begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \end{vmatrix}$

This is known as **Sum Property**

Example :

$$A = \begin{vmatrix} 1+4 & 2 \\ 1+6 & 3 \end{vmatrix}$$

$$|A| = (1+4)3 - 2(1+6) = 15 - 14 = 1 \quad \text{— (i)}$$

Now by using property

$$\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 4 & 2 \\ 6 & 3 \end{vmatrix} = 3-2 + 12-12 = 1 \quad \text{— (ii)}$$

As eq (i) and (ii) are same thus this property exists.